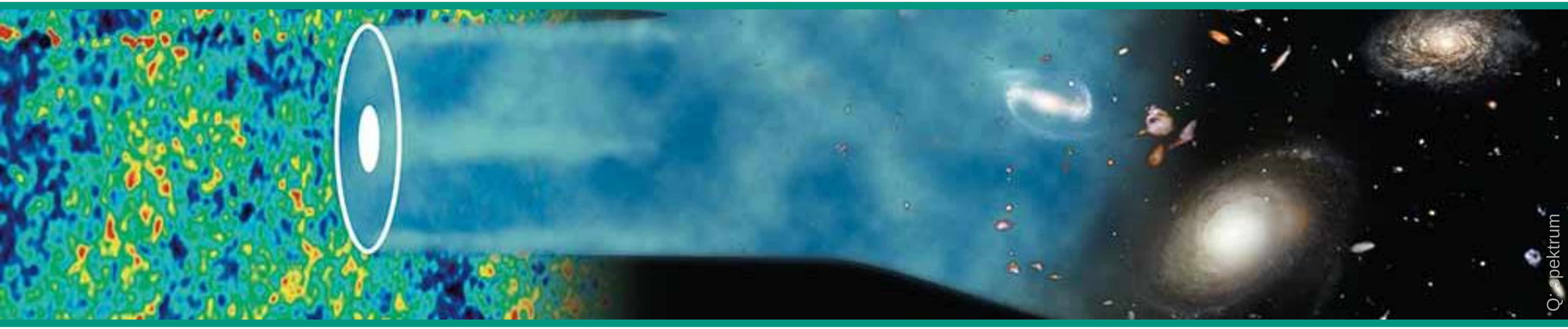
Introduction to Cosmology

Winter term 22/23 Lecture 14 Feb. 14, 2023



Recap of Lecture 13

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- **The observed ´clumpiness´ of the universe: matter power spectrum $P(k)$**

  - strong evidence for DM: after the BAO-phase $\Longrightarrow$ baryons fall into gravity wells formed by evolving DM $\Longrightarrow$ enhanced baryonic density contrast

  - analysis of **density contrast**: as function of **distance** r - galaxy correlation function $\xi(r)$ ⇔ as function of **wave number** k – matter power spectrum $P(k)$

  - DM – modes of specific wave number k evolve independently: important is **time of first causal contact** (radiation- or matter- dominated universe)

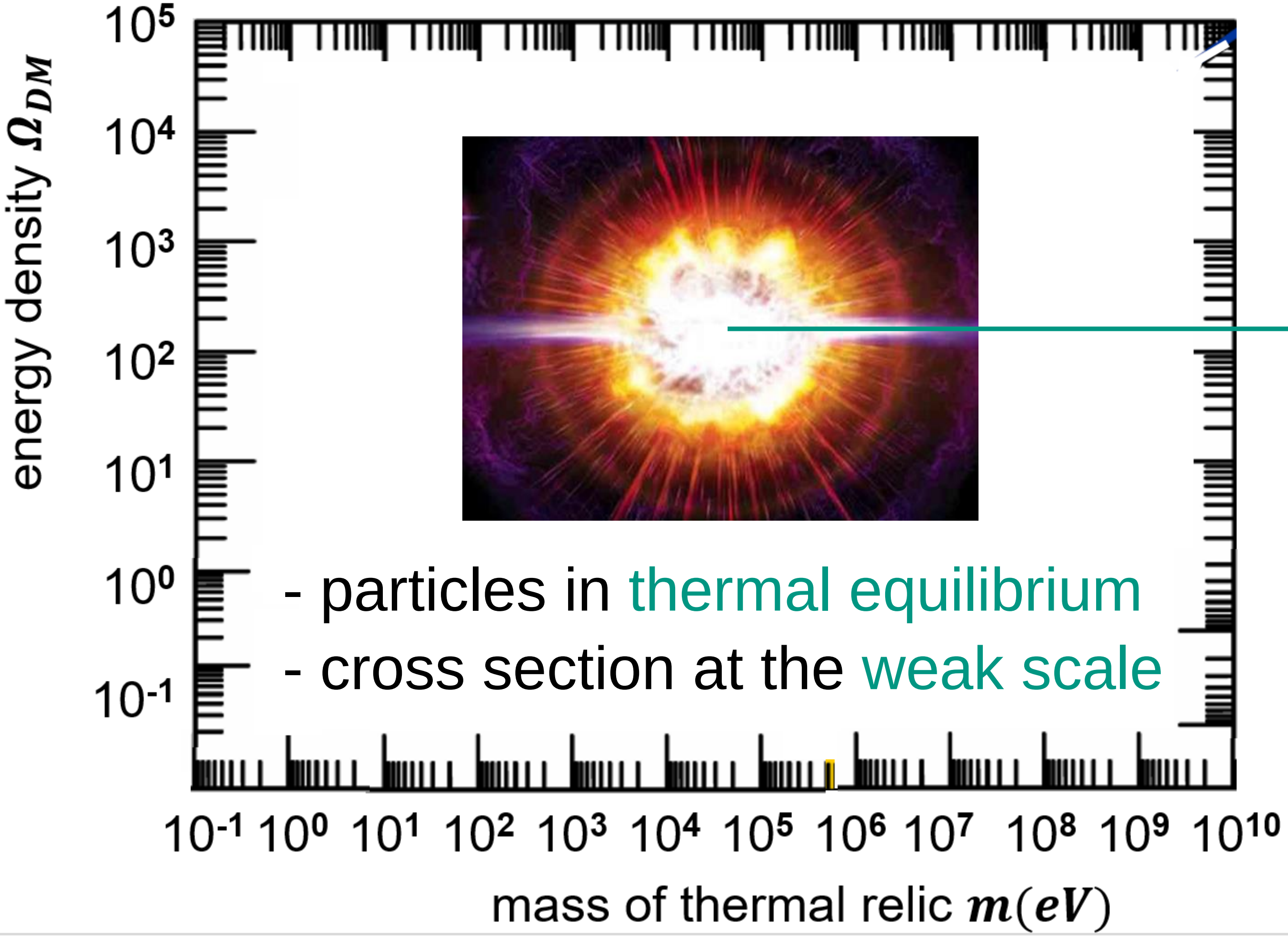
  - **small DM** – mode (large k): growth is delayed in radiation era - $P(k) \sim k^{-3}$
  - **large DM** – mode (small k): growth is not delayed in matter era − $P(k) \sim k$

Lee-Weinberg curve: only HDM or CDM

- **Thermal production of DM: what particle masses can be produced?**

이휘소

Weinberg



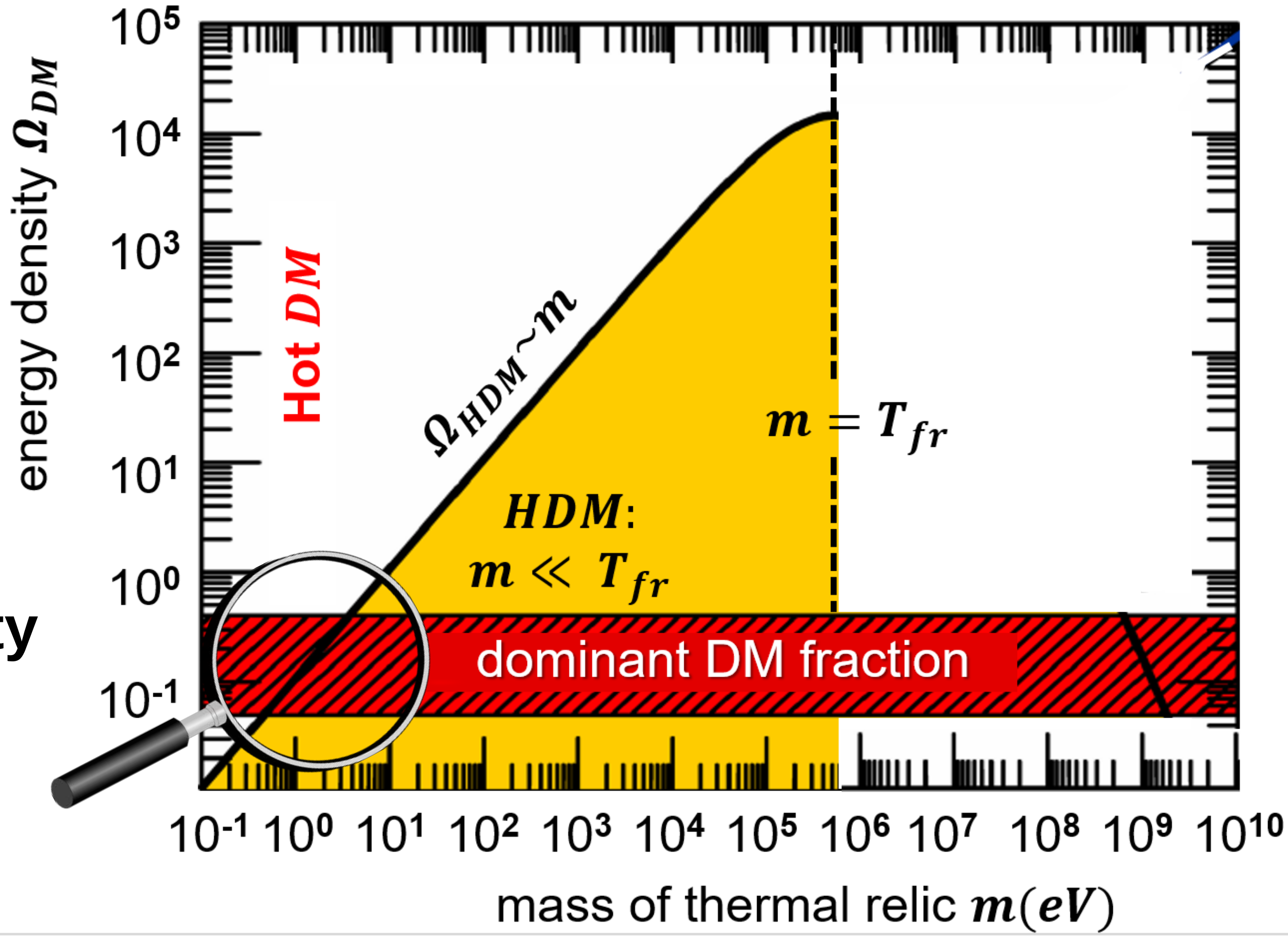
particles in an evolving universe:

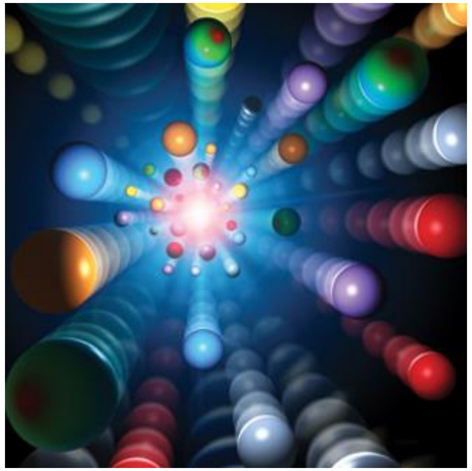
- it cools
- it expands

Lee-Weinberg curve: only HDM or CDM

- Thermal production of DM: relativistic case of neutrinos – no annihilation

⇨ **particle number density not reduced due to $v \approx c$**





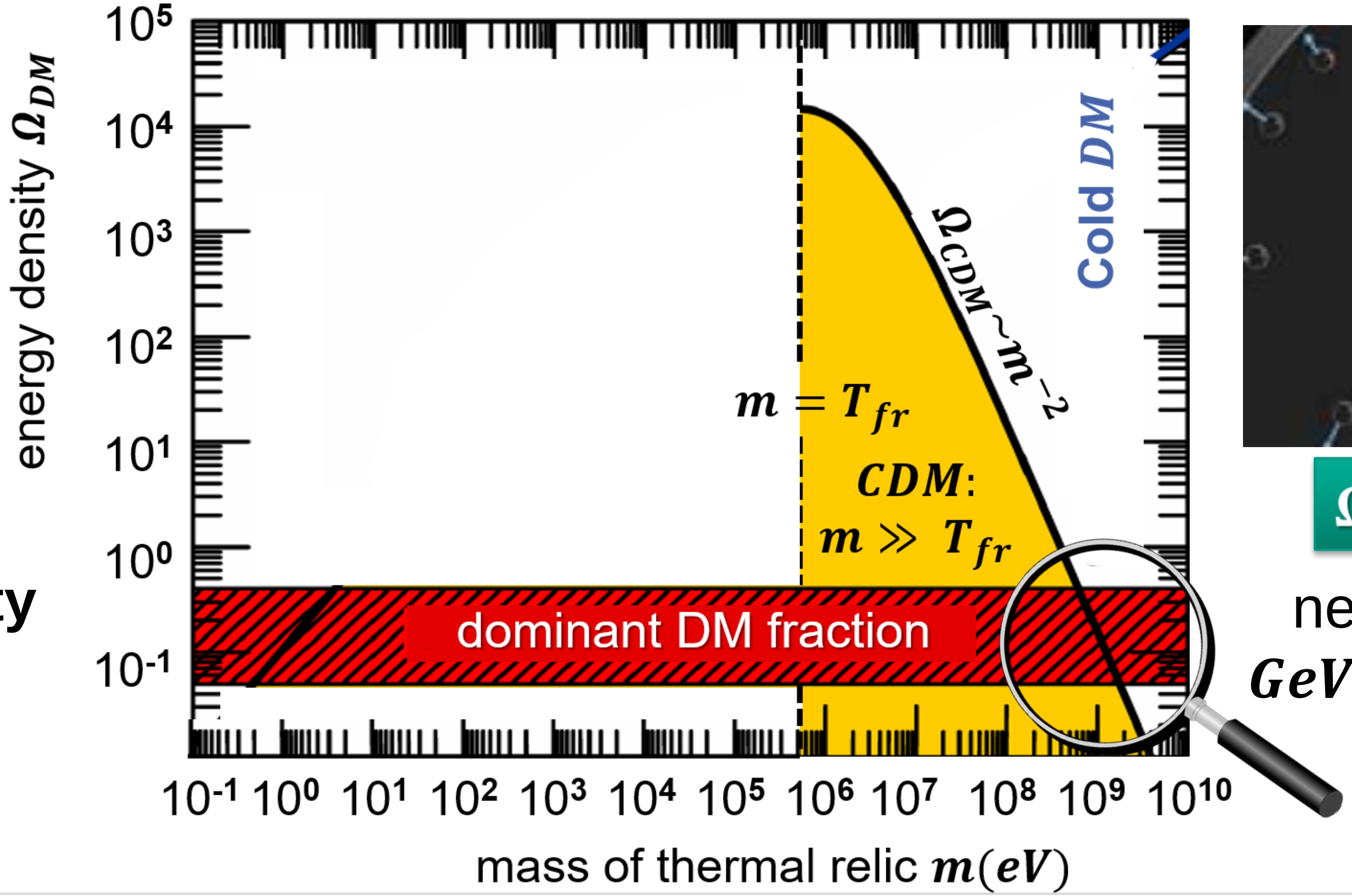
$\Omega_{HDM} \sim m$

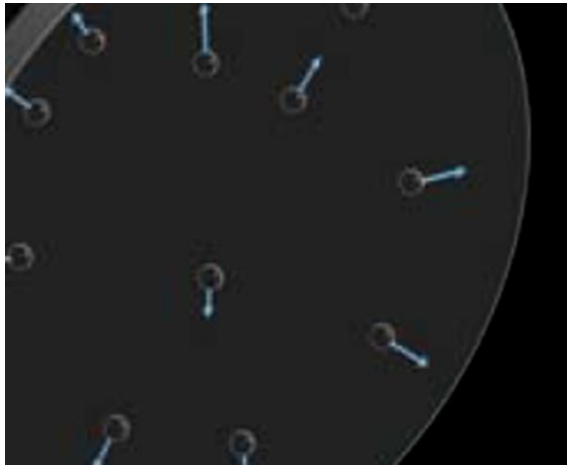
neutrinos with $sub-eV$ masses

Lee-Weinberg curve: only HDM or CDM

- Thermal production of DM: non-relativistic case of neutralinos – annihilation

⇨ **particle number density reduced due to $v \ll c$**





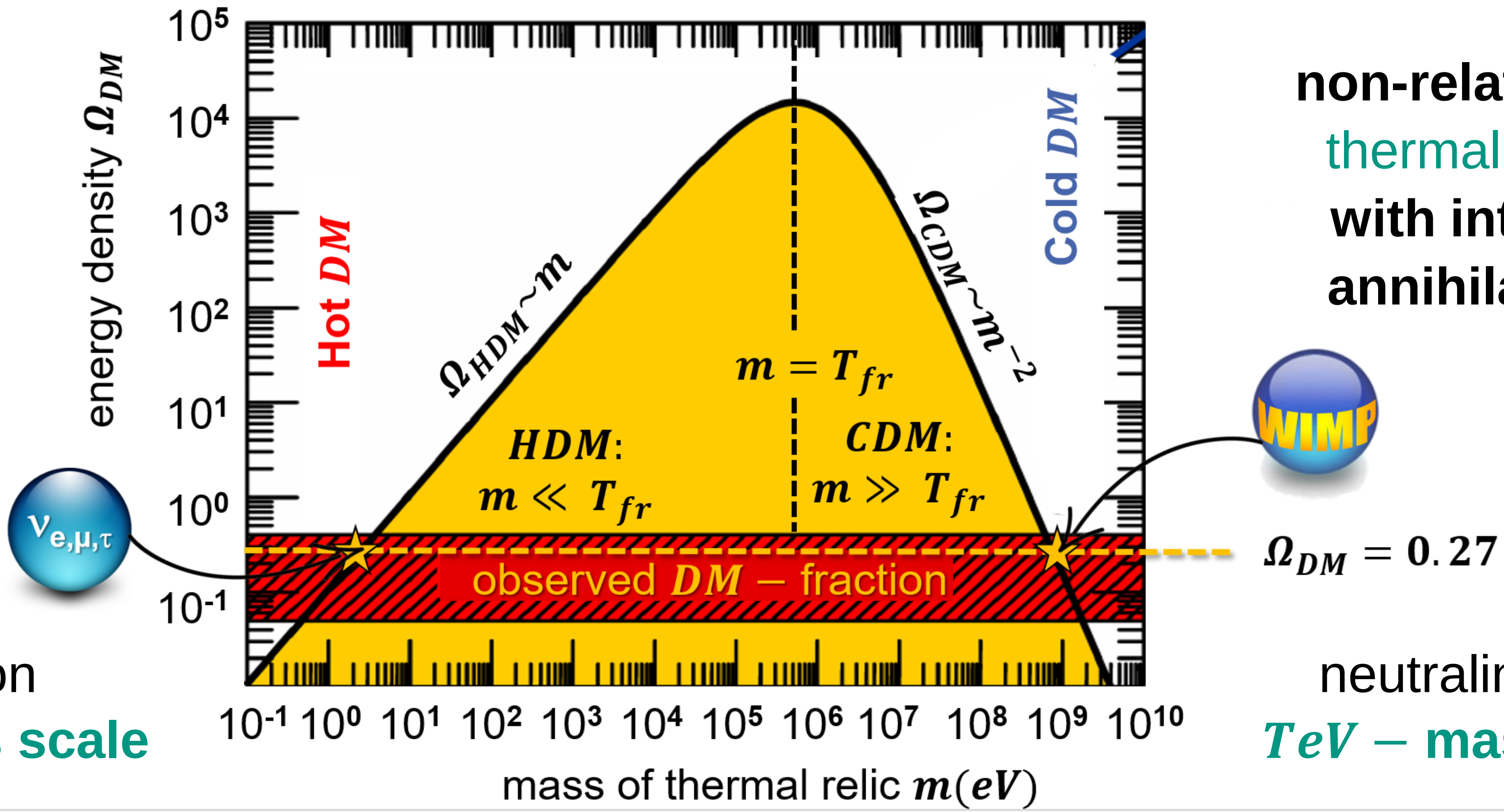
$$\Omega_{CDM} \sim m^{-2}$$

neutralinos with GeV ... TeV masses

Lee-Weinberg curve: only HDM or CDM

- Thermal production of DM: only two rather narrow mass ranges – eV or TeV

relativistic
thermal relics
without annihilation:



non-relativistic
thermal relics
with intense annihilation:

neutrinos on
$sub - eV$ mass scale

neutralinos on
TeV – mass scale

Lee-Weinberg curve: only HDM or CDM

- **Thermal production of DM: only two rather narrow mass ranges – eV or TeV**

  - in order to avoid overclosure of the universe due to thermally produced DM ($\Omega_{DM} \gg 1$) particles in **mass scale $keV \ldots MeV$** or $\ll eV$ are excluded
  ⇨ **weak interaction processes & subsequent freeze-out**

$sub - eV$ scale

T

$\sigma_{ann} = \sigma_{weak}$

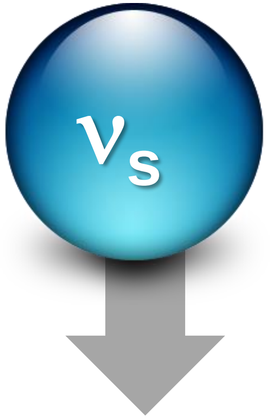
TeV scale

Lee-Weinberg curve: only HDM or CDM

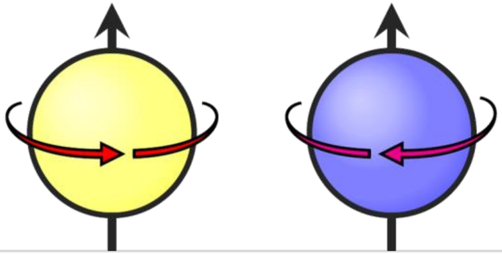
- **Non-thermal production of DM for mass ranges: $m \ll eV$ or $m \sim keV \ldots MeV$**

  - in order to avoid overclosure of the universe due to thermally produced DM ($\Omega_{DM} \gg 1$) particles in **mass scale $keV \ldots MeV$** or $\ll eV$ are excluded
  ⇨ **non-thermal processes**: **ν-oscillations or symmetry breaking**

oscillation from active neutrinos



keV scale



μeV scale

axion: breaking of CP symmetry

Thermal relics: production & annihilation

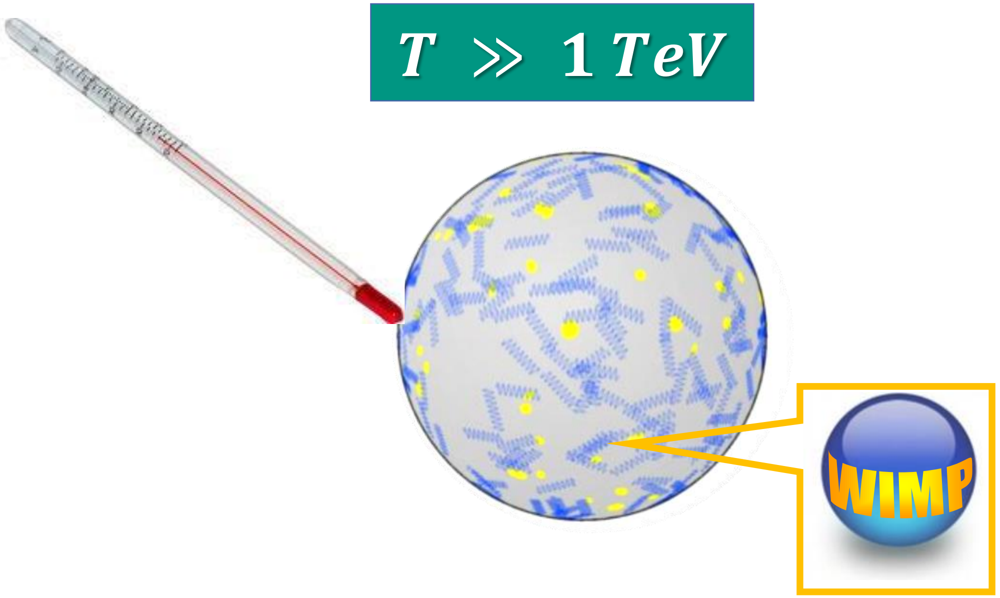
- **The $WIMP$ miracle of DM: phase 1 – thermodnamical equilibrium**

