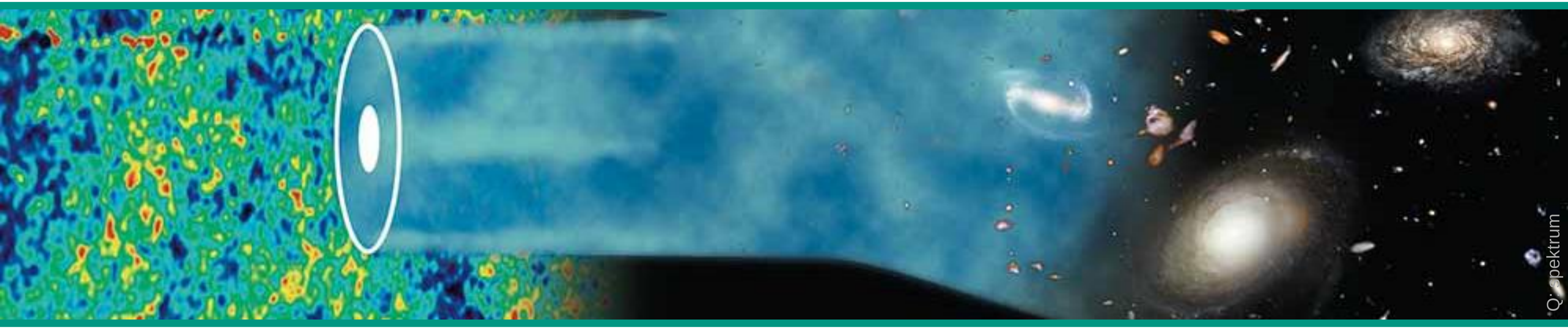
Karlsruhe Institute of Technology

Introduction to Cosmology

Winter term 22/23 Lecture 13 Feb. 7, 2023



Recap of Lecture 12

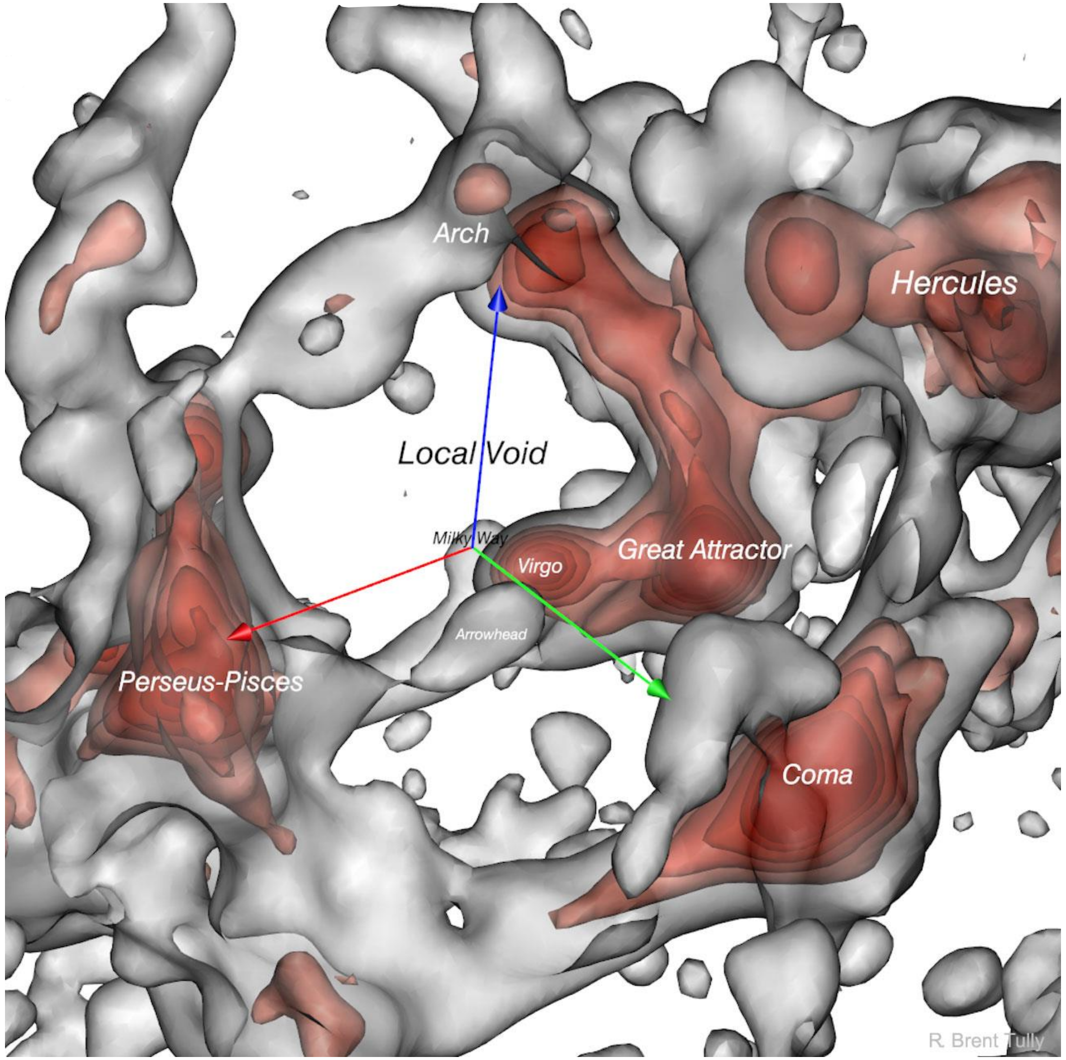
- ***L****arge* ***S****cale* ***S****tructure* (***LSS***) **evolution**

  - galaxy redshift surveys ($\boldsymbol{2dF}$, $\boldsymbol{SDSS}$,…):
    $\boldsymbol{3D}$-distribution of galaxies, reveals
    $\boldsymbol{BAO}$ **– correlation** (rings) at $\boldsymbol{r \sim 140\ Mpc}$

  - linear increase of **density contrast** $\boldsymbol{\delta}$
    of matter: superclusters ($\boldsymbol{\delta} = 2 \dots 3$) &
    supervoids ($\boldsymbol{\delta} = 0.2 \dots 0.3$)

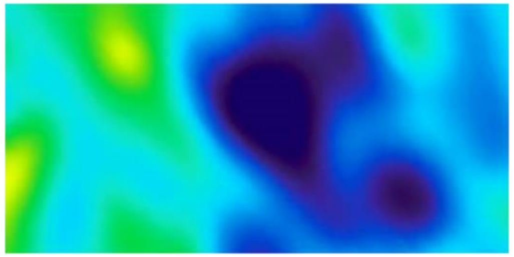
  - ***H****ot* ***D****ark* ***M****atter* (***HDM***): massive
    (few $\boldsymbol{eV}$) neutrinos ⇨ top-down scenario

  - ***C****old* ***D****ark* ***M****atter* (***CDM***): very heavy
    (few $\boldsymbol{TeV}$) neutralinos ⇨ bottom-up scenario



Strucuture formation in an expanding universe

- Linear growth of the density contrast $\delta(t)$ of LSS over the Hubble time t_0



$a(CMB) \sim 10^{-3}$

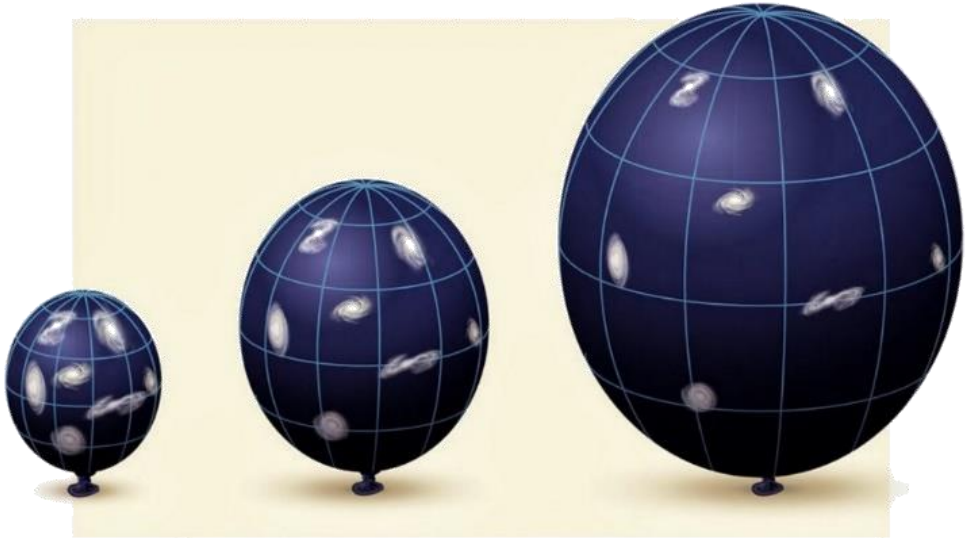
$z(CMB) = 1100$

increase of **scale-factor** by 10^3

$a(t_0) = 1$

$z(t_0) = 0$

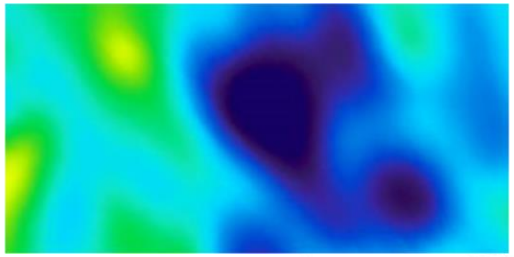
early universe
at $t = 378000\,yr$
CMB decoupling



todays universe
at $t = 13.8 \cdot 10^9\,yr$
present LSS

Strucuture formation in an expanding universe

- **Linear growth of the density contrast $\delta(t)$ of LSS over the Hubble time t_0**



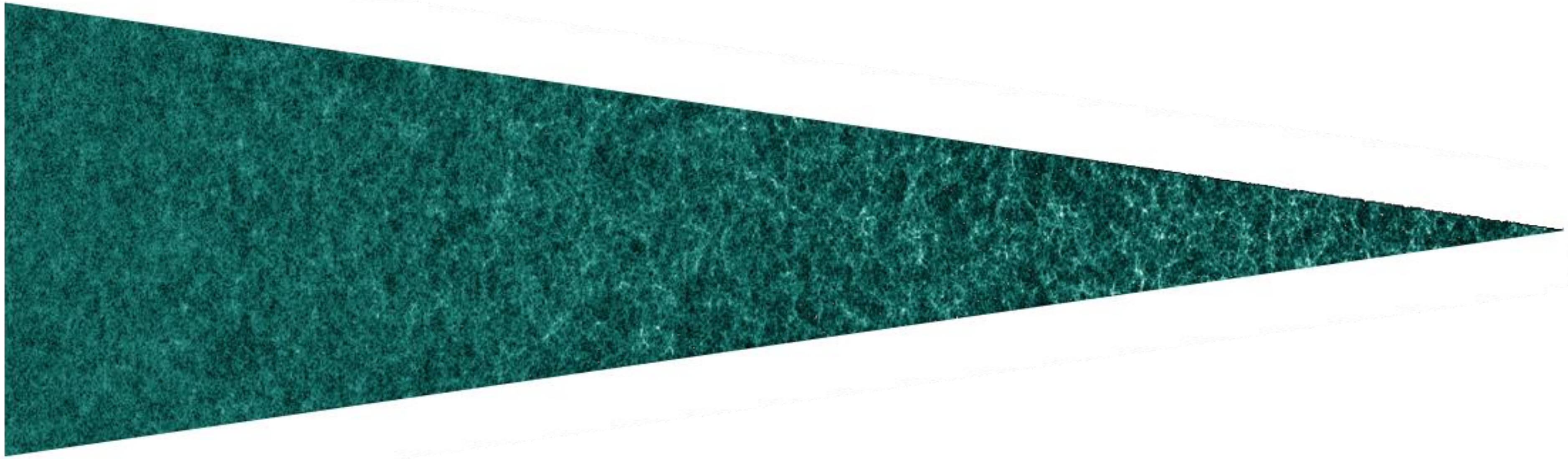
$a(CMB) \sim 10^{-3}$

$\delta(CMB) = 10^{-5}$

increase of **density contrast** by 10^5

$a(t_0) = 1$

$\delta(t_0) = 2 - 3$



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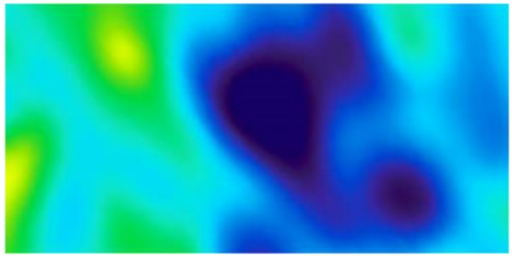


$a(CMB) \sim 10^{-3}$

$\delta(CMB) = 10^{-5}$

increase of **density contrast** by 10^5

$a(t_0) = 1$

$\delta(t_0) = 2 - 3$

Why are these numbers not adding up?

Why has the density contrast increased more?

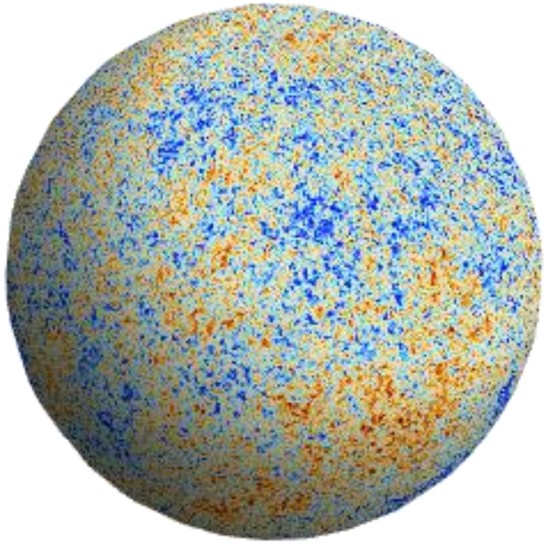
Why is our universe today so clumpy?

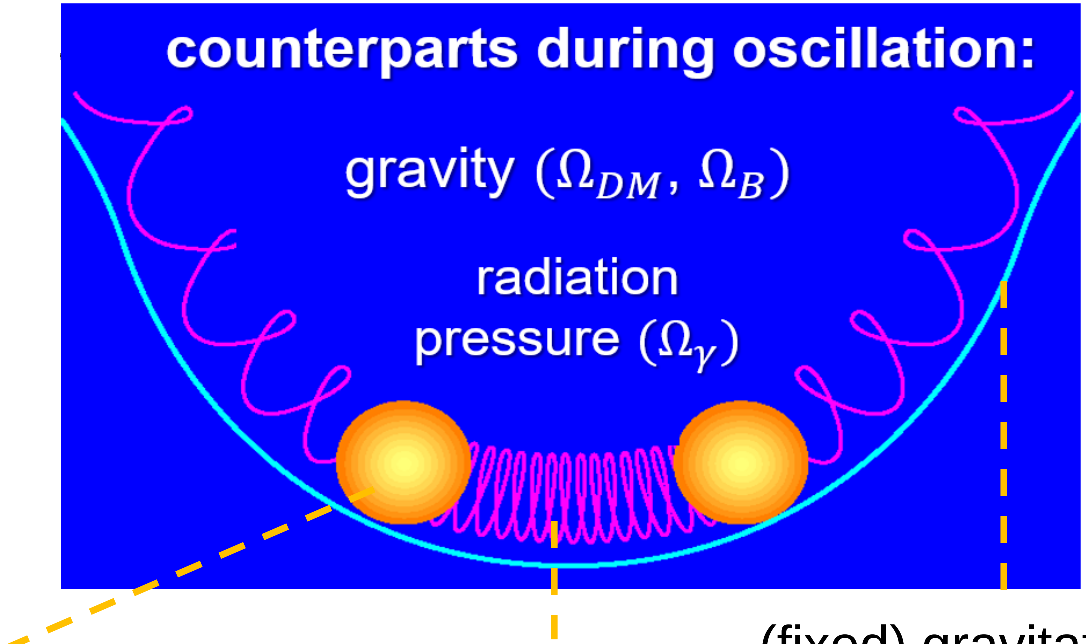
Structure formation: BAO & the DM – seed for LSS

- **Baryon Acoustic Oscillations: an incontrovertible evidence for Dark Matter**

  - the key importance of Dark Matter in structure formation: linear increase of the density contrast ***despite*** the strong coupling of baryons to photons

$$\delta(CMB) = 10^{-5}$$





oscillating mass: **baryons**

photons

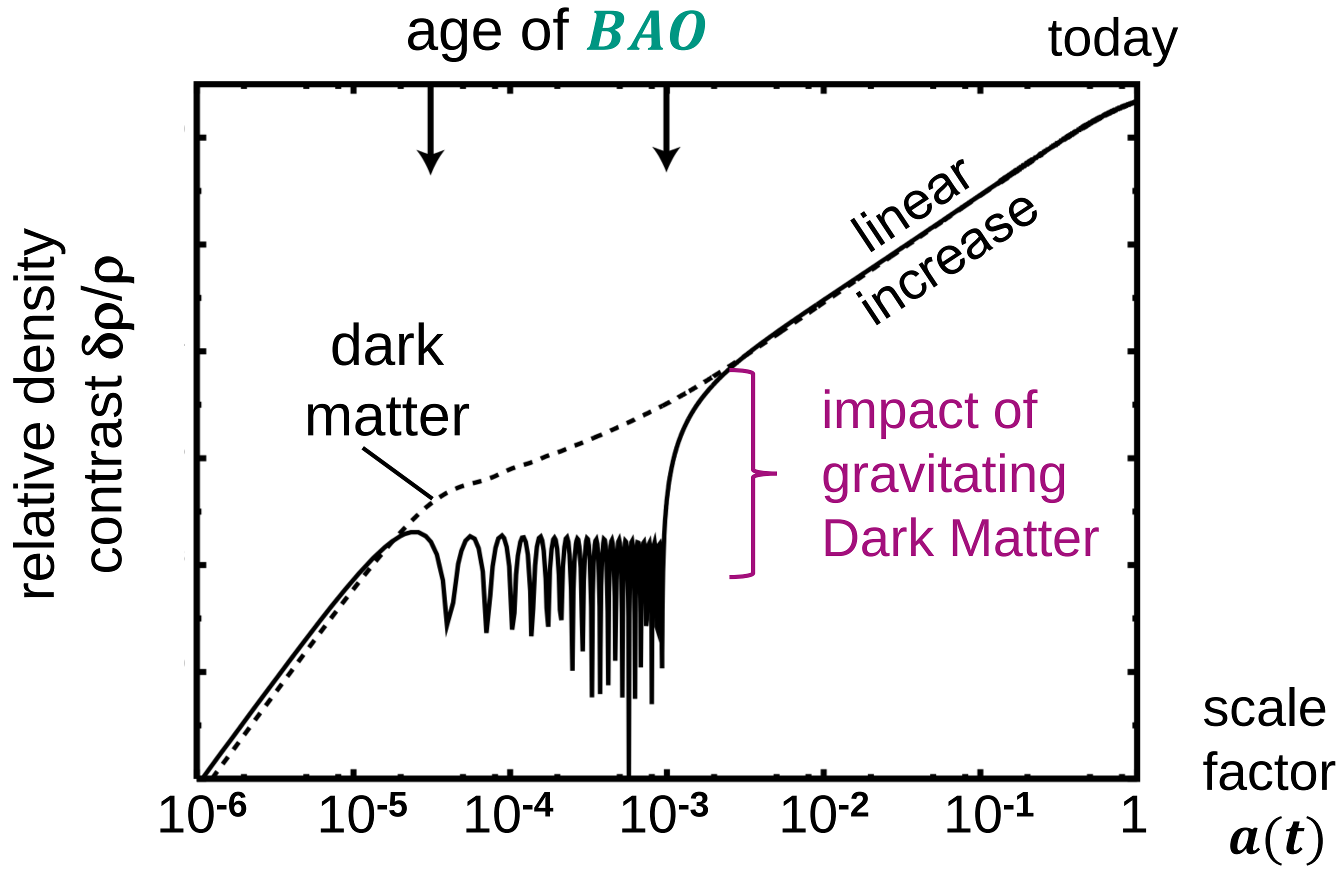
(fixed) gravitational potential **Dark Matter**

Structure formation: $\boldsymbol{BAO}$ & the $\boldsymbol{DM}$ – seed for $\boldsymbol{LSS}$

- **During $\boldsymbol{BAO}$ phase: $\boldsymbol{DM}$ density contrast grows while baryons are stalled**

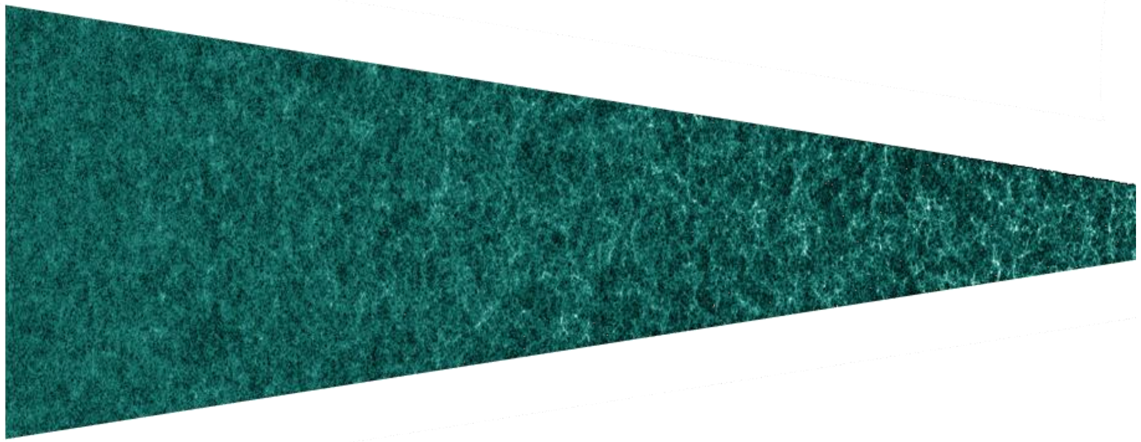
  - in the primodial plasma, the density contrast of baryons cannot further grow and is stalled due to $\boldsymbol{BAO}$

