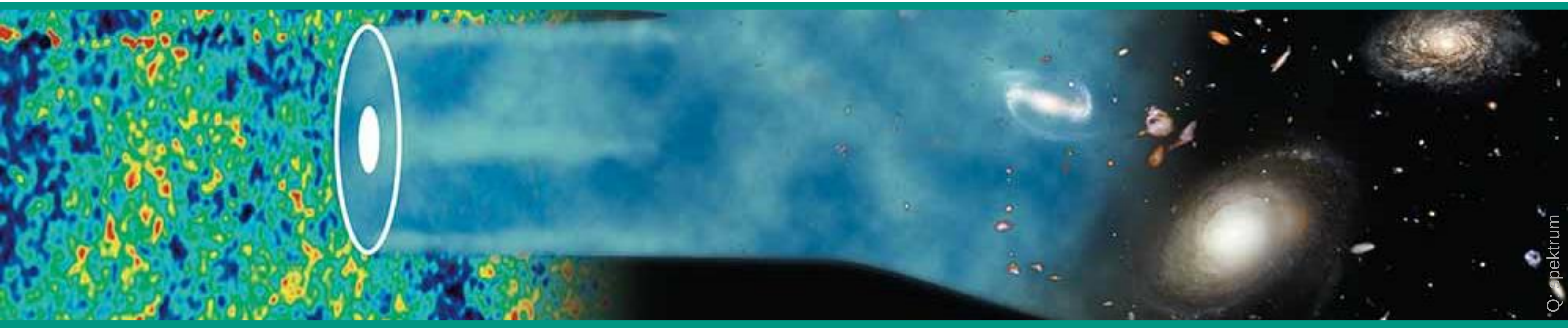
Karlsruhe Institute of Technology

Introduction to Cosmology

Winter term 22/23 Lecture 4 Nov. 29, 2022



Recap of Lecture 3

- **Topology & geometry of the universe**

  - curvature parameter $\boldsymbol{k = -1, 0, +1}$ **(hyperbolic, flat, spherical)**

  - open questions: isotropy, limited/unlimited size, more complex topologies,…

- **Friedmann-Lemaître equation: accleration / braking**

pressure P: important for $a(t), \ddot{a}(t)$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi G\left(\rho(t) + \frac{3P}{c^2}\right)$$

3 cosmological epochs:

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_v(t)$$

  - equation-of-state of vacuum: $\boldsymbol{P_V(t_0) = -1\, \rho_V(t_0) \cdot c^2}$ (anti-gravitation)

Friedmann eq. with cosmological constant Λ

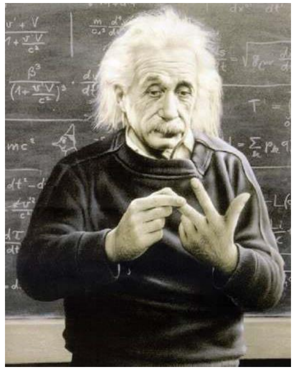
- **Properties ρ_V and P_V of the vacuum ´merged´ into one parameter: Λ**

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi G\cdot\left(\rho_{m,r,V}(t)+\frac{3P_{m,r,V}}{c^2}\right) = -\frac{4}{3}\pi G\cdot\left(\rho_{m,r}(t)+\frac{3P_{m,r}}{c^2}\right)+\frac{\Lambda c^2}{3}$$

matter, radiation & vacuum | matter & radiation | vacuum

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi G\left(\rho_V(t)+\frac{3P_V}{c^2}\right) \Rightarrow\ = \frac{\Lambda c^2}{3}$$

$$\Lambda$$

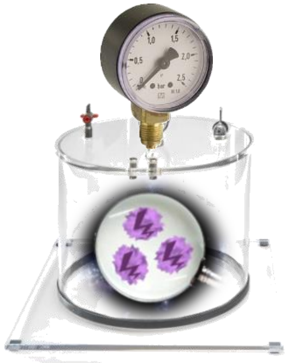


Cosmological constant Λ

- **Recap: properties of the vacuum**

$3x$

as pressure $\boldsymbol{P_V < 0}$
we have $\boldsymbol{\ddot{a}(t) > 0}$



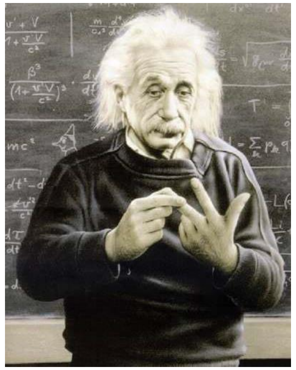
$1x$

as density $\boldsymbol{\rho_V > 0}$
we have $\boldsymbol{\ddot{a}(t) < 0}$

vacuum equation-of-state:

$$\rho_V(t) = -1 \cdot P_V(t)$$

we keep this relation constant, thus we have
⇨ cosmolog. constant



Cosmological constant Λ

- **A very important constant in cosmology**

  - **time-independent, constant** parameter

$$\Lambda = \frac{8\pi G}{c^2} \cdot \rho_V$$

  - positive sign, thus *accelerated* expansion

  - best experimental value at present:

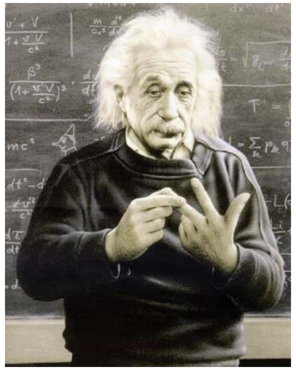
$$\Lambda = [(2.14 \pm 0.13) \times 10^{-3} eV]^4$$

$$\approx 1.1 \cdot 10^{-52}\ m^2 \approx 10^{-29}\ g/cm^3$$

vacuum equation-of-state:

$$\rho_V(t) = -1 \cdot P_V(t)$$

Λ



Cosmological constant Λ

- **Experimental value & theoretical estimate: a ´small´ discrepancy**

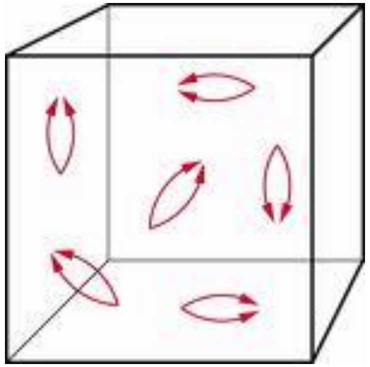
observed: $\rho_V = 3.6\ GeV/m^3$

estimate: $\rho_V = 10^{121}\ GeV/m^3$

zero-point-energy of a quantum field?

- **biggest discrepancy in all of science!**

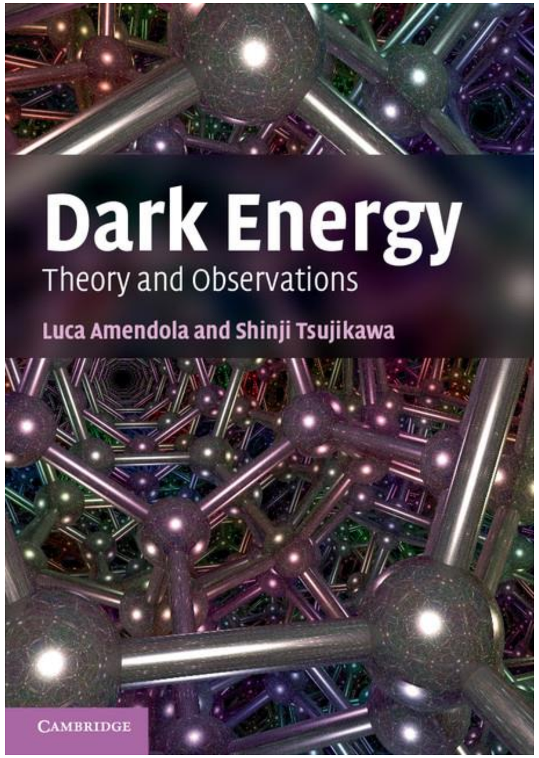
- reduced to ´only´ 60 orders of magnitude in extended models of particle physics



- literature tip: S. Weinberg et al. [Likely Values of the Cosmological Constant]

Cosmological constant Λ

- Popular science: vacuum energy in focus



TOPIC: THE CASIMIR EFFECT

- **An experimental investigation of the strange properties of the vacuum**

  - vacuum is filled with virtual, short-lived particles (Heisenberg uncertainty relation)

  - two parallel metal plates separated by few nm: boundary conditions at the plate surfaces

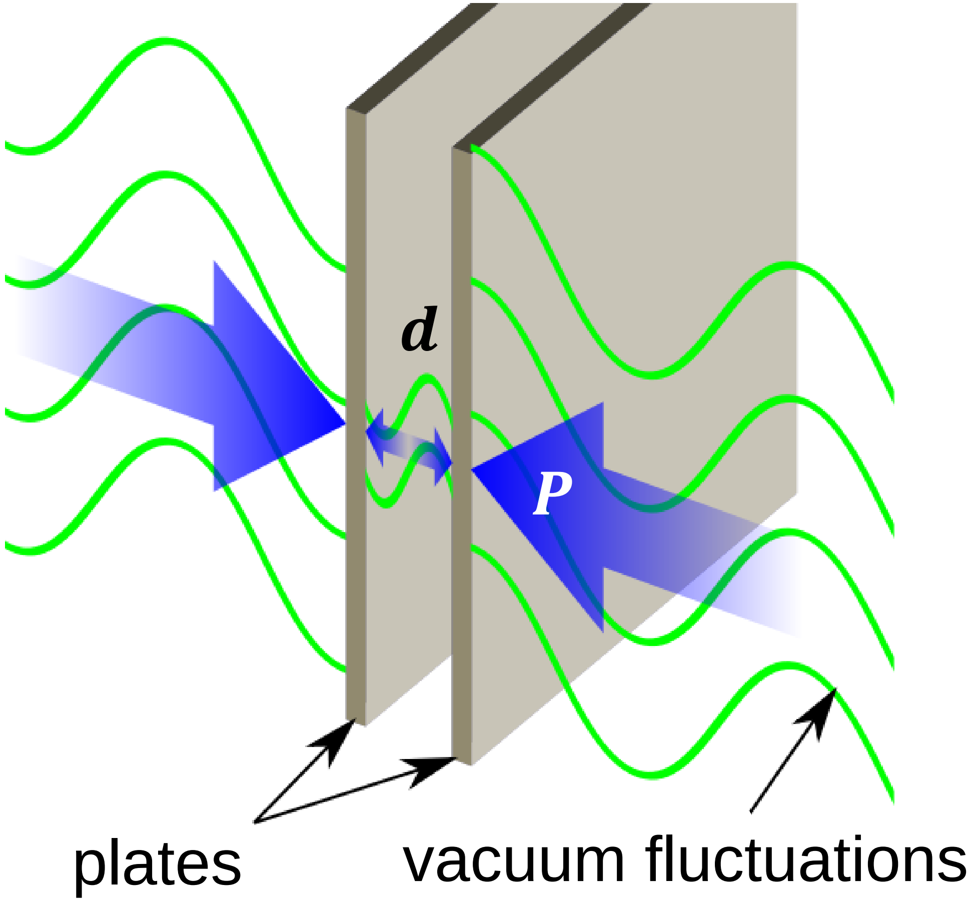
Hendrik Casimir

⇨ impact on electro-magnetic field (**virtual photons**)

⇨ different zero-point energy inbetween

⇨ net force $F \sim 1/d^3$ (dominant at nm)

⇨ experimental observation in 2001



TOPIC: THE CASIMIR EFFECT

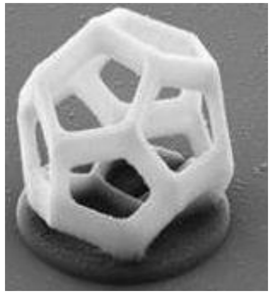
- **An experimental investigation of the strange properties of the vacuum**