  - at $T \gg TeV$: due to their weak interaction, $WIMP$s are in thermodynmical equilibrium

thermodynamical equilibrium

$t < 1\,ns$

$T \gg 1\,TeV$



we consider **co-moving number densities $n_\chi(t)$** where the increase of the scale factor $a(t)$ plays mo further role

Thermal relics: production & annihilation

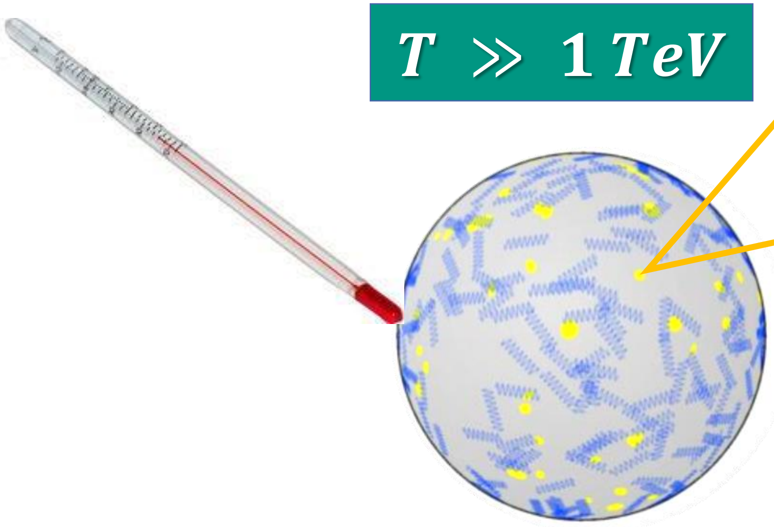
- **The $\boldsymbol{WIMP}$ miracle of $\boldsymbol{DM}$: phase 1 – thermodnamical equilibrium**

  - at $T \gg TeV$: due to their weak interaction, $WIMP$s are in thermodynamical equilibrium

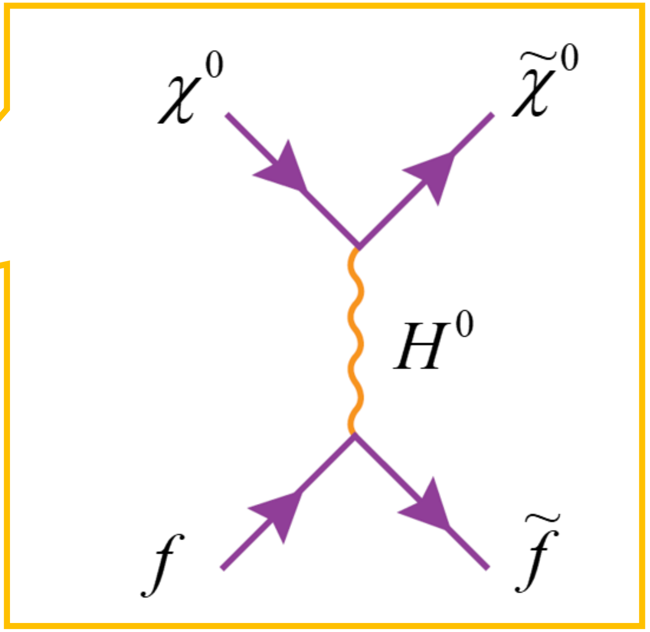
$$\text{rate } (WIMP - \text{pair production}) \equiv \text{rate } (WIMP - \text{pair annihilation})$$

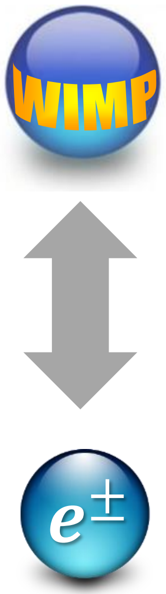
thermodynamical equilibrium

$t < 1\, ns$



$T \gg 1\, TeV$





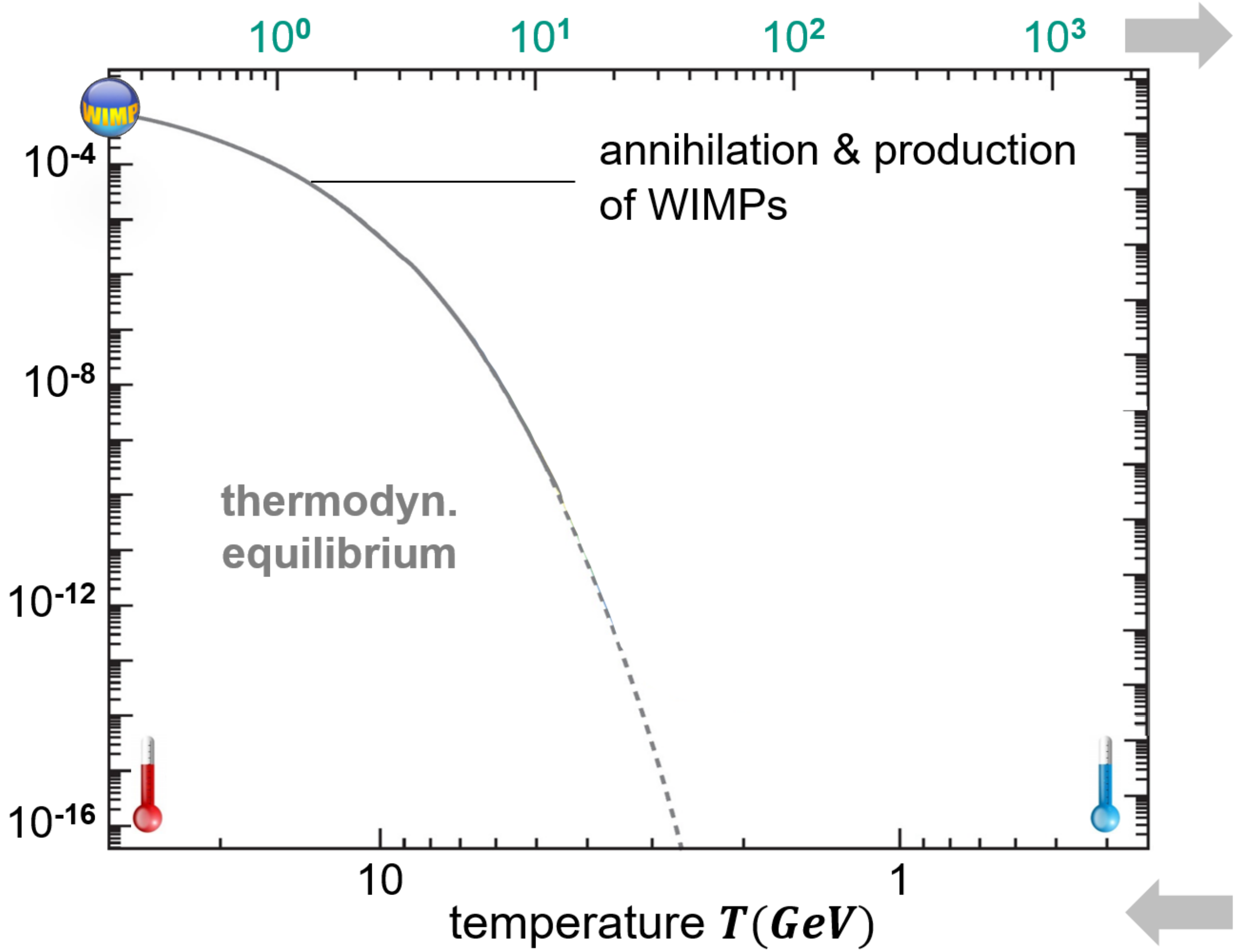
Thermal relics: production & annihilation

- The $WIMP$ miracle of DM: phase 2 – annihilations reduce number density

$$n_\chi(t) \sim e^{-m_{WIMP}/k_B T}$$

exponential decrease of $n_\chi(t)$

$t < 30\, ns$



$m_{WIMP} = 100\ GeV$

$$T(t) \sim \frac{1}{\sqrt{t}}$$

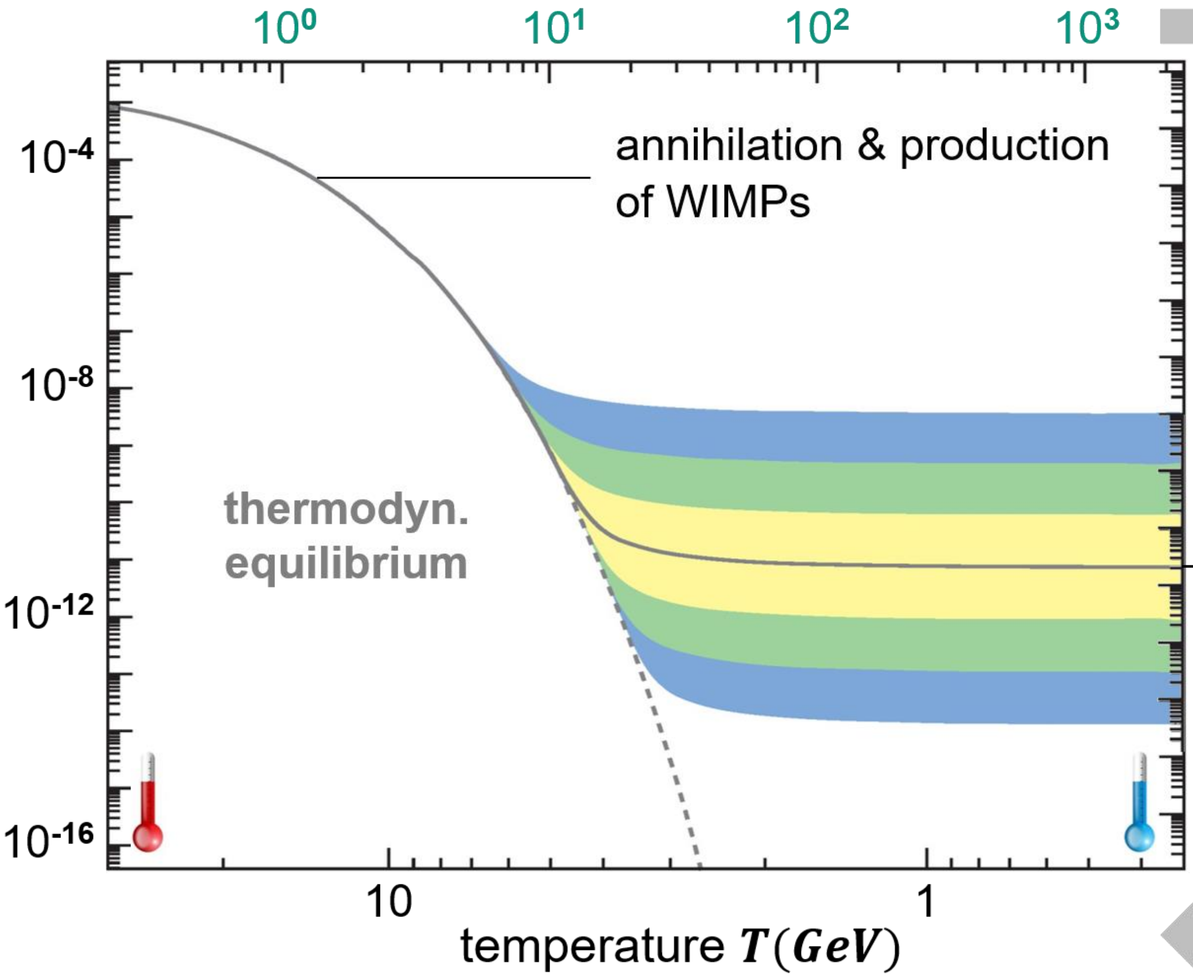
Thermal relics: freeze-out & number density

- **The $WIMP$ miracle of DM: phase 3 – freeze out & decoupling of $WIMPs$**

$$\boldsymbol{\Gamma(t) = H(t)}$$

after **decoupling** of $WIMPs$: constant $n_\chi(t)$

$t > 30\ ns$



$m_{WIMP} = 100\ GeV$

$\Omega_{DM} = const.$

$T(t) \sim \frac{1}{\sqrt{t}}$

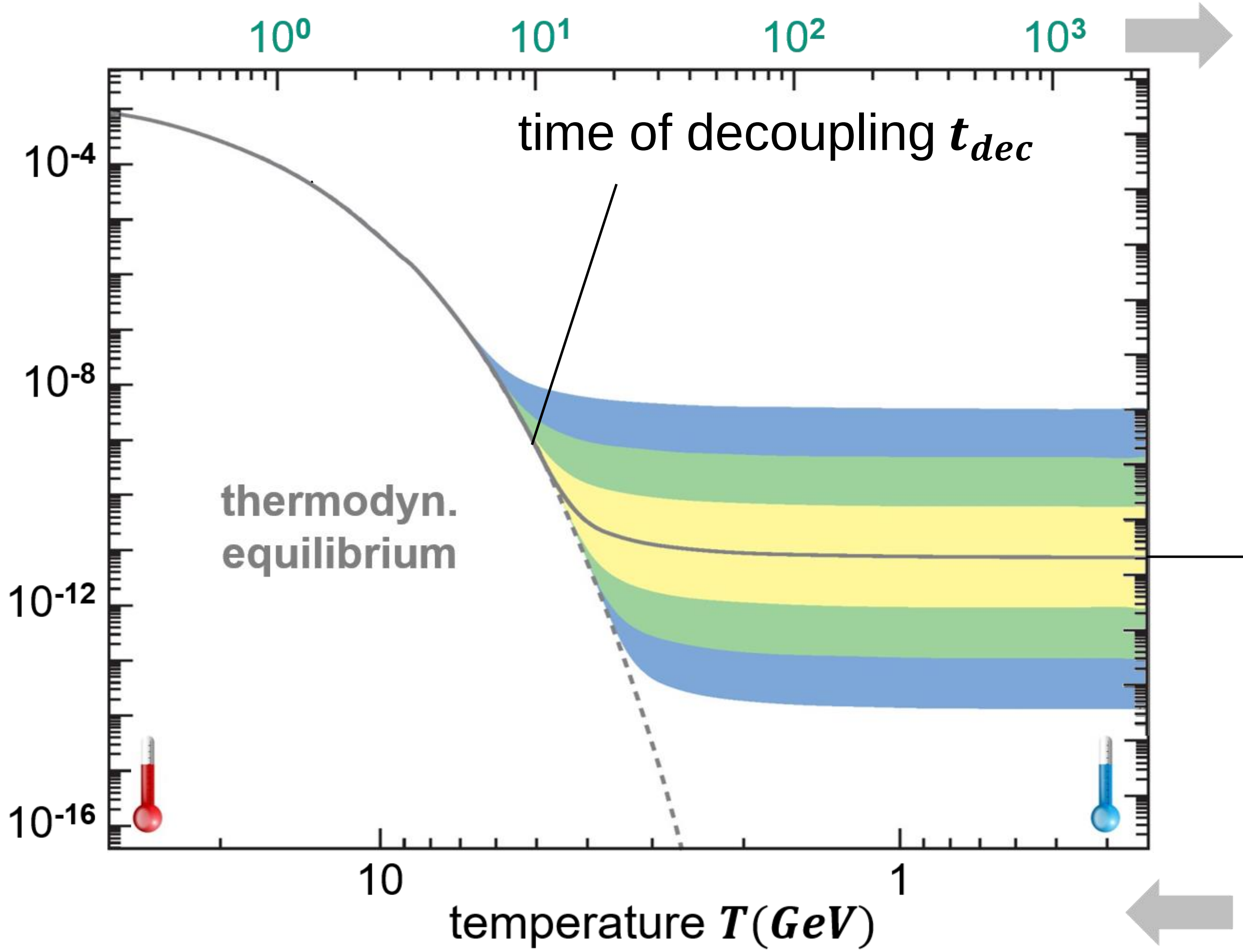
Thermal relics: freeze-out & number density

- The $WIMP$ miracle of DM: phase 3 – freeze out & decoupling of $WIMPs$

$$\boxed{\Gamma(t_{dec}) = H(t_{dec})}$$

after **decoupling** of $WIMP$s: constant $n_\chi(t)$

distance too large for annihilations



$m_{WIMP} = 100\ GeV$

$\Omega_{DM} = const.$

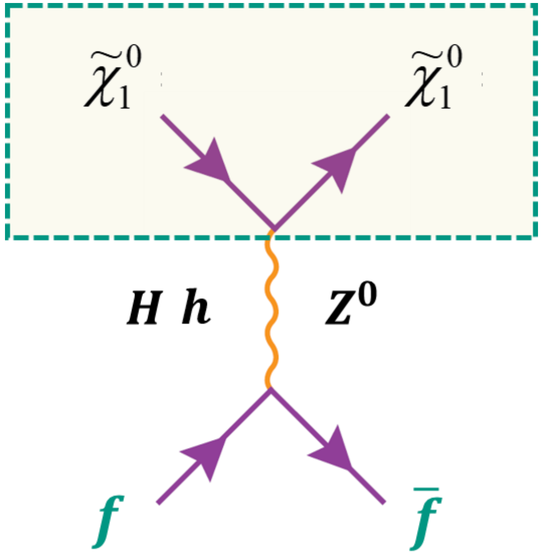
$T(t) \sim \frac{1}{\sqrt{t}}$

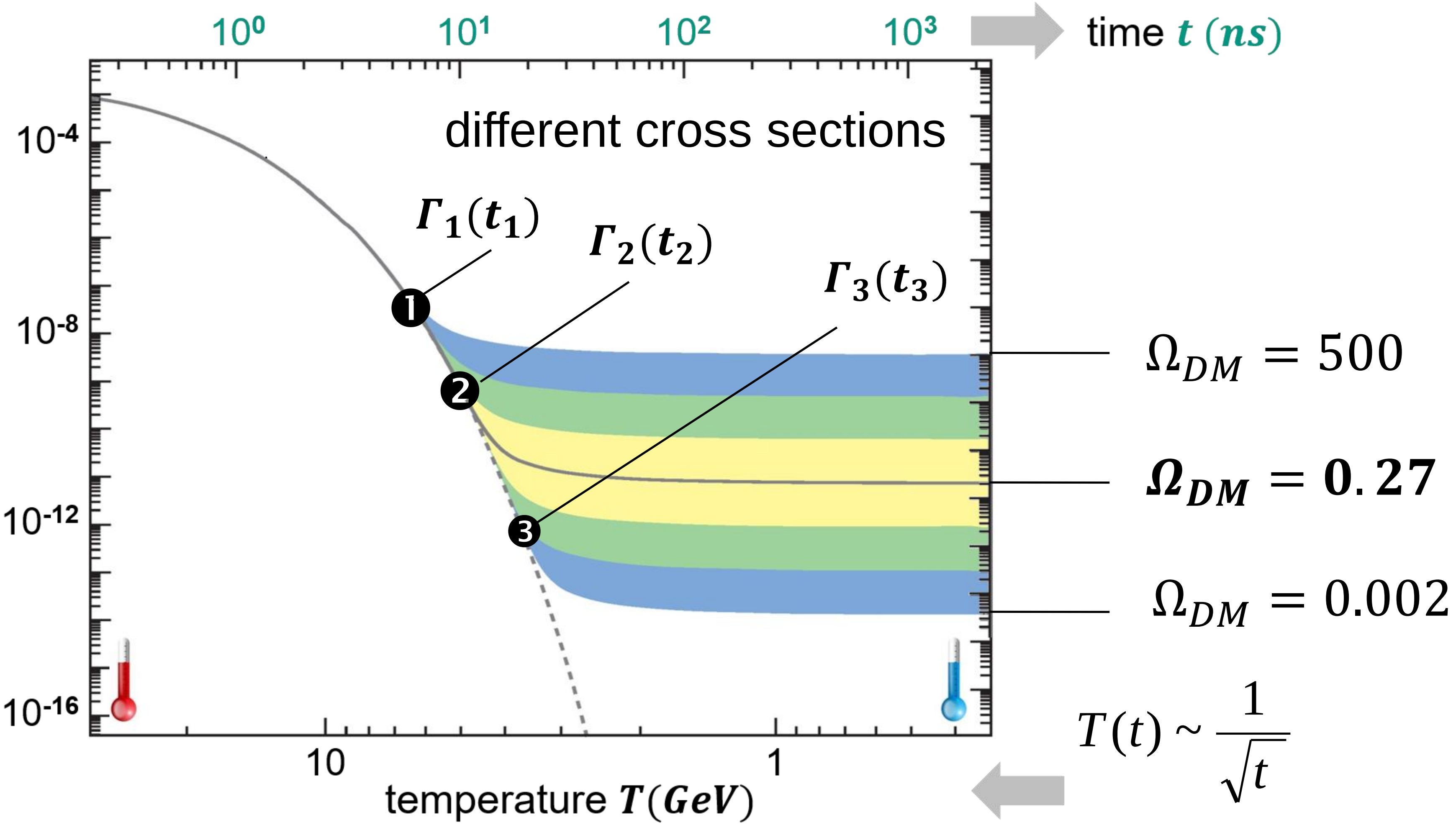
Thermal relics: freeze-out & number density