  - $\boldsymbol{DM}$ is unaffected by $\boldsymbol{BAO}$ and its density contrast continues to grow

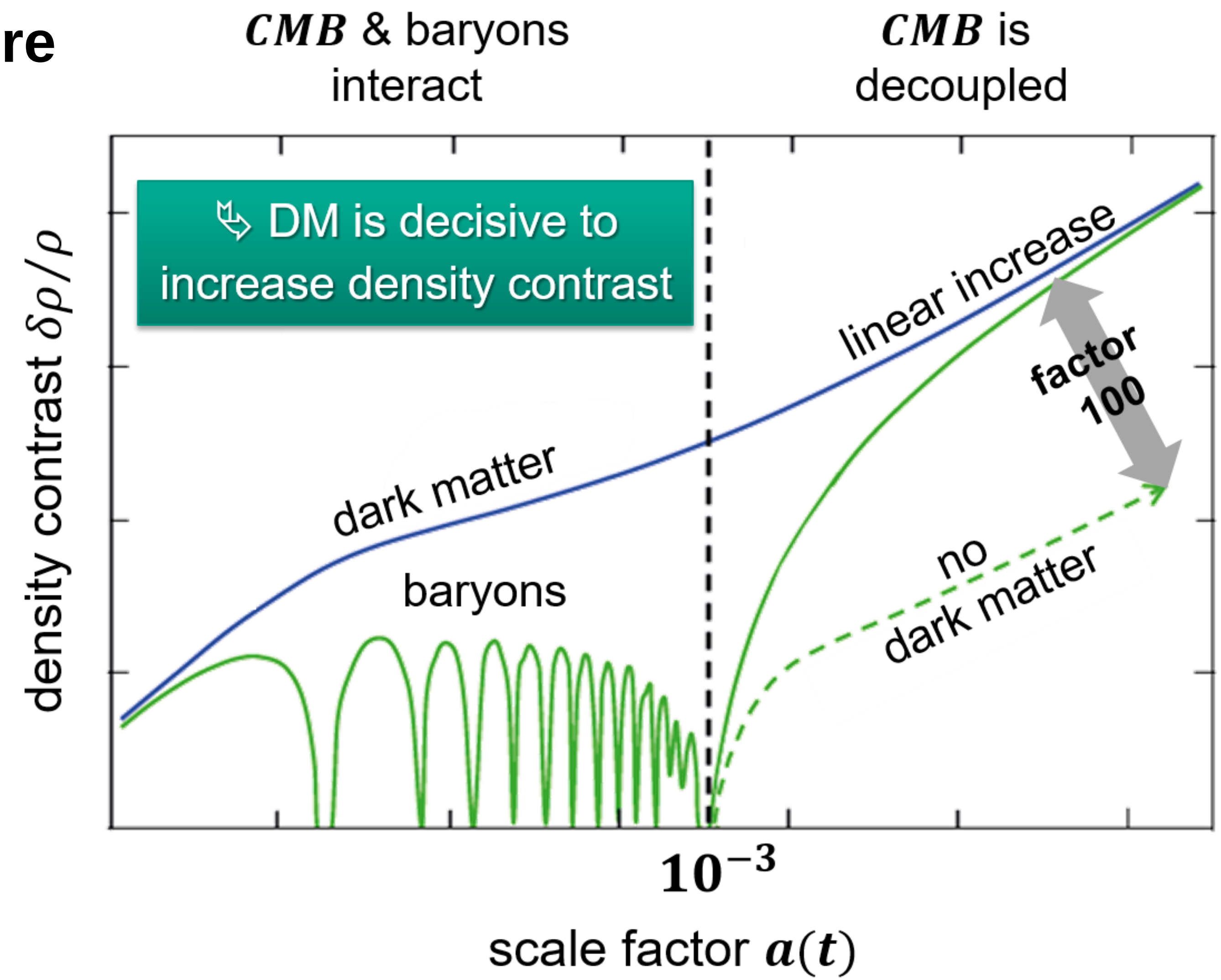


Structure formation: BAO & the DM – seed for LSS

- DM **clumps as seeds for structure**



- DM is unaffected by BAO and its density contrast continues to grow
- no DM during BAO: the universe would be **factor 100** smoother (much less structure)



Structure formation: BAO & the DM – seed for LSS

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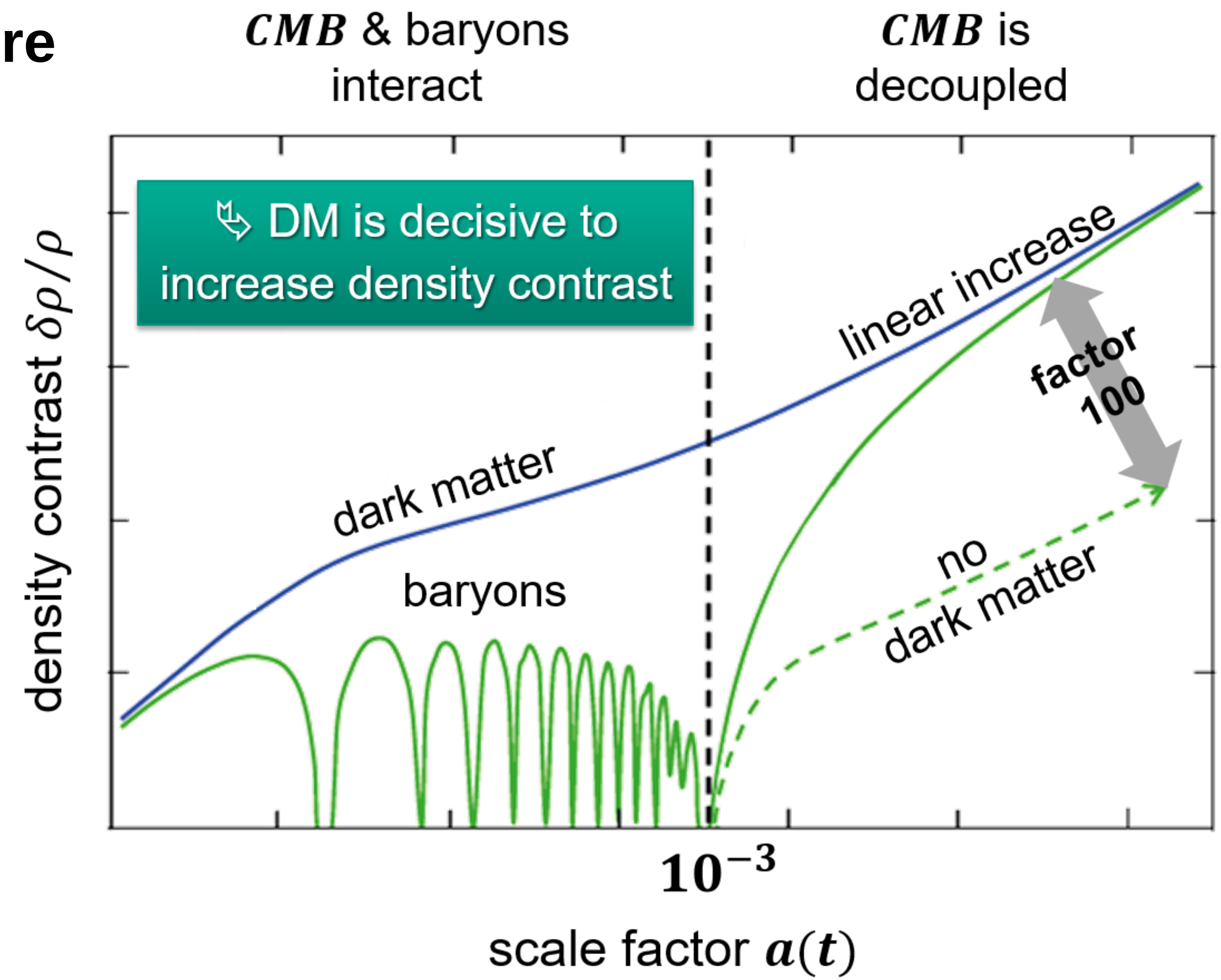
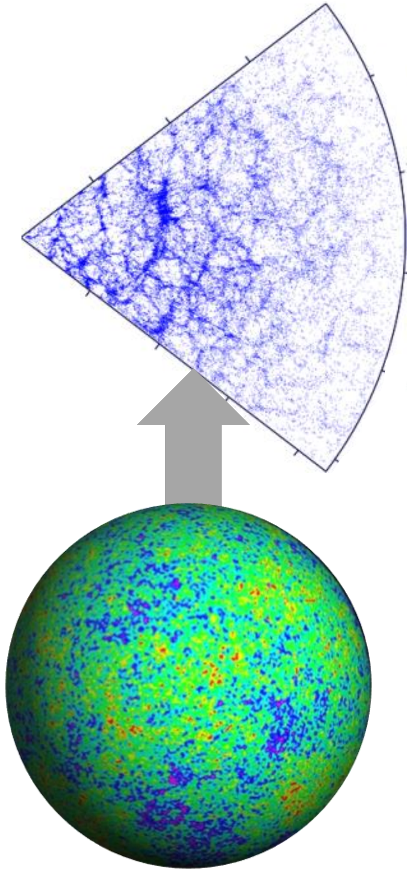
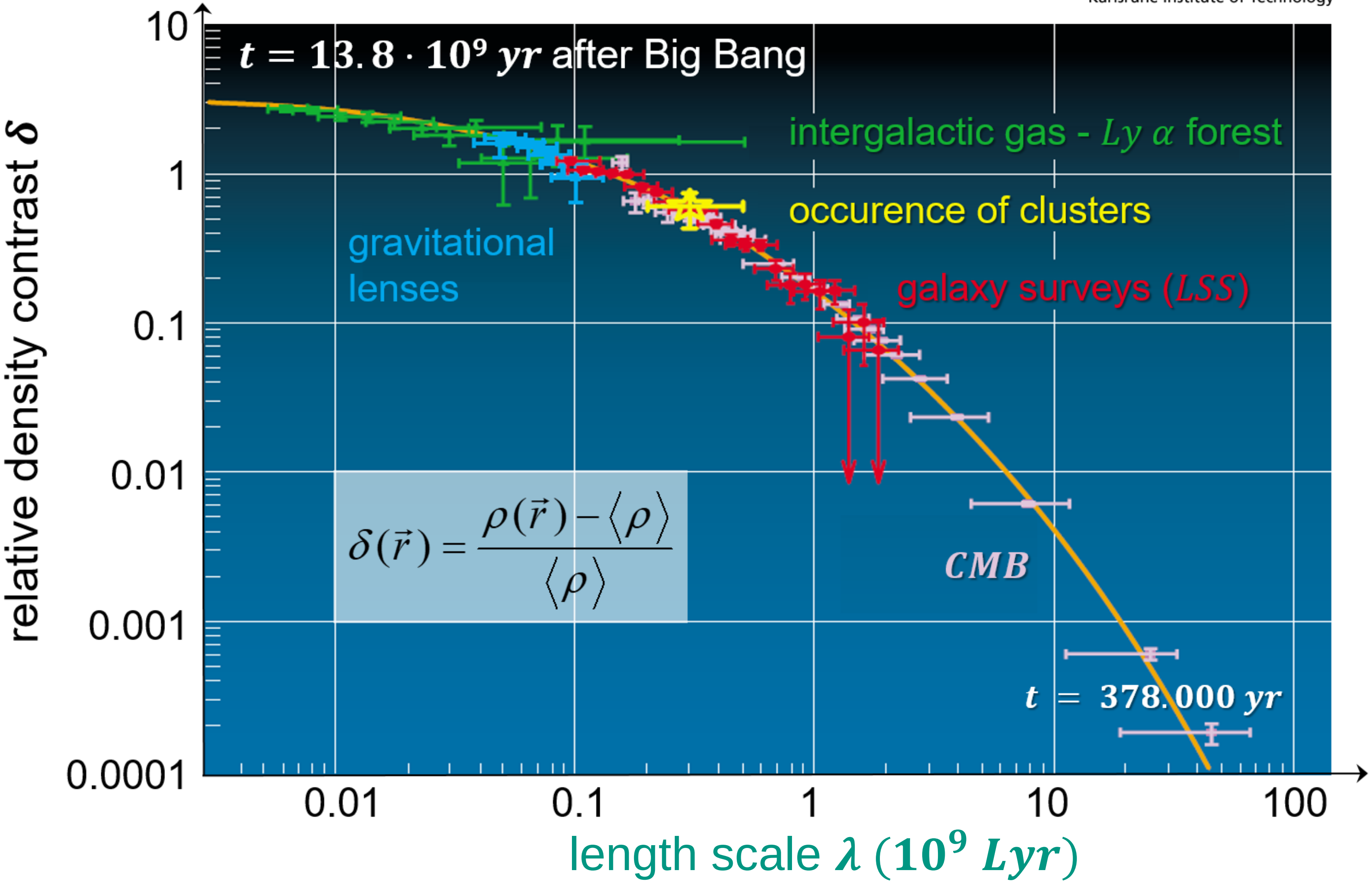


Observational evidence for structure growth

- Measuring the increase of $\delta(t)$





How is the density contrast δ being measured?

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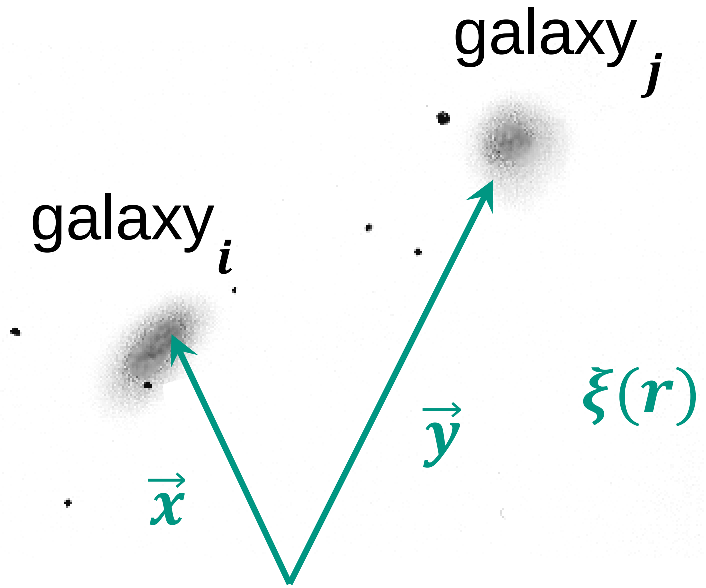
- **James Peebles: measure the distances $\vec{r}$ of galaxy pairs in space**

  - galaxies are <u>not</u> distributed randomly in space:
  use the **galaxy (auto-)** **correlation function $\xi(r)$**
  (for distance $\vec{r}$) to study the
  evolution of large scale structures

$$\delta(\vec{r}) = \frac{\rho(\vec{r}) - \langle\rho\rangle}{\langle\rho\rangle}$$

James Peebles
Nobel price
2019

take an ´average´ galaxy $\boldsymbol{i}$:
determine the probability of finding the next one $\boldsymbol{j}$ at a distance $\vec{r} = |\vec{x} - \vec{y}|$



Galaxy auto-correlation function $\xi(r)$

Making use of galaxy redshift surveys

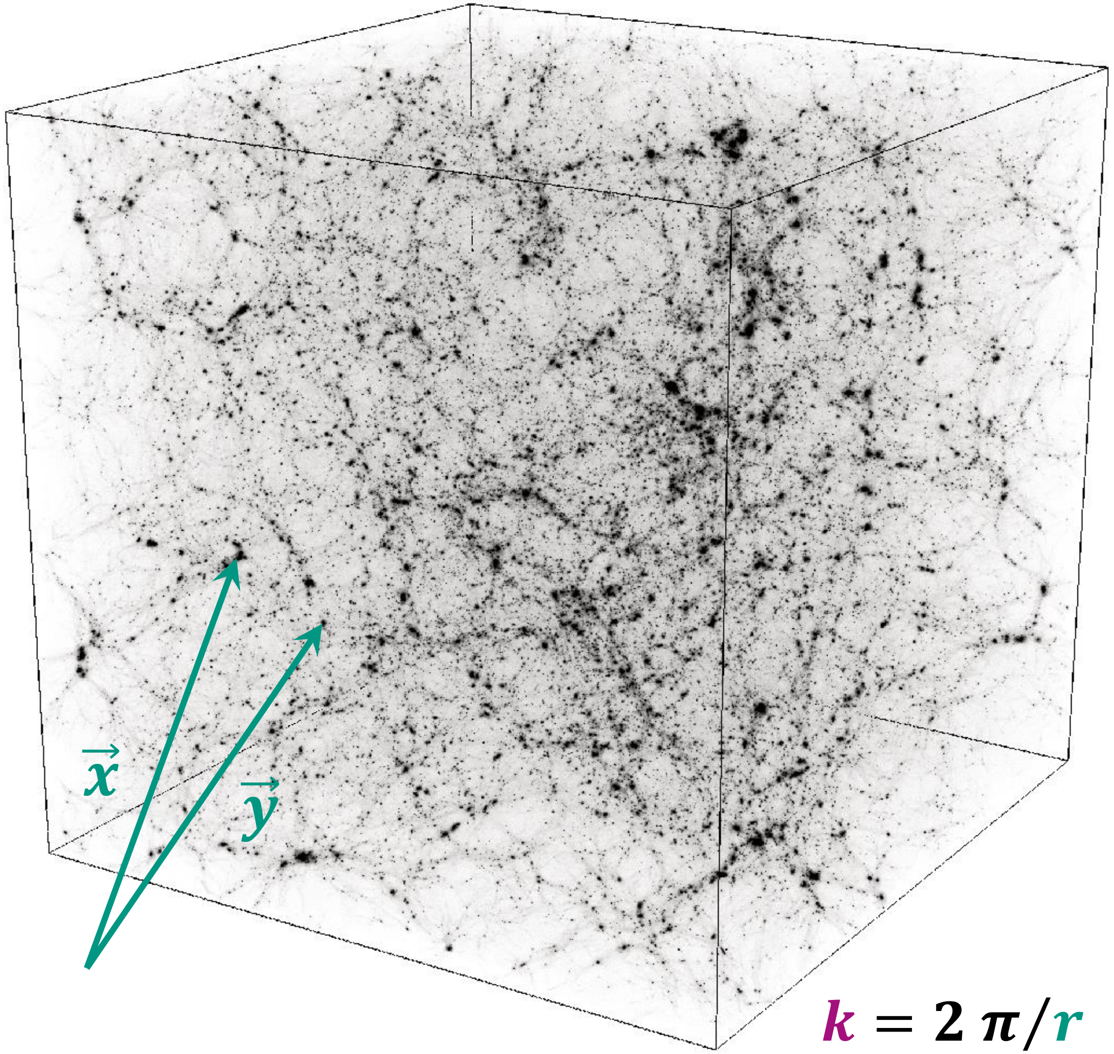
- $3D$ correlation of galaxies only depends on their **relative distance**

$$r = |\vec{r}| = |\vec{x} - \vec{y}|$$

- universe is isotropic & homogenous

- there is no ´average´ galaxy
- large statistical ensemble yields a simple **power-law** distribution $\sim r^{-1.8}$

$$\xi(r) \sim \left(\frac{r}{5\, h^{-1}\, Mpc} \right)^{-1.8}$$



$k = 2\,\pi / r$

RECAP: the ´revealing power´ of Fourier transform

- **Example in domains of time t and frequency ω**

- $\mathcal{F}$: Fourier transform integral - definition

$$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t) \cdot e^{-i \cdot 2\pi \cdot \omega \cdot t} \cdot dt$$