  - vacuum is filled with virtual, short-lived particles (Heisenberg uncertainty relation)

  - Casimir force can now be measured by integrated silicon chips (US-Chinese team)

at $d = 11\ nm$

$\Rightarrow P = 1\ bar$



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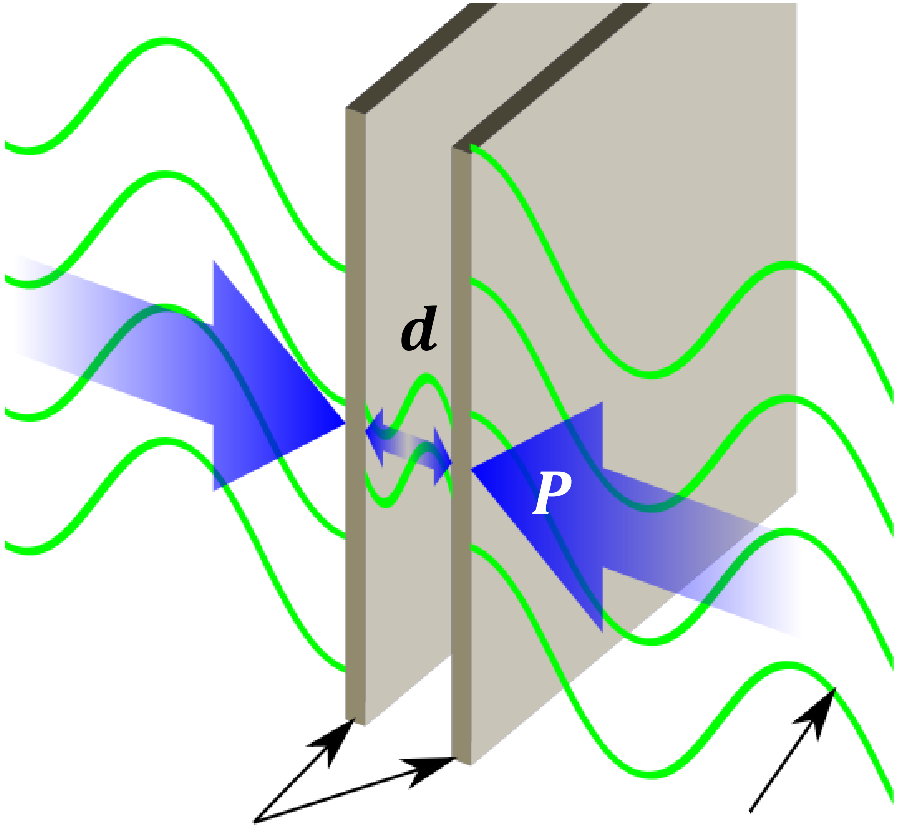
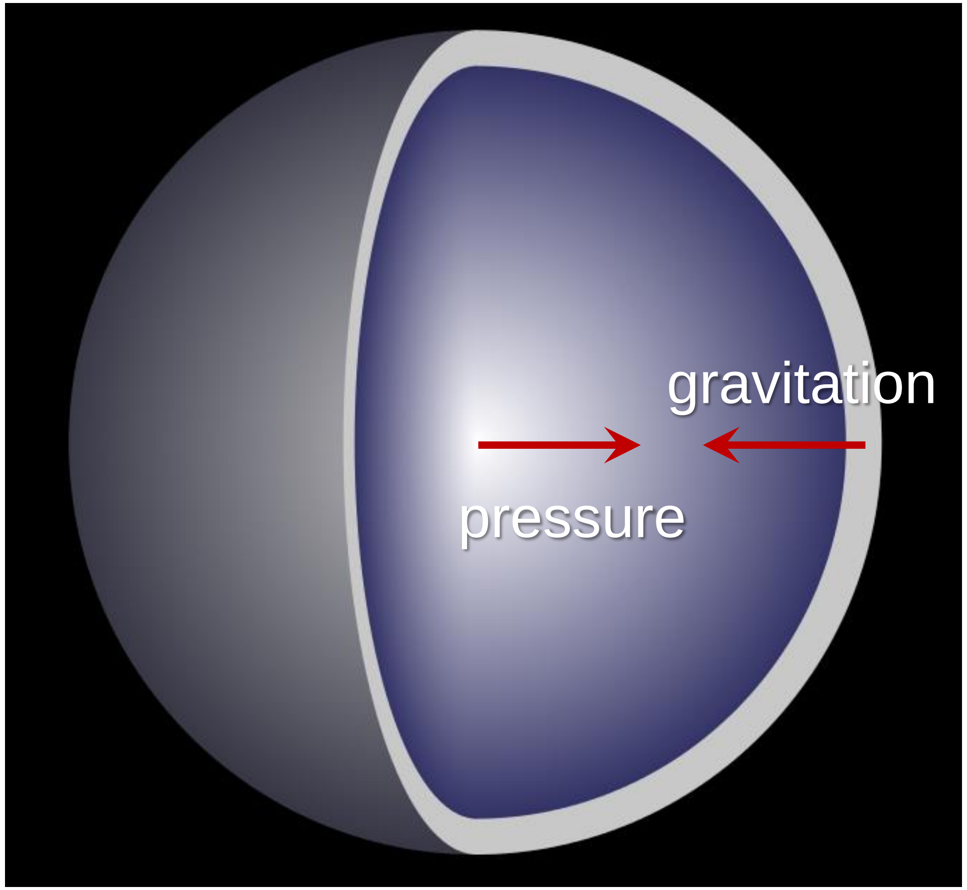


*objects manufactured at μm – scale

TOPIC: PRESSURE AND GRAVITY

- **Extreme pressures inside a compact object (neutron star)**

  - **neutron stars***: extremely compact objects
  - radius $R \sim 10 - 20\ km$, mass $M < 2 - 3\ M_\odot$
  - very high density $\rho \sim (6 - 8) \times 10^{17}\ kg/m^3$



- ´degeneracy´ pressure of neutrons counteracts gravity, but is itself a source of the gravitational field

⇨ limited masses of neutron stars

Robert Oppenheimer

*s. ATP-II (Summer Term 2023)

Friedmann-Lemaître Equations

- **2 fundamental equations to describe dynamics of cosmological expansion**

  - expansion rates governed by: matter, radiation, vacuum

  ⇨ **total energy density & topology of the universe**

Aleksandr Friedmann
(1888 – 1925)

Georges Lemaître
(1894 – 1966)

Friedmann-Equations

- **Recap: acceleration of a homogenous & isotropic universe**

  - we will now start to **integrate** our well-know acceleration equation to obtain a relation for parameter $\dot{a}(t)$

$\dot{a}(t)$

time-dependent energy densities

time-dependent pressure values

cosmological constant

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi G \cdot \left(\rho_{m,r,V}(t) + \frac{3P_{m,r,V}}{c^2} \right) = -\frac{4}{3}\pi G \cdot \left(\rho_{m,r}(t) + \frac{3P_{m,r}}{c^2} \right) + \frac{\Lambda c^2}{3}$$

$P_m = 0$

Friedmann-Equations: let´s do some maths…

- **Integration to obtain the second expansion equation**

$$\ddot{a}(t) = -\frac{4}{3}\pi \cdot G \cdot \rho(t) \cdot a(t)$$

$$\rho(t) = \frac{\rho_0}{a^3(t)}$$

$$\ddot{a}(t) = -\frac{4}{3} \cdot \pi \cdot G \cdot \rho_0 \cdot \frac{1}{a^2(t)} \quad | \cdot 2 \cdot \dot{a}(t)$$

$$\ddot{a}(t) \cdot 2 \cdot \dot{a}(t) = -\frac{2 \cdot 4}{3} \cdot \pi \cdot G \cdot \rho_0 \cdot \frac{\dot{a}(t)}{a^2(t)}$$

integration

$$\dot{a}^2(t) = -\frac{8}{3} \cdot \pi \cdot G \cdot \rho_0 \cdot \left(\frac{-1}{a(t)}\right) - \boldsymbol{k}c^2$$

k: integration constant

Friedmann-Equations: we´re (almost) done…

- **Second Friedmann Expansion Equation**

$$\dot{a}^2(t) = -\frac{8}{3}\cdot\pi\cdot G\cdot\rho_0\cdot\left(\frac{-1}{a(t)}\right) - kc^2$$

re-use: $\rho_0 = \rho(t)\cdot a^3(t)$

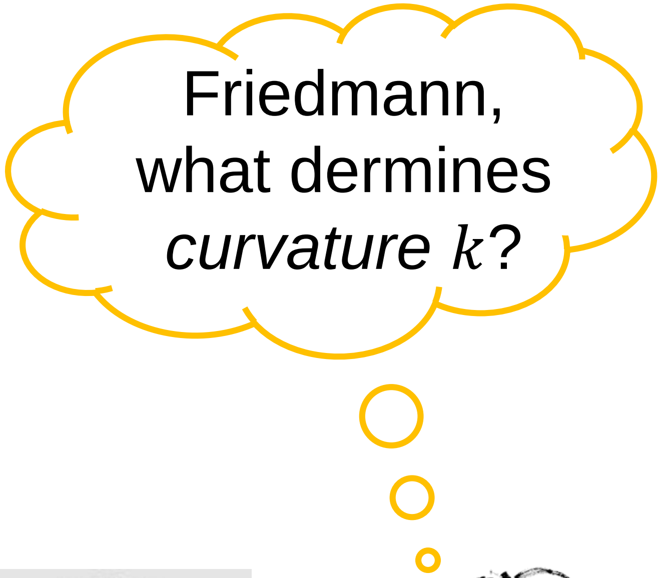
$$\dot{a}^2(t) = \frac{8}{3}\cdot\pi\cdot G\cdot\rho(t)\cdot a^2(t) - kc^2 \qquad \mid\ : a^2(t)$$

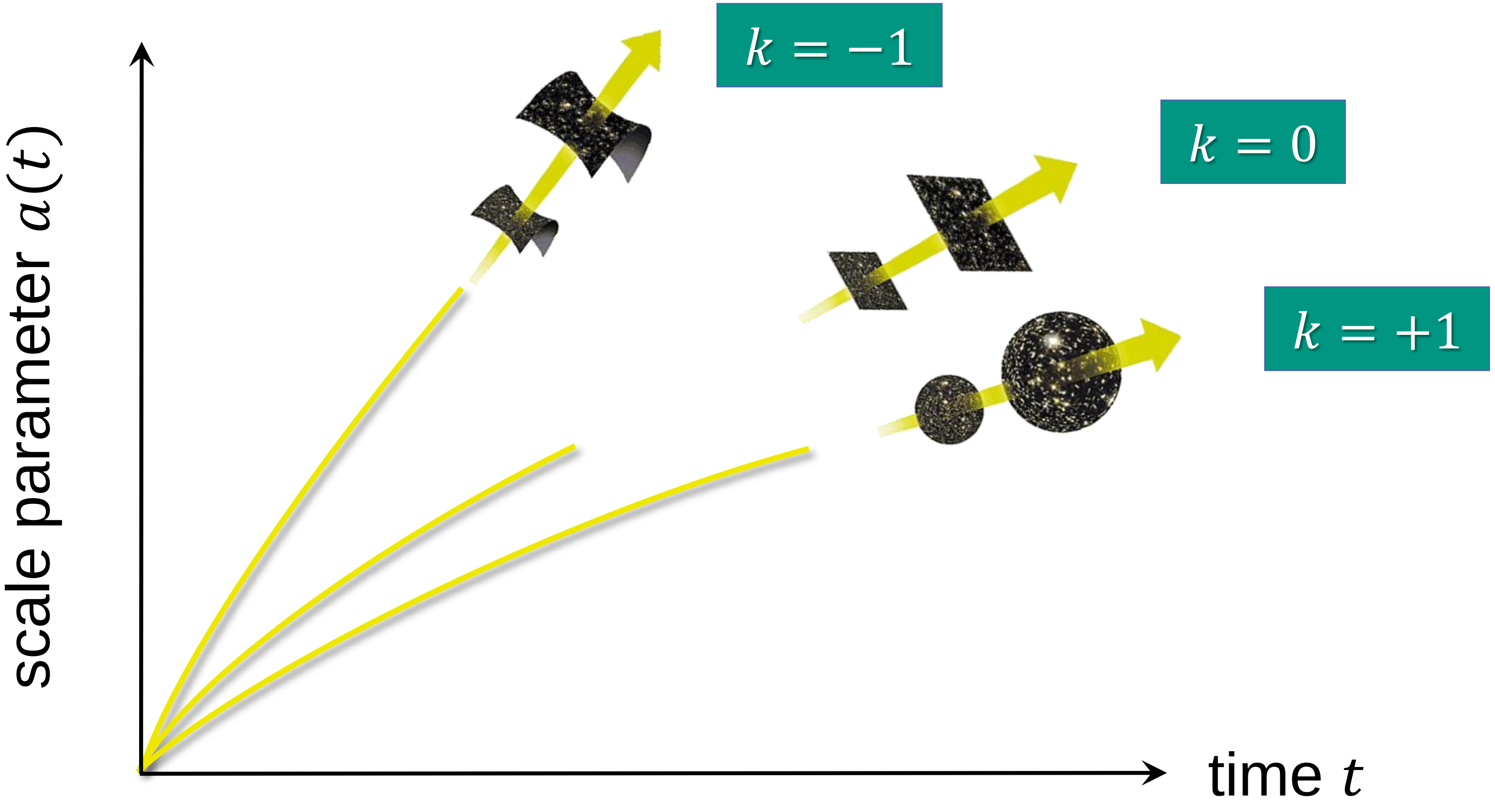
$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8}{3}\pi G\rho(t) - \frac{kc^2}{a^2(t)}$$