- The $WIMP$ miracle of DM: phase 3 – freeze out & decoupling of $WIMPs$

$$\boxed{\Gamma_i(t_{dec,i}) = H(t_{dec,i})}$$

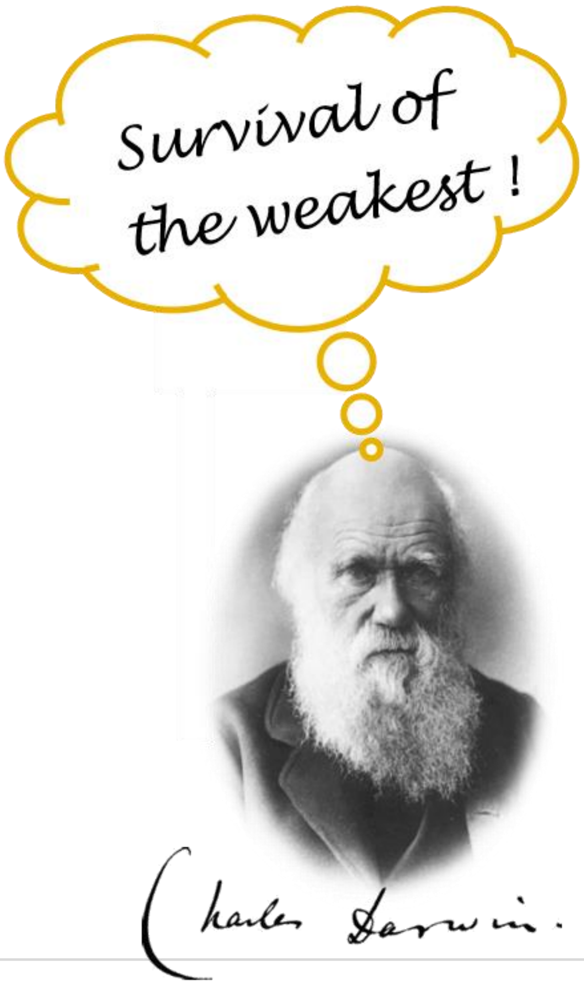
Γ_i = annihilation xsecs of $SUSY-WIMPs$

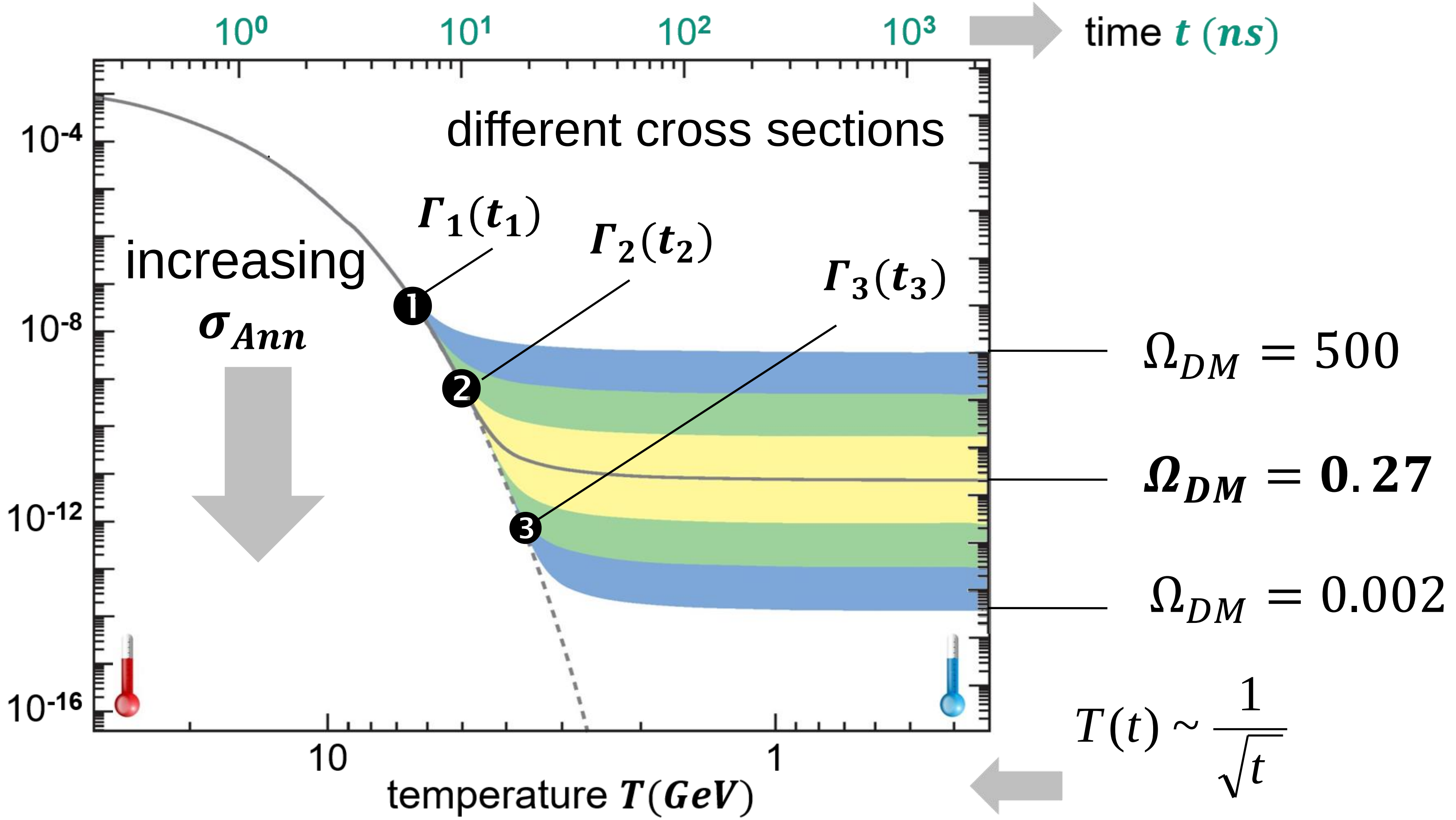




Thermal relics: freeze-out & number density

- **The $WIMP$ miracle of DM: phase 3 – freeze out & decoupling of $WIMPs$**

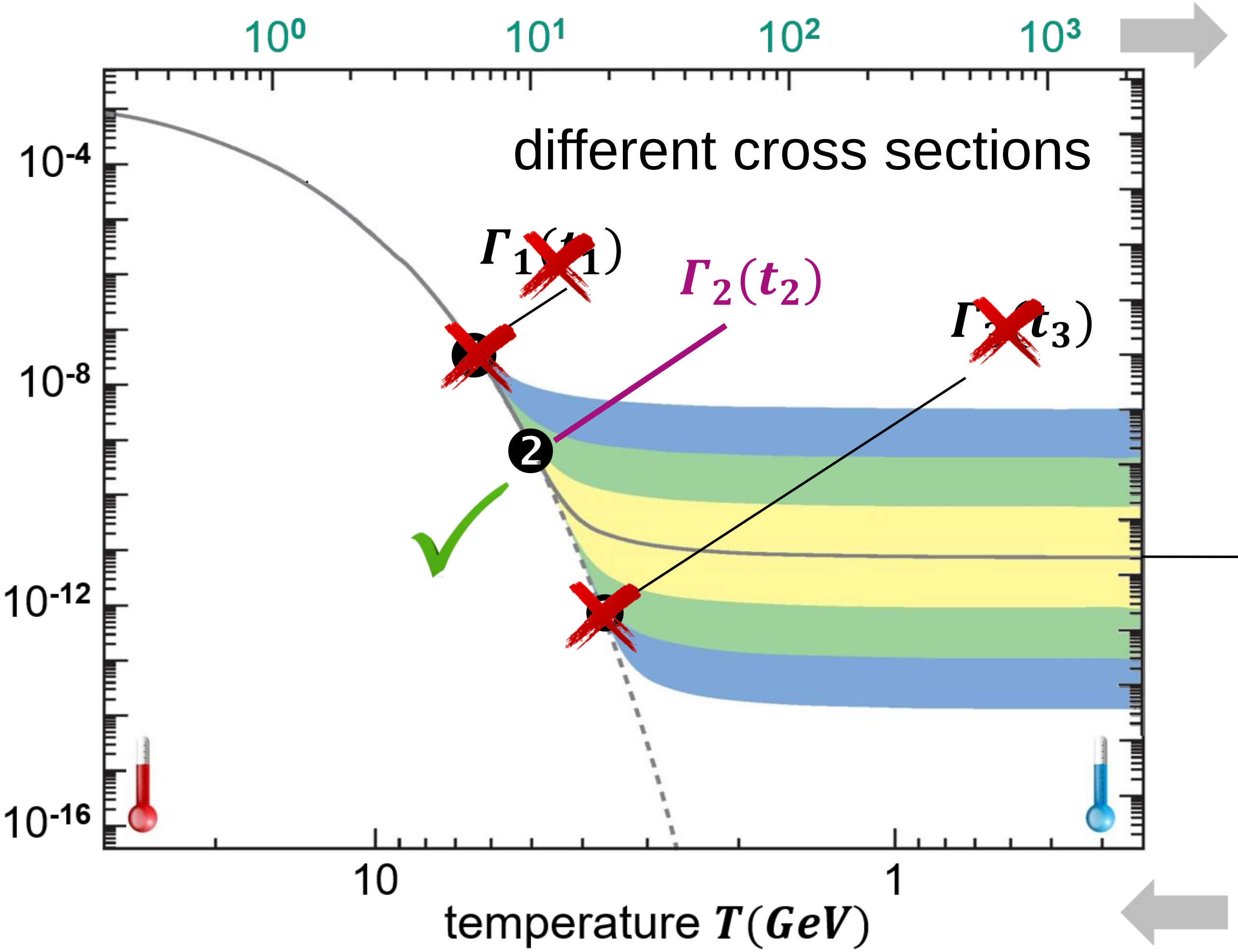


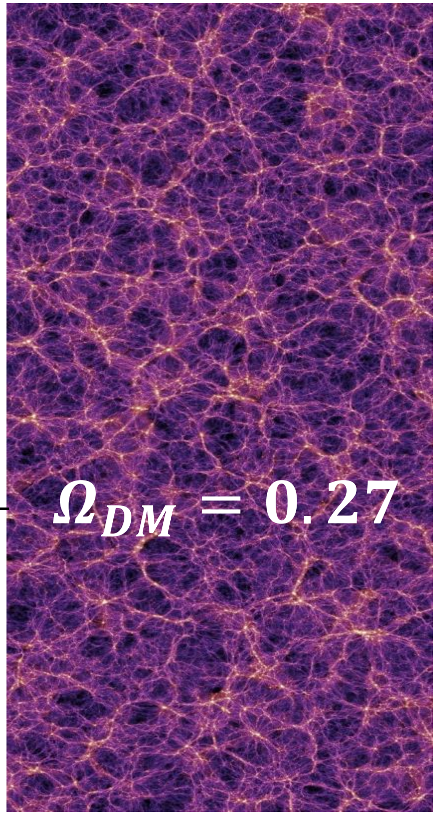


Thermal relics: freeze-out & number density

- **The $WIMP$ miracle of DM: phase 3 – freeze out & decoupling of $WIMPs$**

- is the observed DM – content of the universe of $\Omega_{DM} = 0.27$ just a chance value?



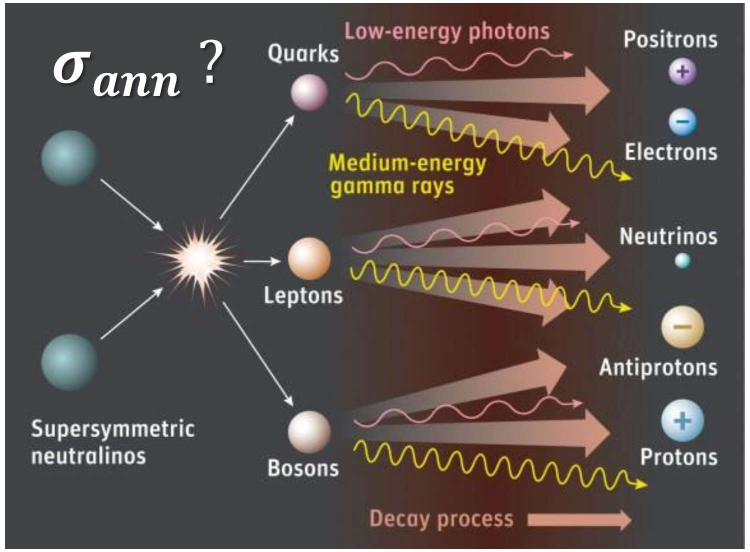


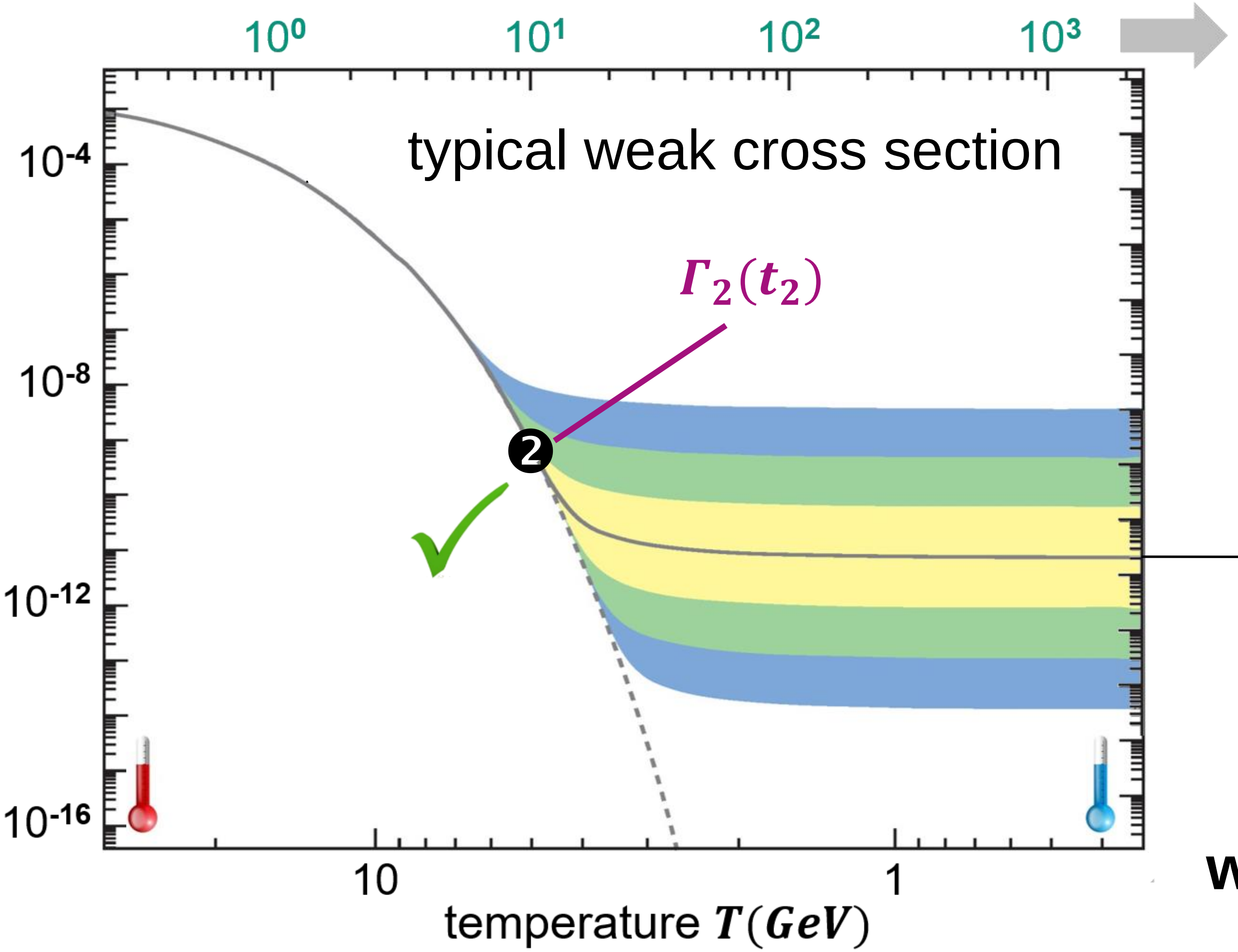
Thermal relics: freeze-out & number density

- The $WIMP$ miracle of DM: we ´automatically´ obtain Ω_{DM} if $\sigma_{ann} \approx 1\,pb$

- DM – fraction today:

$$\Omega_{DM}(t = t_0) \sim \frac{1}{\langle \sigma_{ann} \cdot v \rangle}$$





$\Omega_{DM} = 0.27$

$\sigma_{ann} \approx 1\,pb$

weak interaction

v: WIMP velocity at decoupling

Thermal relics: freeze-out as Cold Dark Matter

- The $WIMP$ miracle of DM: we ´automatically´ obtain **Cold Dark Matter**

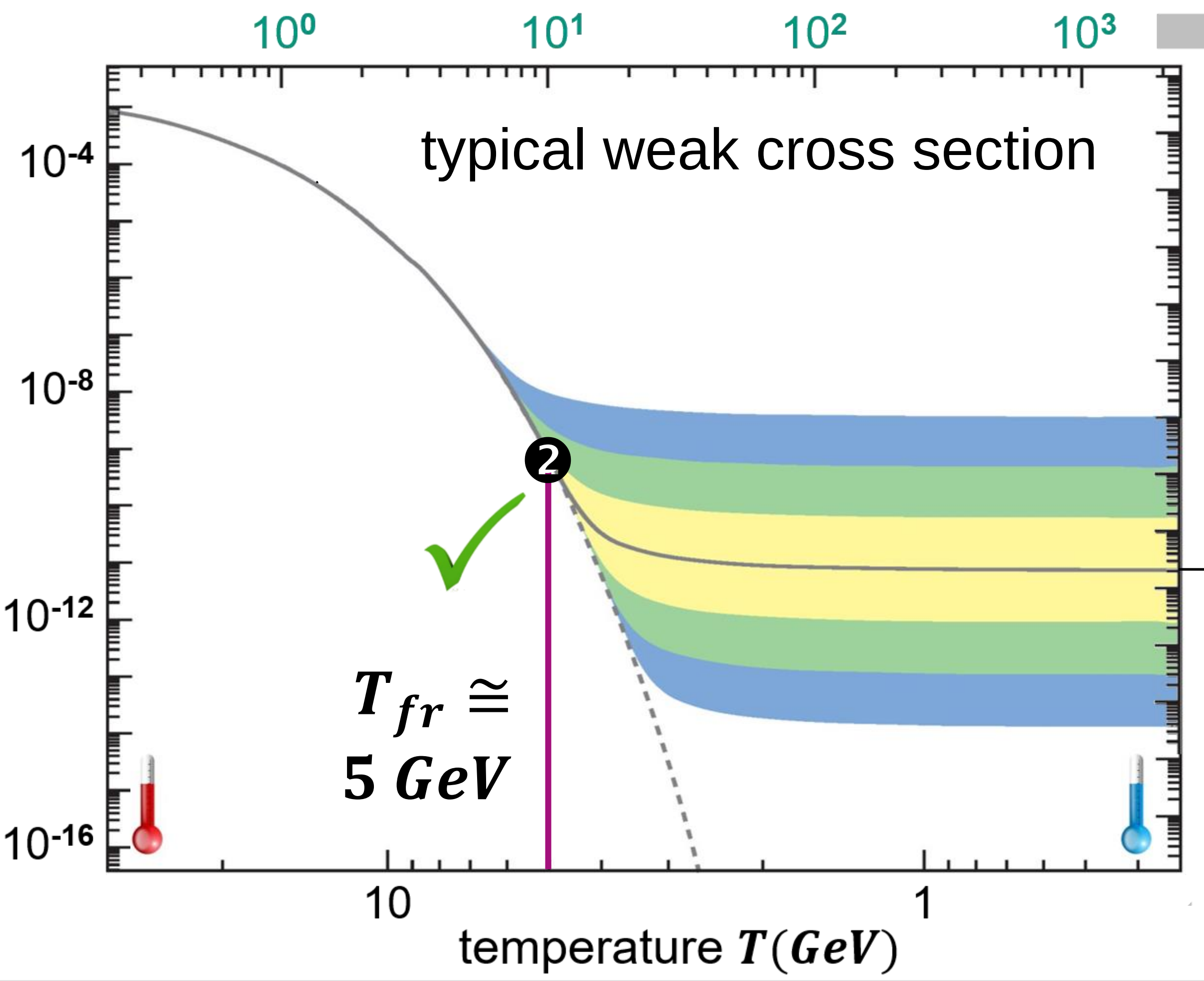
- $WIMP$ – velocity

$$x_{fr} = \frac{T_{fr}}{M_{WIMP}} = \frac{1}{20}$$

is **non-relativistic**

due to relation:
$T_{fr} \ll M_{WIMP}$

CDM



time $t\ (ns)$

$\Omega_{DM} = 0.27$

$m_{WIMP} =$
$100\ GeV$

v: WIMP velocity at decoupling

Thermal Relicts: relativistic neutrinos

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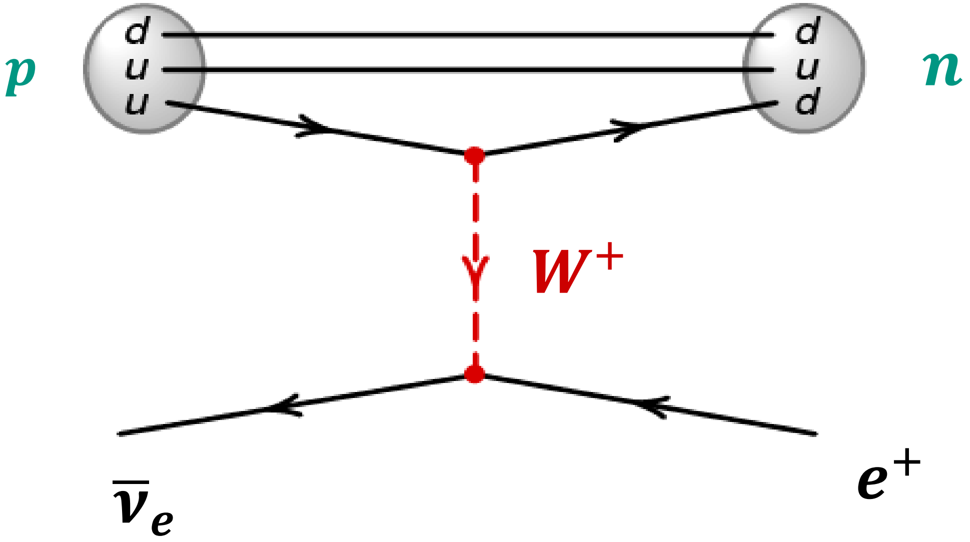
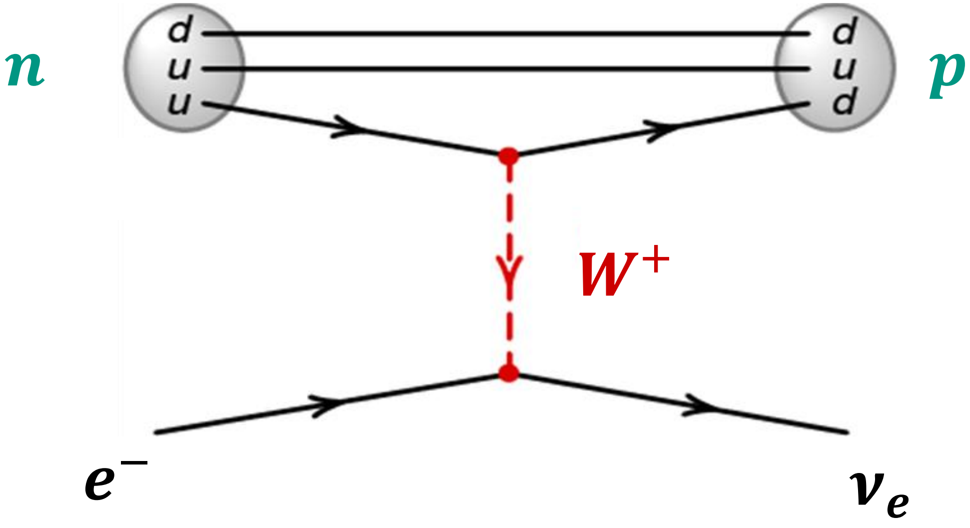
- **RECAP: neutrinos remain in thermal equilibrium until $t = 1\ s$**