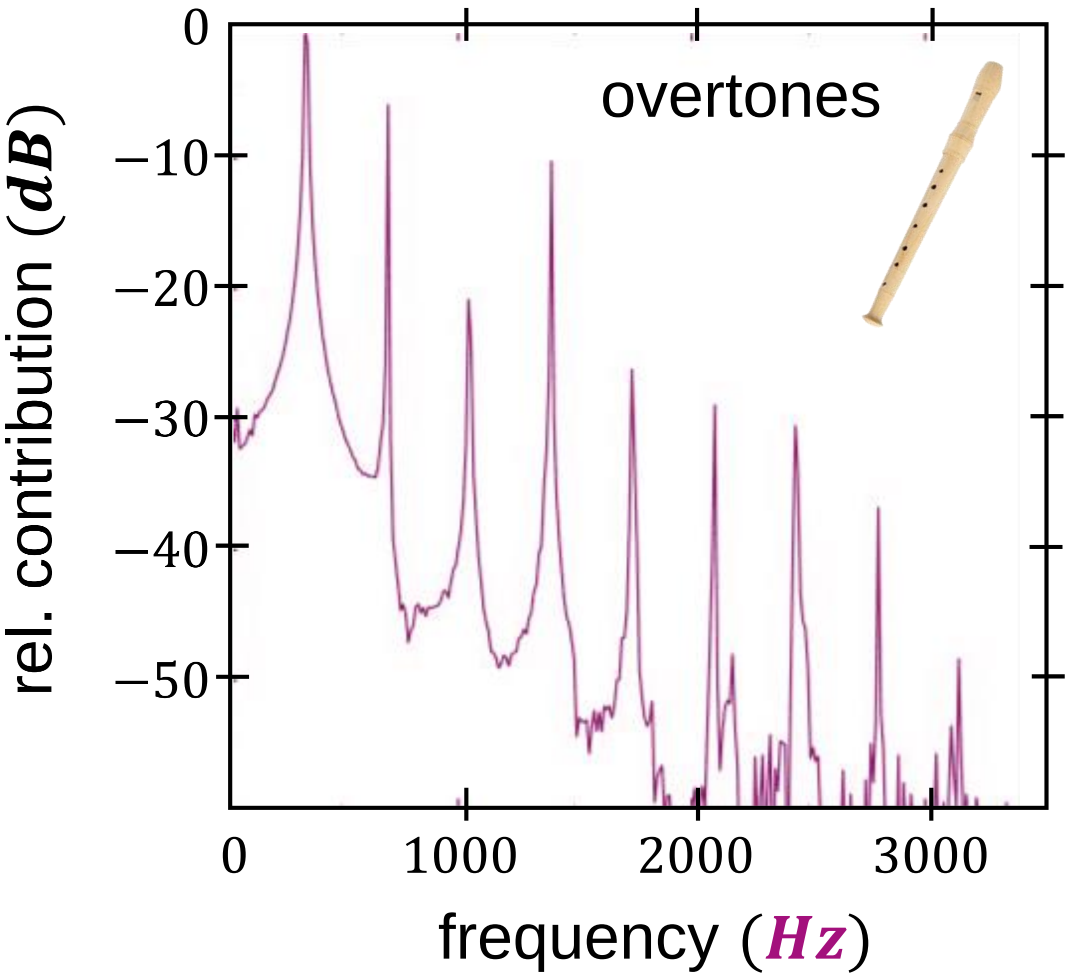
frequency → $Hz = s^{-1}$

time → s

$$\hat{f}(\omega) \overset{\mathcal{F}}{\Leftrightarrow} f(t)$$

frequency domain — time domain

FFT of flute playing



density contrast $\delta(\vec{k})$ in wave number k – space

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- $\delta(\vec{k})$ **is the Fourier transformation of** $\delta(\vec{r})$

- as we now are working with wave numbers $k = |\vec{k}|$

$$\delta(\vec{r}) = \frac{\rho(\vec{r}) - \langle\rho\rangle}{\langle\rho\rangle}$$

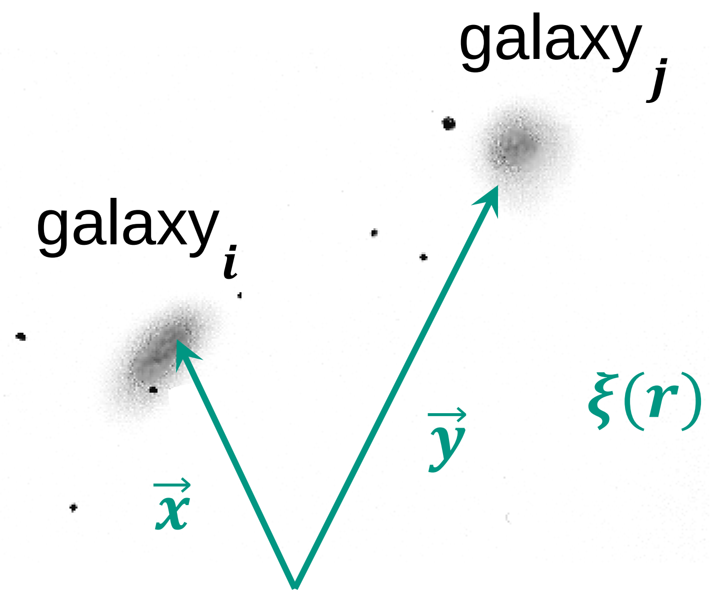
$$\delta(\vec{k}) = \sum \delta(\vec{r}) \cdot e^{-i \cdot \vec{k} \cdot \vec{r}}$$

wavenumber position

- this allows to derive the **density contrast**

$$\langle|\delta(\vec{k})|^2\rangle$$

from the **galaxy correlation** function $\xi(r)$



Galaxy correlation & matter power spectrum

- **Relation between correlation function $\xi(r)$ and power spectrum $P(k)$**

  - the key ´observable´ for cosmology is the **matter power spectrum $P(k)$**, which is a measure of the **density contrast of matter** as function of a **wave number k**

$$\xi(r) = \frac{1}{2\pi^2} \cdot \int k^2 \cdot P(k) \cdot \frac{\sin(kr)}{kr} \cdot dk$$

galaxy correlation — **matter power spectrum**

$$P(k) \sim \langle |\delta(\vec{k})|^2 \rangle$$

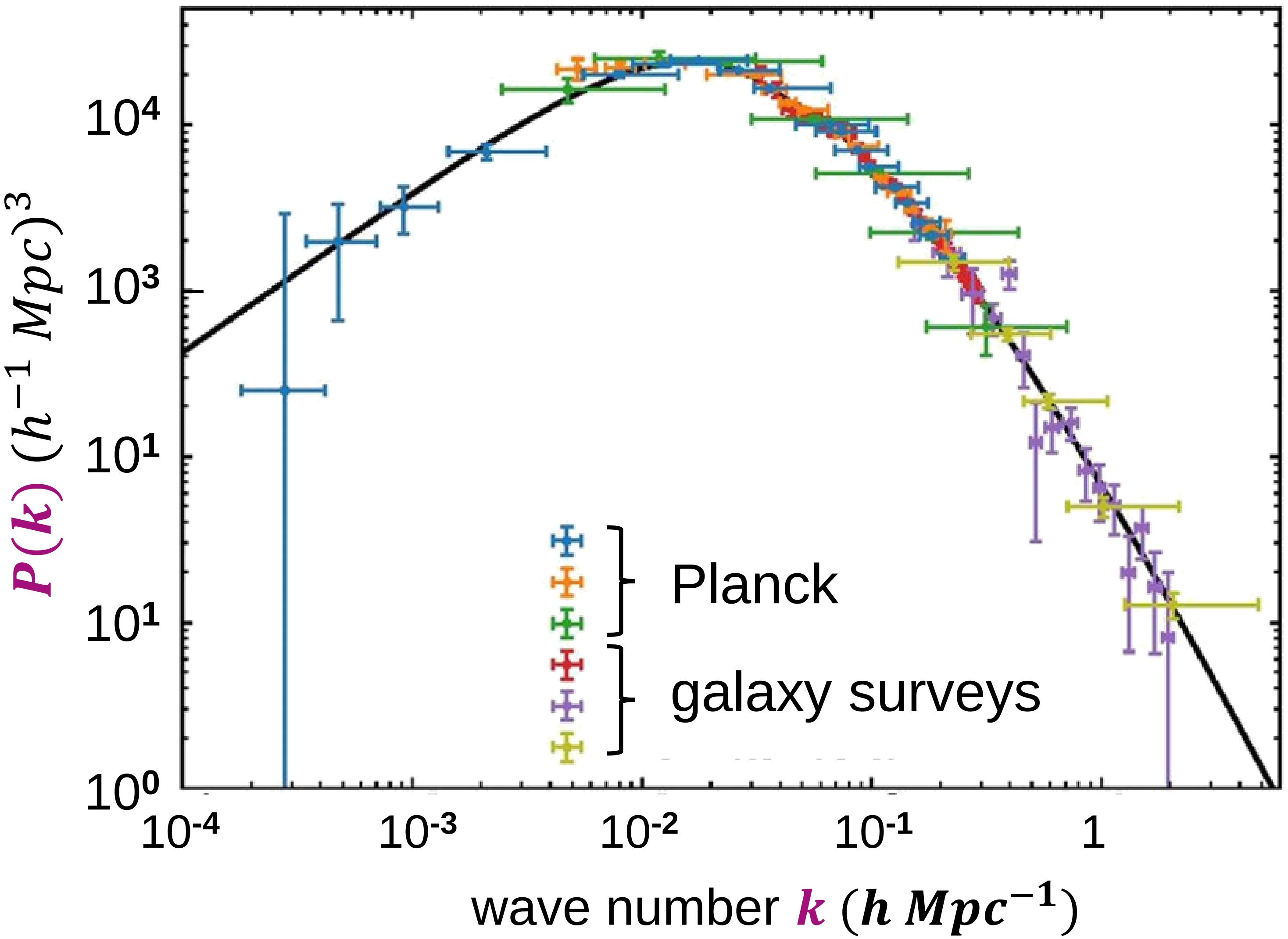
Observed matter power spectrum

- **Compilation of data from early universe (CMB) & the present universe (LSS)**

- it covers a factor $\sim \mathbf{10^6}$ in length scales

small k: huge length scale r

large k: small length scale r



Observed matter power spectrum

- **Compilation of data from early universe (CMB) & the present universe (LSS)**

- it covers a factor ~ $\mathbf{10^6}$ in length scales

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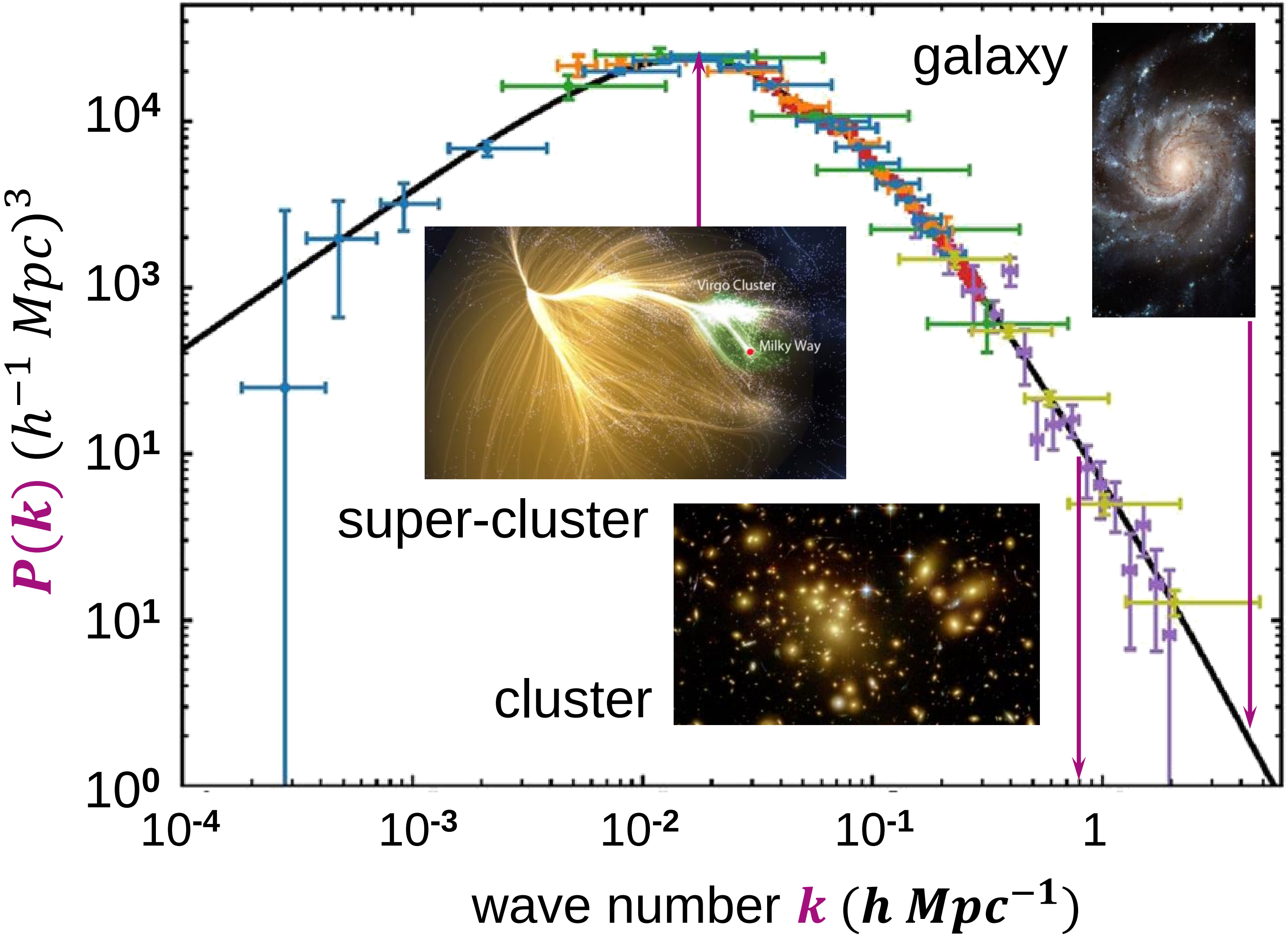

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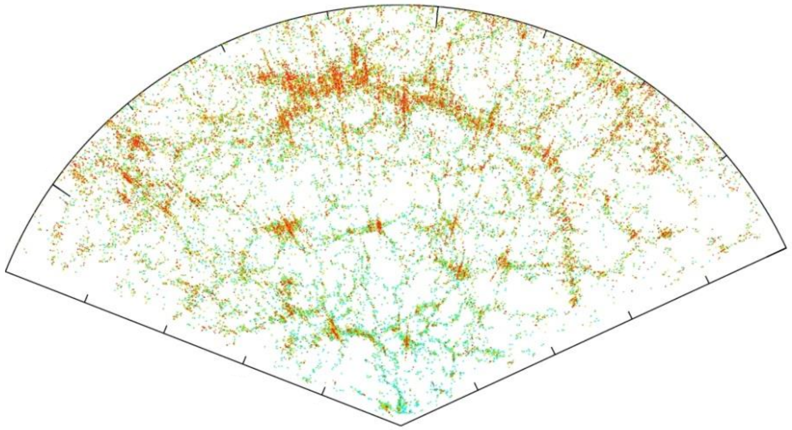
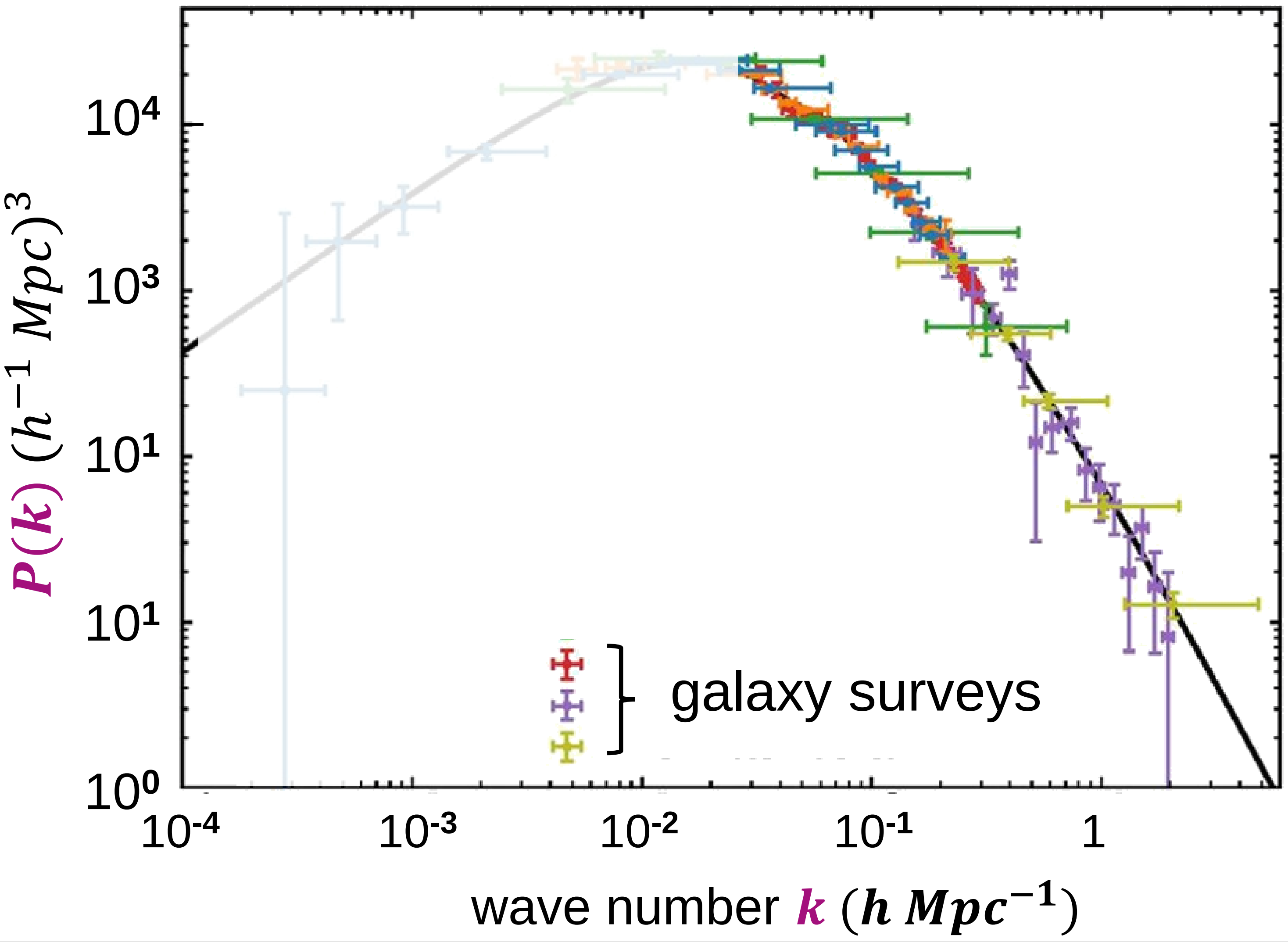


large k: small length scale r



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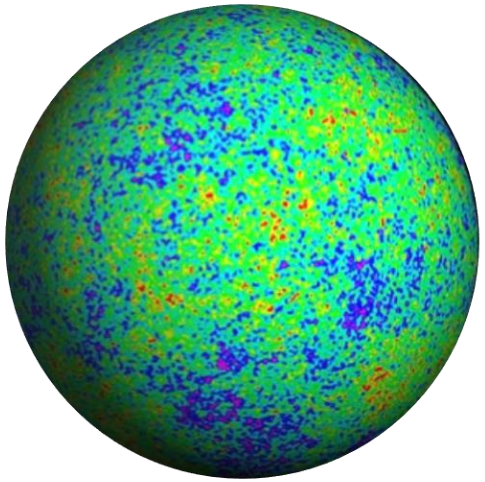