- allows to calculate Hubble parameter $H^2(t)$ for different epochs

Topology and overall energy density

- **Curvature parameter k ´from integration´: impact on scale parameter $a(t)$**





Topology and overall energy density: some math

- To see which parameter is determining k let´s use comoving coordinates x

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8}{3}\pi G\rho(t) - \frac{kc^2}{a^2(t)} \qquad \Big| \cdot \frac{a(t)^2}{2}$$

$$\rho(t) = \frac{\rho_0}{a^3(t)}$$

$$a(t) = \frac{r(t)}{x}$$

$$\dot{a}(t) = \frac{\dot{r}(t)}{x}$$

$$\frac{\dot{a}(t)^2}{2} - \frac{4}{3}\cdot\pi\cdot G\cdot\rho(t)\cdot a(t)^2 = -\frac{k\cdot c^2}{2}$$

$$\frac{\dot{r}(t)^2}{2\cdot x^2} - \frac{4}{3}\cdot\pi\cdot G\cdot\rho_0\cdot\frac{x}{r(t)} = -\frac{k\cdot c^2}{2}$$

Topology and overall energy density: some math

- To see which parameter is determining k let´s use comoving coordinates

$$\frac{\dot{r}(t)^2}{2 \cdot x^2} - \frac{4}{3} \cdot \pi \cdot G \cdot \rho_0 \cdot \frac{x}{r(t)} = -\frac{k \cdot c^2}{2} \qquad \Big| \cdot x^2$$

$$\frac{\dot{r}(t)^2}{2} - G \cdot \frac{M(x)}{r(t)} = -\frac{k \cdot c^2}{2} \cdot x^2$$

$$M(x) = \frac{4}{3} \cdot \pi \cdot \rho_0 \cdot x^3$$

for unit sphere

$x \equiv 1$

$M = M(x)$

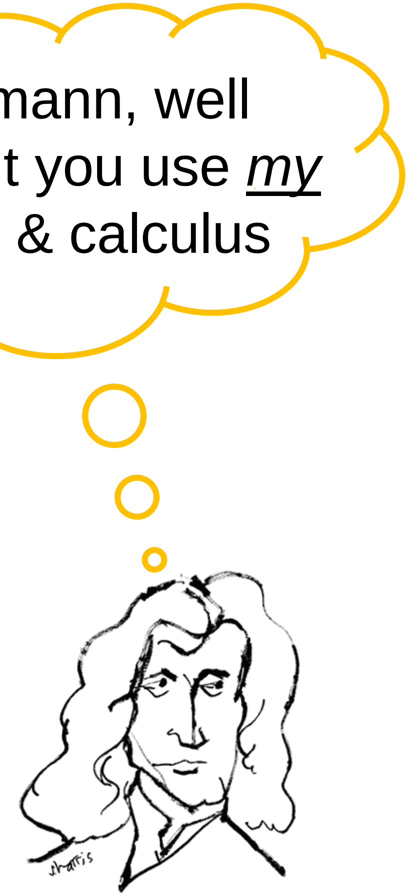
$$\underbrace{\frac{\dot{r}(t)^2}{2}}_{E_{kin}} \underbrace{- G \cdot \frac{M}{r(t)}}_{E_{pot}} = \underbrace{-\frac{k \cdot c^2}{2}}_{E_{tot}}$$

$$E_{kin} + E_{pot} = E_{tot}$$

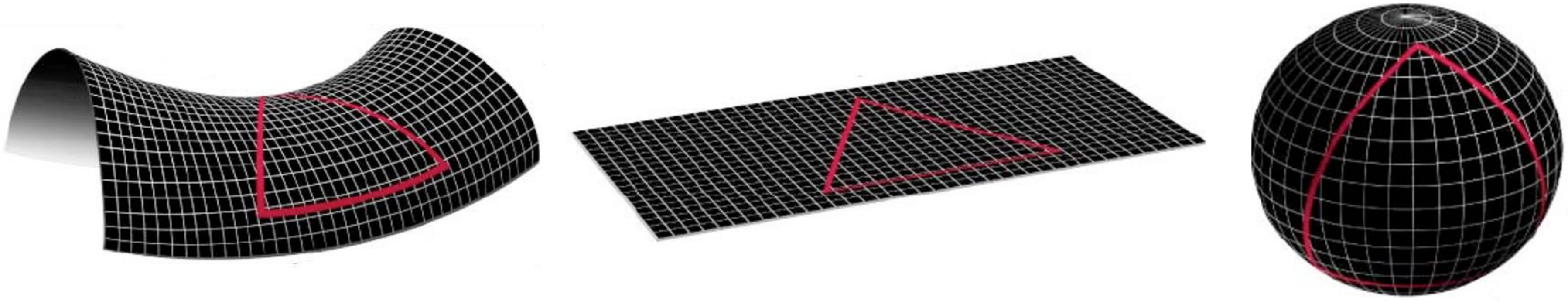
Topology and overall energy density: curvature k

- **Curvature k of the universe is determined by ist total energy E_{tot}**

constant k	curvature	topology	total energy
$k = -1$	hyperbolic	open	$E_{tot} > 0$
$k = 0$	euclidean	flat	$E_{tot} = 0$
$k = +1$	spherical	closed	$E_{tot} < 0$



$k = -1$ $\qquad$ $k = 0$ $\qquad$ $k = +1$

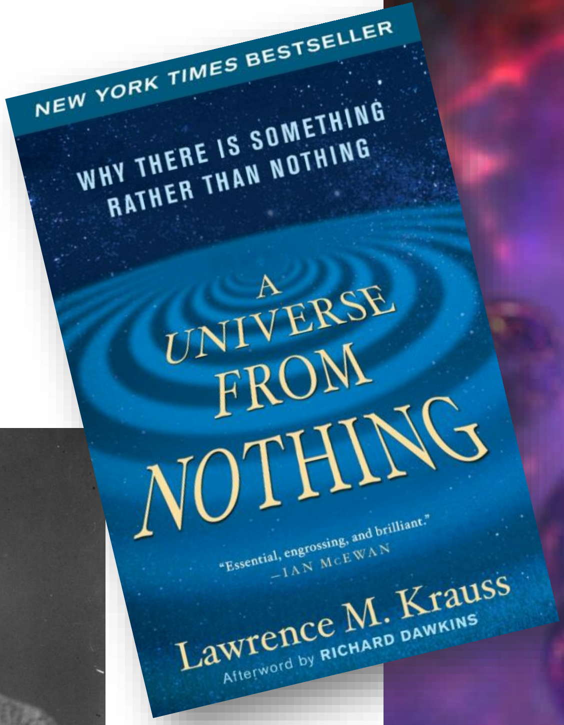


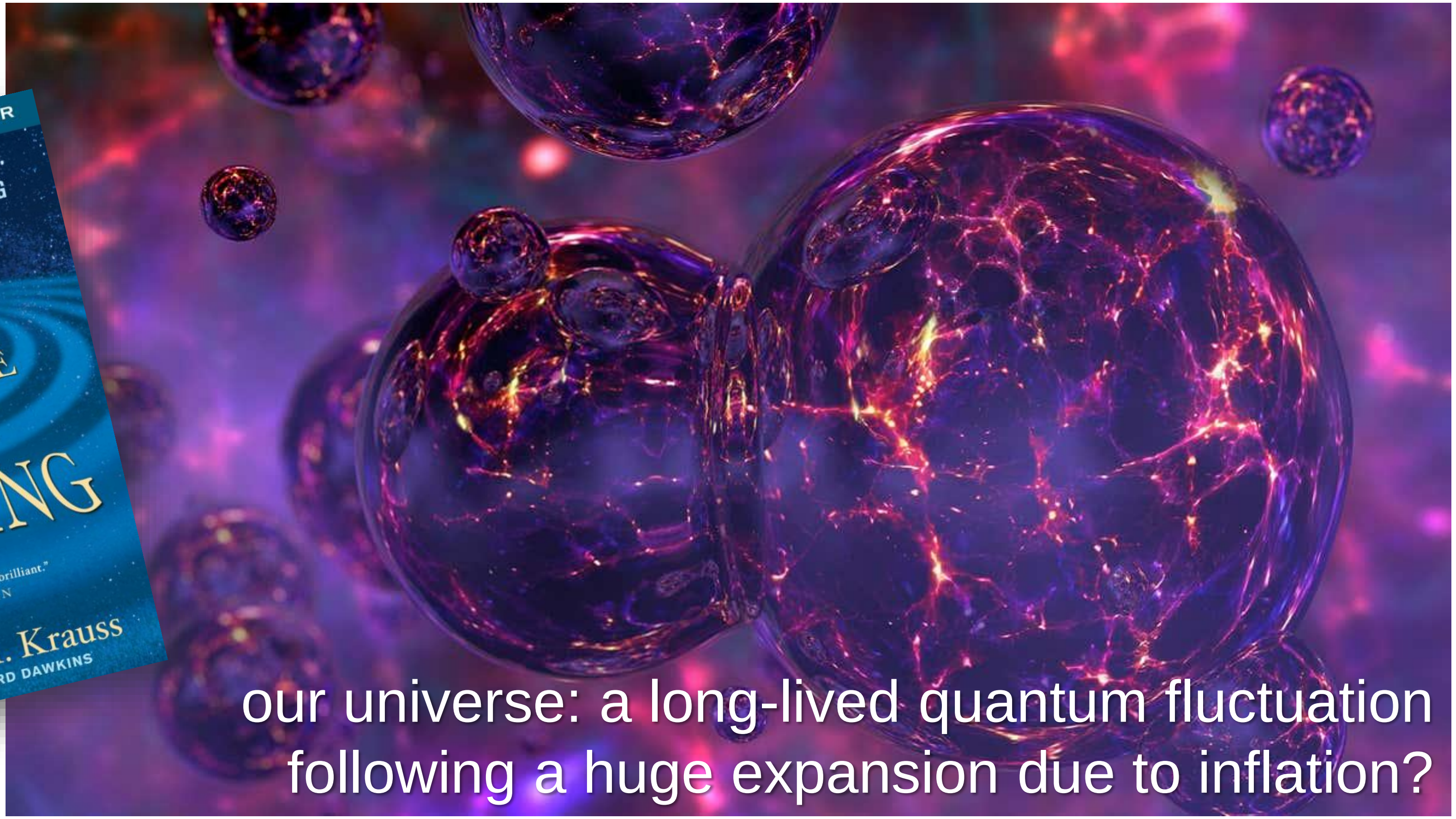
$E_{tot} > 0$ $\qquad$ $E_{tot} = 0$ $\qquad$ $E_{tot} < 0$

Topology and overall energy density

- Heisenberg uncertainty relation in view of the total energy of the universe

$$\underbrace{\Delta E}_{\approx 0}\,\underbrace{\Delta t}_{\approx \infty} \leq \frac{h}{2\pi}$$





our universe: a long-lived quantum fluctuation following a huge expansion due to inflation?