  - semi-leptonic reactions with protons, neutrons via CC (charged current) and NC (neutral current) processes: important to fix $n/p-$ **ratio** for BBN

$$p + \bar{\nu}_e \leftrightarrow n + e^+$$

$$p + e^- \leftrightarrow n + \nu_e$$





*s. Chap. 3.1

Thermal Relicts: relativistic neutrinos as HDM

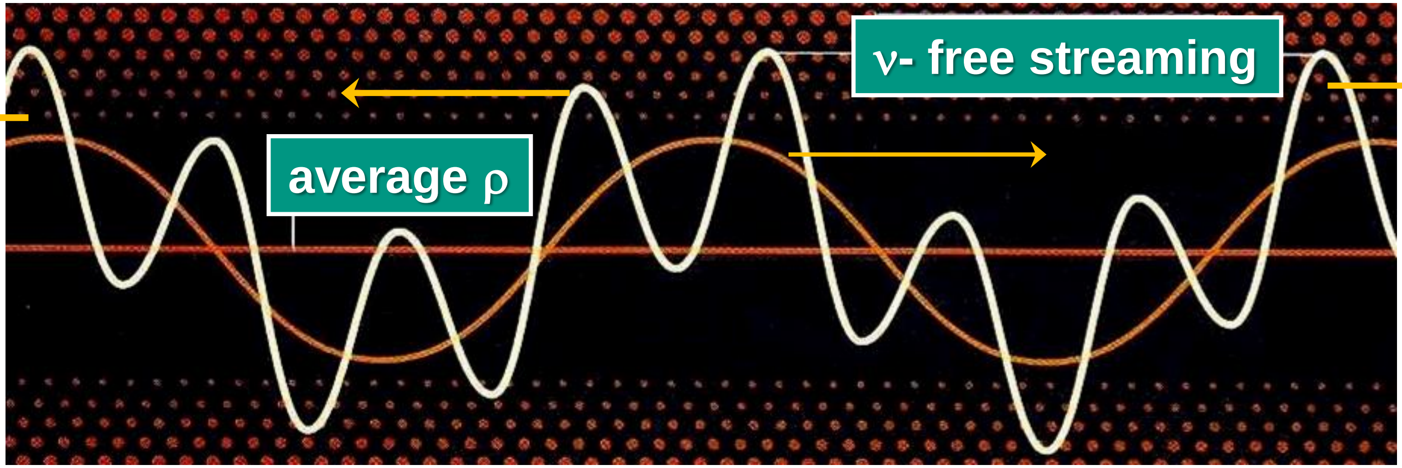
- **neutrinos free-stream in evolving universe over distances $d \sim Gpc$**
  - neutrinos decouple after $t = 1\ s$ at temparture $T_{fr} \sim MeV$

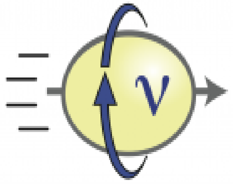
    $sub - eV$ mass (KATRIN 2022: $m(\nu) < 0.8\ eV\ (90\%CL)$)

    ⇨ resulting **Lorentz-$\gamma = 10^6 \dots 10^7$**

    ⇨ **free-streaming distance $d \sim Gpc$**

several Gpc





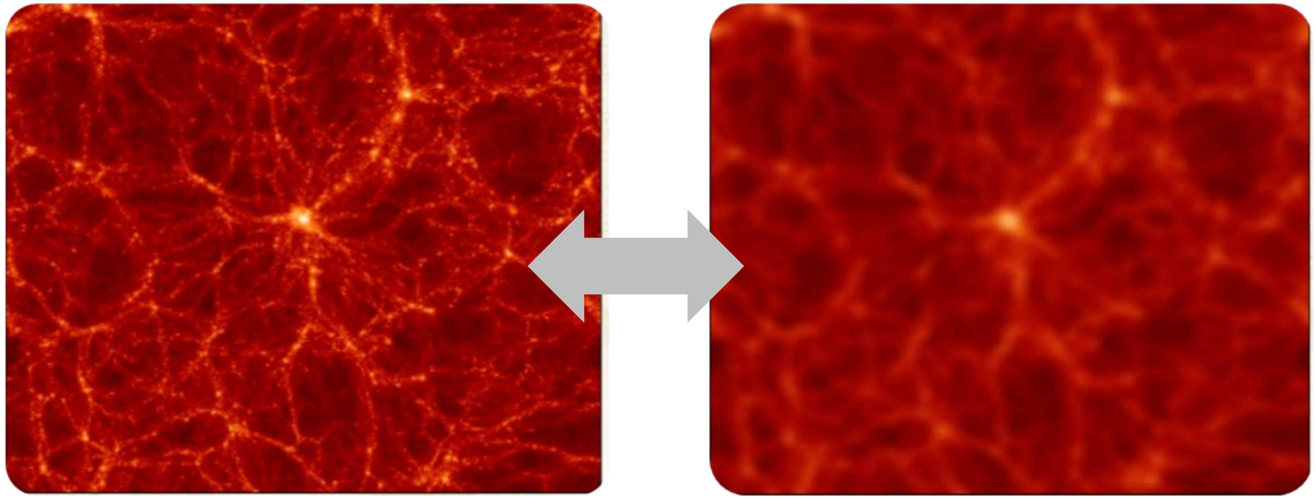
HDM

Thermal Relicts: relativistic neutrinos as HDM

- neutrinos **free-stream** in evolving universe over distances $d \sim Gpc$

$m(\nu) = 0\ eV$

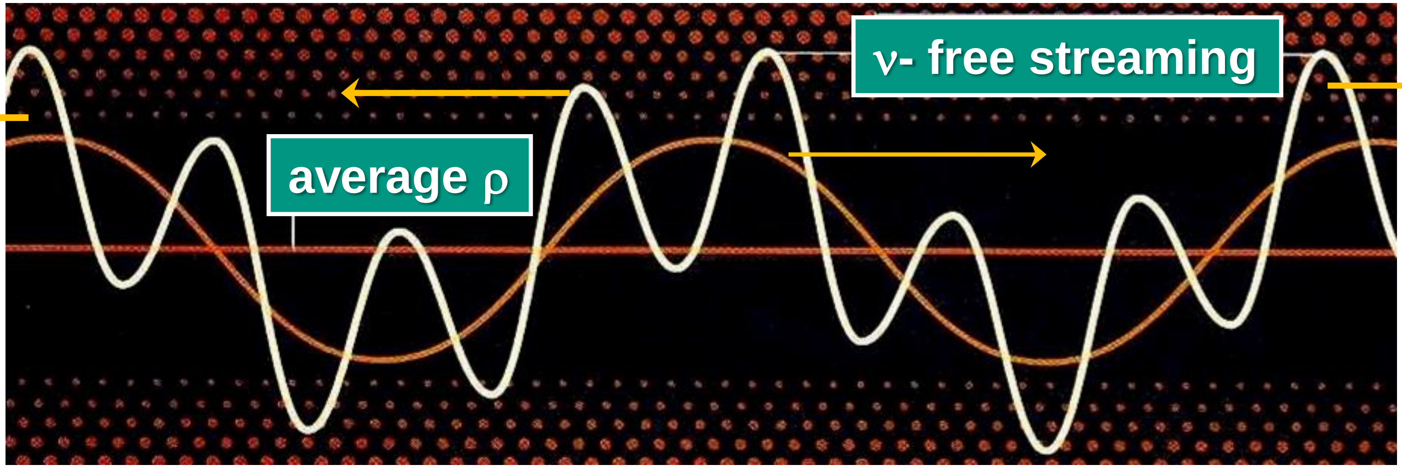
only CDM:
bottom-up
scenario

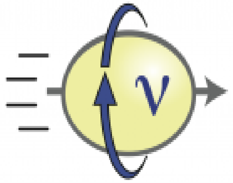


$m(\nu) = few\ eV$

mostly HDM:
top-down
scenario

several Gpc





HDM

Thermal Relicts: relativistic neutrinos as HDM

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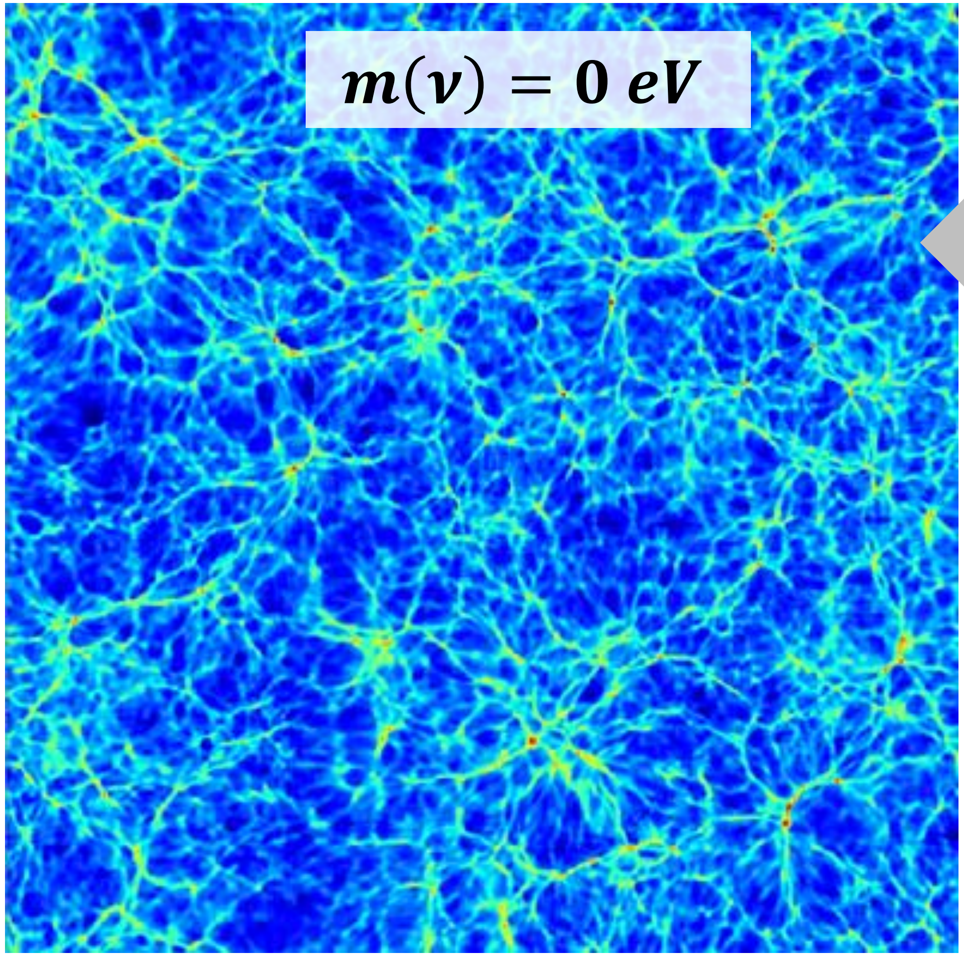
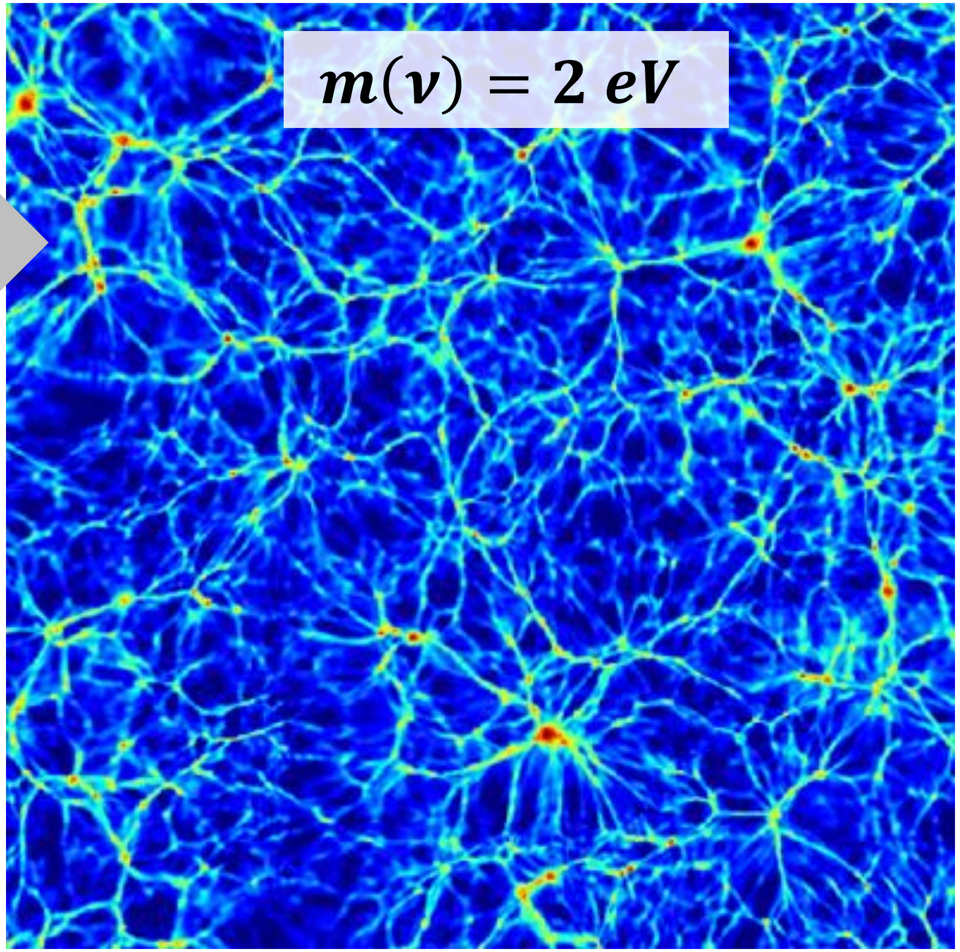
- neutrinos free-stream: wash out of small-scale structures in the universe

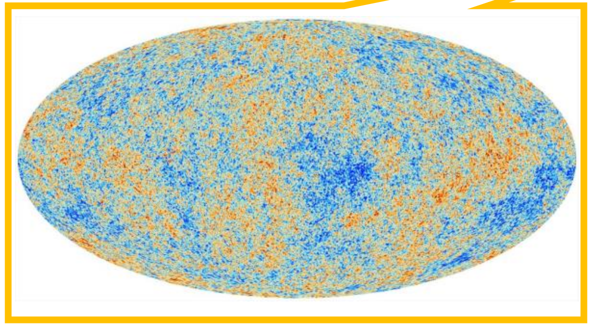


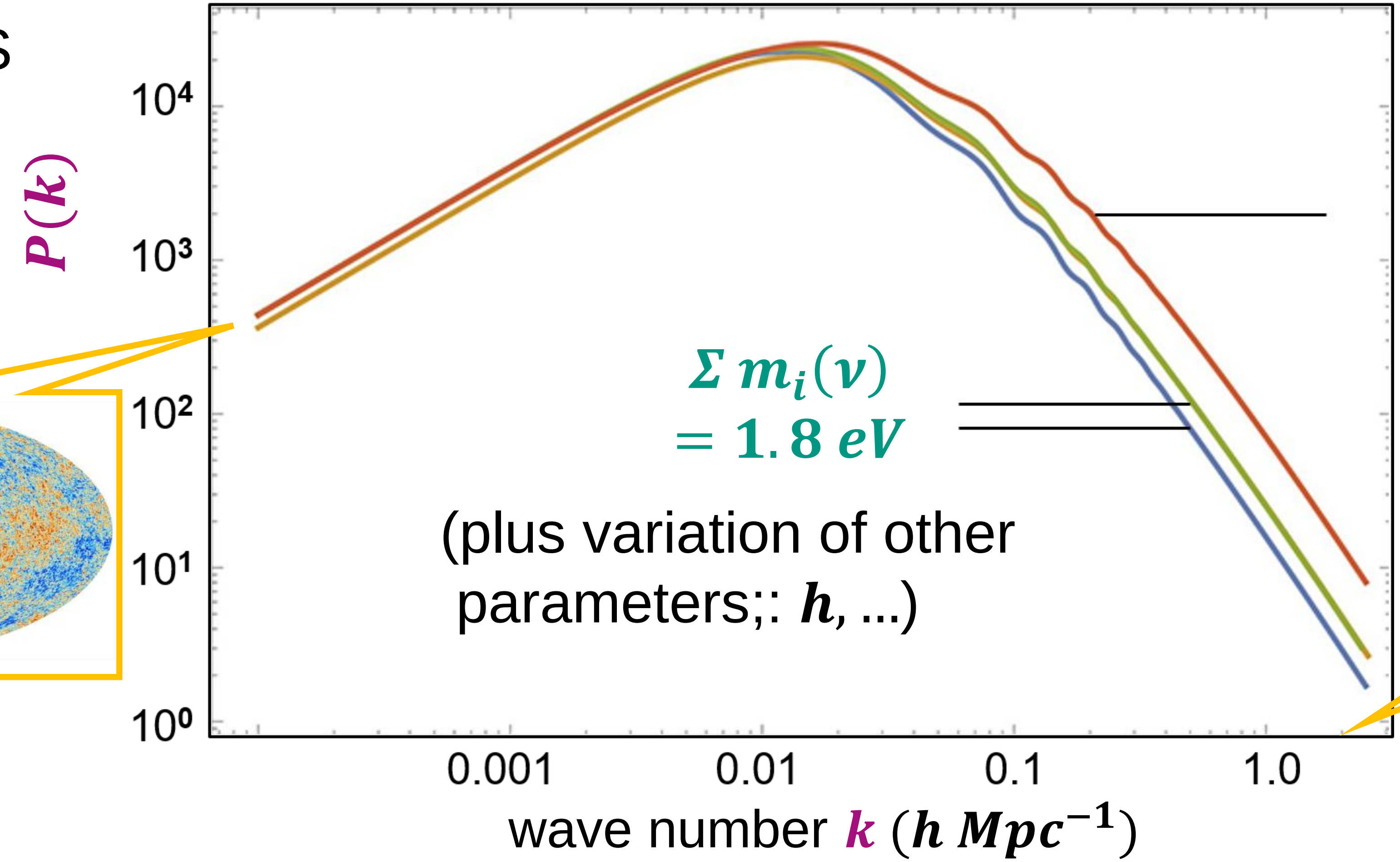


Massive ν´s (HDM) & matter power spectrum $P(k)$

- **Imprint of massive neutrinos on large wave numbers k of spectrum $P(k)$**

- CMB+LSS





minimal:

$\Sigma\, m_i(\nu)$
$= 0.06\ eV$

Massive ν´s (HDM) & matter power spectrum $P(k)$

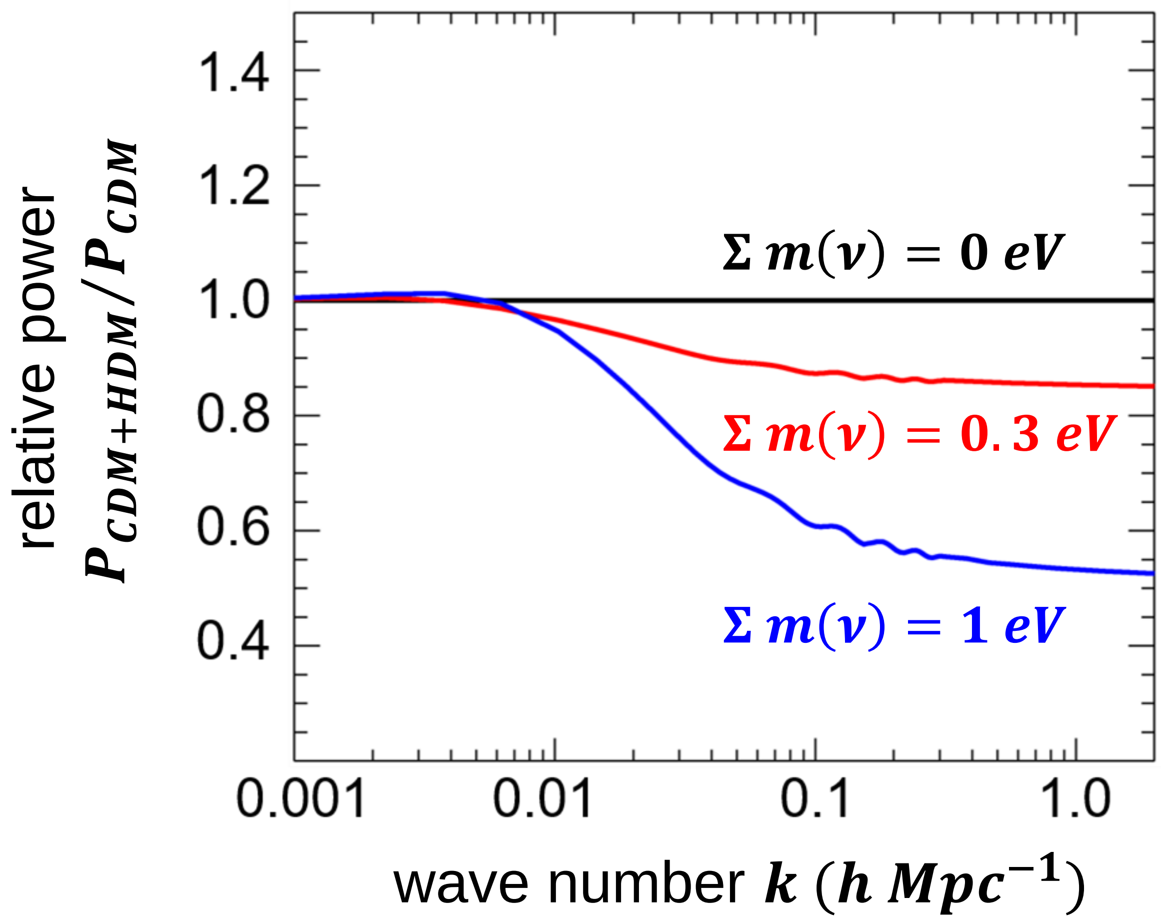
- **Imprint of massive neutrinos on large wave numbers k of spectrum $P(k)$**

- adding small amounts of $\boldsymbol{HDM}$ to evolution of $\boldsymbol{LSS}$ **reduces** $\boldsymbol{P(k)}$

large factor

$$\boxed{P(k) \sim P_{CDM}(k) \cdot \left(1 - \frac{8 \cdot \Omega_\nu}{\Omega_M}\right)}$$

- example: $\boldsymbol{\Sigma\, m(\nu) = 0.3\ eV}$
  $\sim \mathbf{15\%}$ reduction of power
  for ´small´ structures at
  $\boldsymbol{k = 1\ h\ Mpc^{-1}}$