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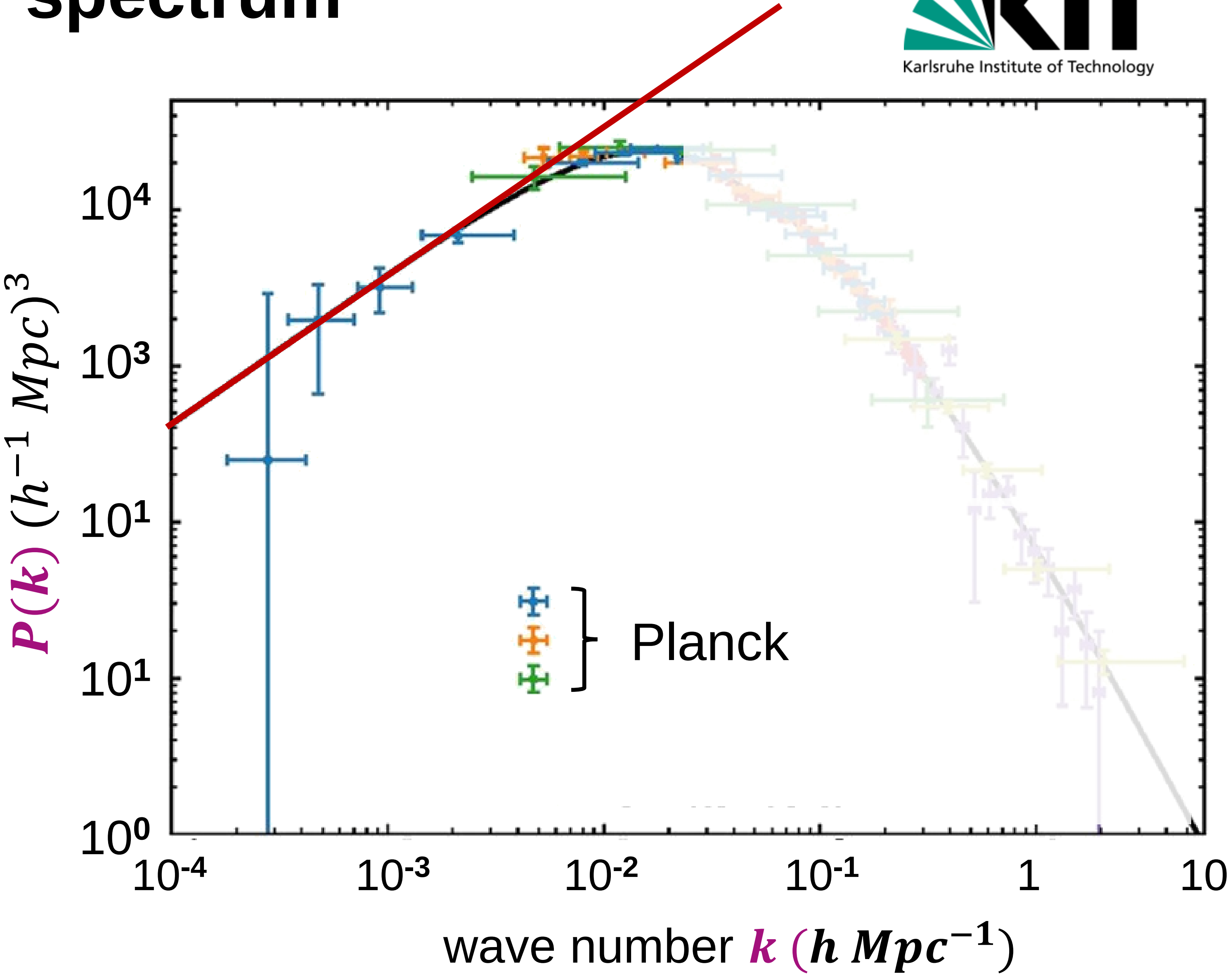
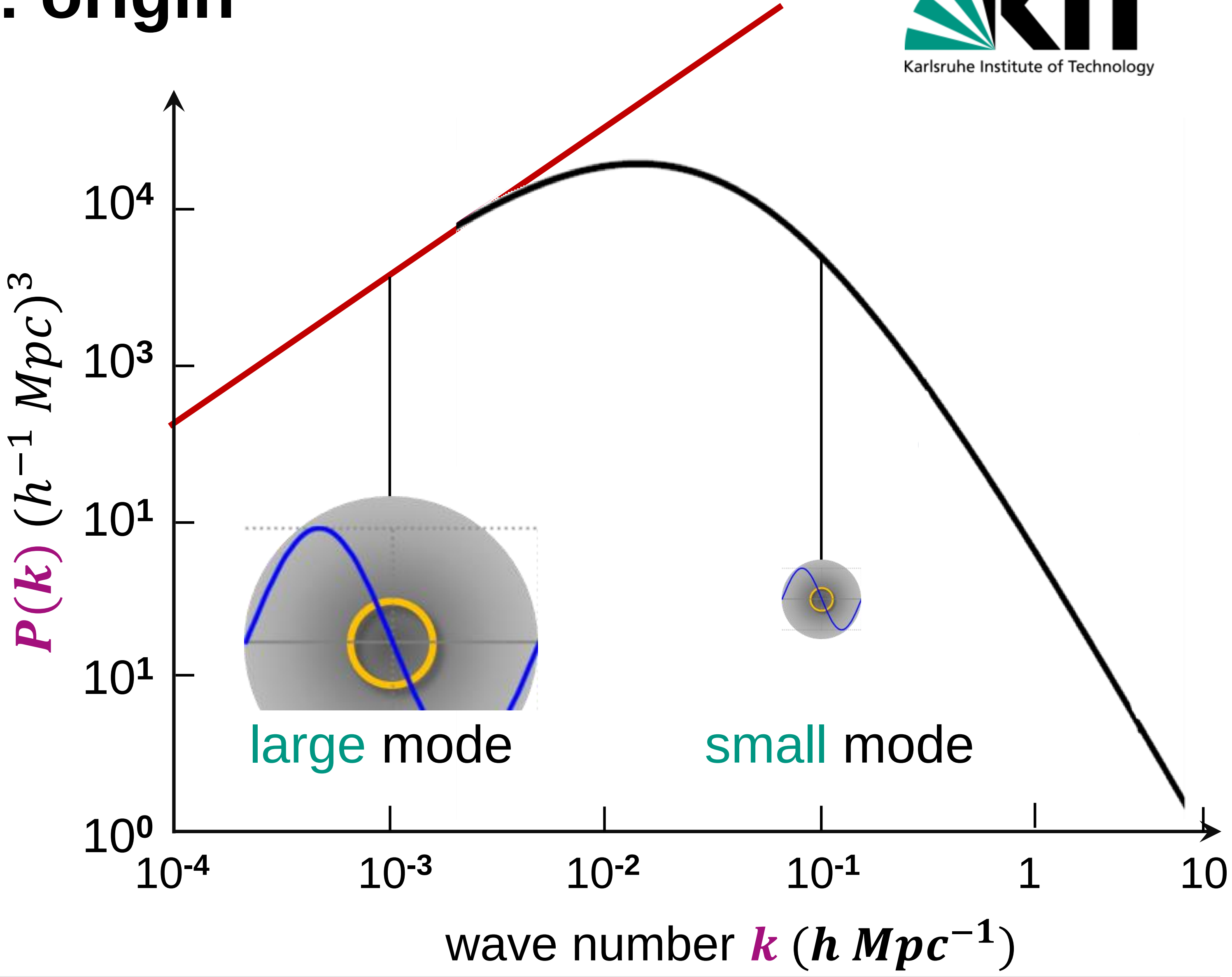


Matter power spectrum: origin

- **Density contrast & $\boldsymbol{Dark}$ $\boldsymbol{Matter}$ ($\boldsymbol{DM}$) modes**

- $P(k)$ arises from the superposition of a large number of DM – **modes**, which grow **independently** from each other

- **single DM – mode**:
  wave number k
  length scale r



Matter power spectrum: discussion

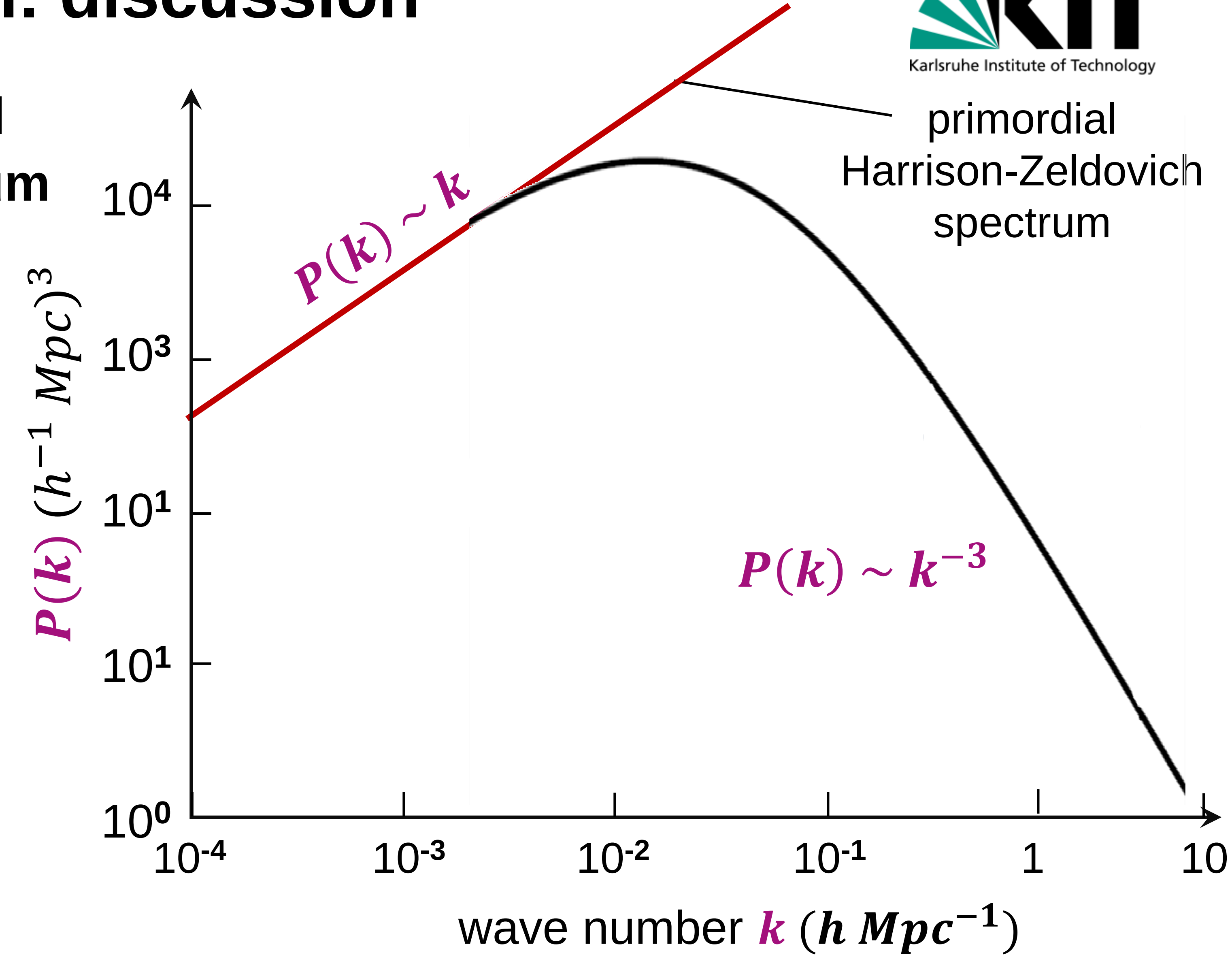
- CMB: **we see the primordial Harrison-Zeldovich spectrum**

largest scales:
´scale-invariant´ spectrum with **linear growth** of density contrast

$$P(k) \sim k$$

smaller scales:
strong reduction in power with

$$P(k) \sim k^{-3}$$



Matter power spectrum: discussion

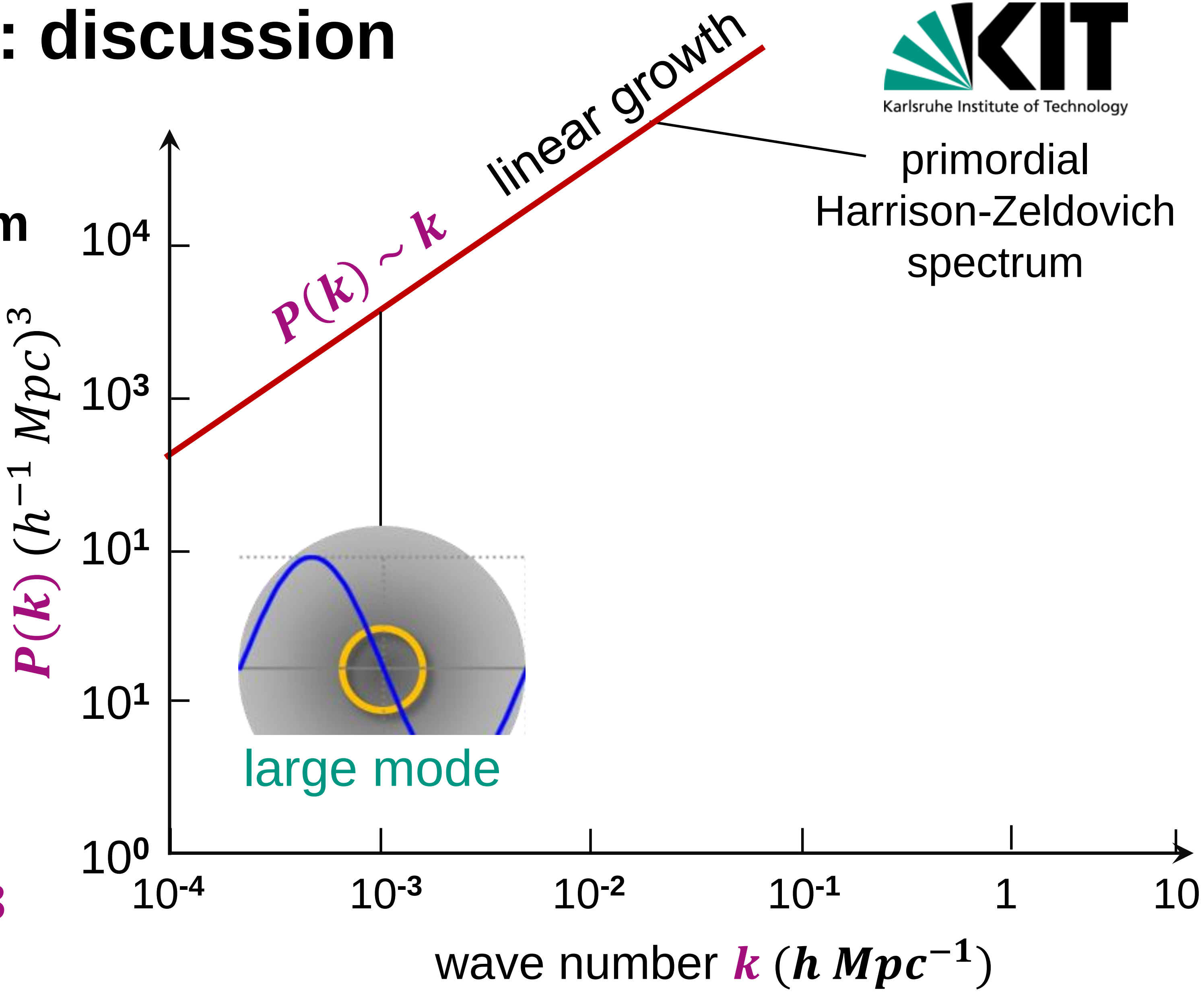
- $\boldsymbol{CMB}$: **we see the primordial Harrison-Zeldovich spectrum**

- expectation from **inflation**:

$\boldsymbol{DM}$ −**modes** increased to scale-invariant Harrison-Zeldovich power spectrum $\boldsymbol{k^n}$

$\boldsymbol{n}$ = (scalar) spectral index

theory: $\boldsymbol{n = 0.92 \dots 0.98}$
Planck: $\boldsymbol{n = 0.9603 \pm 0.0073}$

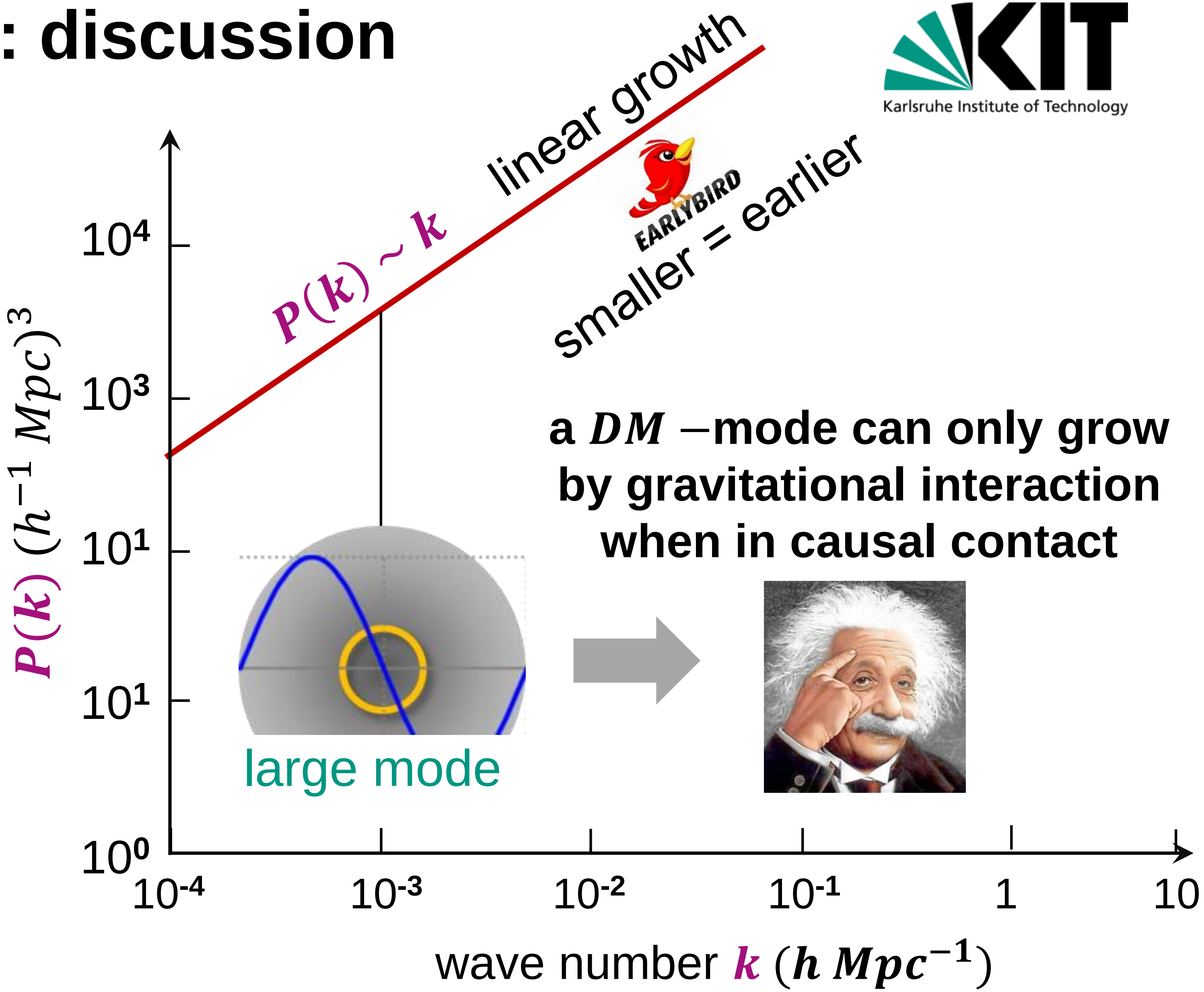


Matter power spectrum: discussion

- **Density contrast of smaller modes increases**

- $\boldsymbol{DM}$ –modes need to be in causal contact to start gravitational interaction against cosmological expansion
then: linear growth regime

smaller $\boldsymbol{DM}$ –modes:
earlier in causal contact
& longer in linear growth

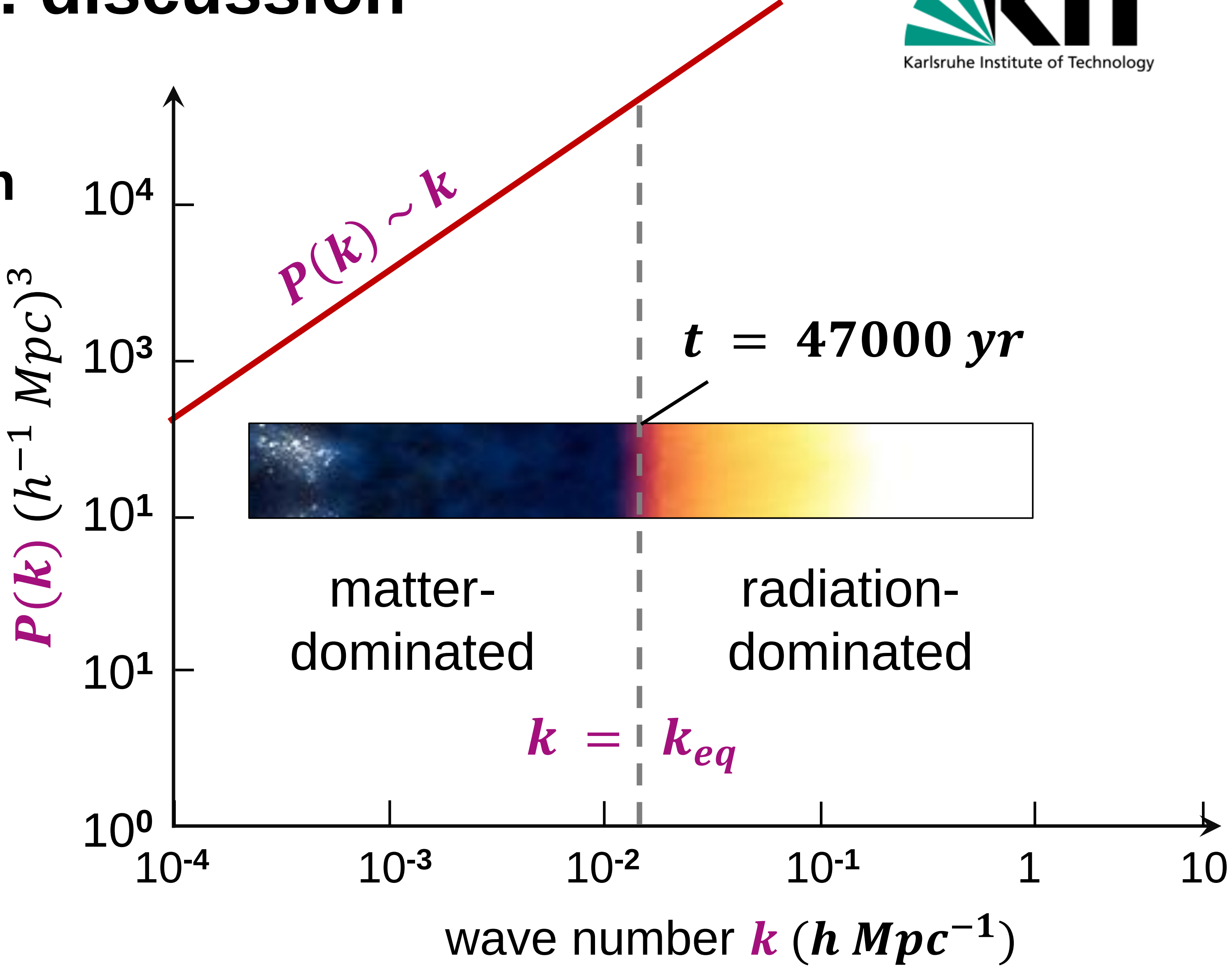


Matter power spectrum: discussion

- **Very small modes: causal contact in the early radiation dominated epoch***

- we need to investigate what happens to very small DM – modes that come into causal contact at $t < 47000\ yr$ in radiation dominated epoch

very small DM – modes: during radiation dominated epoch: no linear growth (**stopped**)