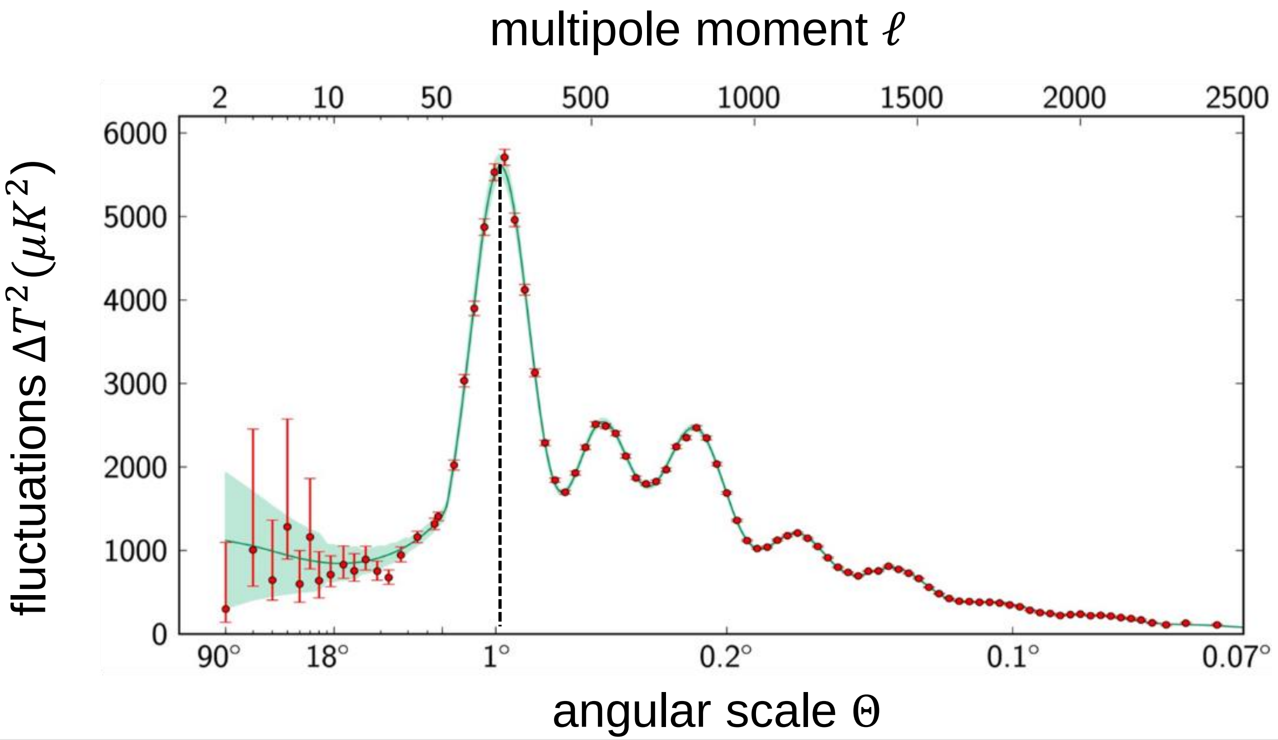
Topology and overall energy density

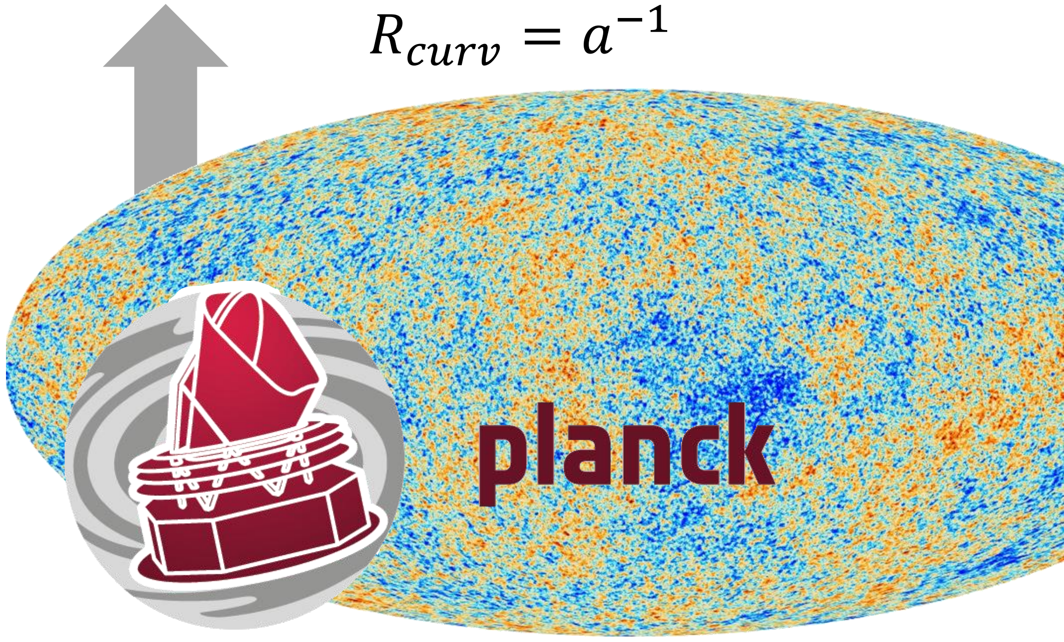
- **2015 findings of the Planck satellite mission: a universe *without* curvature**

  - analysis of the CMB multipole distribution*
    curvature $k = 0$ (Ω_k) from 1. peak at $\ell \sim 200$

$$\Omega_k = \frac{-kc^2}{H_0^2 R_{curv}^2} = 0.000 \pm 0.005$$

$$R_{curv} = a^{-1}$$





*see lectures 8 & 9

overall energy density & inflationary cosmology

- **2015 findings of the Planck satellite & expectation from the theory of inflation**

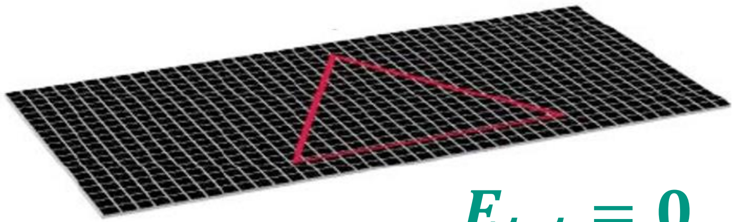
  - **inflation**: exponential increase of the size $a(t)$ of universe from time $t = 10^{-36}s \ldots 10^{-32}s$ due to a scalar field, typ. expansion factor $\gg 10^{26}$

    ⇨ flat space, no curvature

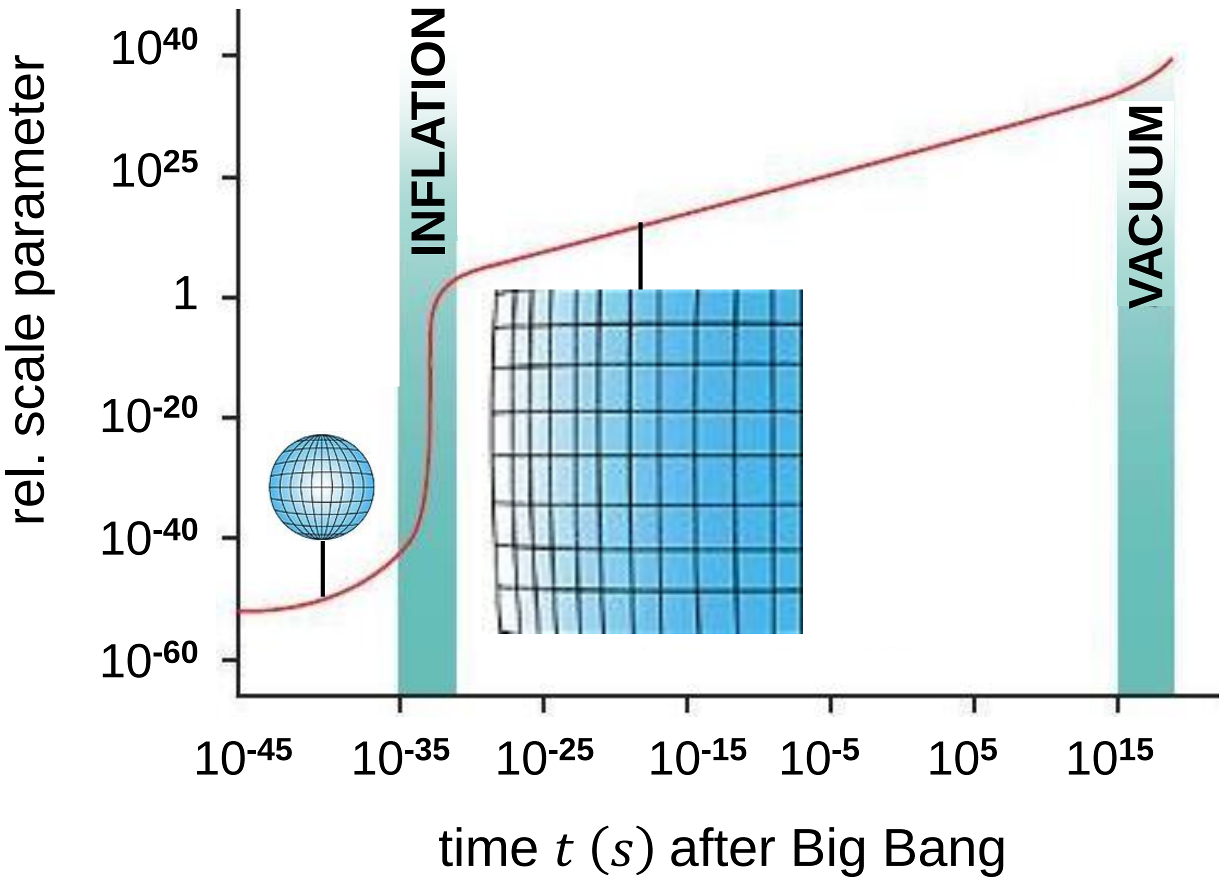
  - **observation**: space is flat to $\sim 0.5\%$ (Planck, 2015)

Euclid

$$\boldsymbol{k = 0}$$



$$\boldsymbol{E_{tot} = 0}$$



Topology and overall energy density

- **2018 findings of the Planck satellite mission: a universe *with* curvature??**