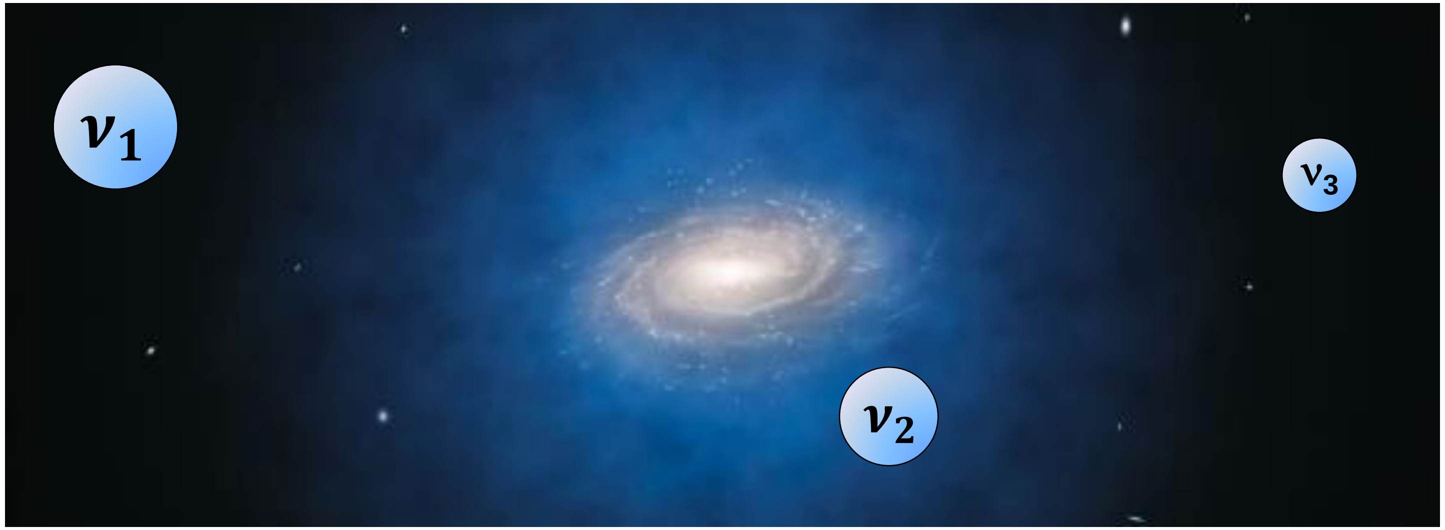


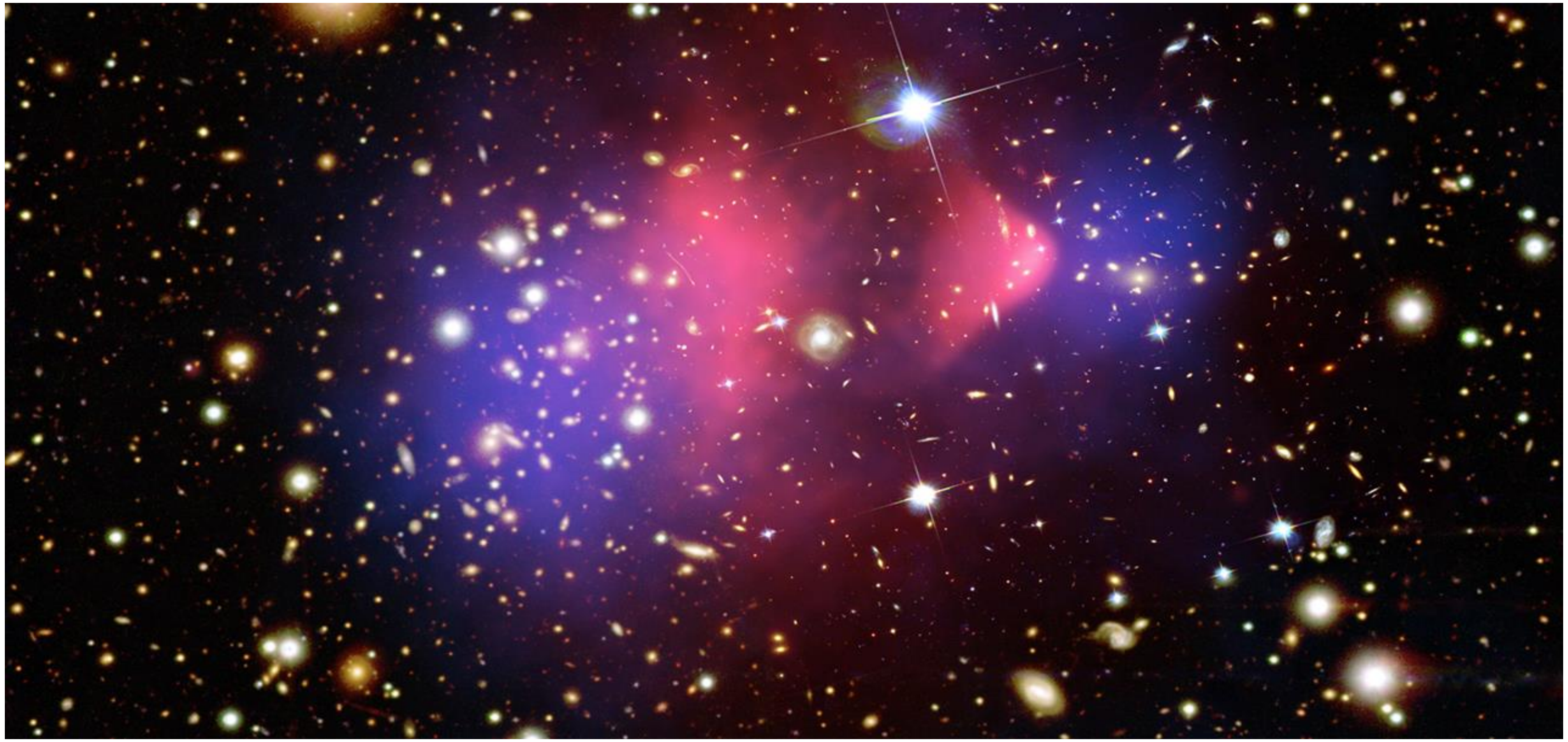
Massive ν´s (HDM): mass eigenstates $m_{1,2,3}$

- **primordial ν´s have cooled down to $T = 1.9\ K$ in todays´ universe**

  - neutrinos from Big Bang with masses $m \approx 50\ meV$ today are bound gravitationally in galaxy clusters (i.e. on scales $d \approx 50\ Mpc$)

  - **flavour states $\nu_{e,\mu,\tau}$** produced up to $t = 1\ s$ today have fully ´decoupled´ to **mass eigenstates $\nu_{1,2,3}$** (long *de Broglie* wavelengths)





CHAPTER 5 – DARK UNIVERSE

5.1 Evidences for Dark Matter

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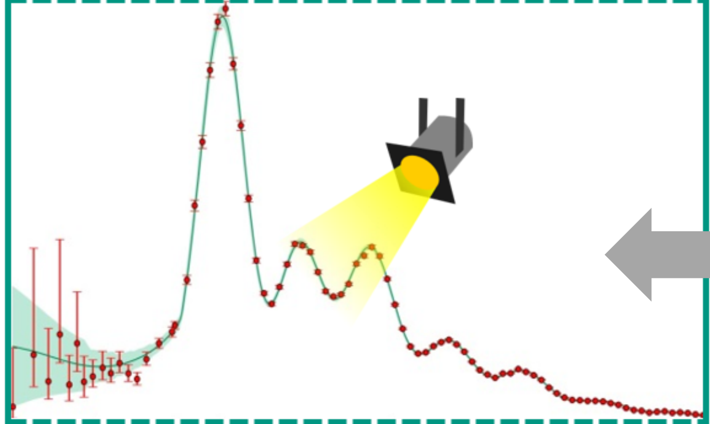
cosmological scale

method:
- 2./3. acoustic peak of $\boldsymbol{CMB}$ multipoles



result:
- (Cold) Dark Matter is dominant $\boldsymbol{\Omega_{DM} \sim 0.27}$

$\boldsymbol{Gpc}$ ↔ $\boldsymbol{Mpc}$

galaxy clusters

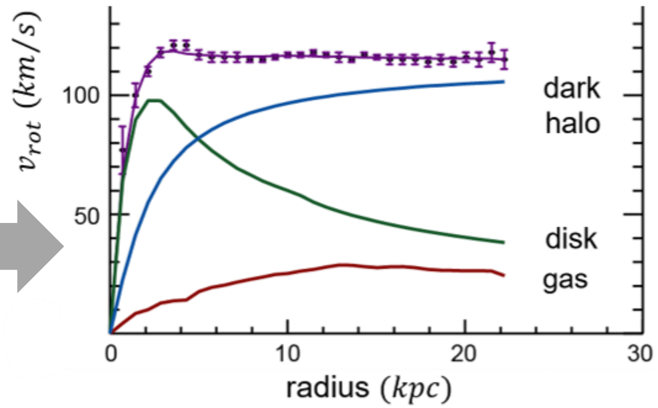
method:
- virial theorem (*Zwicky*)
- hot gas, gravit. lensing

result:
- extended $\boldsymbol{DM}$ – halos in galaxy clusters

$\boldsymbol{Mpc}$ ↔ $\boldsymbol{kpc}$

galaxy (local)

method:
- rotation curves of galaxies (*V. Rubin*)



result:
- dark galactic halo
- local value $\boldsymbol{\Omega_{DM}}$

Dark Matter & galaxy clusters

- ***Fritz Zwicky*** **proposes the existence of Dark Matter (from the Coma cluster)**

  - observation: (too) high peculiar velocities of single galaxies in the very large **Coma cluster of galaxies** !

Virial theorem:

$$\langle E_{kin} \rangle = -\frac{1}{2} \langle U_{pot} \rangle$$

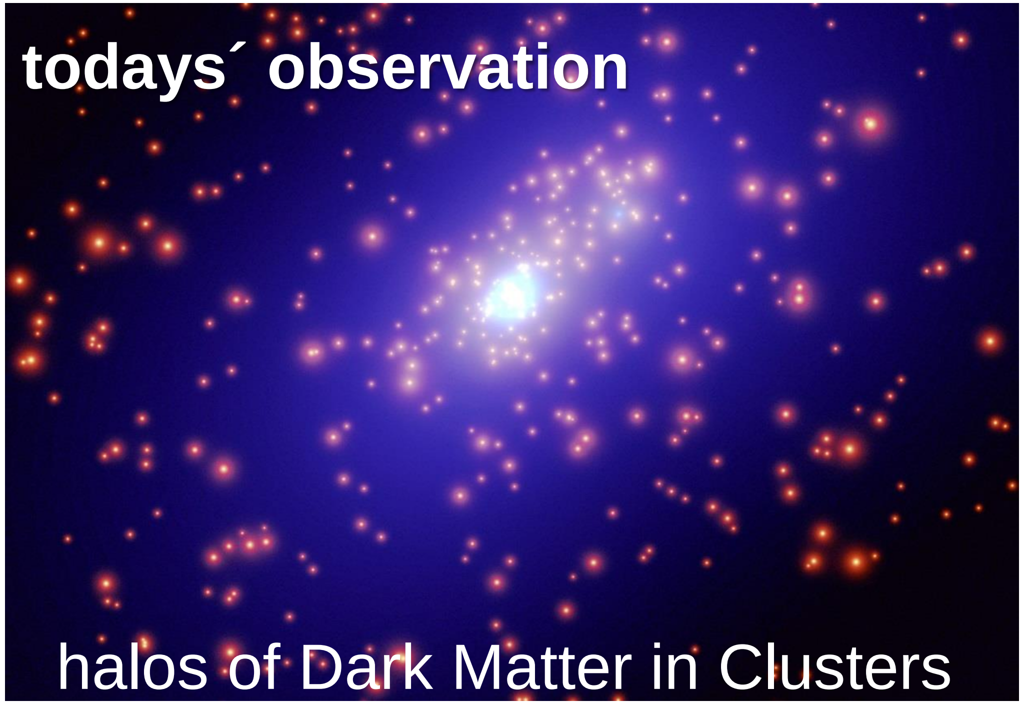
nichtleuchtende Materie
~ 90% der Masse
im Coma-Cluster…

F. Zwicky
Helv. Phys. Acta **6** 110-127 (1933)
´Die Rotverschiebung von extragalaktischen Nebeln´

Dark Matter & galaxy clusters

- ***Fritz Zwicky*** **proposes the existence of Dark Matter (from the Coma cluster)**

  - explanation: non-luminous form of matter (´Dark Matter´) which interacts only via **gravitational potential**!



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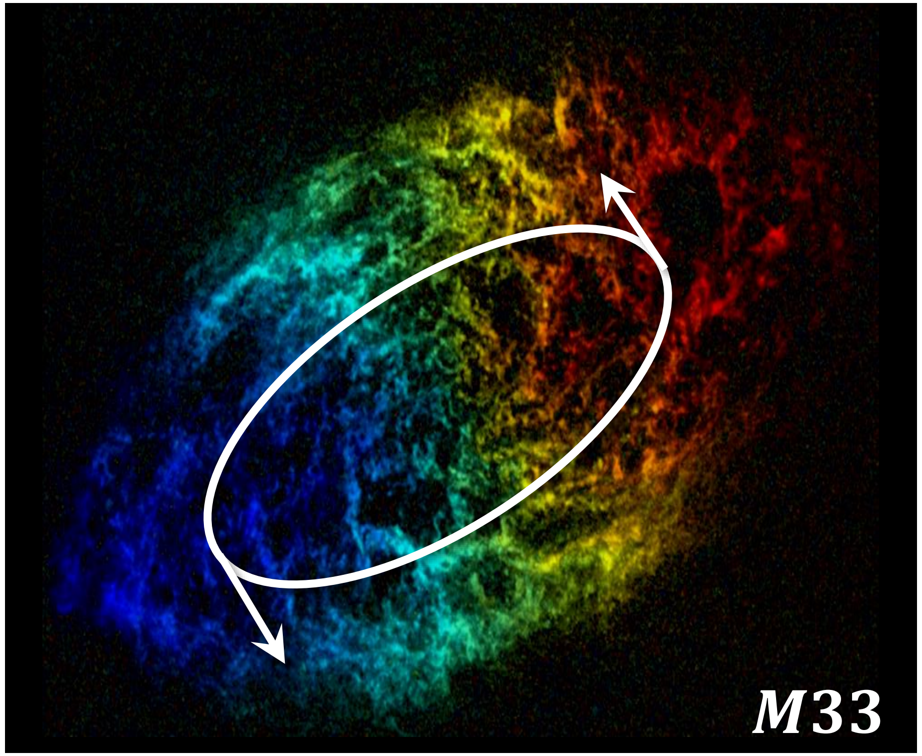
´Die Rotverschiebung von extragalaktischen Nebeln´

Dark Matter & rotational curves of galaxies

- ***Vera Rubin*** **observes flat rotational profiles of galaxies**

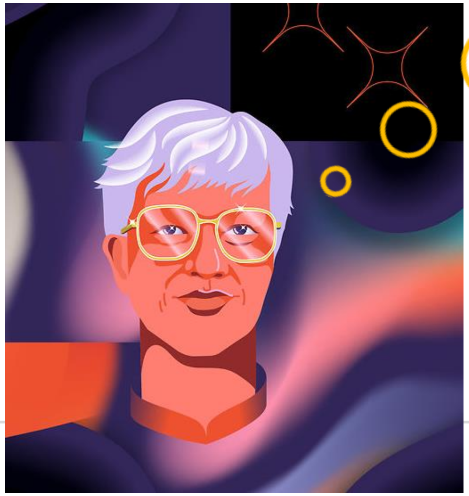
  - observation: (too) high velocities of single stars & gas clouds in the very large **Andromeda spiral galaxy**!



Newton:

$$F = \frac{G M_r m}{r^2} = m \cdot a$$

non-luminous matter
~90% of the mass
in galaxies...



V. Rubin et al.,
ApJ **159** 379 (1970)
´Rotation of the Andromeda Nebula´

Dark Matter & rotational curves of galaxies

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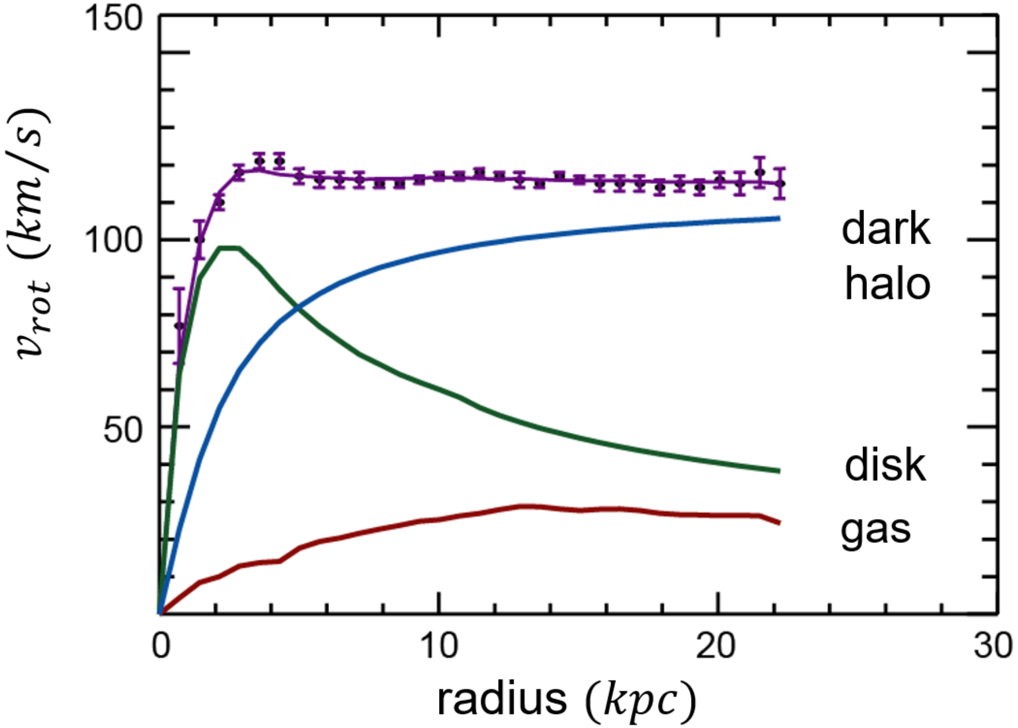
  - explanation: non-luminous form of matter (´Dark Matter´) which interacts only via **gravitational potential**!



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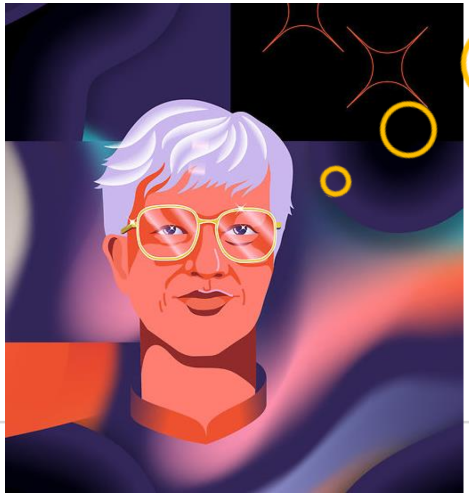
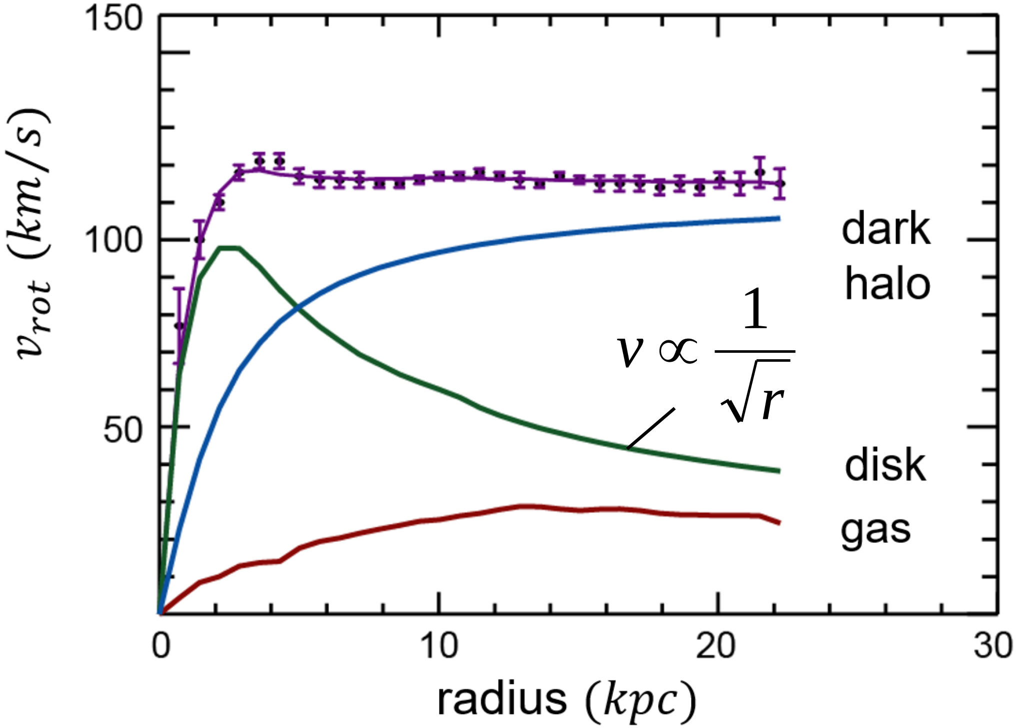
~90% of the mass
in galaxies…



