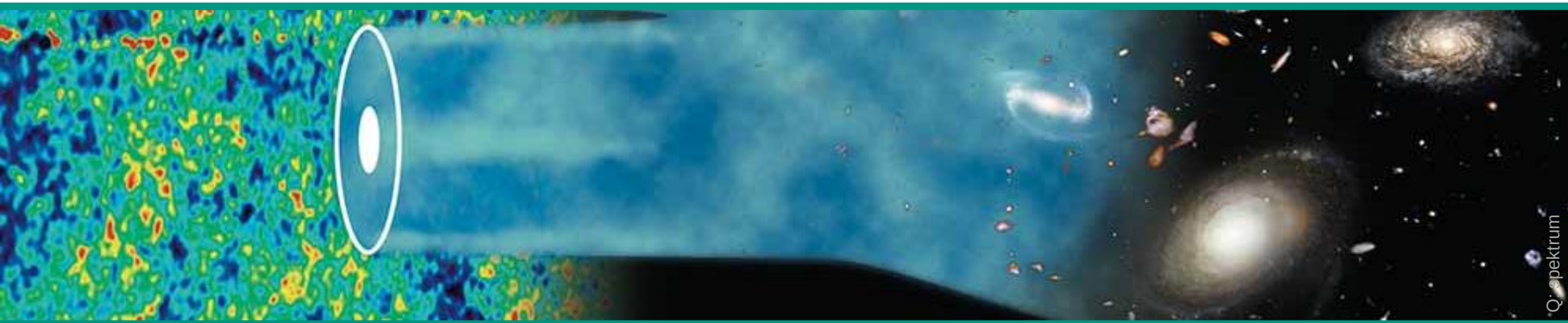


Introduction to Cosmology

Winter term 22/23

Lecture 5

Dec. 1, 2022



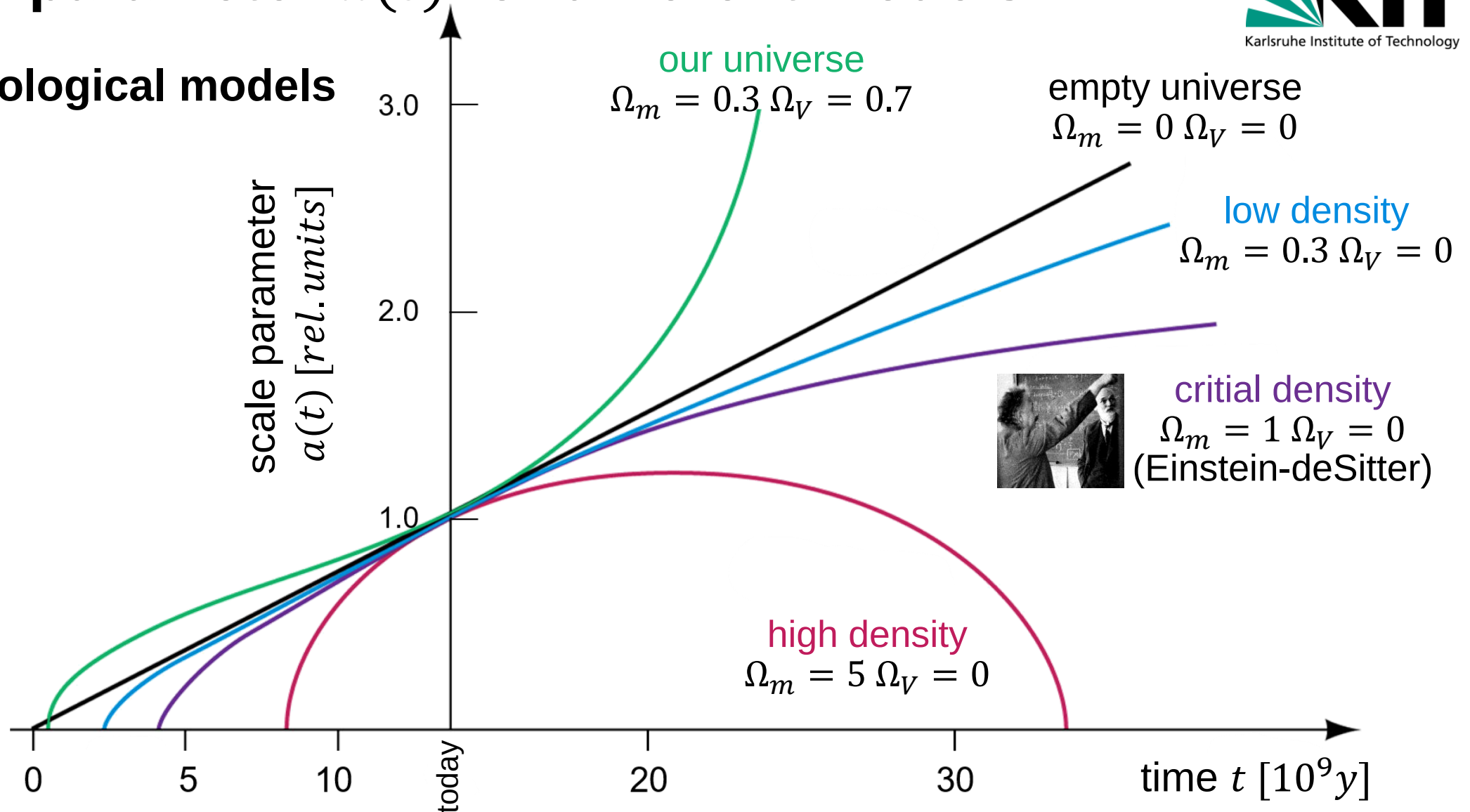
Recap of Lecture 4

■ Friedmann equations for expanding universe

- cosmological constant $\Lambda \neq 0$ ($3.6 \text{ GeV}/m^3$ vs. $10^{121} \text{ GeV}/m^3$)
- integration of acceleration eq.: **Hubble param.** $H^2(t) = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8}{3} \pi G \rho(t) - \frac{kc^2}{a^2(t)}$