  - analysis of CMB radiation using lensing effect*

$$\Omega_k = -0.007 \ldots - 0.095 \quad (99\% CL)$$

Article | Published: 04 November 2019

Planck evidence for a closed Universe and a possible crisis for cosmology

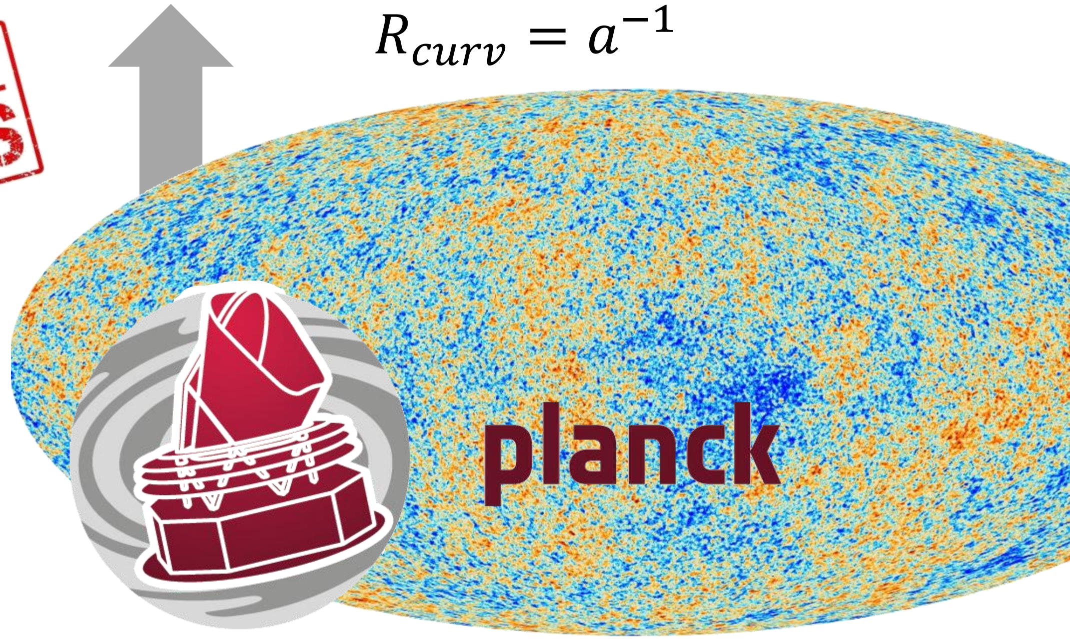
Eleonora Di Valentino, Alessandro Melchiorri & Joseph Silk

Nature Astronomy (2019) | Cite this article

Abstract

The recent Planck Legacy 2018 release has confirmed the presence of an enhanced lensing amplitude in cosmic microwave background power spectra compared with that predicted in the standard Λ cold dark matter model, where Λ is the cosmological constant. A closed Universe can provide a physical explanation for this effect, with the Planck cosmic microwave background spectra now preferring a positive curvature at more than the 99% confidence level. Here, we further investigate the evidence for a closed Universe from Planck, showing that positive curvature naturally explains the anomalous lensing

$$R_{curv} = a^{-1}$$



*see lectures 13 & 14

inflationary cosmology: acclerated masses

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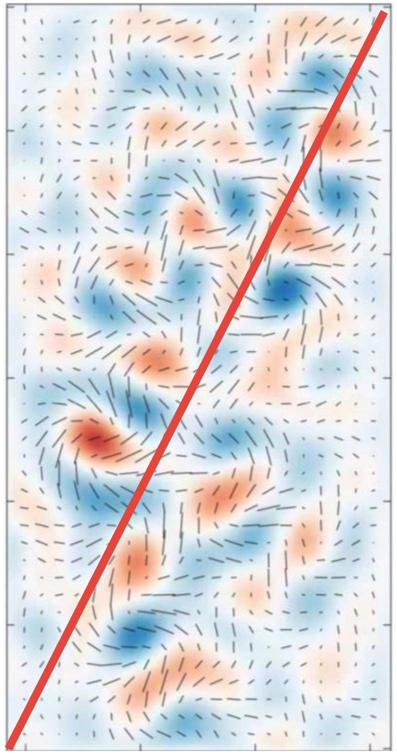
- **2021 update from BICEP3 & expectation from the theory of inflation**

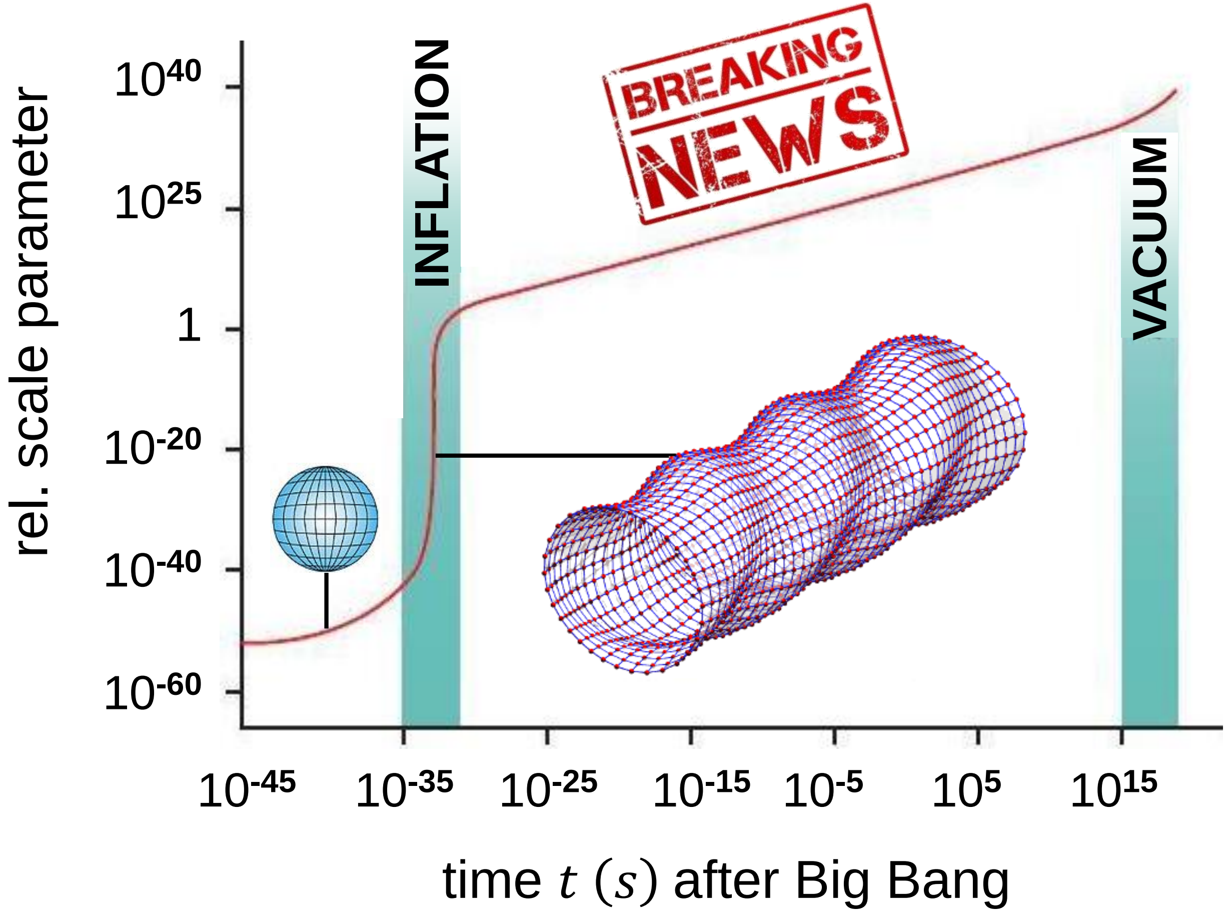
- inflation should have produced a specific GW* signal: but no detection!

BICEP3 tightens the bounds on cosmic inflation

10/26/21 | By Nathan Collins

A new analysis of the South Pole-based telescope's observations has all but ruled out several popular models of inflation.





*Gravitational Waves, see ATP-2 (summer 2023)

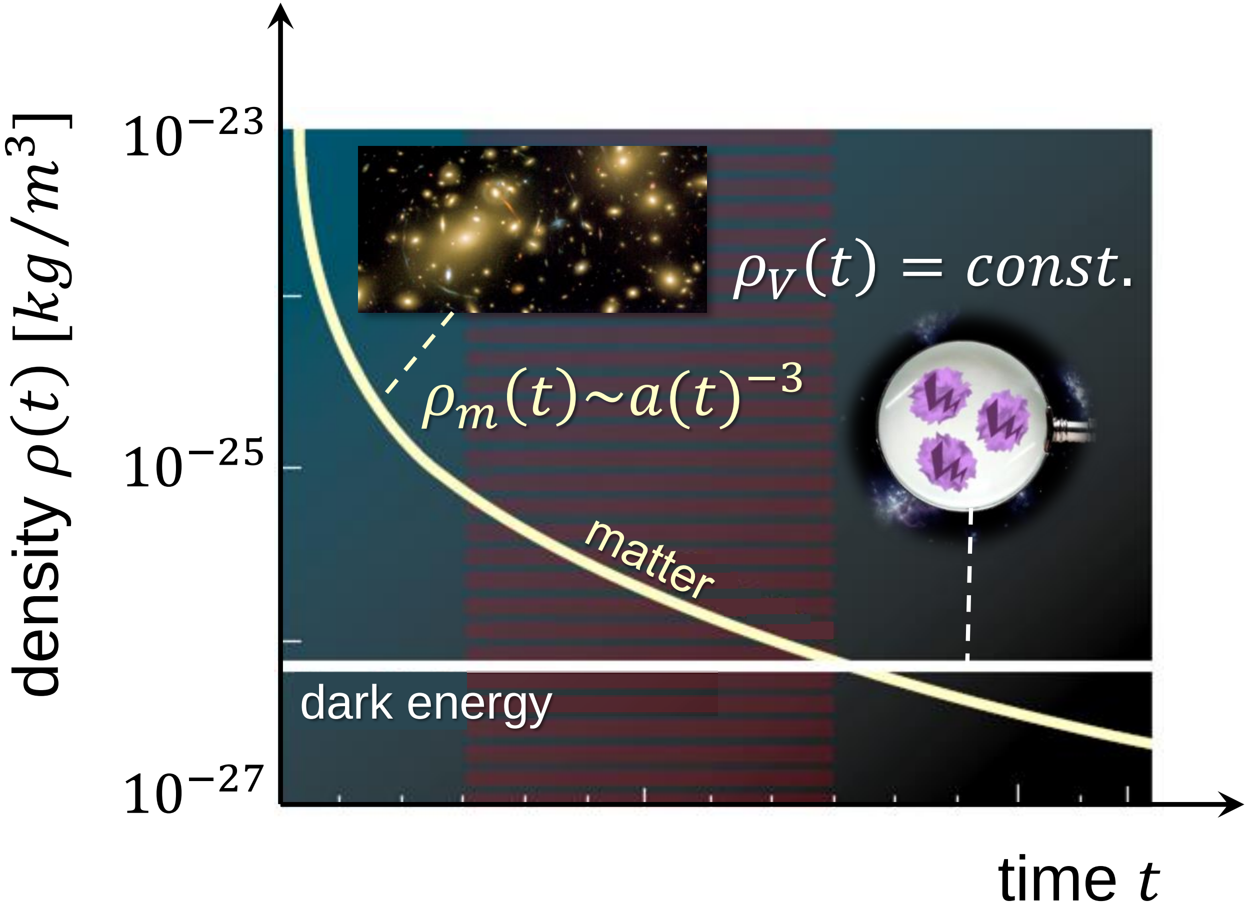
Friedmann equations for a flat universe

- **Second expansion equation: development of $\rho(t)$ over cosm. time scales t**