V. Rubin et al.,
ApJ **159** 379 (1970)
´Rotation of the Andromeda Nebula´

RECAP: Newton´s Law & rotational curves

- **Sir Isaac: rotational velocity profile of a galaxy should fall off as $v_{rot} \sim 1/\sqrt{r}$**
  - explanation: there is missing mass in the galaxy, so we need a **halo of Dark Matter**



galactic mass enclosed in r

$$F = \frac{G M_r m}{r^2} = m \cdot a$$

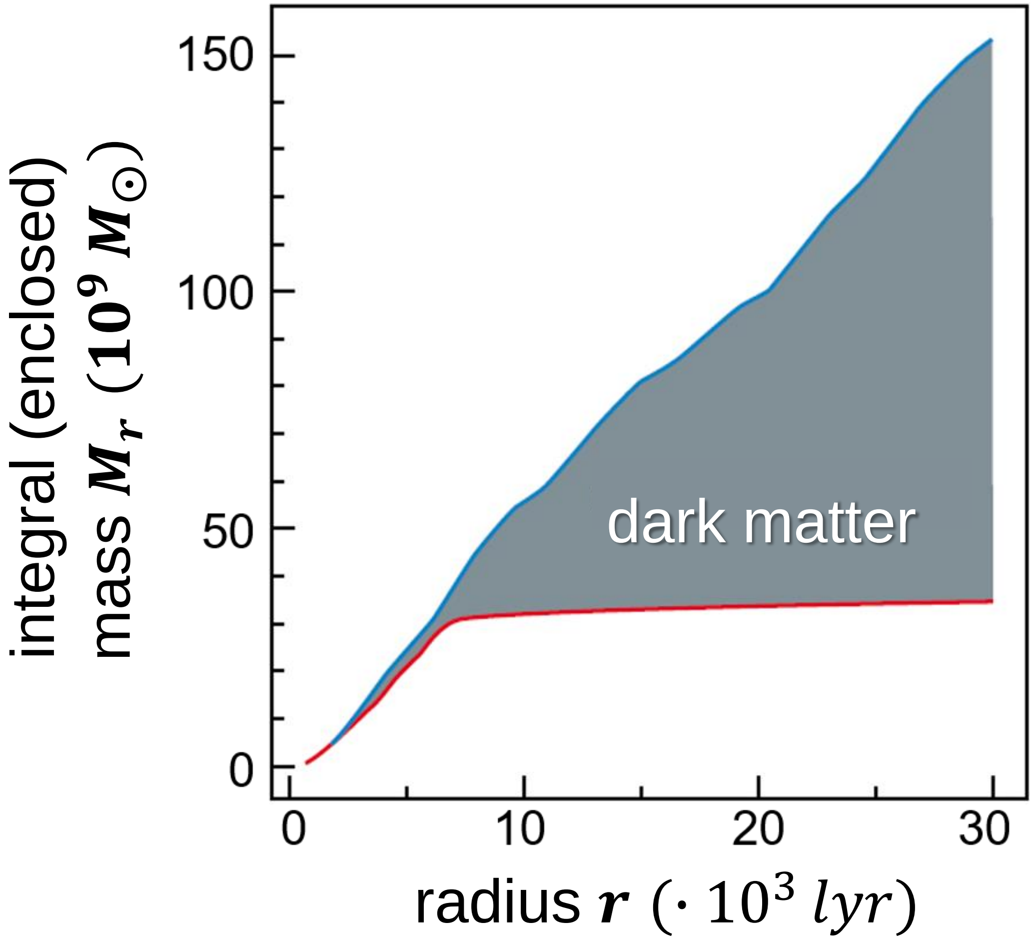
´test mass´ (star,…)

$$a = \frac{v_{rot}^2}{r} = \frac{G \cdot M_r}{r^2}$$

$$\Rightarrow v_{rot}(r) = \sqrt{\frac{G \cdot M_r}{r}}$$

Rotational curves reveal a Dark Matter halo

- Todays´ observations: linear increase of enclosed mass M_r up to $r = 50\ kpc$



- DM – halo with $(80 \ldots 90)\%$ of entire mass

$$F = \frac{G M_r m}{r^2} = m \cdot a$$

galactic mass enclosed in r

´test mass´ (star,…)

$$M(r) \propto r \Rightarrow \rho(r) \propto \frac{1}{r^2}$$

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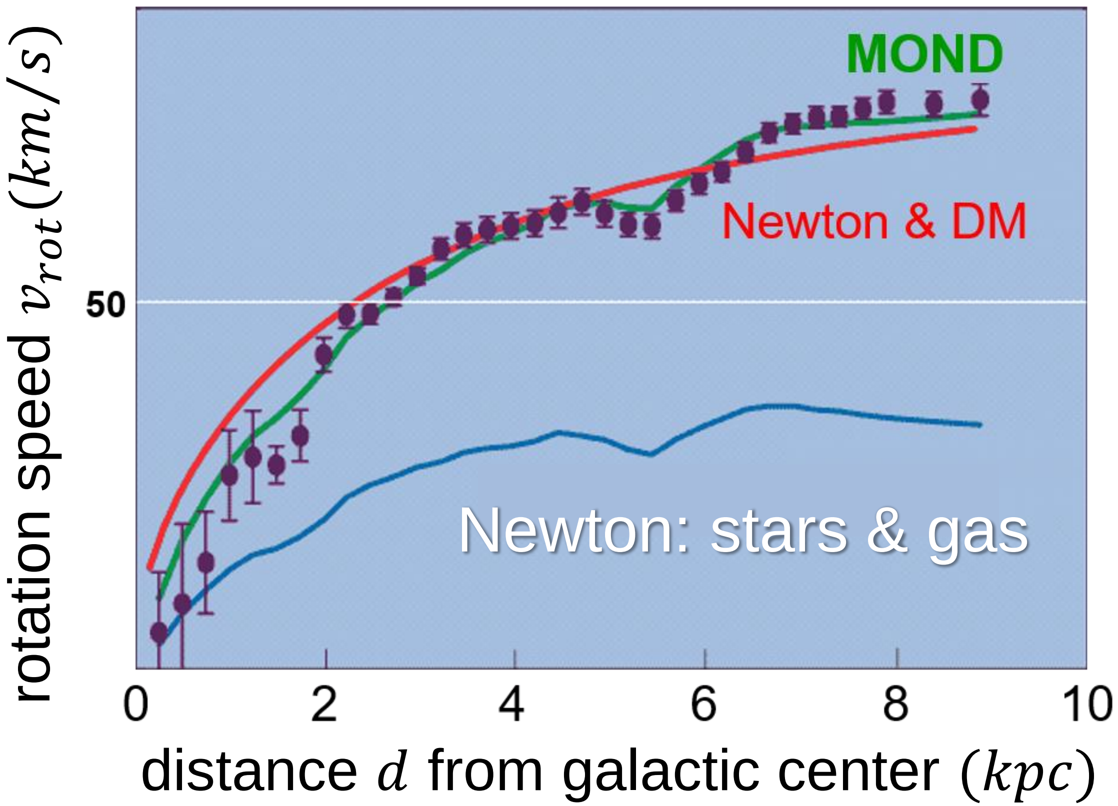
Rotational Curves & the ´ad hoc´ MOND theory

- **MOND: Modified Newton Dynamics as an ´alternative´ to Dark Matter**

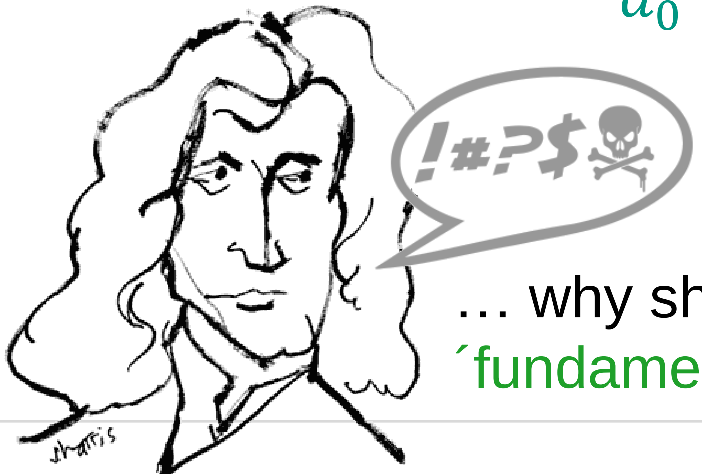
  - galactic rotational profiles can be ´reproduced´ by modifying Newtonian gravity



- $a/a_0 \ll 1$: $\mu = \dfrac{a}{a_0}$ else: $\boldsymbol{\mu} = \mathbf{1}$

$$\boxed{\vec{F} = m \cdot \mu\left(\frac{a}{a_0}\right) \cdot \vec{a}}$$

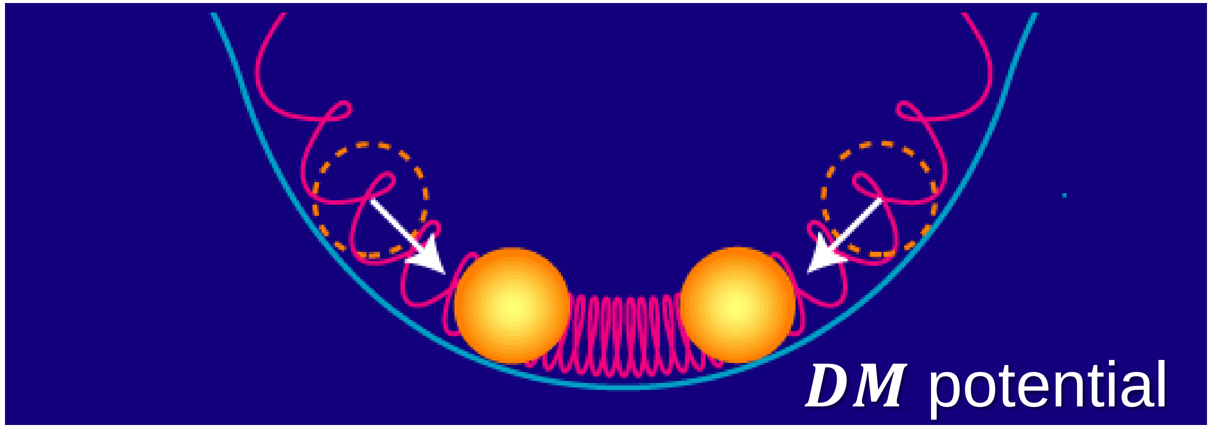
$a_0 \approx 1.2 \cdot 10^{-10}\, m\, s^{-2}$

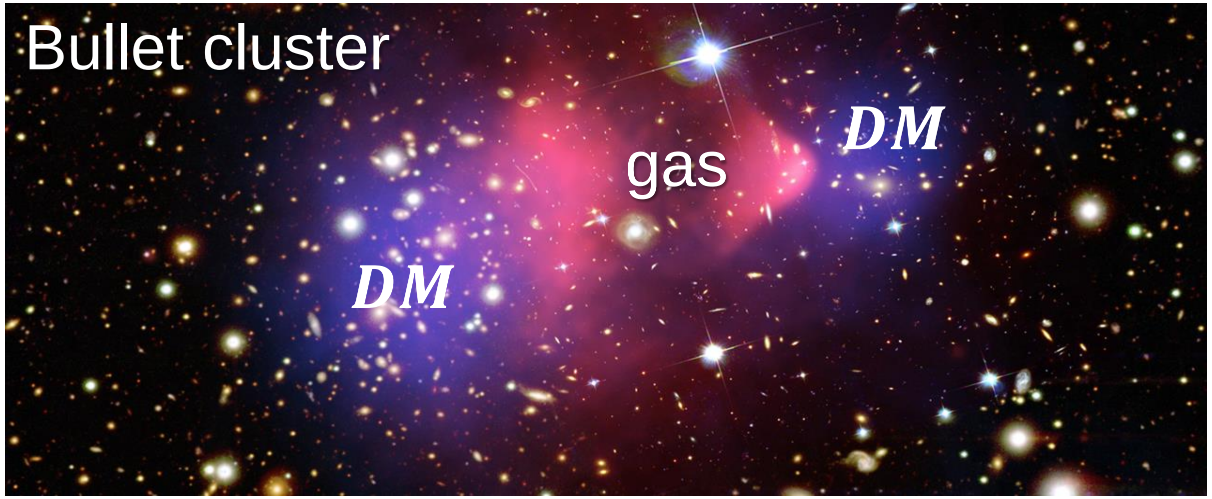


… why should there be a ´fundamental´ acceleration a_0 ?...

Rotational Curves & BAO: irrefutable proof of DM

- **MOND theory: you may fit rotation curves but fail to describe BAO, clusters,…**





MOND theory not compatible with

a) **Baryon Acoustic Oscillations** via gravitational potential by **Dark Matter**

b) **Bullet cluster**
collision of two galaxy clusters: separation of baryons (hot cluster gas) from **Dark Matter** (made visible by gravitational lensing)

5.2 Gravitational Lenses

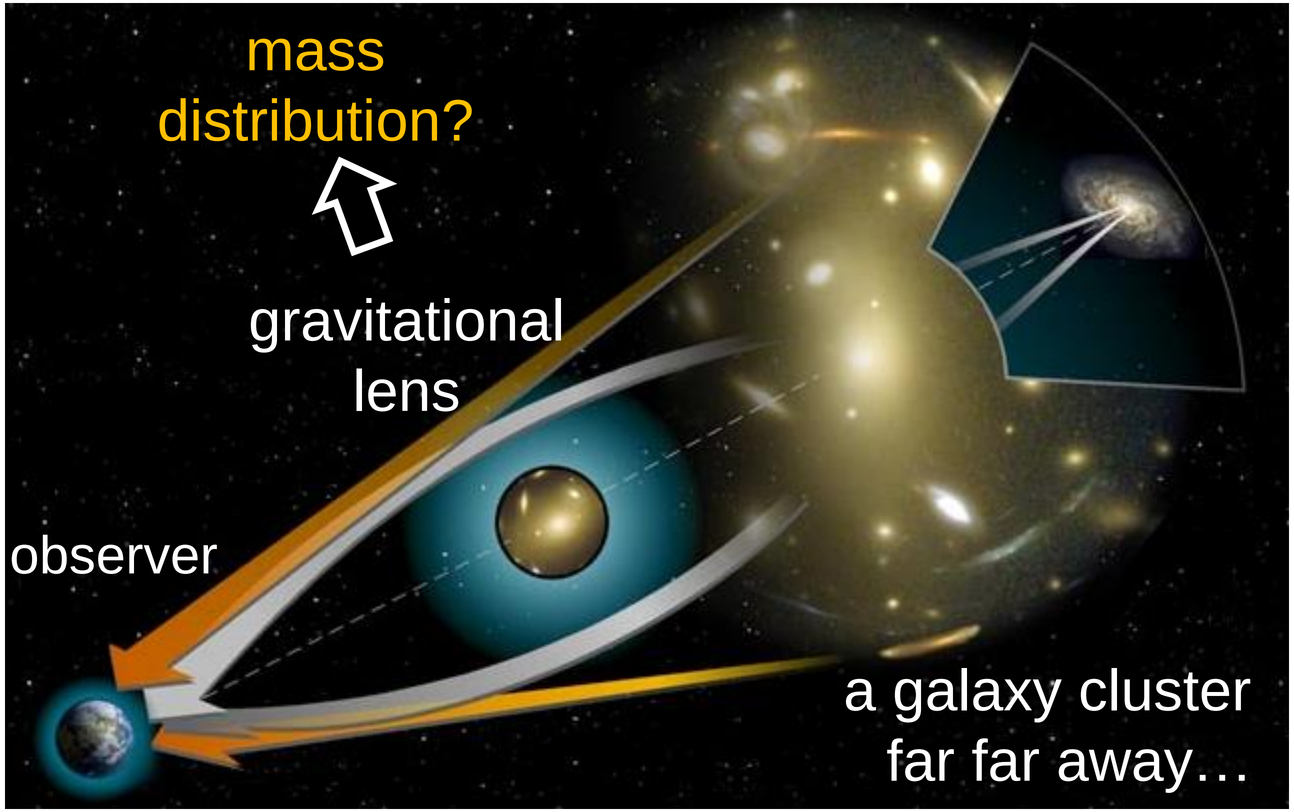
■ **Revealing the presence of DM via the process of gravitational lensing**

- A. Einstein: light is propagating along geodesic lines

- **gravitational lenses**:
  distortion of the optical imaging due to gravitational potentials can be used to dervie **mass distribution of large objects** (galaxy clusters,…)

- **strong lensing**: arcs, rings, multiple images of far-off galaxies/quasars

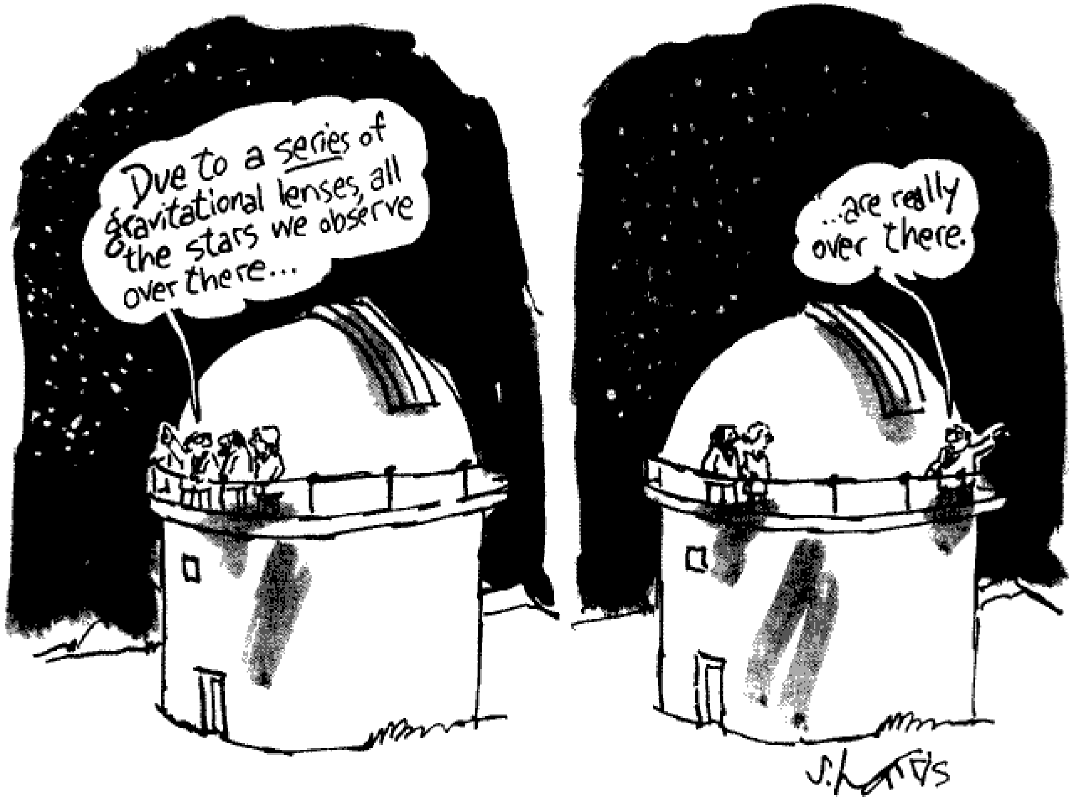
- **weak lensing**: statistical distortion of images of single galaxies

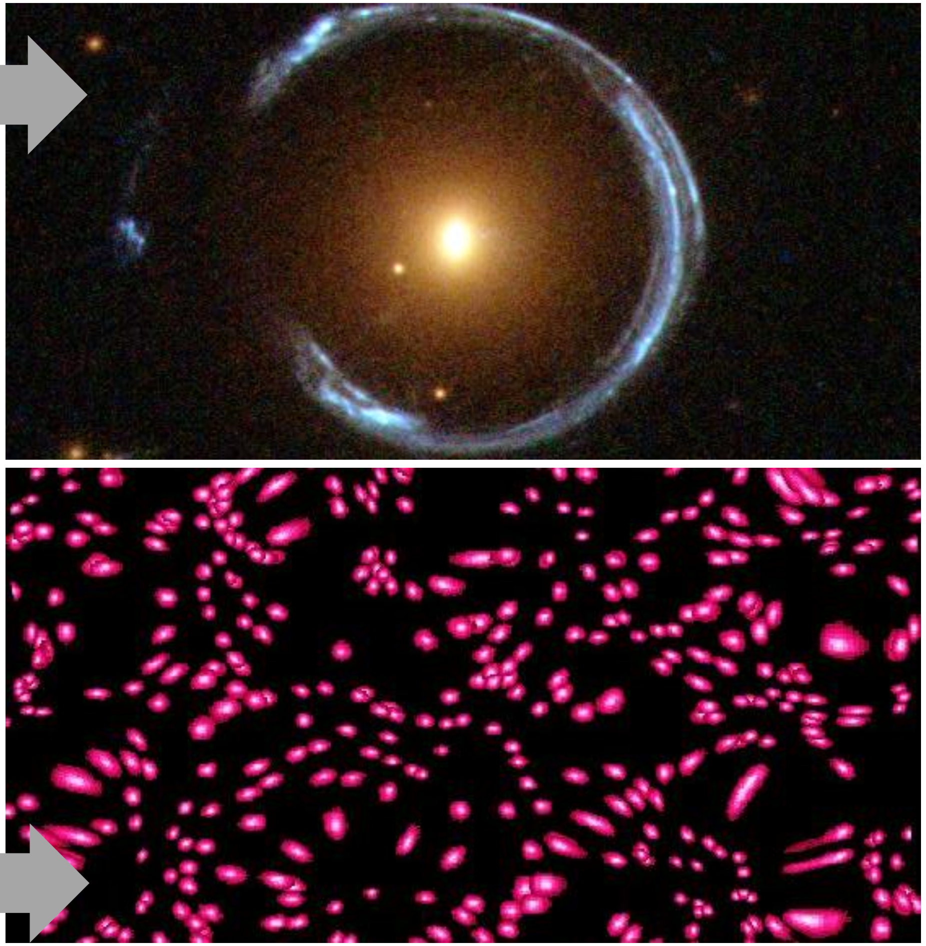


Strong and weak gravitational lensing

- **Important techniques to map out regions of Dark Matter**

  - we can ´see´ where Dark Matter is





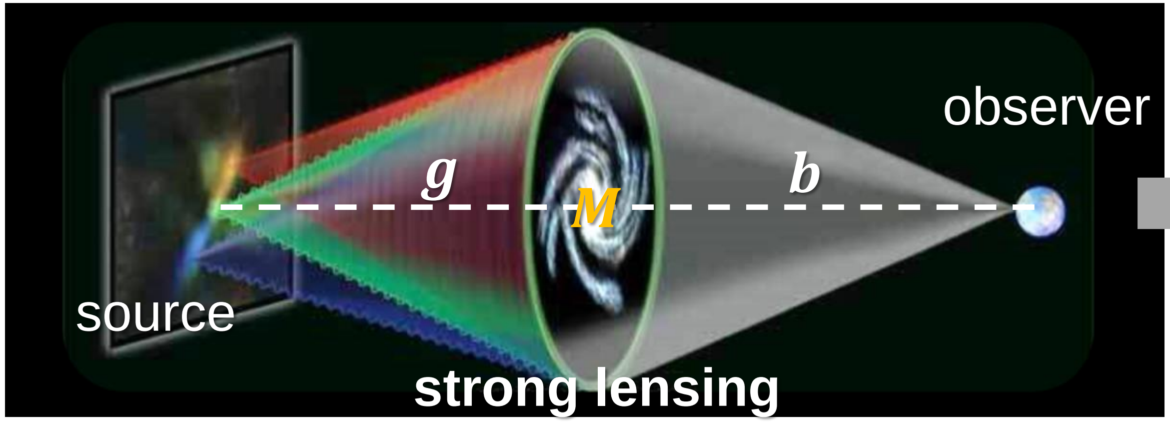
strong

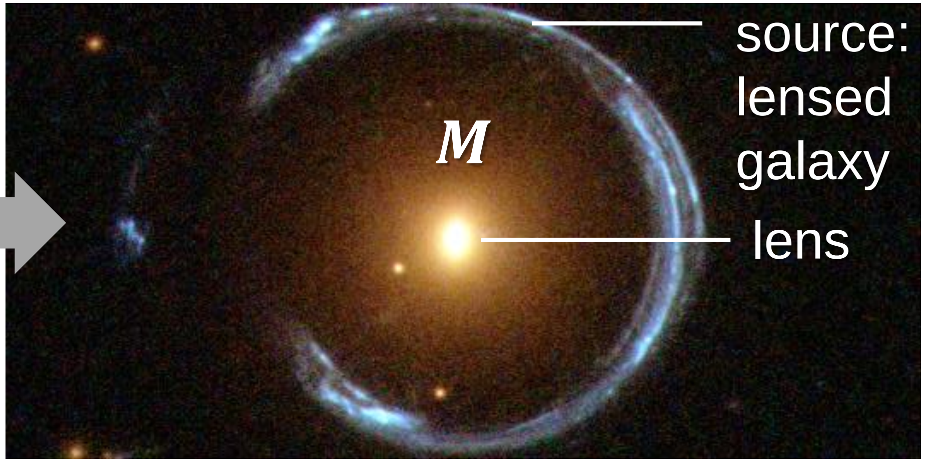
lensing

weak

Strong gravitational lensing

- An ideal method to map the spatial distribution of DM on galactic scales





thin lens formula

$$\boxed{\frac{1}{g} + \frac{1}{b} = \frac{1}{f}}$$

- perfect alignment of source, lens & observer: we see an Einstein ring with opening angle θ_E