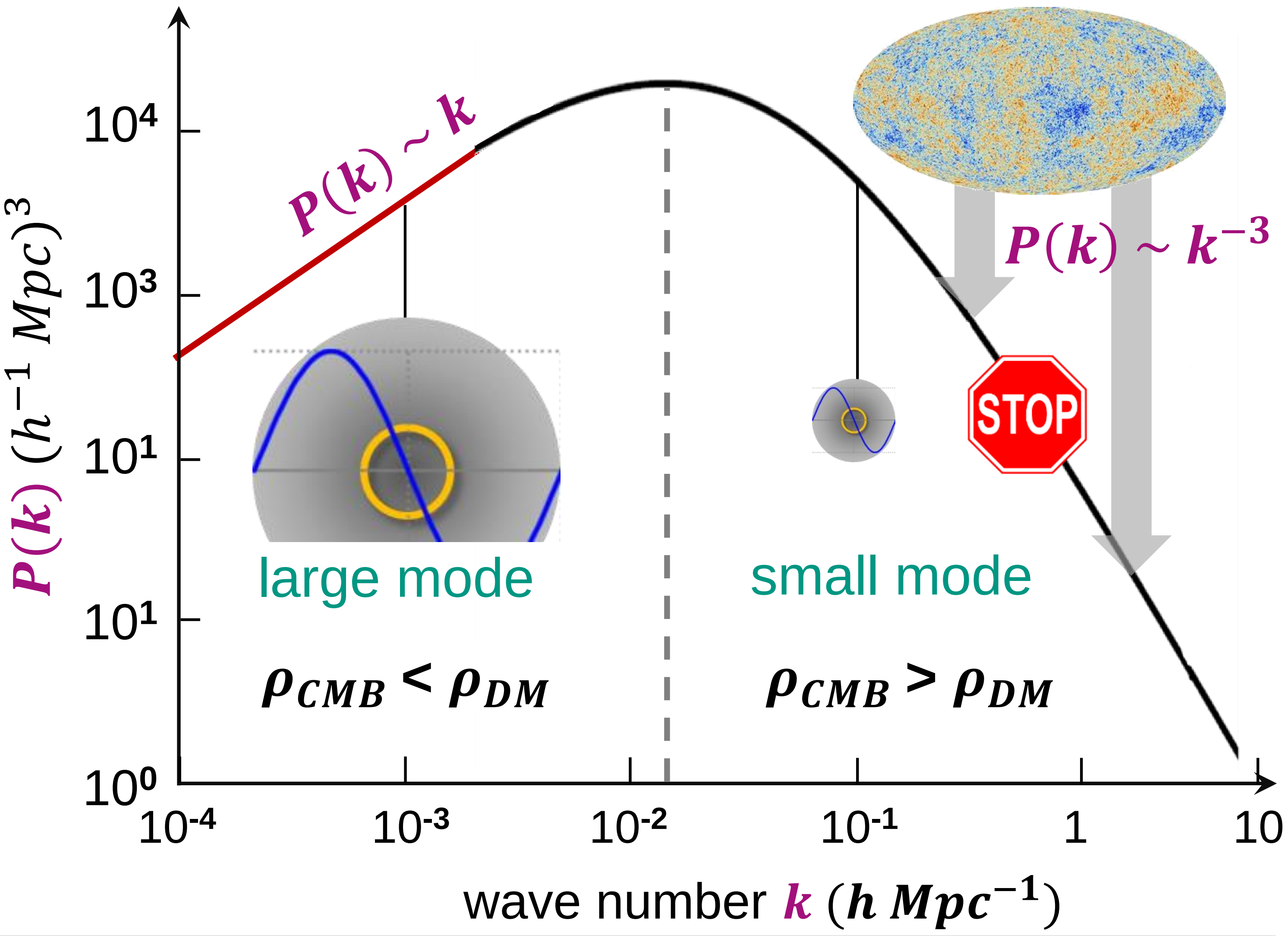
*see Lecture 3 p. 38

Matter power spectrum: discussion

- The **size of a DM – mode** determes its causal horizon & thus its density contrast

- relativistic photons inhibit the growth of the density contrast of small DM – modes during $t < 47000\ yr$

very small DM – modes: during radiation dominated epoch: **photons diffuse**



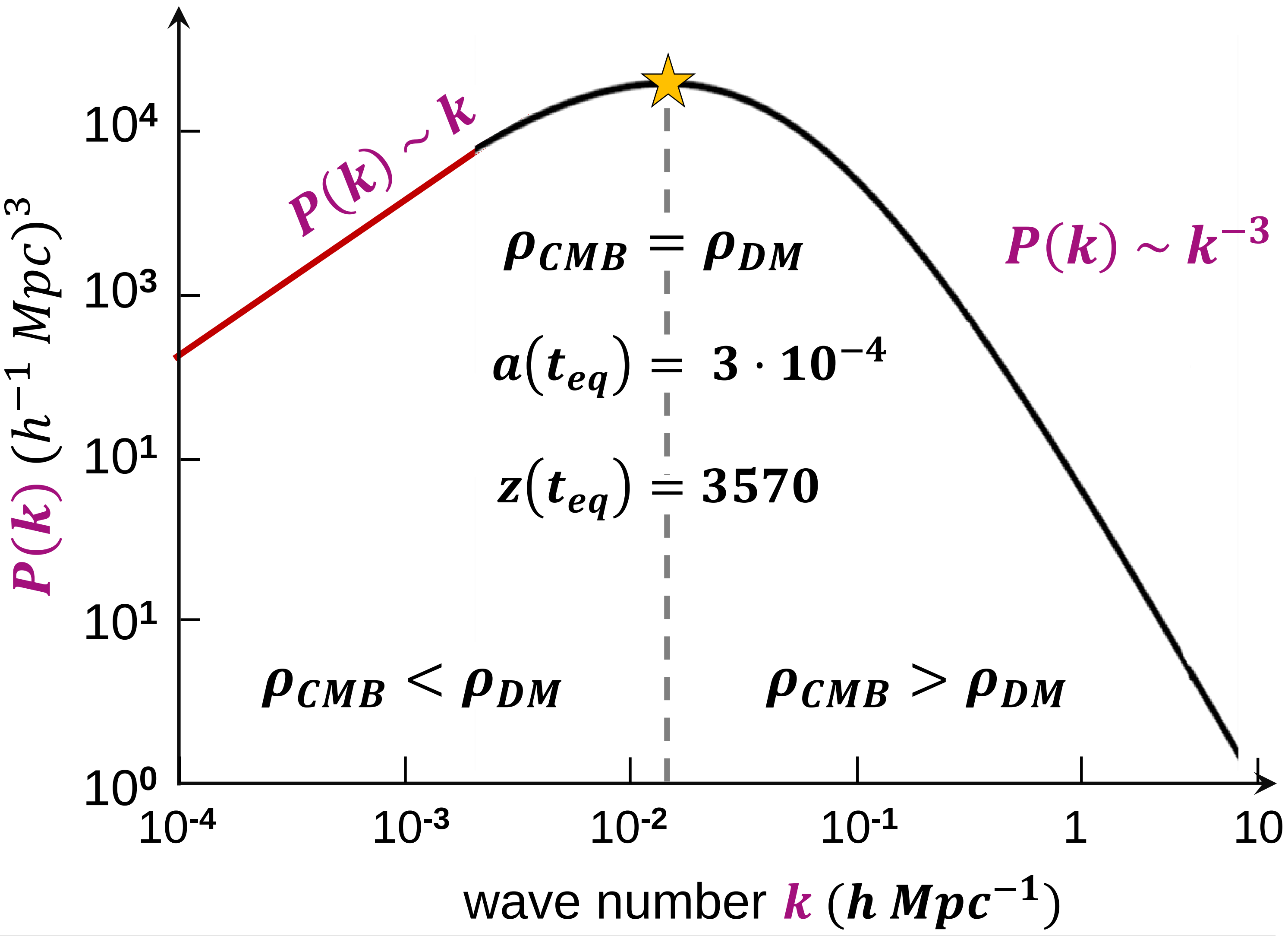
Matter power spectrum: discussion

- **The power spectrum $P(k)$ is maximal at $k = k_{eq}$**

- ´optimum´ case for $P(k)$ is reached for all DM – modes $k = k_{eq}$ which come into causal contact at $t = t_{eq}$

´optimum´ DM – mode:

wavelength $\lambda_{eq} = 350\ h^{-1}Mpc$



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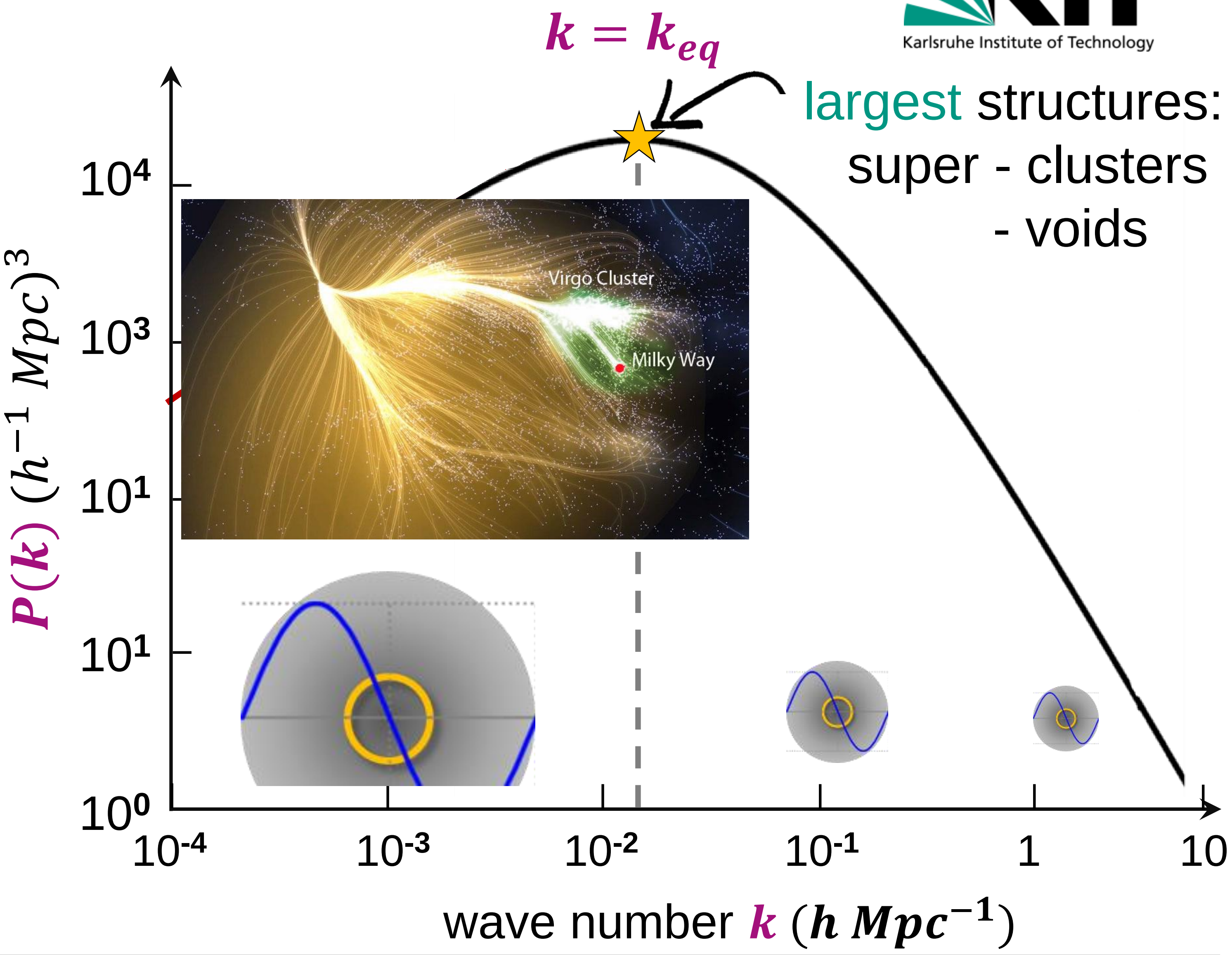

Comparing CMB & LSS analyses

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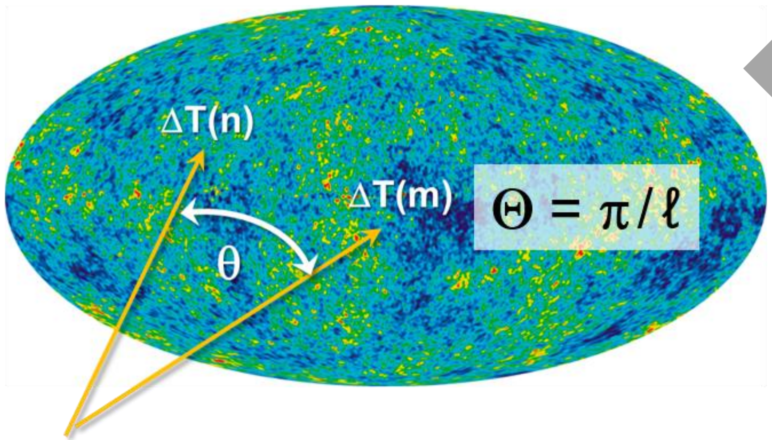
- **What we can learn from CMB multipoles and from matter power spectrum**

CMB-power spectrum: coefficiants C_ℓ

- **2 dim. :** on surface of a sphere
- angle Θ ⇔ multipole ℓ
- correlation function:

$$C(\theta) = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_\ell P_\ell(\cos\theta)$$

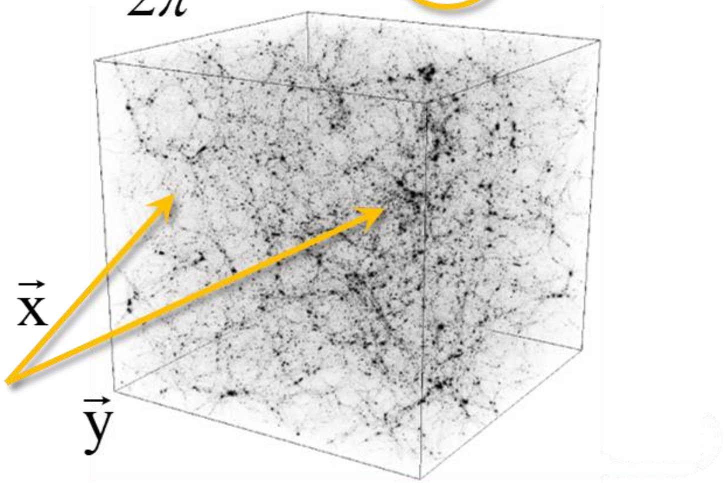


same origin

matter-density contrast δ

- **3 dim. :** within volume of sphere
- distance ***r*** ⇔ wave number ***k***
- correlation function density contrast:

$$\langle \delta^2 \rangle = \frac{1}{2\pi^2} \int k^2 \cdot P(k) \cdot dk$$



Comparing CMB & LSS analyses

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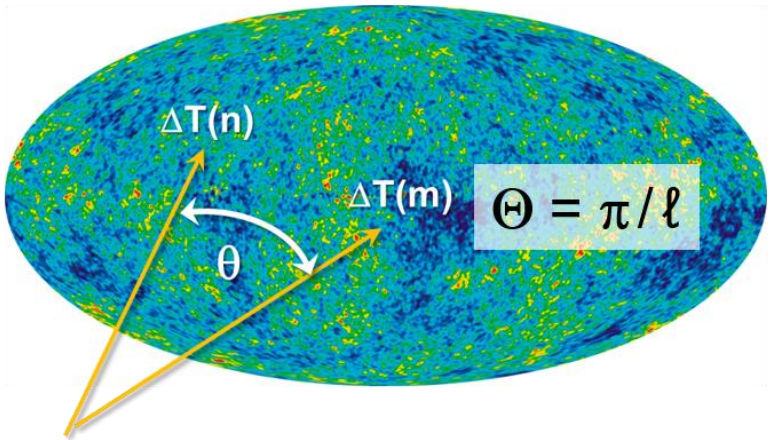