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8}{3}\pi G \rho_{m,r,V}(t)$$

$$= \frac{8}{3}\pi G \rho_{m,r}(t) + \frac{\Lambda c^2}{3}$$

$$H(t) \sim \sqrt{\rho_{m,r}(t)} + const.$$



Friedmann equations for a flat universe

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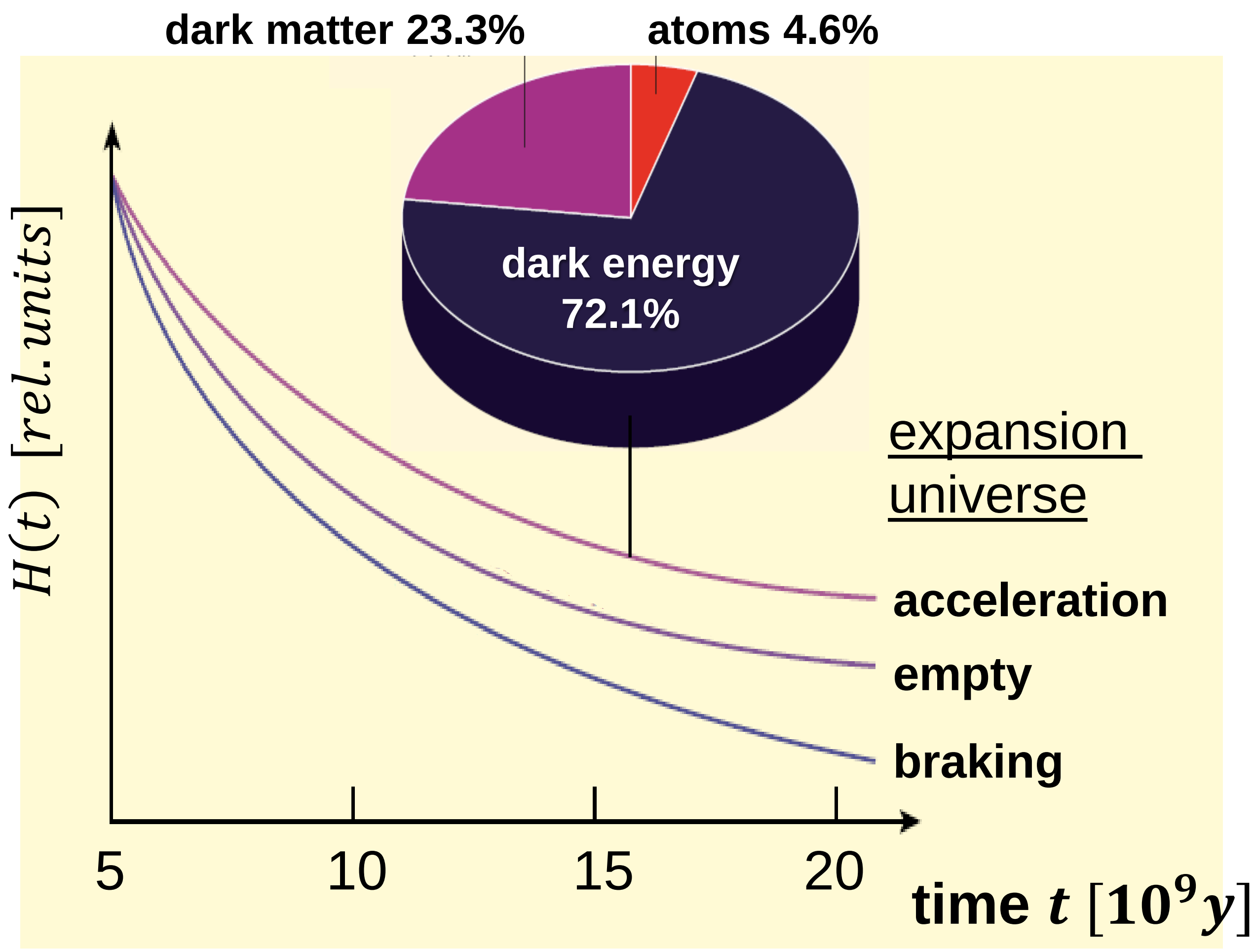
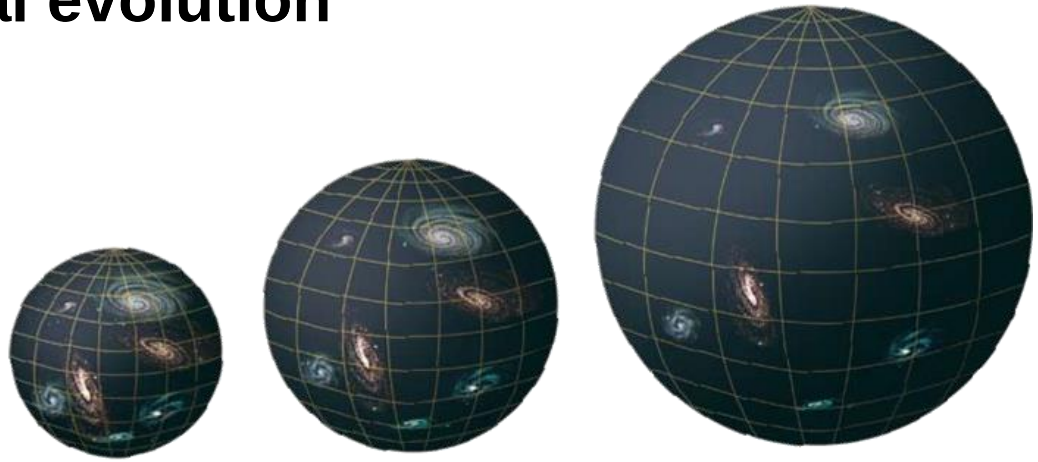


RECAP: Friedmann-Lemaître Equations

- The two equations governing cosmological evolution

expansion equation with curvature *k*

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8}{3}\pi G \rho(t) - \frac{kc^2}{a(t)^2}$$



acceleration equation

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi G\left(\rho(t) + \frac{3P}{c^2}\right)$$

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_v(t)$$

Aleksandr Friedmann

Georges Lemaître

RECAP: Friedmann-Lemaître Equations

- **The two equations governing cosmological evolution using Λ**

expansion equation for $k = 0$ with Λ

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8}{3}\pi G\rho(t) + \frac{\Lambda c^2}{3}$$

acceleration equation for $k = 0$ with Λ

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi G\left(\rho(t) + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3}$$

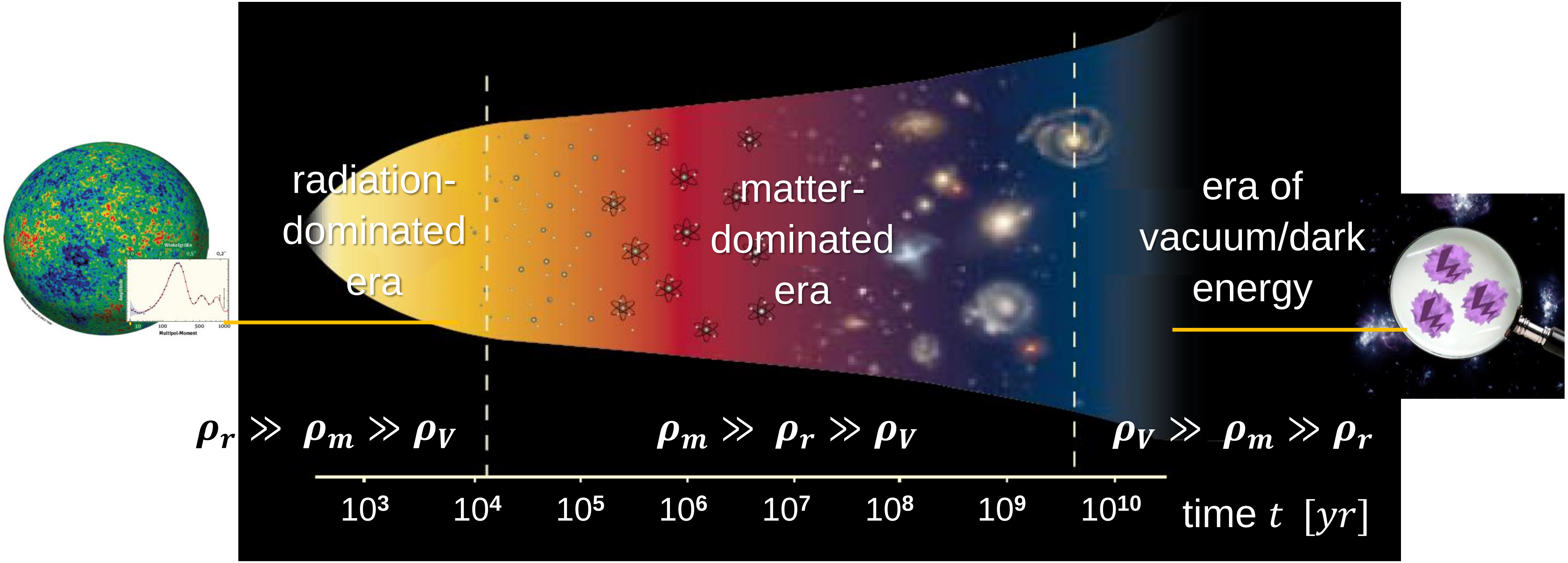
$$\rho(t) = \rho_m(t) + \rho_r(t)$$

Aleksandr Friedmann

Georges Lemaître

Different cosmological epochs & $a(t)$

- Radiation / matter / vacuum energy – dominated cosmological ages



Different cosmological epochs & $a(t)$

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- **Radiation / matter / vacuum energy – dominated cosmological ages**

  - evolution of scale parameter $a(t)$ calculated with Friedmann equations

dominant part	equation-of-state	density	scale parameter
radiation	$P_r = +1/3 \cdot \rho_r c^2$	$\rho_r \sim a^{-4}$	$a(t) \sim t^{1/2}$

Λ

constant density

$\boldsymbol{\rho_V = 3.6\ GeV/m^3}$

$\alpha = \sqrt{\Lambda/3}$

exponential increase

Matter-dominated model of cosmology

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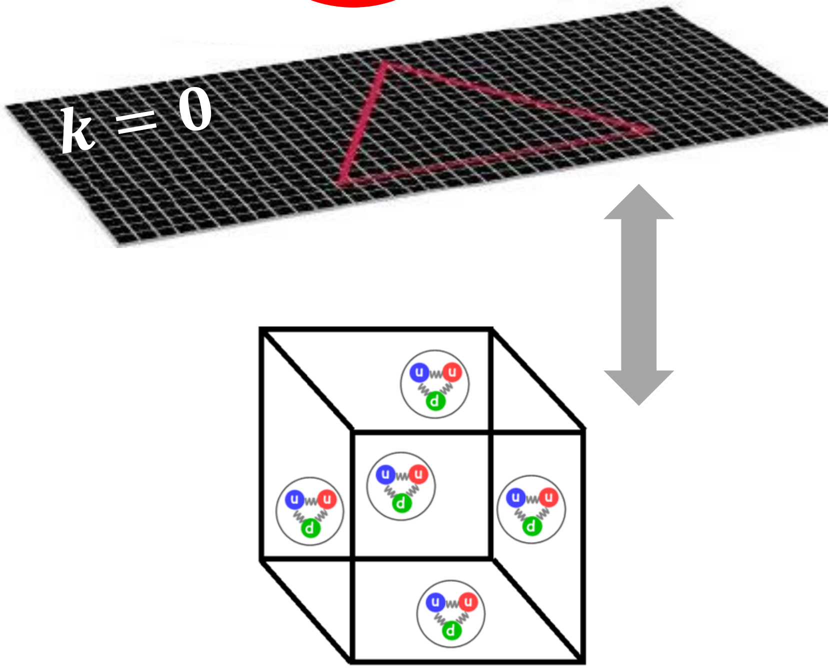
- **Hypothetical assumption: present, flat universe that contains only baryons**

  - flat universe ($k = 0$), no vacuum energy ($\Lambda = 0$)

  - critical energy density ρ_c for flat universe, baryons only:

$$\rho_c = \frac{3}{8\pi G} H_0^2 = 9.2 \times 10^{-27} \frac{kg}{m^3}$$

$= 5.1\ GeV/m^3$ (i.e. ~ 5 protons per m^3)



- **Our present universe has a baryon density ρ_b**

$= 0.2\ GeV/m^3$

(i.e. $\rho_b < 5\%$ of ρ_c)