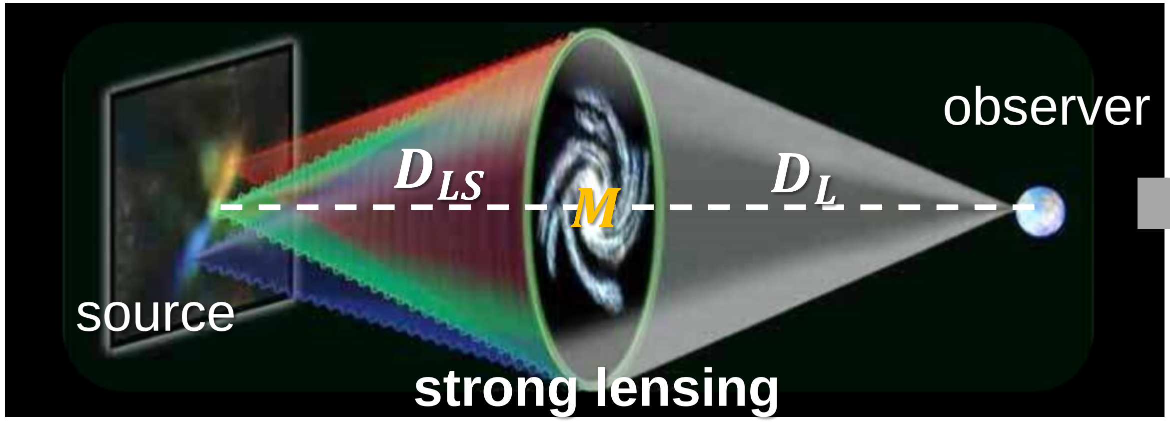
g : distance of source

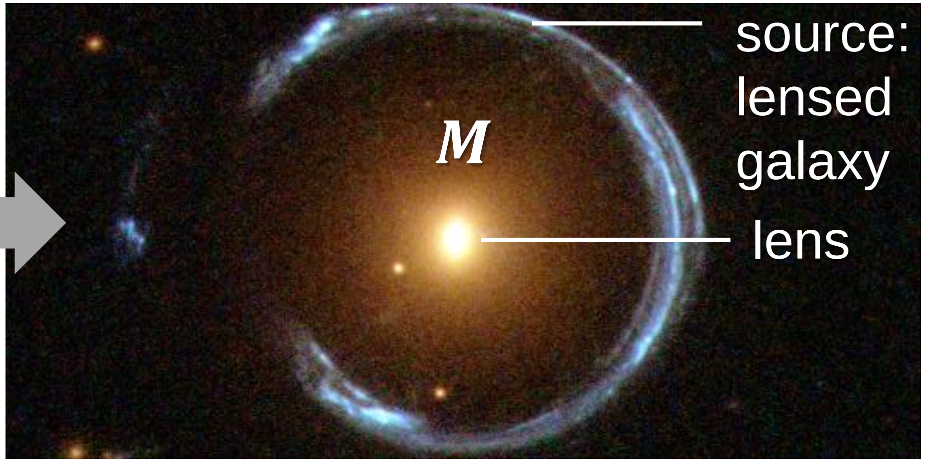
b : distance of observer

f : focal length

Strong gravitational lensing

- An ideal method to map the spatial distribution of DM on galactic scales





lensing formula

$$\theta_E = \sqrt{\frac{4GM}{c^2} \cdot \frac{D_{LS}}{D_S D_L}}$$

$\boldsymbol{M}$: Mass of the lens (shows presence of $\boldsymbol{DM}$)

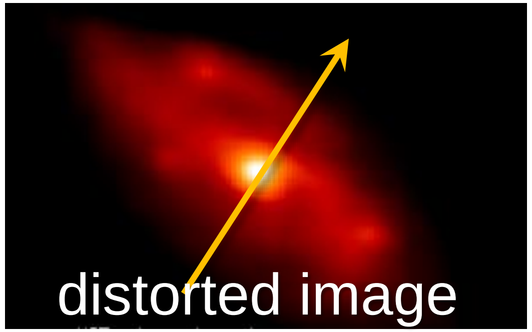
$\boldsymbol{D_S = D_{LS} + D_L}$ (source – observer)

- perfect alignment of source, lens & observer: we see an Einstein ring with opening angle $\boldsymbol{\theta_E}$

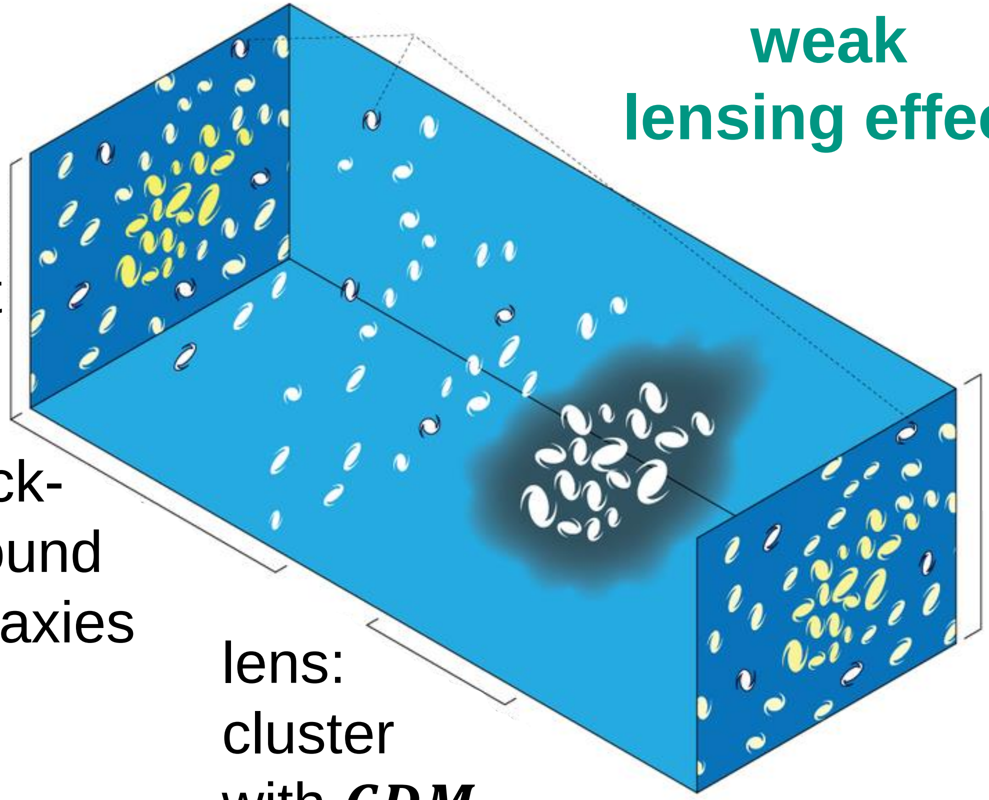
- ~70 Einstein rings/arcs observed: always considerable amount of $\boldsymbol{DM}$

Weak gravitational lensing

- **Small (statistical) strechting of galaxy images: large-scale imaging of DM**
  - **weak gravitational lensing** due to extended lensing galaxy cluster with DM
    ⇨ statistical **strechting** (factor ~ 1%) of the images of galaxies in the background
    ⇨ perform statistical analysis



´**stretching**´ of a galaxy by factor 1%

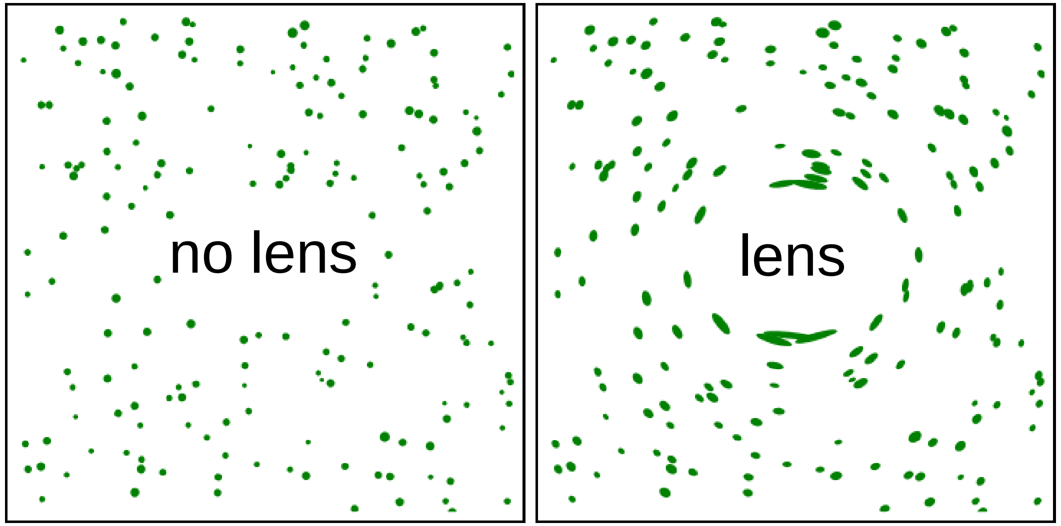


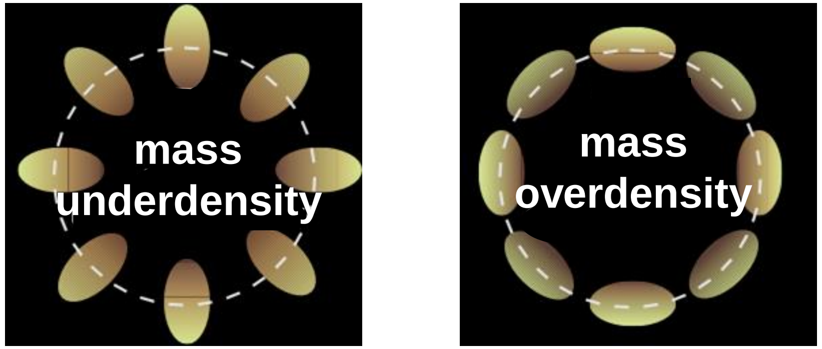
Weak gravitational lensing

- **Small (statistical) strechting of galaxy images: large-scale imaging of DM**

  - primary ratio of the semi-axes (major to minor) of a galaxy image is unknown:
    ⇨ **stretching of image due to weak lens** has to be analysed **statistically**

  - signature of a **void** (under-dense region): major axes align **radially** to void
    signature of a **cluster** (over-dense region): major axes align in a **ring-like** form





radially ring-like

Weak gravitational lensing

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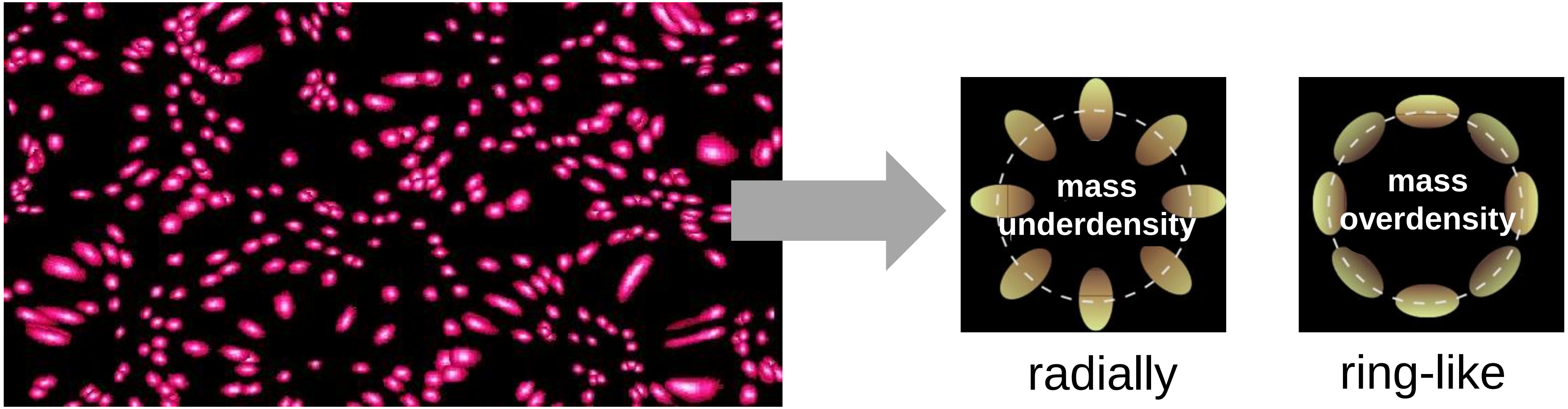
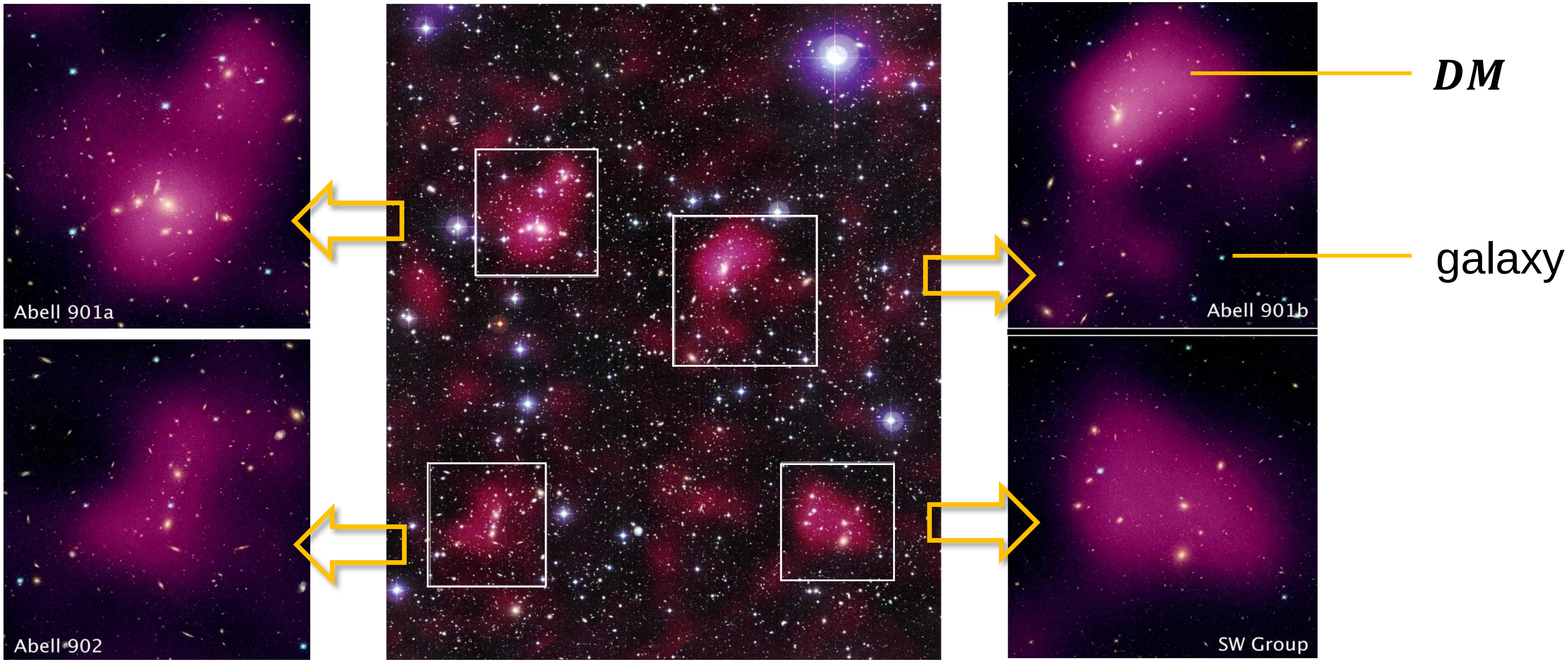


Weak lensing: distribution of DM in a cluster

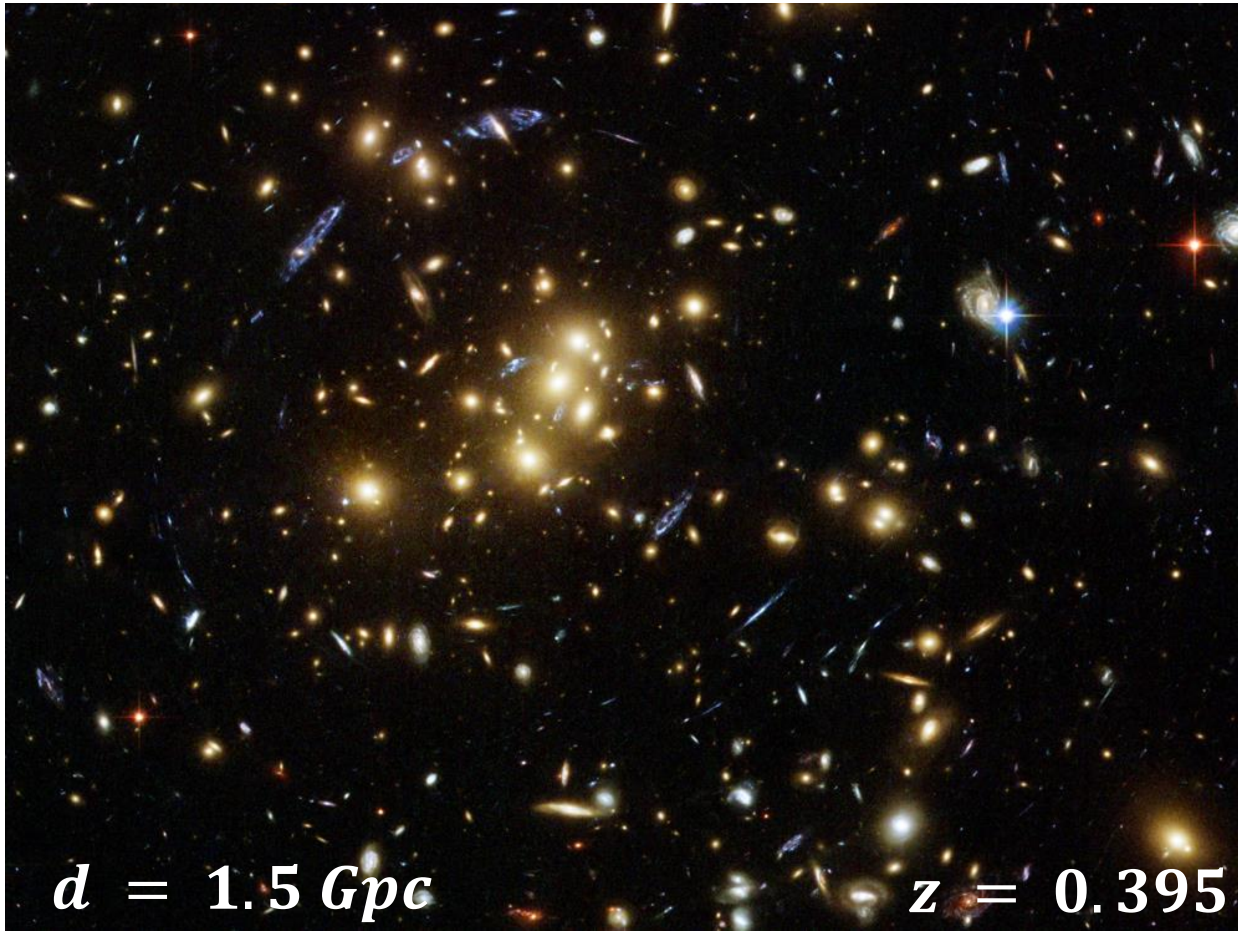
- Example of $DM-$ distribution in galaxy clusters $Abell\ 901/902$



Rare: weak & strong lensing in the same picture

- **Combination of the two effects in galaxy cluster $CL0025 + 1654$**

- observation of ***strong lensing:***
  several blue arcs of lensed images of
  far-off background galaxies



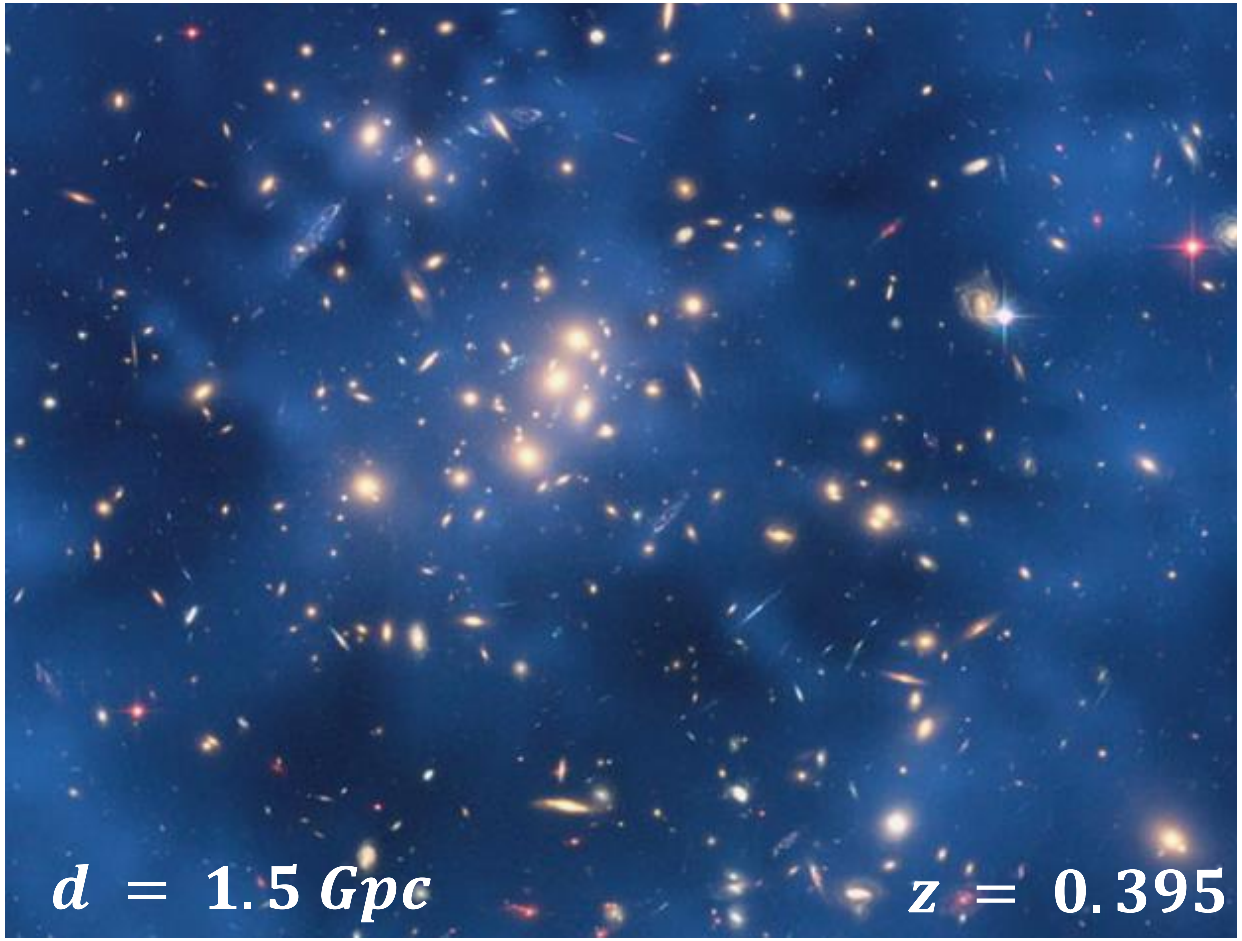
weak lensing: distribution of dark matter