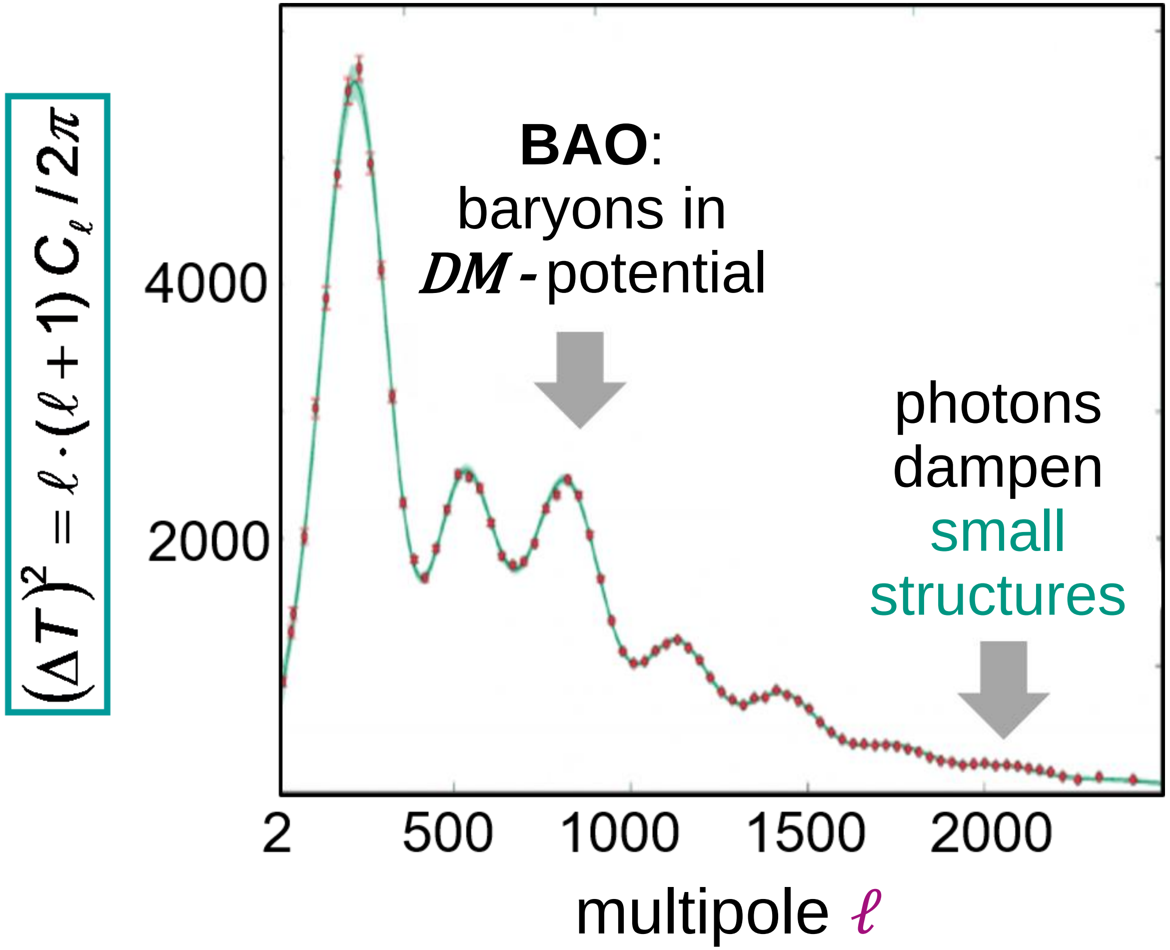

Comparing CMB & LSS analyses

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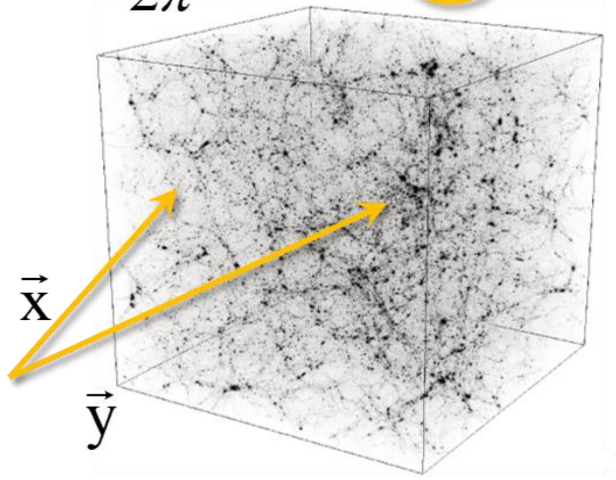
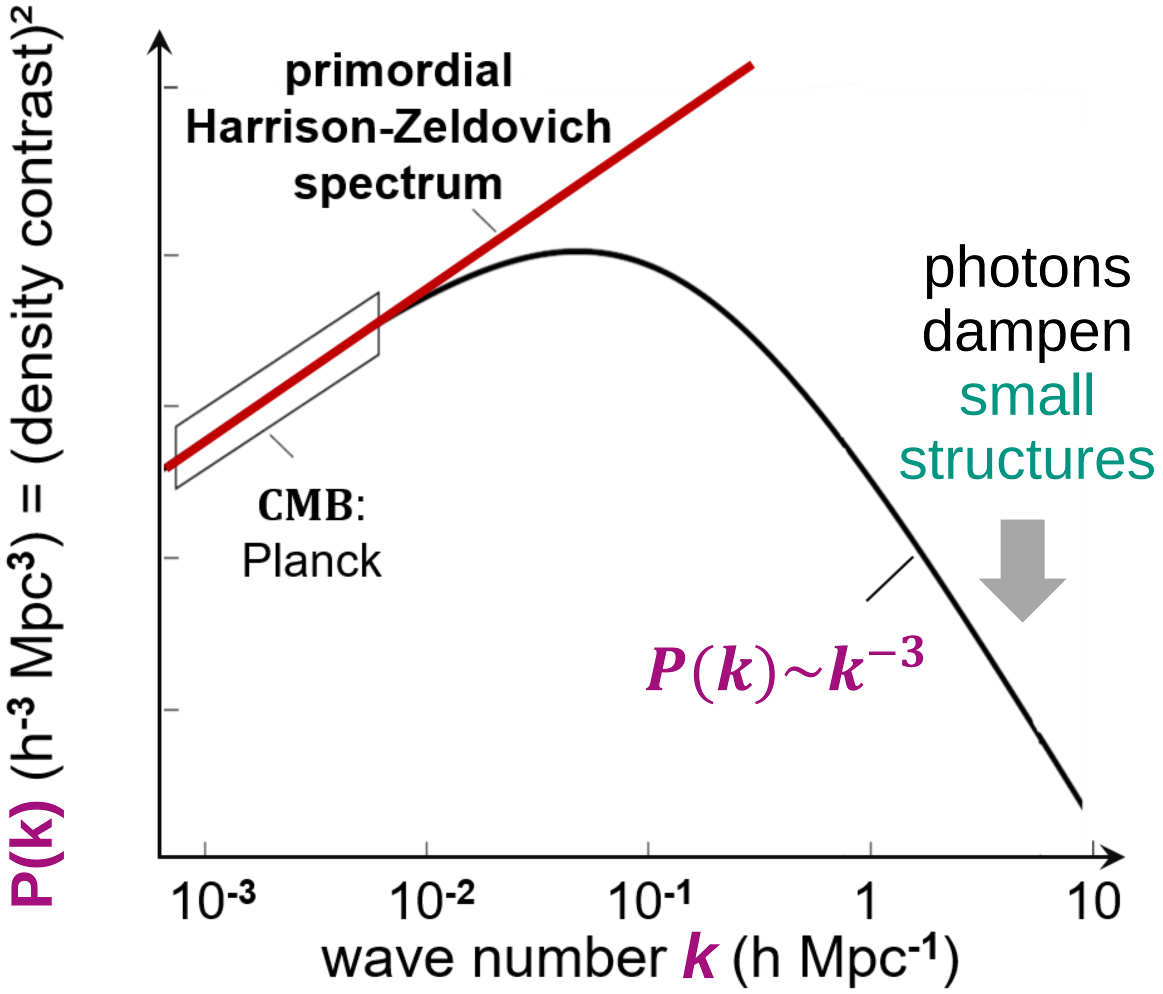


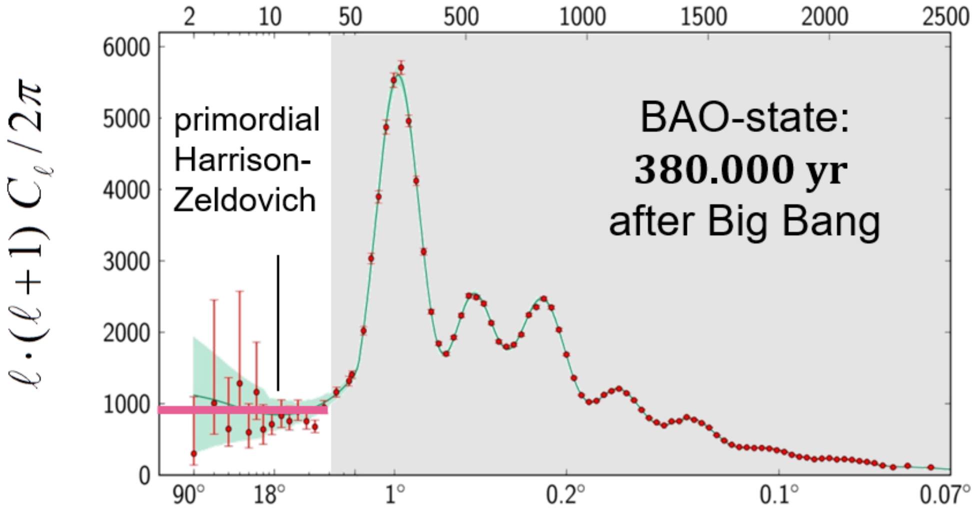


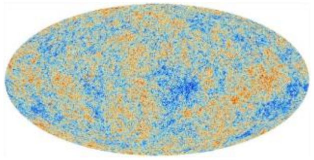
Comparing CMB & LSS analyses

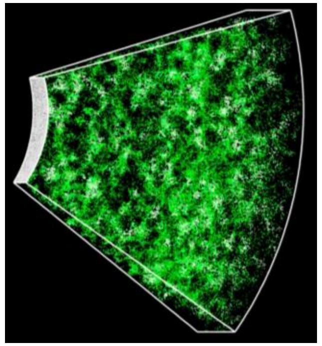
- **What we can learn from CMB multipoles and from matter power spectrum**

CMB - power spectrum C_ℓ

- C_ℓ: structure information on surface of sphere up to decoupling of CMB

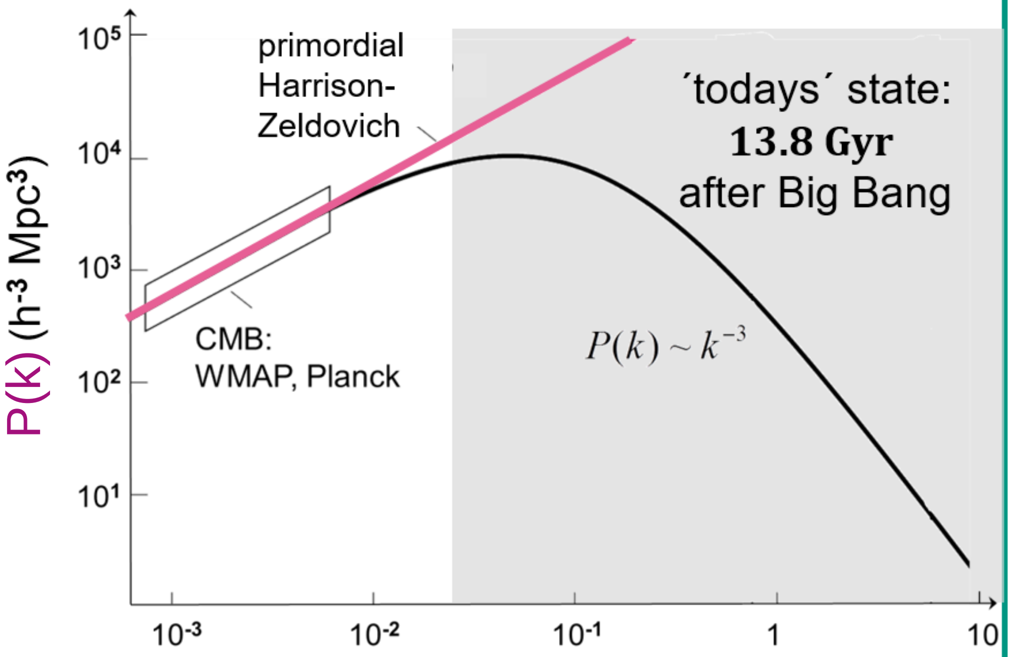






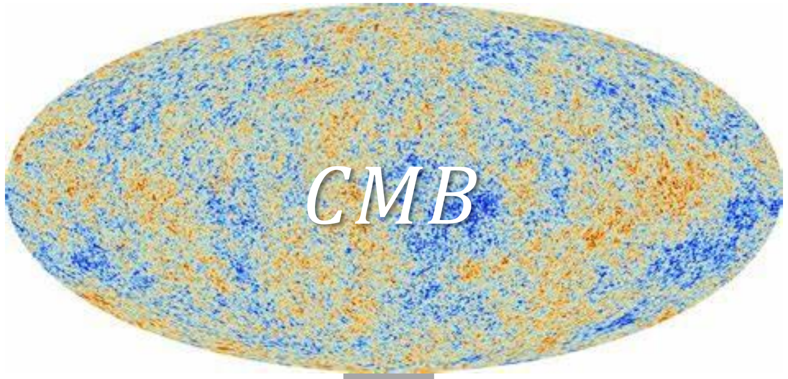
galaxy- power spectrum $P(k)$

- $P(k)$: structure information in volume of sphere up to present universe



Matter power spectrum & relativistic particles

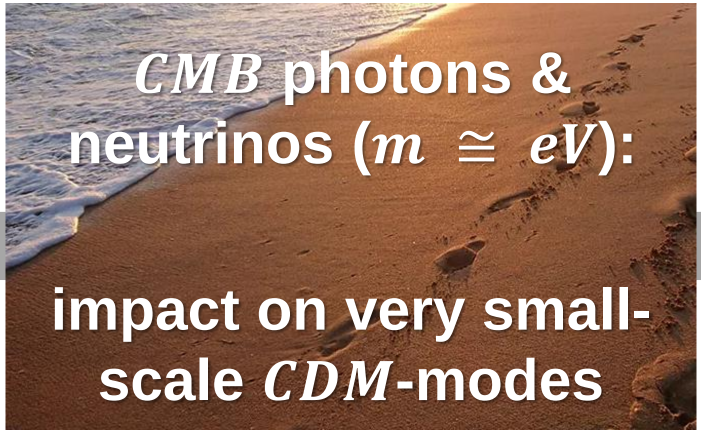
- $P(k)$ and the wash-out of structures at small scales due to photons & ν´s

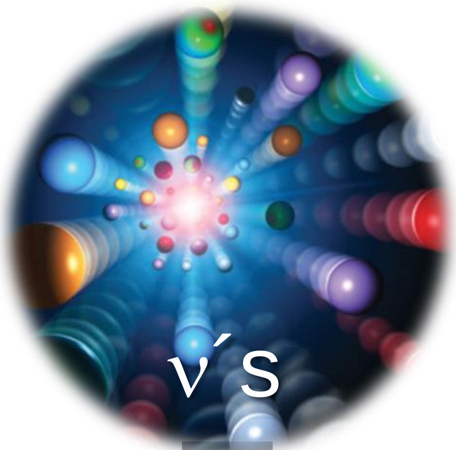


relativistic BAO &

CMB-photons: **Silk-damping**

(dark) matter:
$P(k)$ reduced due to gravitational interaction





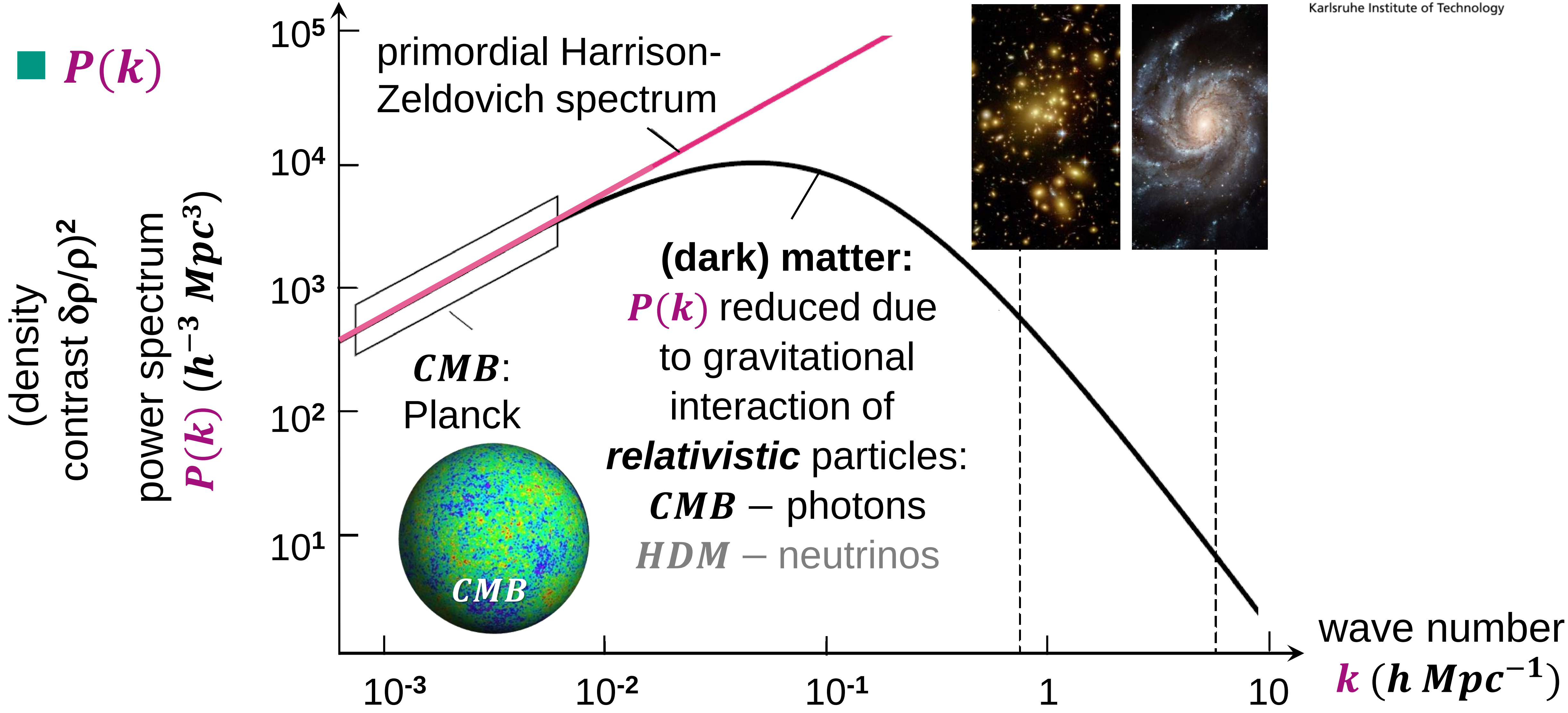
neutrinos with **´freestreaming´**

eV –masses: over Gpc

wash-out of small-scale structures

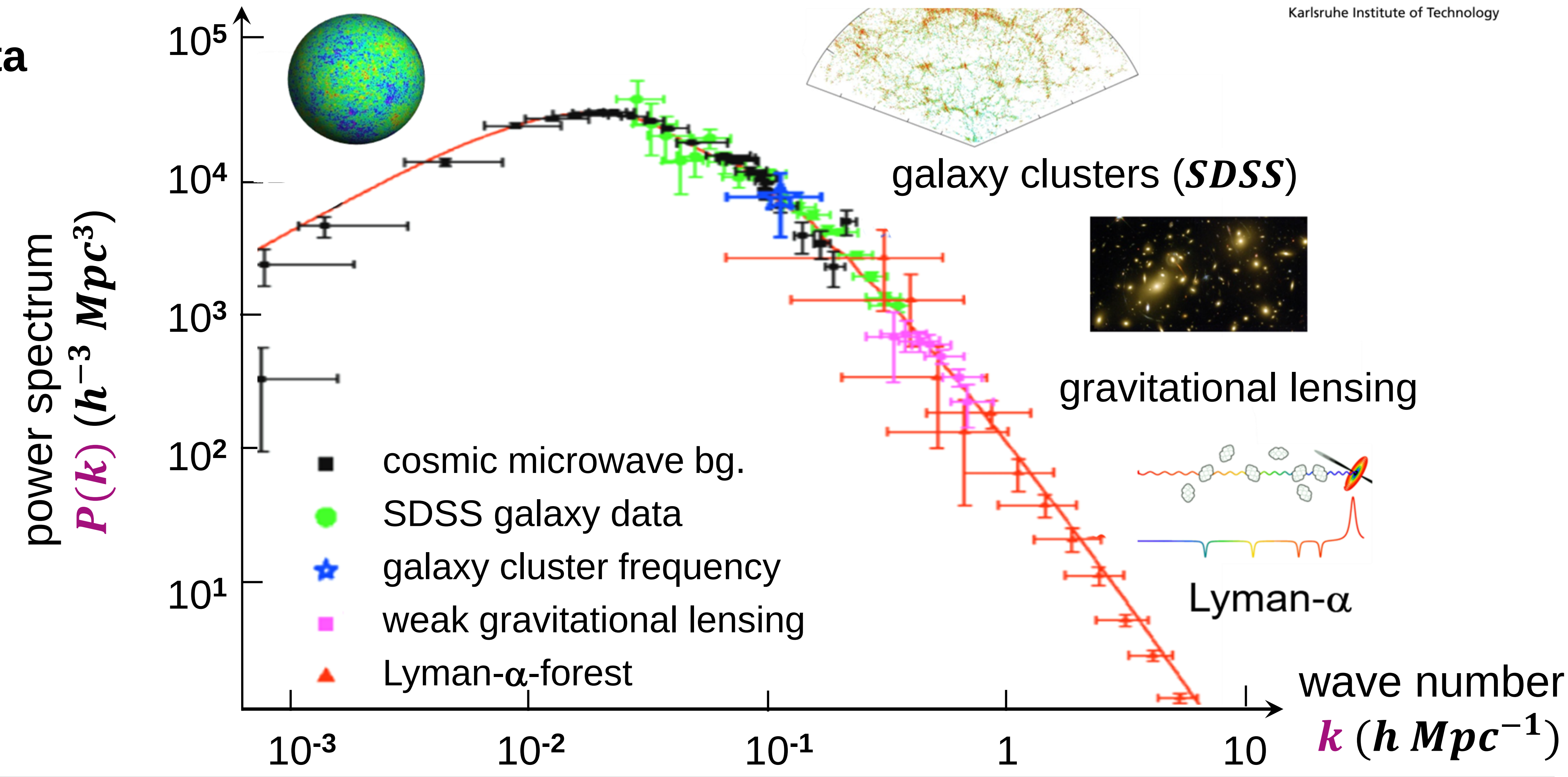
Matter power spectrum & scales

- $P(k)$



Matter power spectrum & measurements

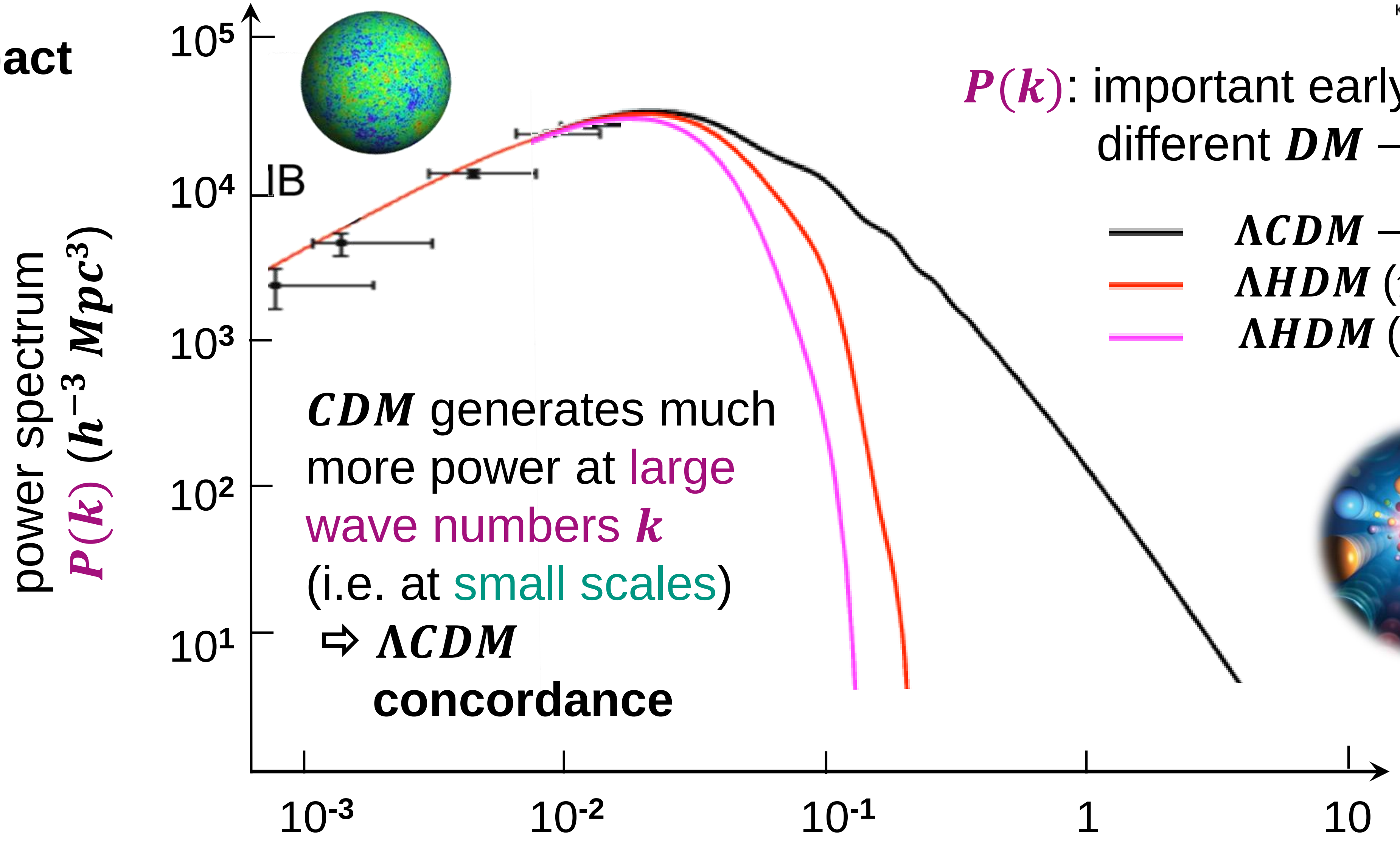
- data



Matter power spectrum: CDM vs. HDM only

- **impact**

$P(k)$: important early test of different DM – models



CDM generates much more power at large wave numbers k (i.e. at small scales)