Building a Standard Model of cosmology

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- **Dimensionless density parameters Ω_i**

  - Ω_i is a dimensionless parameter, given by ratio of actual density ρ_i relative to critical critical density ρ_c for a flat universe

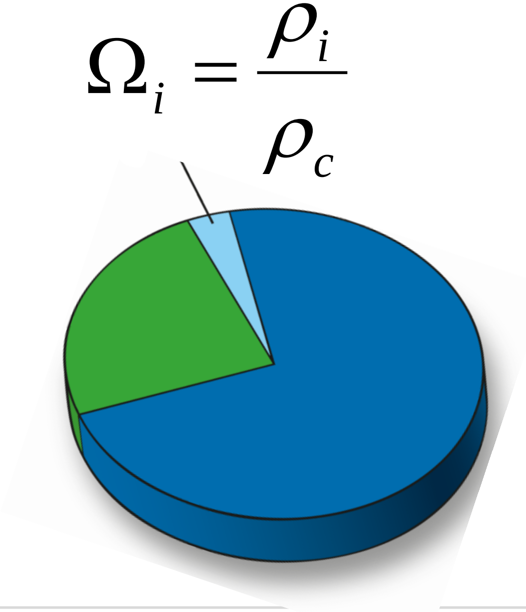
$$\Omega_i = \frac{\rho_i}{\rho_c} = \frac{8\pi \cdot G}{3\,H_0^2} \cdot \rho_i$$

  - $\Omega_{tot} = \sum \rho_i = 1$ for a flat universe with $\boldsymbol{E_{tot} = 0}$

- **Summing up all density parameters Ω_i**

  - contributions: matter – radiation – vacuum – curvature $k \neq 0$

$$\boldsymbol{\Omega_{tot} = \Omega_m + \Omega_r + \Omega_V + \Omega_k}$$



Hubble time t_H: definition & relation to H_0

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- **Hubble time t_H is based on a scenario with uniform expansion rate H_0**

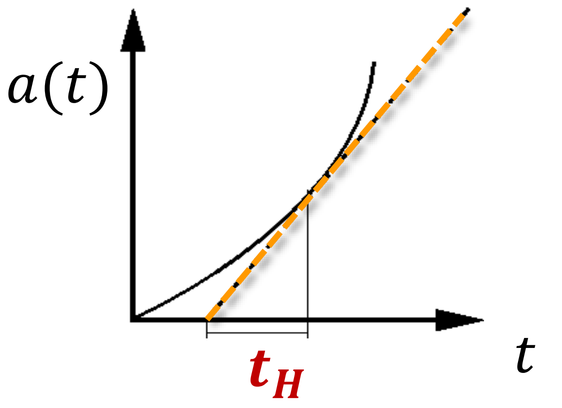
$$H(t) = const. = H_0$$

$$t_H = \frac{1}{H_0}$$

$$H_0 = \frac{72\,km/s}{Mpc} = 2.3\times10^{-18}\,s^{-1}$$

$$t_H = \frac{1}{2.3\times10^{-18}}\,s$$

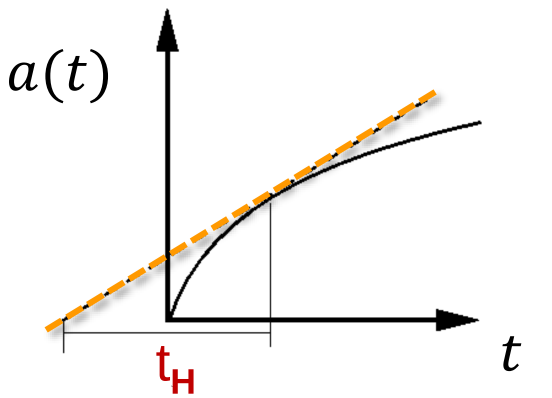
$$t_H = 13.8\cdot10^9 y$$



accelerated expansion

$\rho_{tot} < \rho_c$

$t_H < t_{real}$



braked expansion

$\rho_{tot} > \rho_c$

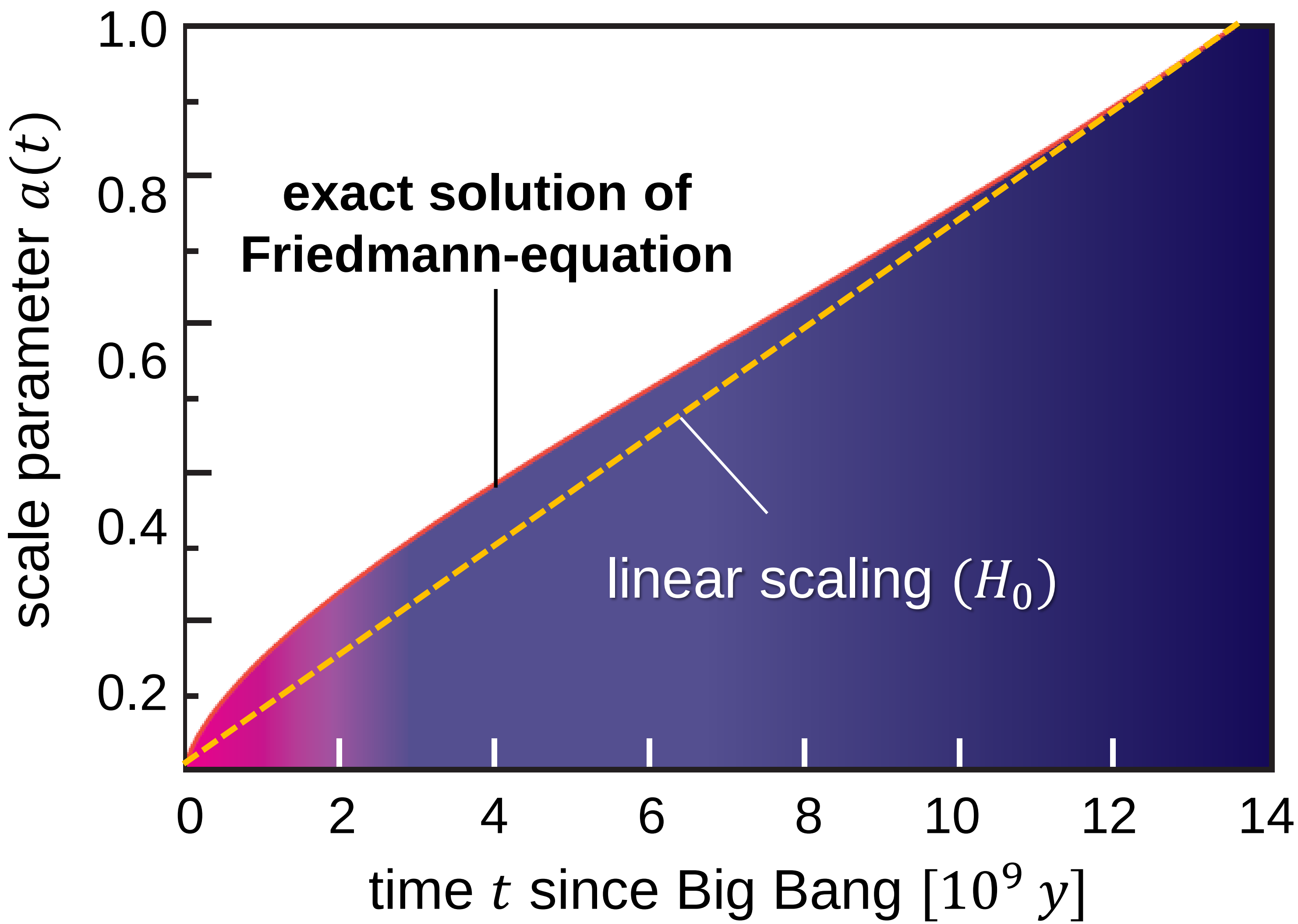
$t_H > t_{real}$

Hubble time t_H and actual expansion rate

- **Linear and acutal expansion rate of our universe**

- surprise:
  rather good approximation of $a(t)$ by a linear increase using *present* value of H_0

- exact Friedmann solution:
  at first braked expansion $(\ddot{a}(t) < 0)$, now accelerated expansion with $\ddot{a}(t) > 0$



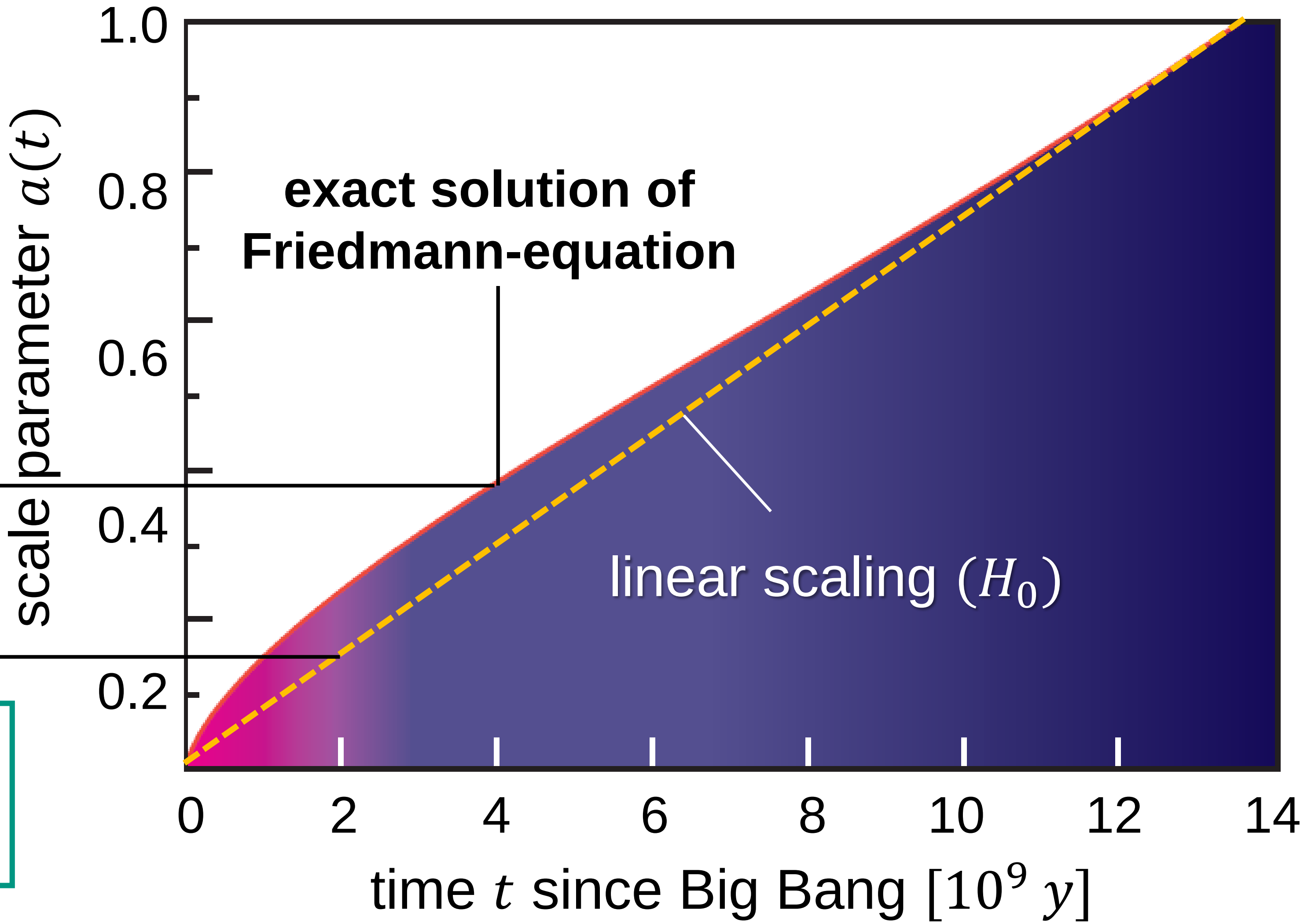
Hubble time t_H and actual expansion rate