- **Combination of the two effects in galaxy cluster $CL0025 + 1654$**

- observation of ***strong lensing:***
  several blue arcs of lensed images of
  far-off background galaxies

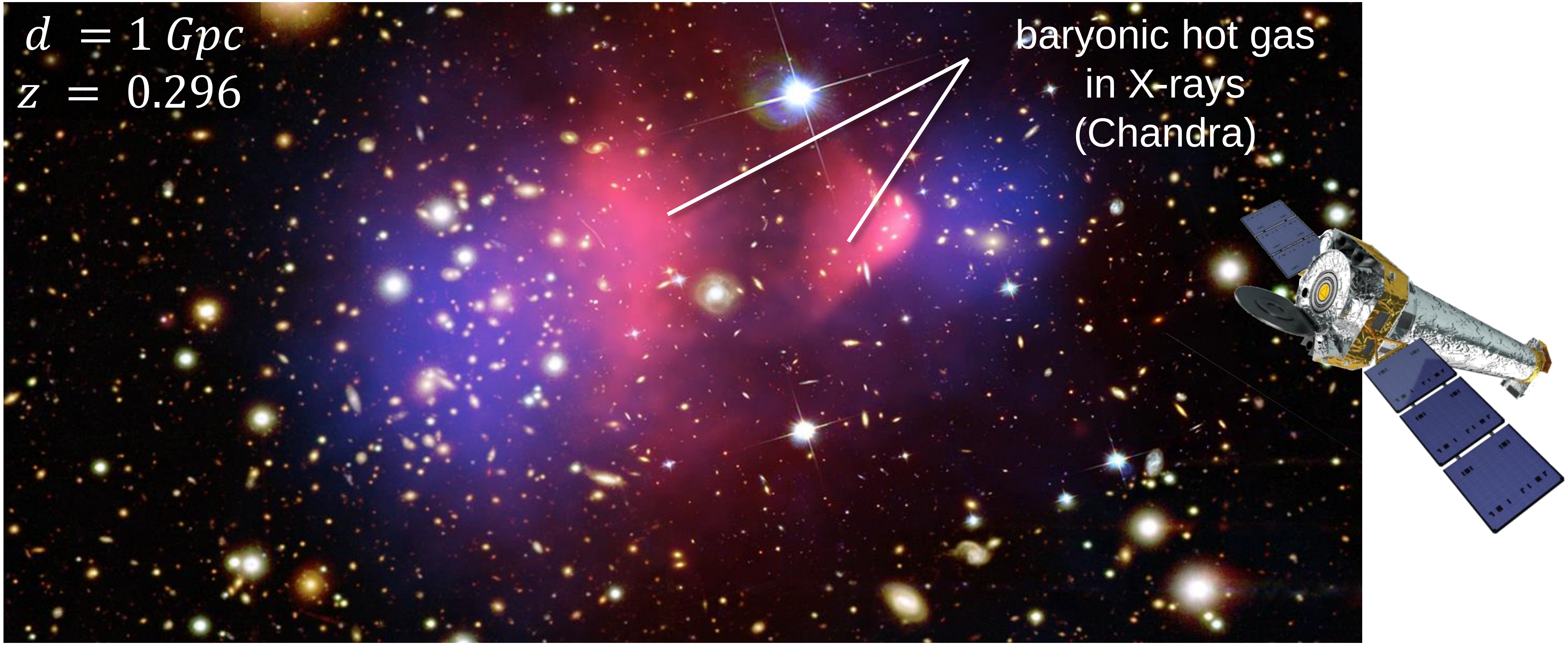
- observation of **weak lensing**:
  statistical distortion of the images of
  7000 background galaxies

- allows to map distribution of $\boldsymbol{DM}$ of the
  in-between cluster CL0025+1654
  which acts as weak gravitational lens



Weak lensing: the famous Bullet cluster

- $DM-$ distribution in galaxy: separation from baryons after collision



Weak lensing: the famous Bullet cluster

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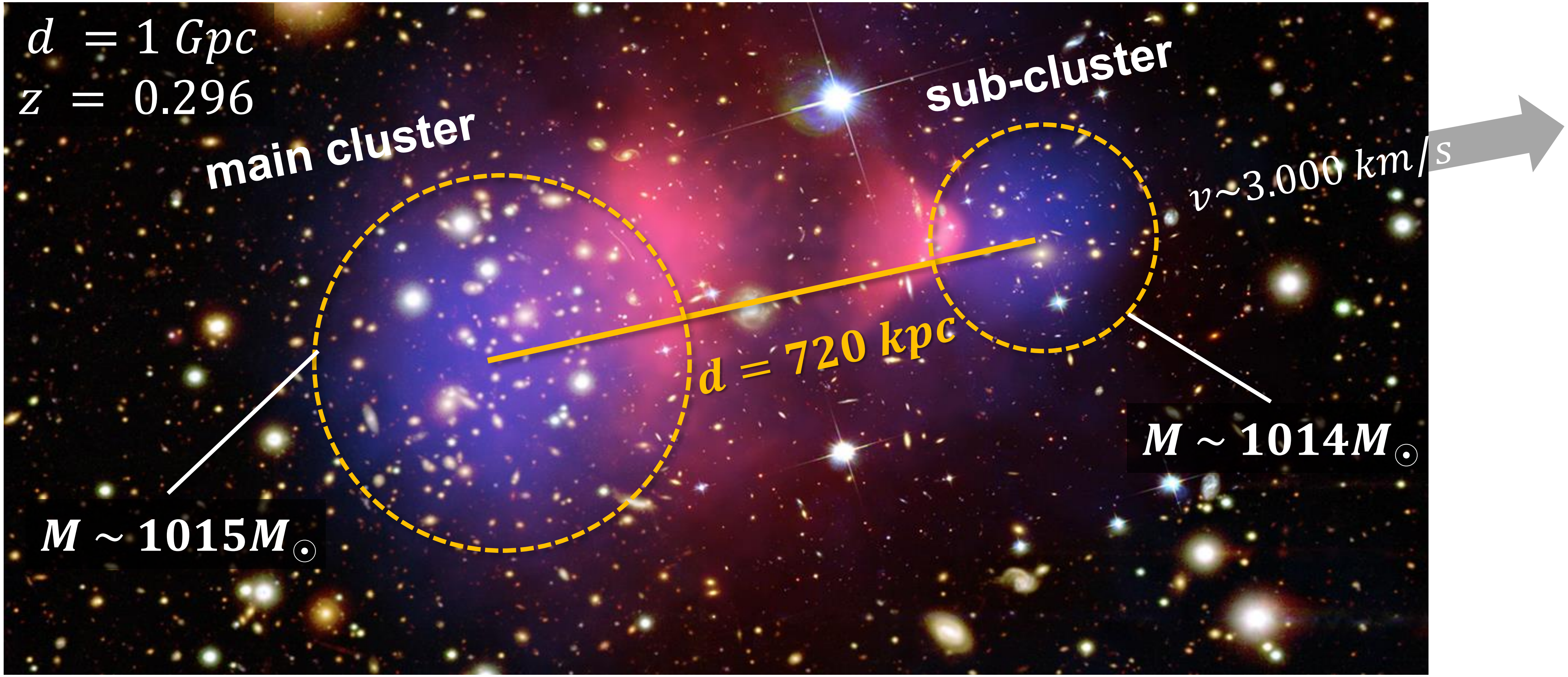


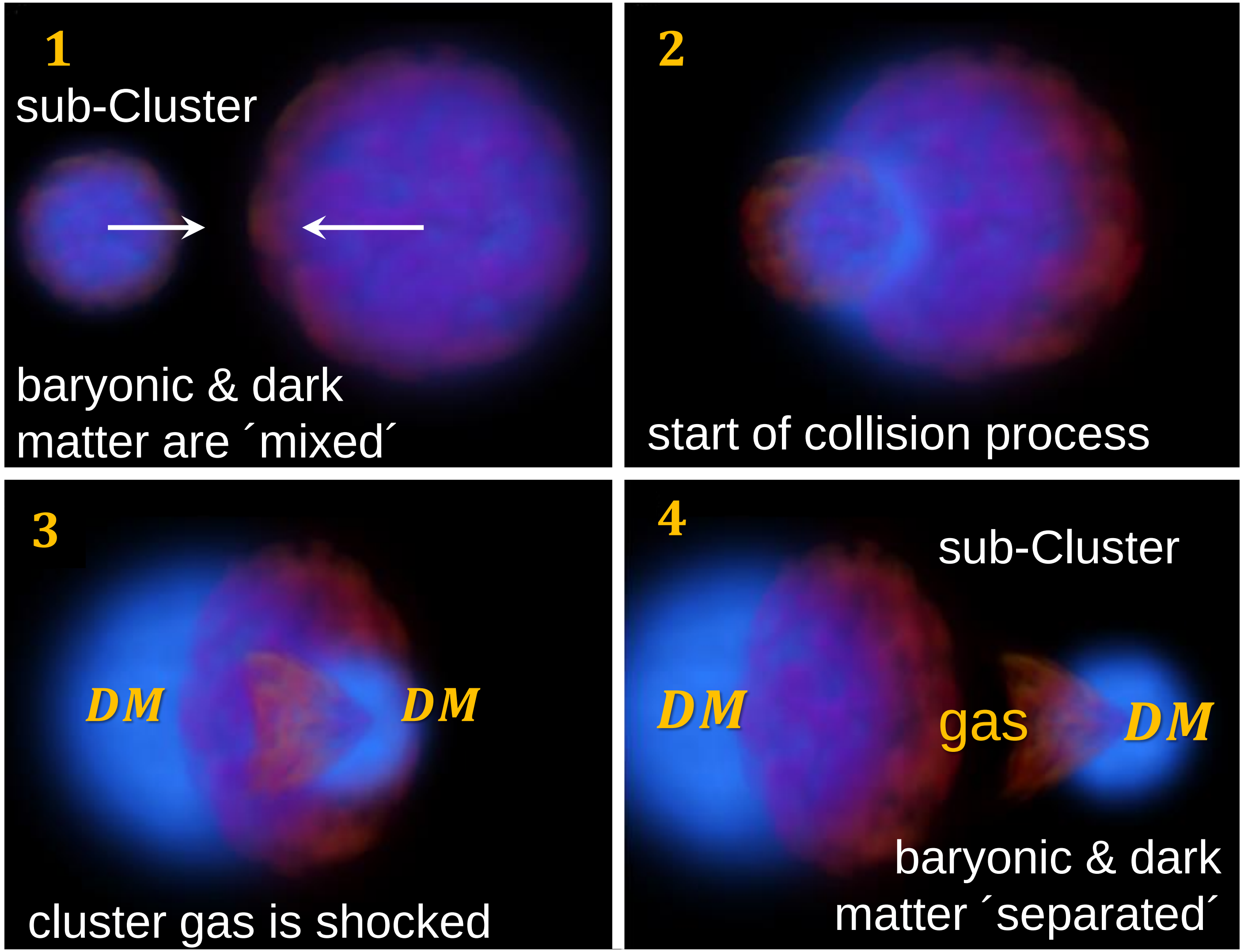
Weak lensing: the famous Bullet cluster

- **Phases of collision process between 2 galaxy clusters – evidence for *DM***

- **Dark Matter**:
  no dissipation, no interaction processes during collision

DM & gas separated

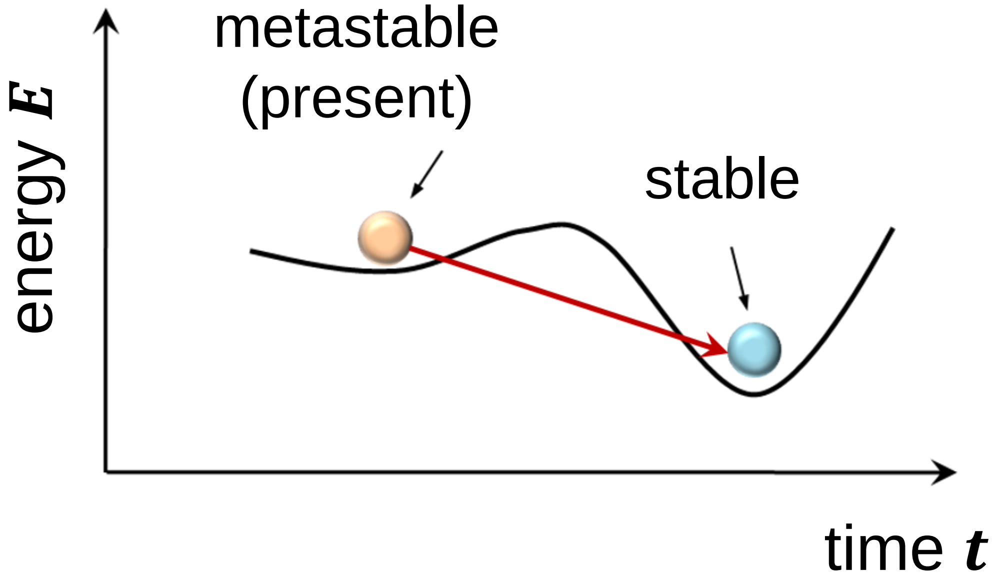
- **Baryonic gas**:
  during collision: gas is shocked & strongly heated due to very intense interactions (dissipation)

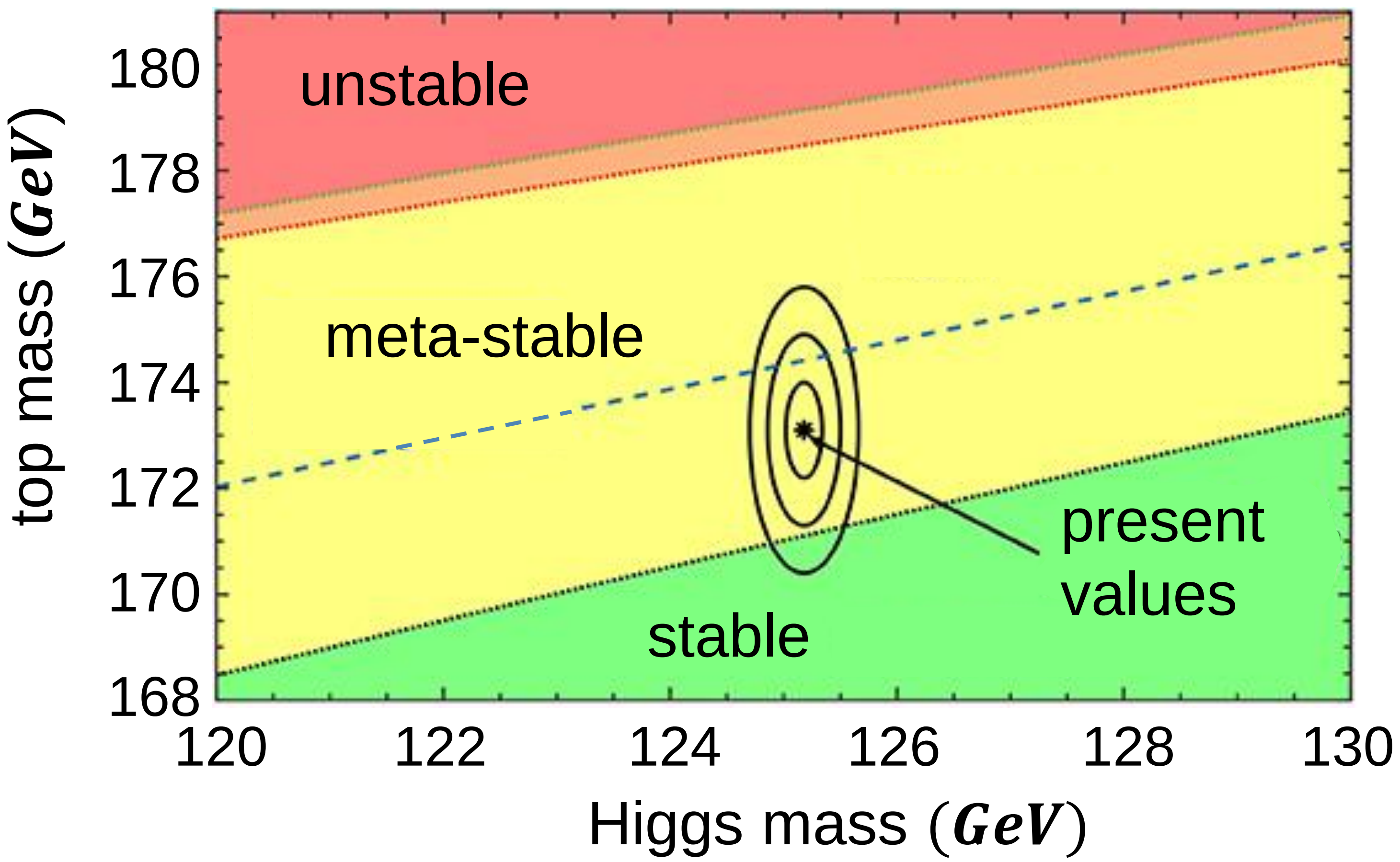


Cosmology: ´final´ open questions

- **Is the vacuum (electroweak ground state) stable over very long times?**

  - spontaneous decay via a **tunneling process** to the ´true´ vacuum state…





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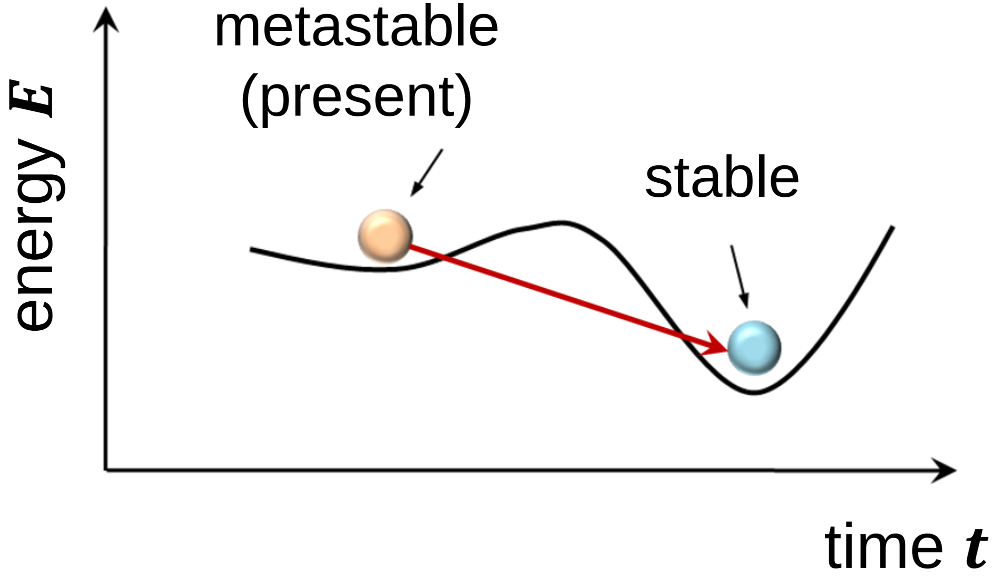
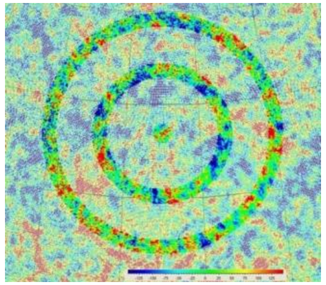


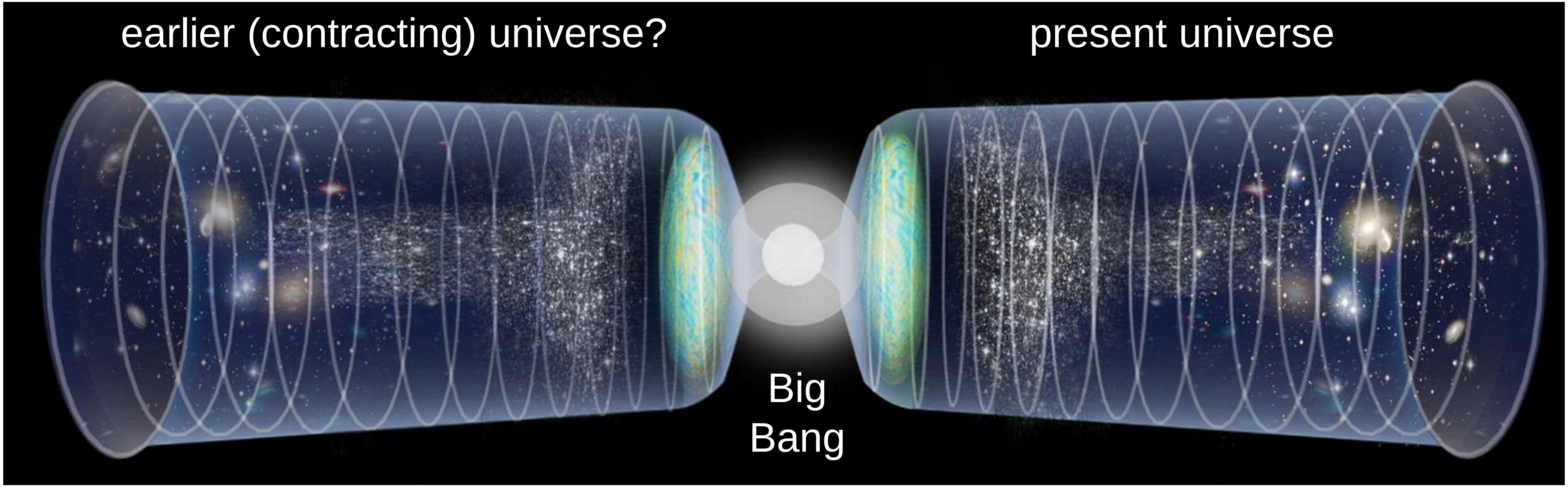
a ***new*** Standard Model….

Cosmology: ´final´ open questions

- **Is the universe cyclic? Did it emerge from the ´Big Bounce´?**

  - signature of a possible earlier collapsed universe:
    observation of rings in the CMB





Cosmology: ´final´ question

- **Where did it all come from?**

THANK YOU…