⇨ ΛCDM **concordance**

Matter power spectrum: $sub - eV$ limits for m_ν

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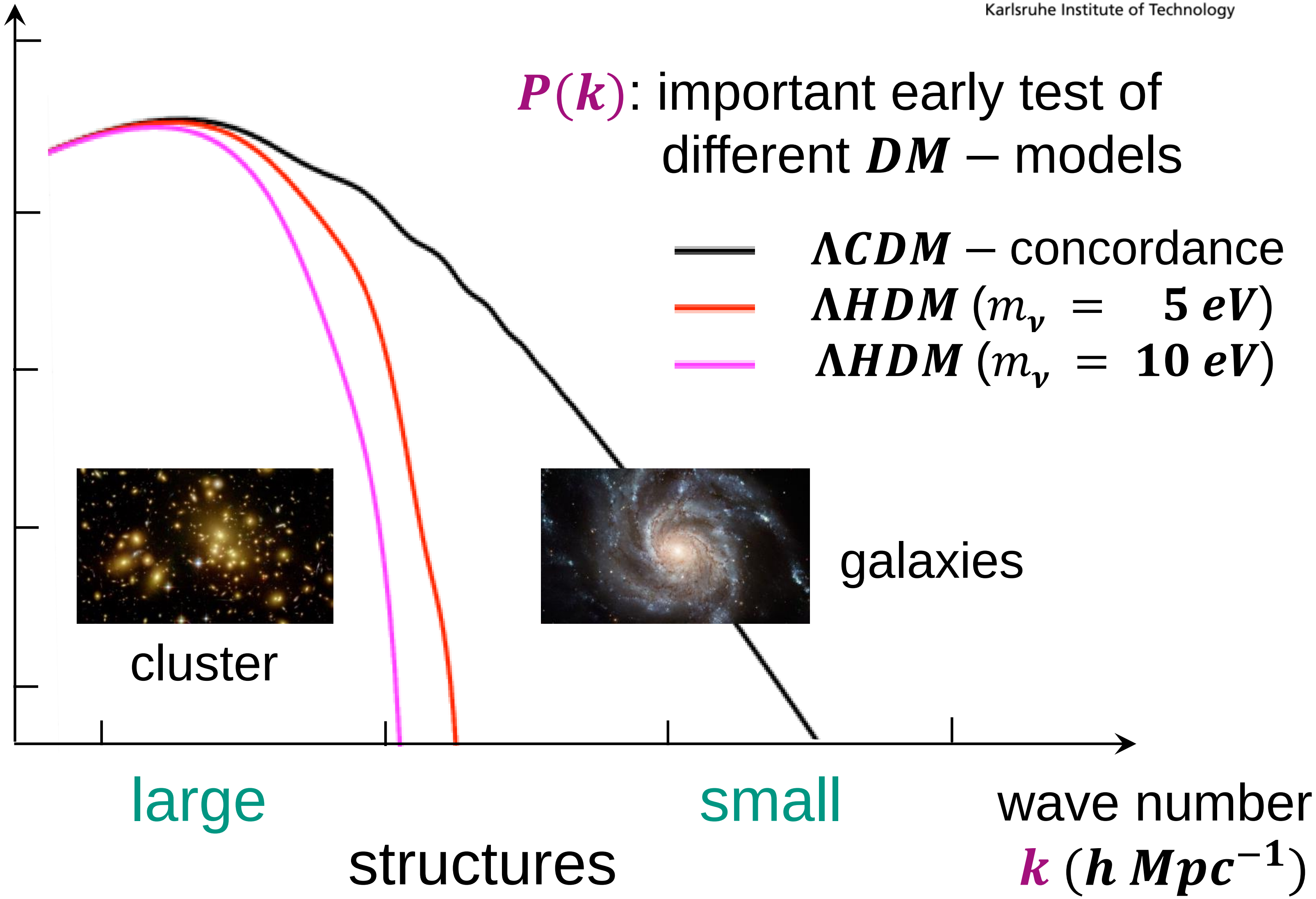
- **Cosmological limits for neutrino masses**

- interplay of LSS **data** from **cosmology** & **elementary particle physics**!

$P(k)$: important early test of different DM – models



Matter power spectrum: stop press…

- **Cosmological limits for neutrino masses**

- interplay of ***LSS* data** from **cosmology** & **elementary particle physics**!

Astronomy

'Less clumpy' universe may suggest existence of mysterious forces

Survey could mean there is a crucial component missing from so-called standard model of physics

[Joint analysis of Dark Energy Survey Year 3 data and CMB lensing from SPT and Planck. III. Combined cosmological constraints (aps.org)]

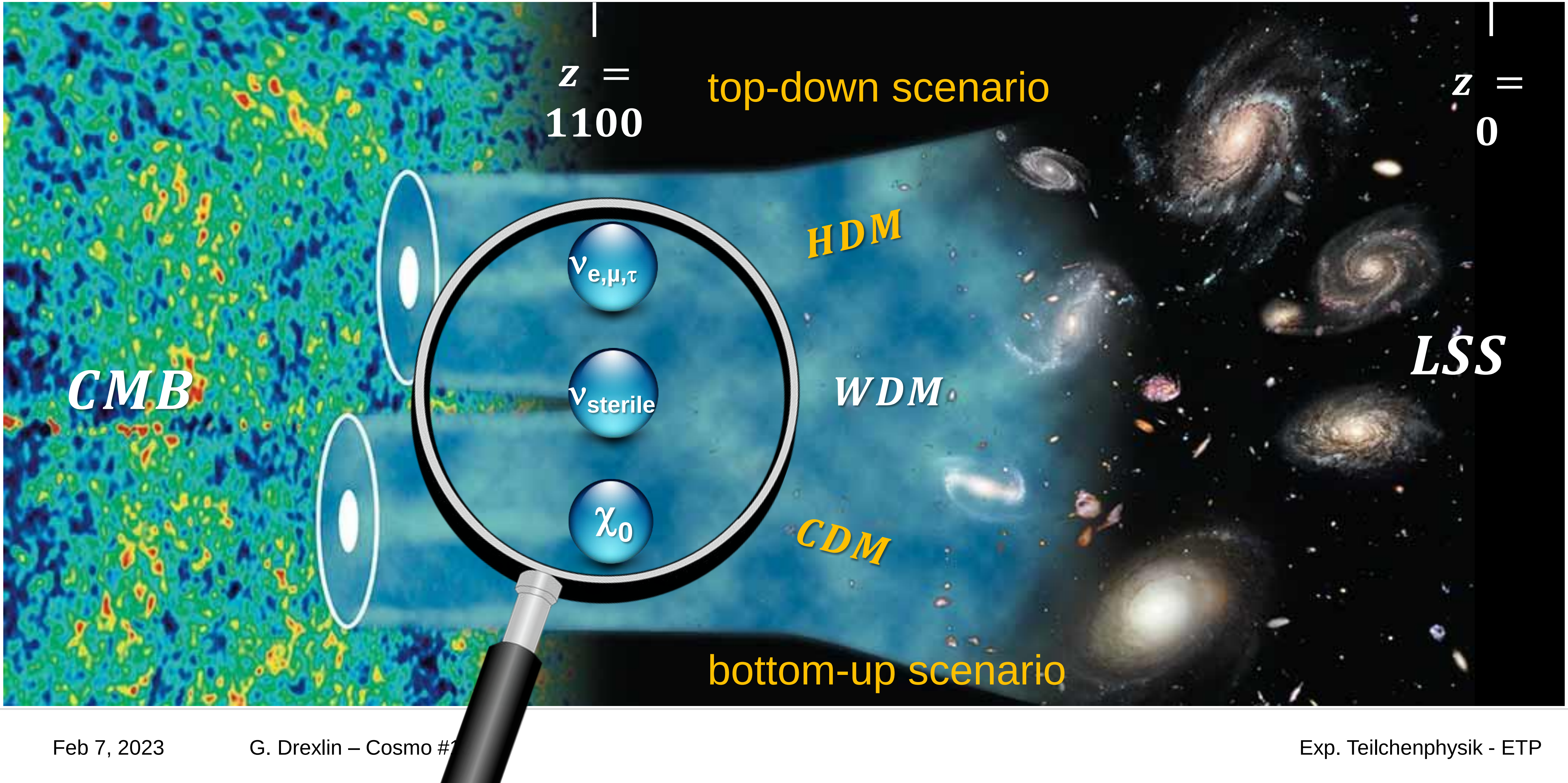
Hannah Devlin *Science correspondent*

@hannahdev

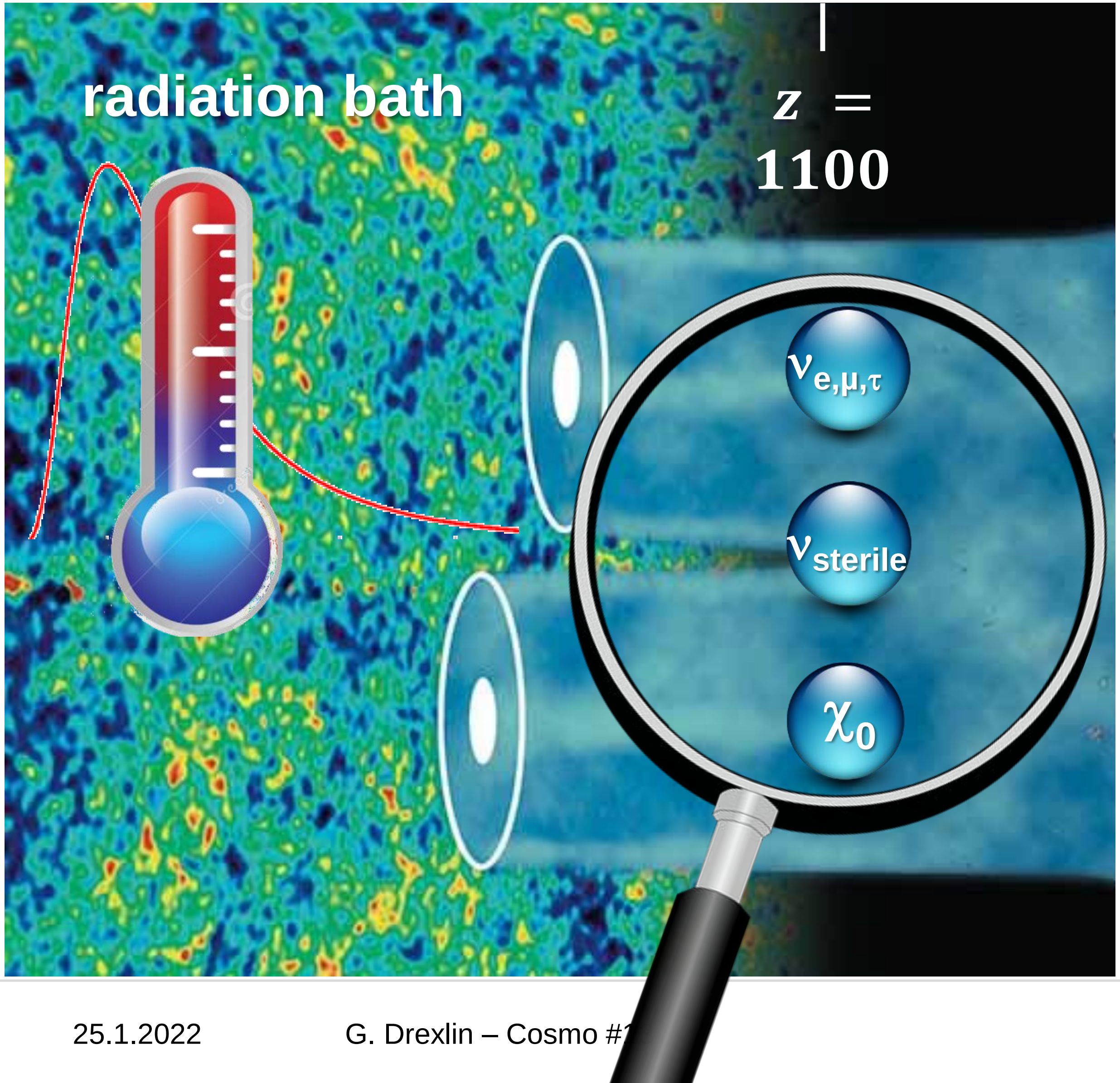
Tue 31 Jan 2023 13.22 GMT

One of the most precise surveys of the structure of the universe has suggested it is "less clumpy" than expected, in findings that could indicate the existence of mysterious forces at work.

A closer look at properties of CDM, WDM & HDM



thermal DM relics vs. non-thermal DM production



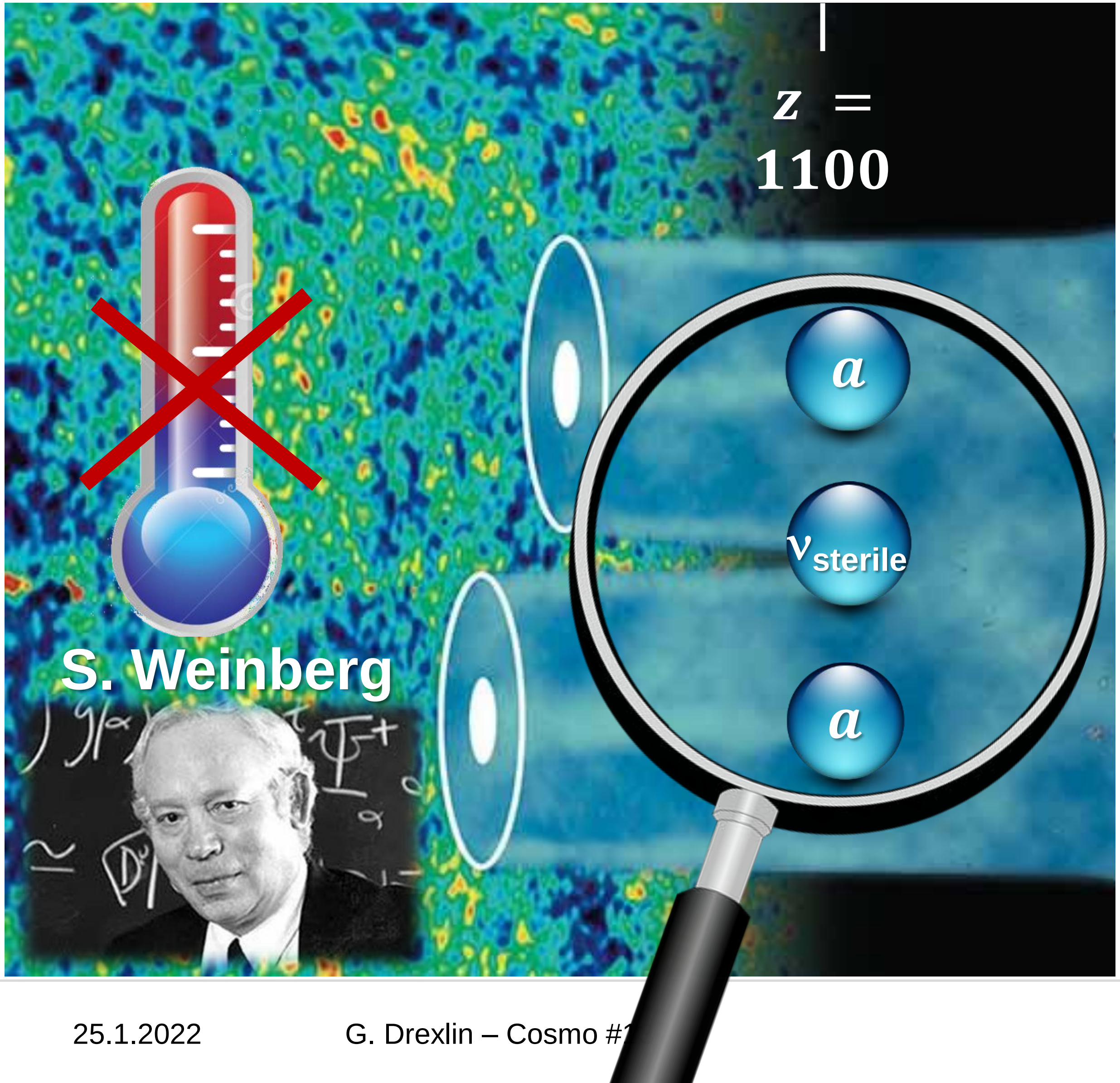
DM – particles as **thermal relics**:
produced via thermal processes in radiation bath of early universe

ultra-relativistic → HDM: $T_{fr} \gg m$

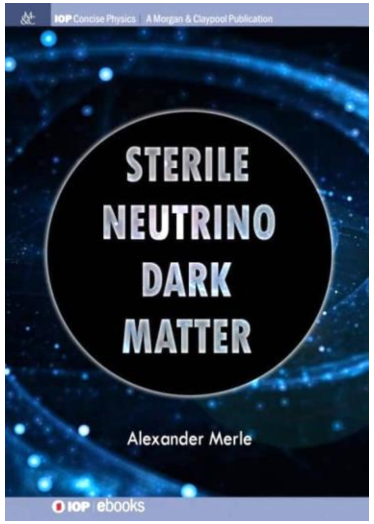
non-relativistic → CDM: $T_{fr} \ll m$

⇨ characteristic freeze-out temperature T_{fr}

thermal DM relics vs. non-thermal DM production

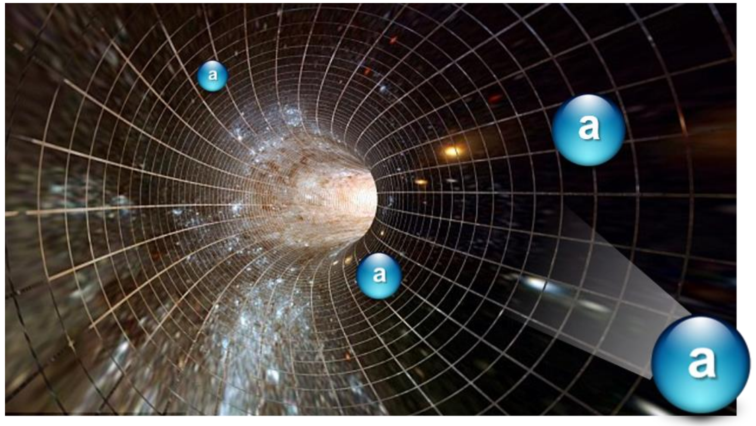


DM – particles as **non-thermal relics** via **symmetry breaking** or due to **ν-oscillations** (Stephen Weinberg et al.)