- **Linear and acutal expansion rate of our universe**

  - 4 contributions to $\Omega_{tot} = 1$

$$\frac{H^2}{H_0^2} = \Omega_r \cdot a^{-4} + \Omega_m \cdot a^{-3} + \Omega_V + \Omega_k \cdot a^{-2}$$

$$t_H = \frac{1}{2.3\times10^{-18}}\,s = 13.8\cdot10^9\ a$$



Hubble expansion $H(t)$: CosmoCalc – a useful app

Karlsruhe Institute of Technology

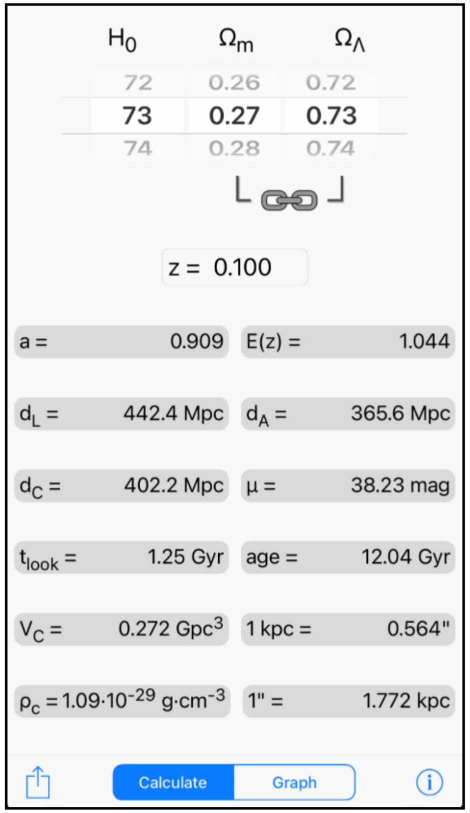
- **Cosmological parameters & their implications: an app for your smartphone**

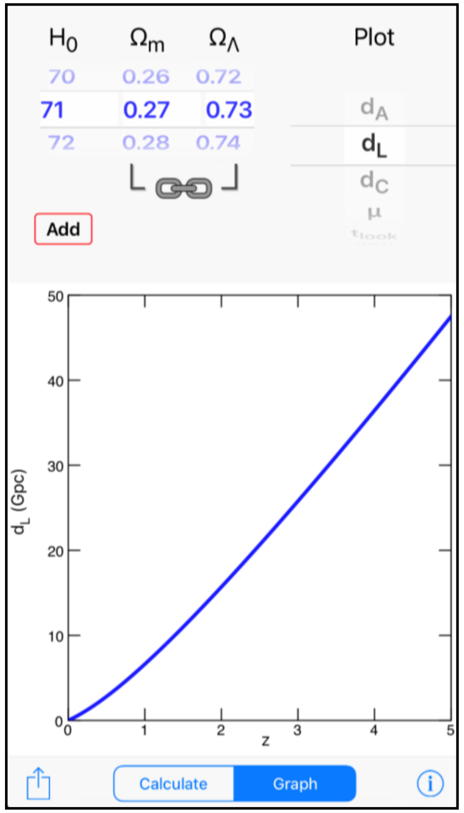
  - select your model universe & see its properties

CosmoCalc App für iOS

$$H(t)^2 = H_0^2 \cdot \left[\Omega_m(t) + \Omega_r(t) + \Omega_V(t) + \Omega_k(t)\right]$$

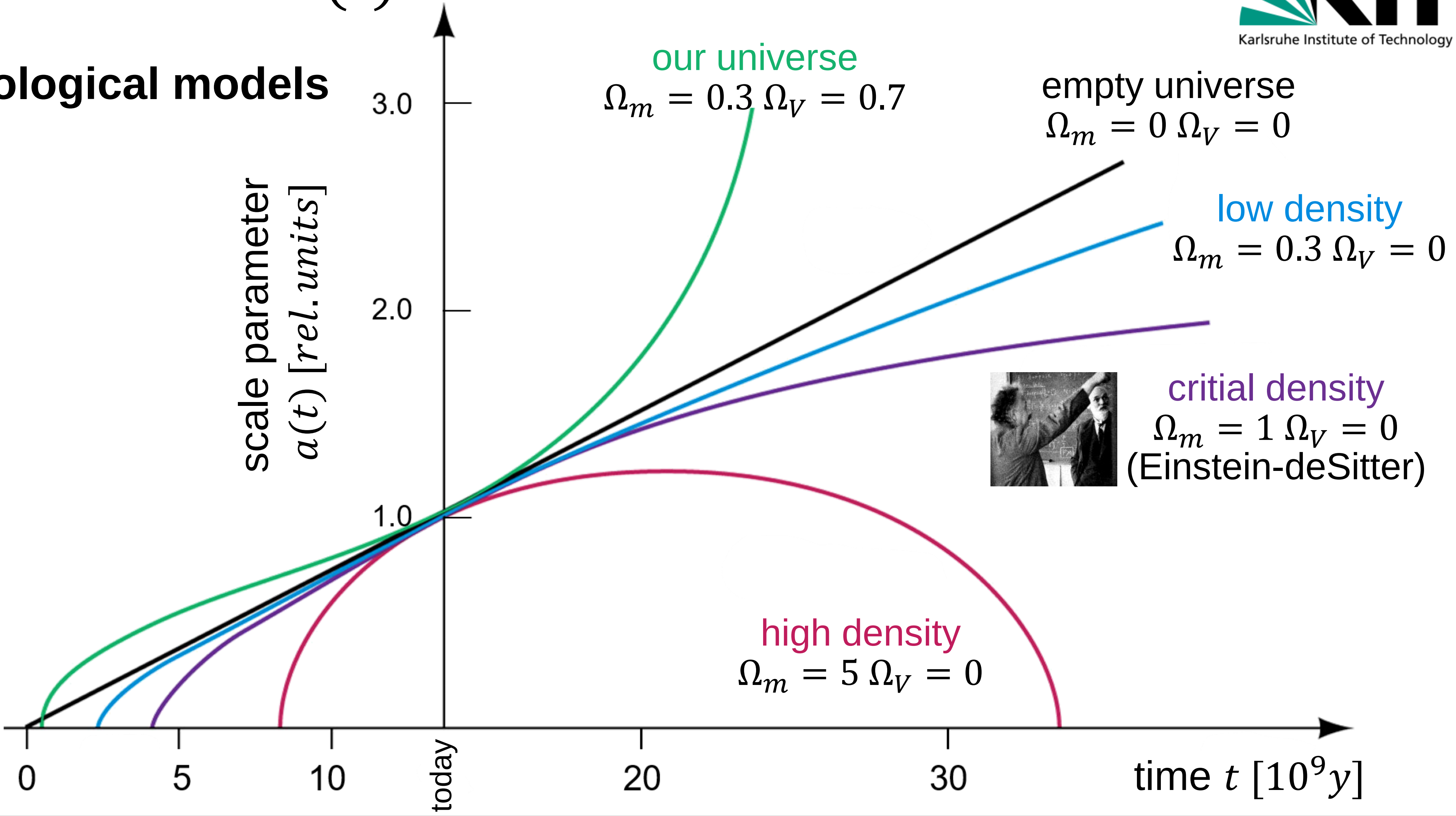
$$H(t)^2 = H_0^2 \cdot \begin{bmatrix} \Omega_m(0)\cdot(1+z)^3 \\ +\Omega_r(0)\cdot(1+z)^4 \\ +\Omega_V(0) \\ +\Omega_k(0)\cdot(1+z)^2 \end{bmatrix} \qquad \begin{matrix} \sim 1/a^3 \\ \sim 1/a^4 \\ const. \\ \sim 1/a^2 \end{matrix}$$





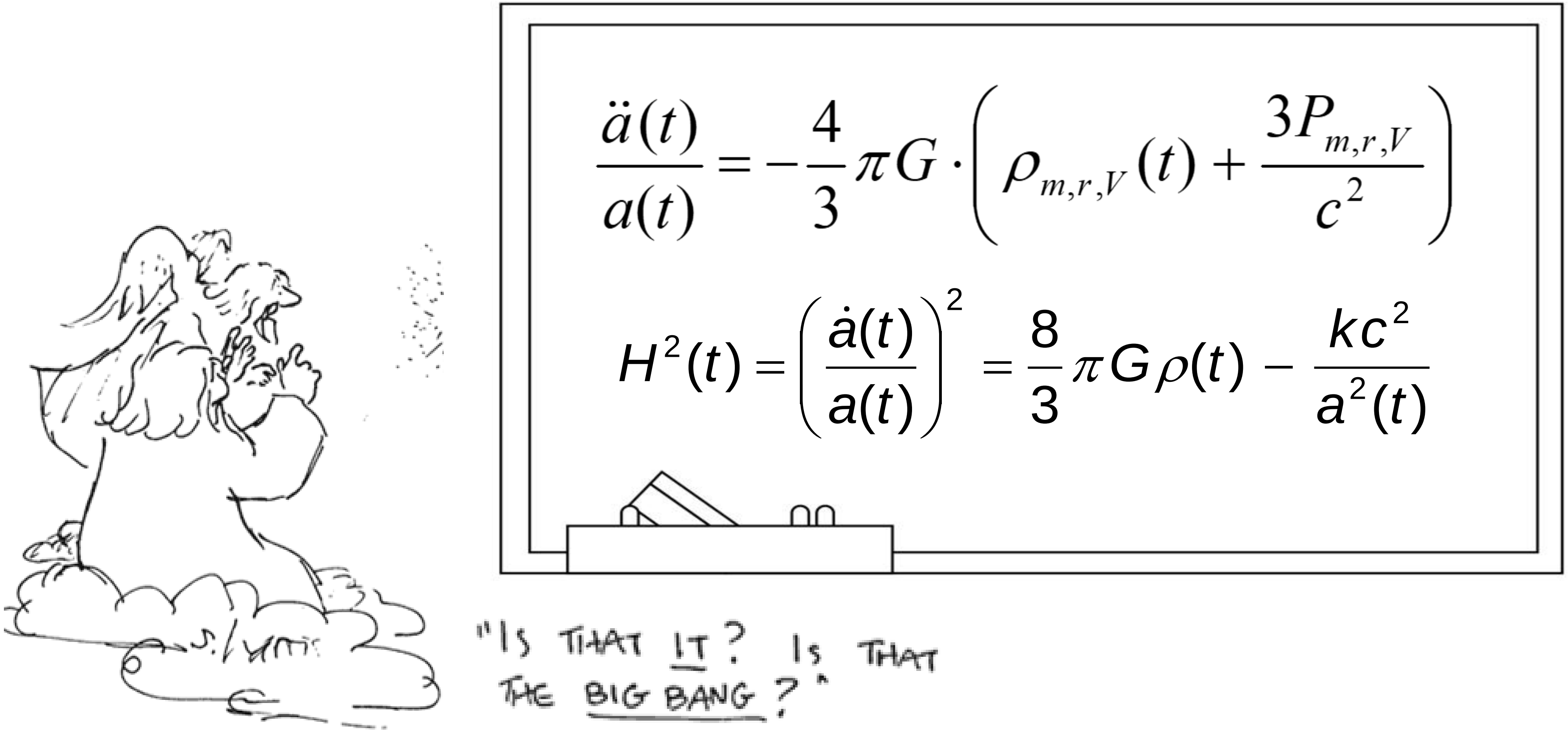
Scale parameter $a(t)$ for different models

- **Cosmological models**



The end (of todays´ lecture)

- **Friedmann equations revisited**



$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi G \cdot \left(\rho_{m,r,V}(t) + \frac{3P_{m,r,V}}{c^2} \right)$$

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8}{3}\pi G \rho(t) - \frac{kc^2}{a^2(t)}$$