WDM: sterile neutrinos

HDM, CDM: axions



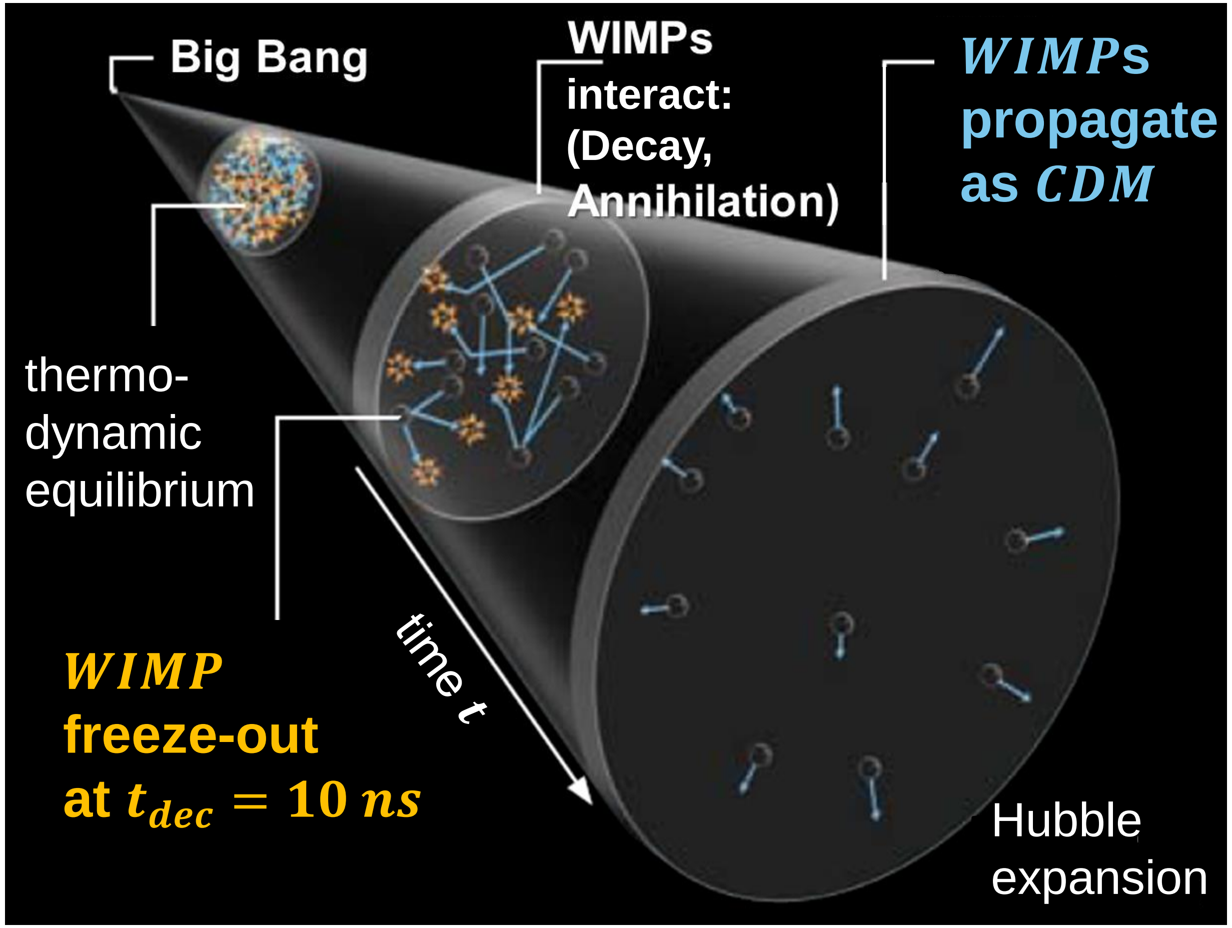
CDM: $WIMPs$ as thermal relics

Thermal production of CDM: $WIMPs$*

- $WIMPs$: massive, non-baryonic thermal relics left over from Big Bang, wich interact via **weak interaction** only (+ **gravity)**

- only pair production / pair annihilation of $WIMPs$ (≡ Majorana particles with conserved $SUSY$ quantum number R_P)

- huge mass (TeV – scale):
  ⇨ vast phase space in case of decay



*Weakly Interacting Massive Particle

CDM: $WIMPs$ as thermal relics

Annihilation rate of $WIMPs$

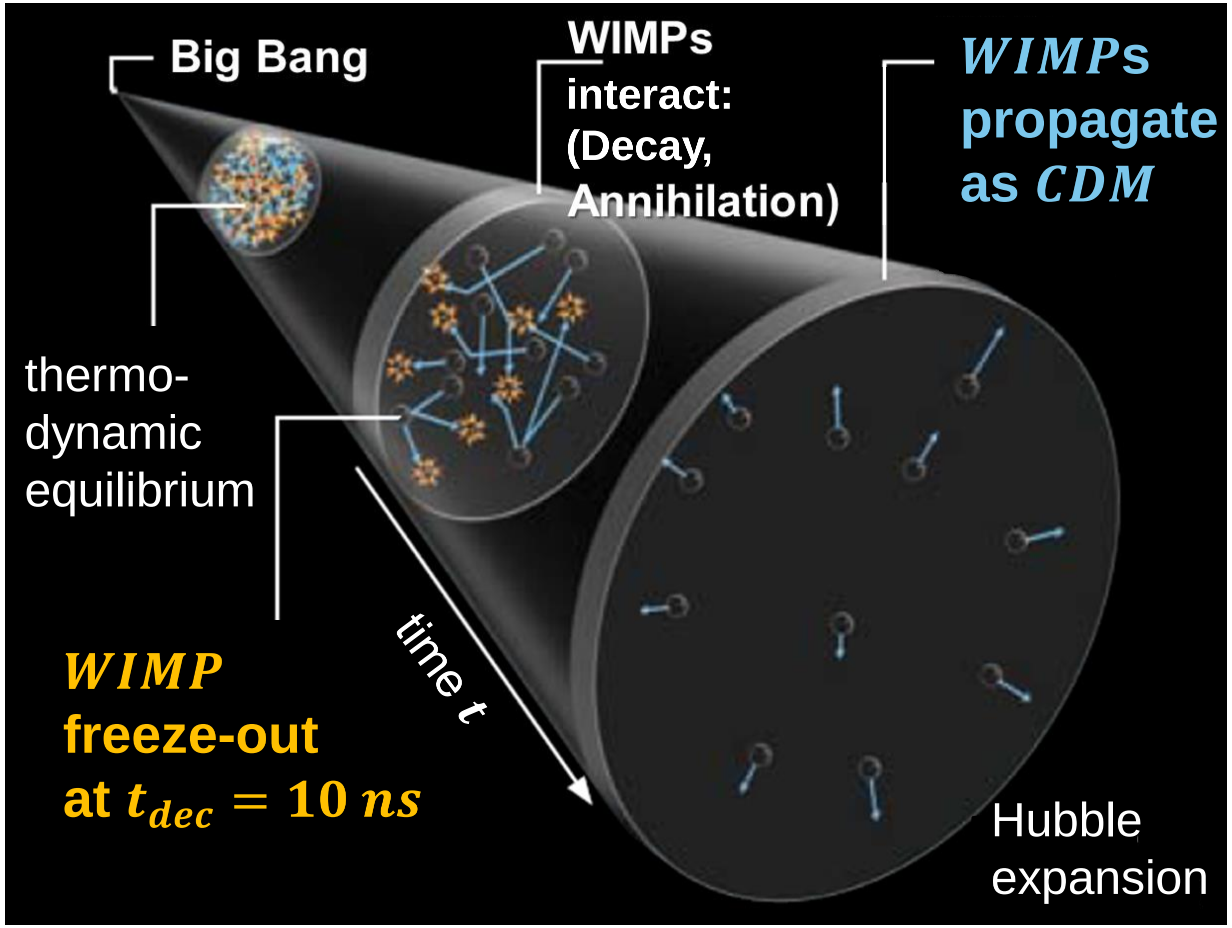
- $WIMPs$: annihilation rate given by typical **weak interaction strength**

$$\sigma_{ann} = \sigma_{weak}$$

- decoupling time from radiation bath is $t_{dec} \approx 10\ ns$

$$\sigma_{ann}(t_{dec}) = H(t_{dec})$$

- $WIMP$ **miracle** (will be discussed later)



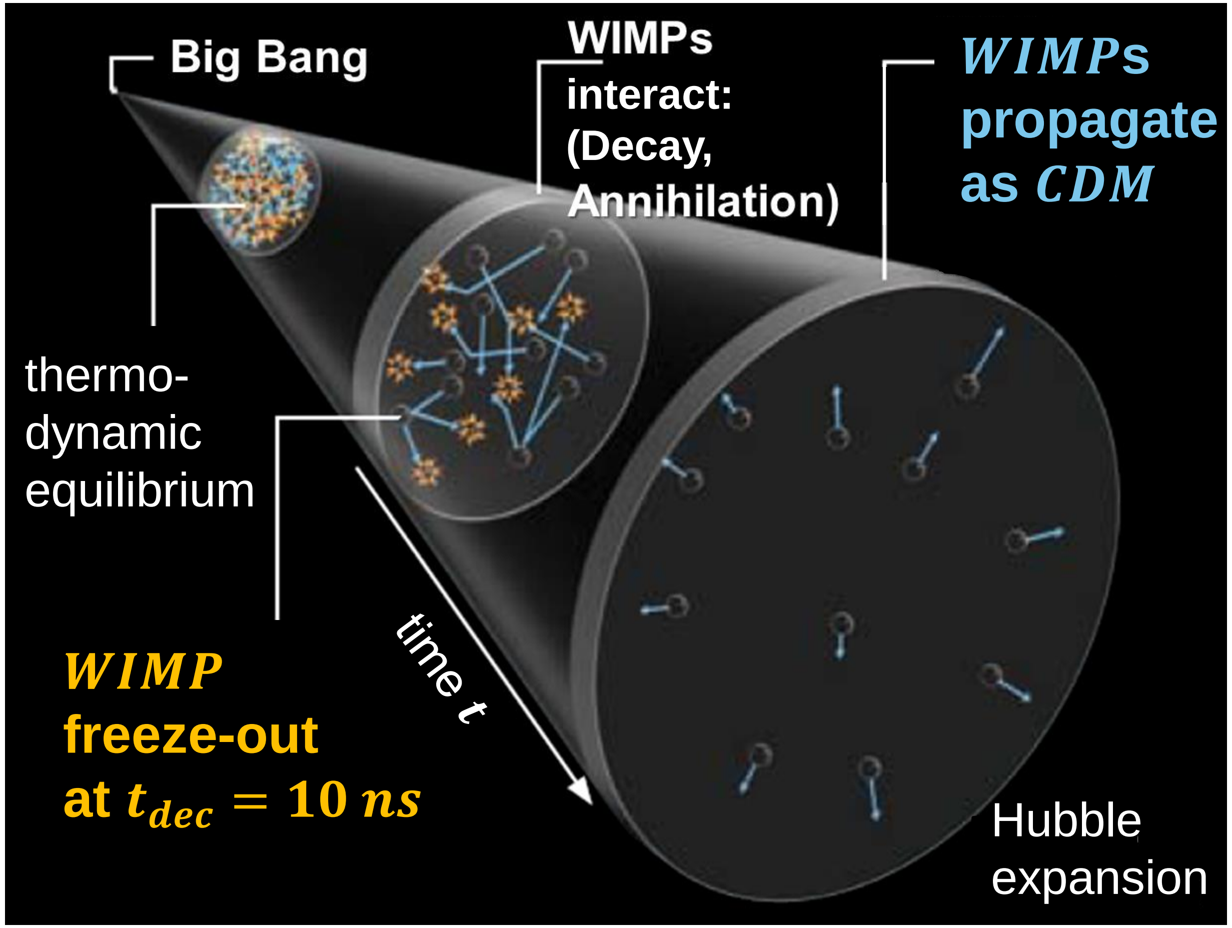
CDM: $WIMPs$ as thermal relics

Annihilation rate of $WIMPs$

- $WIMPs$: non-relativistic propagation with very limited free-streaming range as

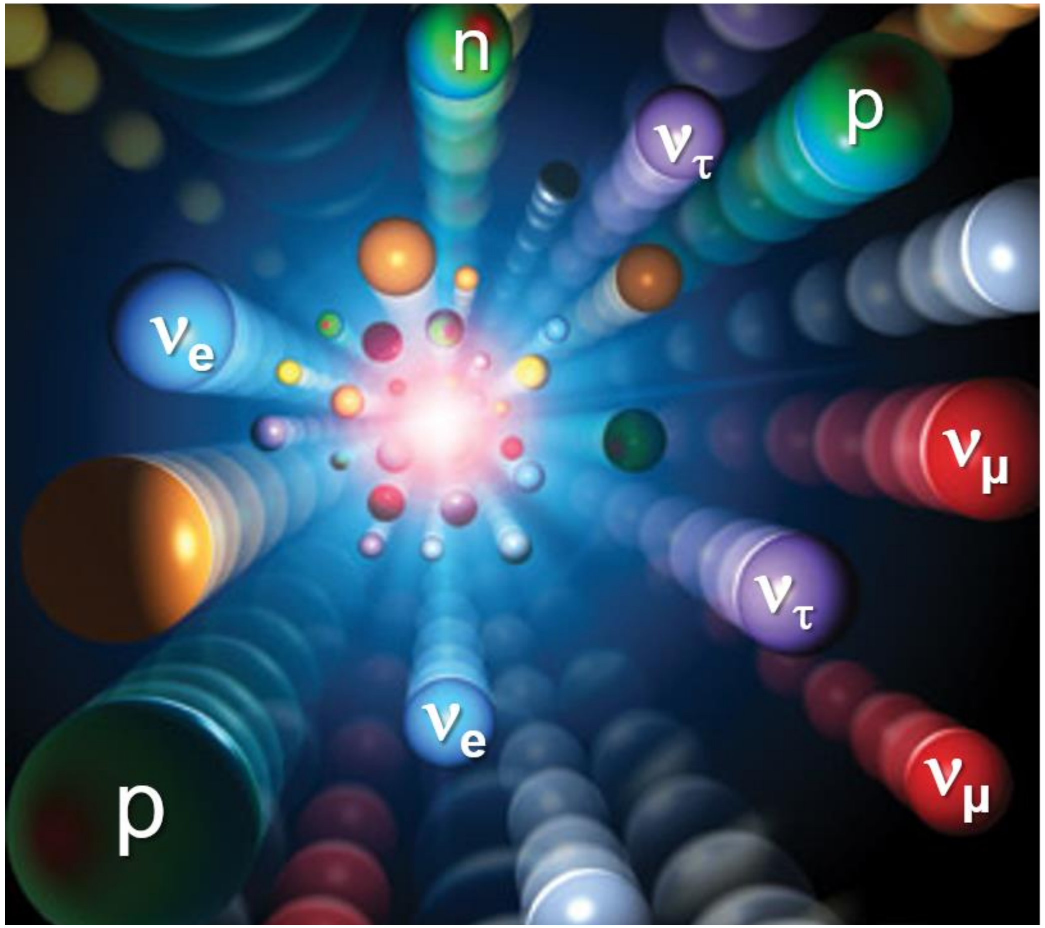
$$E_{kin}(WIMP) \approx 0.05 \cdot m(WIMP)$$

- after decoupling: stable over long, cosmological time scales due to intrinsic $SUSY$ – based R – parity R_P



HDM: ν´s as light thermal relics

- **Hot dark matter: relativistic free-streaming**



- $\boldsymbol{\nu}$´**s**: relativistic propagation over an exceedingly long free-streaming range of $> \; Gpc$

$$E_{kin}(\nu) > 10^6 \cdot m(\nu)$$

- after decoupling: stable over long, cosmological time scales due to conserved lepton number $\boldsymbol{L}$

HDM: ν´s as light thermal relics

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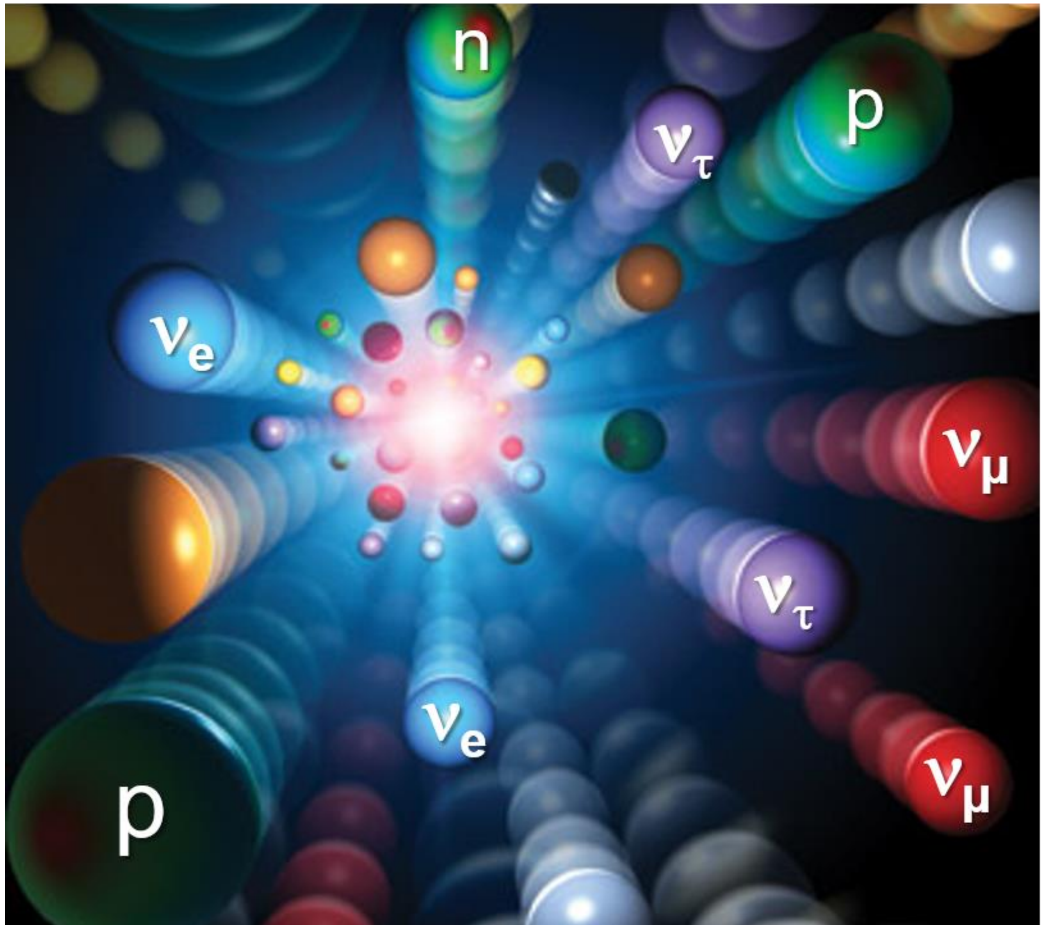
■ Hot dark matter: relativistic free-streaming



- **ν´s**: relativistic propagation suppresses annihilation in the early universe (also: no lighter leptons in the SM)
  weak interaction only with matter

- decoupling at $t \cong 1\ s$ and $T \cong 1\ MeV$
  (see 3.1 Big Bang Nucleosynthesis, lecture #5 $p.51$)

Dark Matter: hot, warm or cold ($HDM/WDM/CDM$)

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- Generic particle models for cosmological $DM -$ density $\rho_{DM} \cong 1\, GeV/m^3$

Hot Dark Matter	Warm Dark Matter	Cold Dark Matter
particle candiate: ***active*** neutrinos $\nu_{e,\mu,\tau}$ $m \sim 0.05 \dots 0.8\, eV$	particle candiate: ***sterile*** neutrinos ν_s $m \sim 1 \dots 20\, keV$	particle candiate: ***SUSY*** neutralinos χ^0 $m \sim 0.1 \dots 10\, TeV$
number density: N(active): $339/cm^3$ decoupling: $T_{fr} = 2 - 3\, MeV$ $T_{fr}/m \sim 10^6 \dots 10^7$	number density: N(sterile): $< 1/cm^3$ decoupling: no thermal process, but via ν –oscillations	number density: $N(\chi^0)$: $< 10^{-9}/cm^3$ decoupling: $T_{fr} = GeV \dots TeV$ $T_{fr}/m \sim 1/20$
impact on LSS: wash-out of structure on scales $\lambda \leq 1\, Gpc$	impact on LSS: wash-out of structure on scale $\lambda < 0.1\, Mpc$	impact on LSS: wash-out of structure on scales $\lambda < 0.1\, pc$

Dark Matter: hot, warm or cold ($HDM/WDM/CDM$)

- Generic particle models for cosmological $DM-$ density $\rho_{DM} \cong 1\,GeV/m^3$

Hot Dark Matter

particle candiate:

active neutrinos $\nu_{e,\mu,\tau}$

$m \sim 0.05 \ldots 0.8\ eV$

impact on LSS:

wash-out of structure on scales $\lambda \leq 1\,Gpc$

Warm Dark Matter

particle candiate:

sterile neutrinos ν_s

$m \sim 1 \ldots 20\ keV$

impact on LSS:

wash-out of structure on scale $\lambda < 0.1\,Mpc$

Cold Dark Matter

particle candiate:

SUSY neutralinos χ^0

$m \sim 0.1 \ldots 10\ TeV$

impact on LSS:

wash-out of structure on scales $\lambda < 0.1\,pc$

Dark Matter: hot, warm or cold ($HDM/WDM/CDM$)

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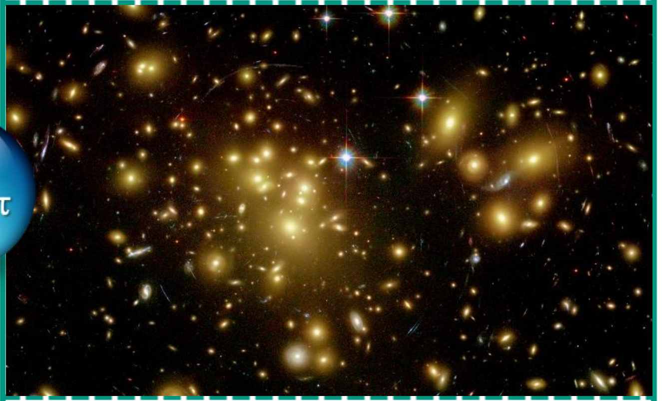
- **Generic particle models: pre-view of Lee-Weinberg curve**

Hot Dark Matter

particle candiate:

active neutrinos $\nu_{e,\mu,\tau}$

$m \sim 0.05 \ldots 0.8\ eV$



impact on LSS:

wash-out of structure

on scales $\lambda \ \leq \ 1\ Gpc$

Lee-Weinberg curve

이휘소

Weinberg

for thermal production & subsequent reduction due to annihilation processes: only **two narrow mass regions** to explain $\Omega_{DM} \sim 0.25$

Cold Dark Matter

particle candiate:

SUSY neutralinos χ^0

$m \sim 0.1 \ldots 10\ TeV$

impact on LSS:

wash-out of structure

on scales $\lambda < \ 0.1\ pc$