- universe appears to be **flat**: **curvature** $k = 0$ (brief inflationary epoch?)
- three cosmological epochs: **radiation** / **matter** / **vacuum**
- total energy density $\Omega_{tot} = \Omega_m + \Omega_r + \Omega_V + \Omega_k$ ($= 1$, if critical density)
- definition of **Hubble time** $t_H = (H_0)^{-1} = 13.8 \cdot 10^9 \text{ yr}$ (uniform expansion)

Recap: parameter $a(t)$ for different models

■ Cosmological models



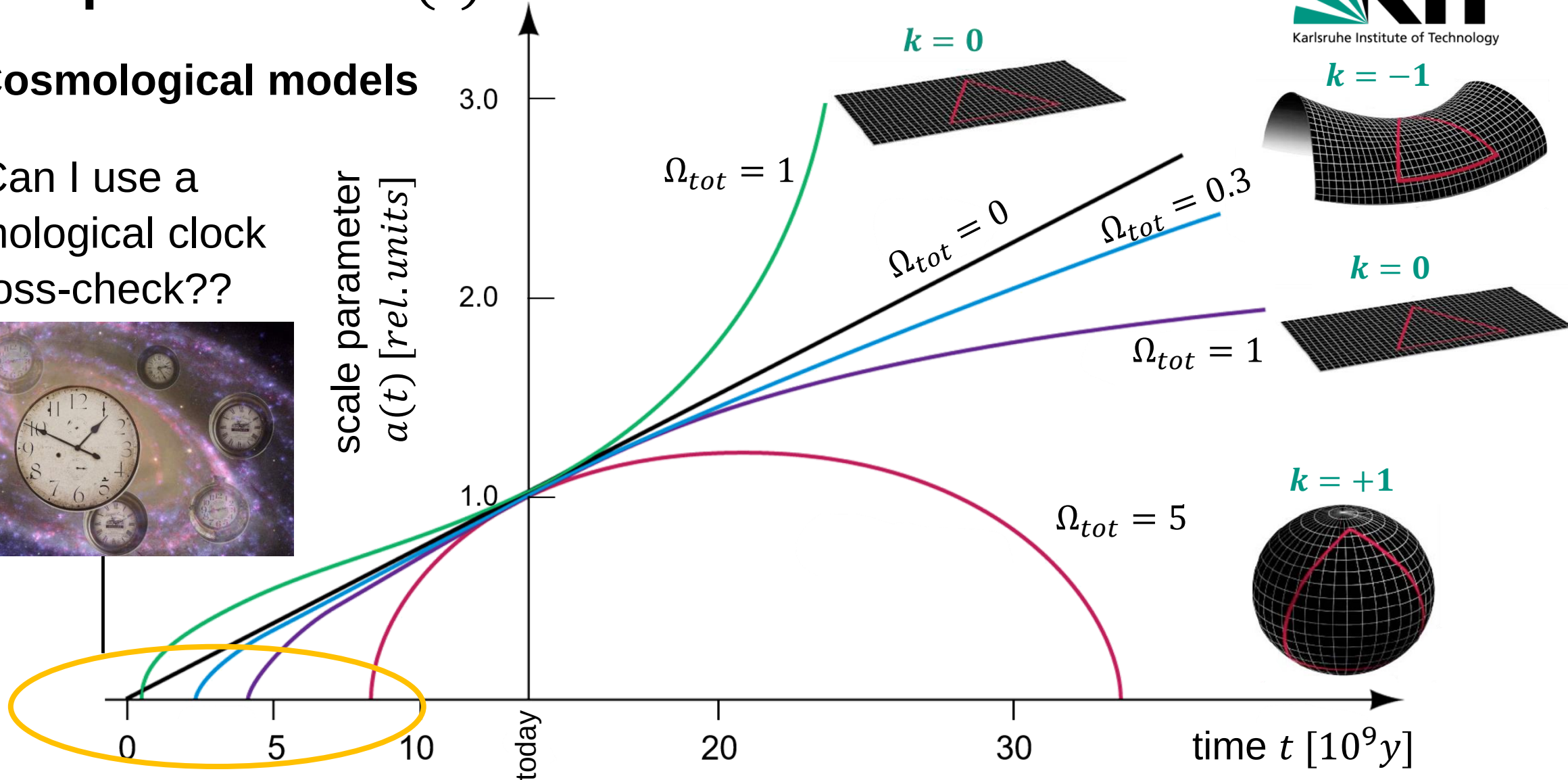
Scale parameter $a(t)$ for different models

■ Cosmological models

Can I use a cosmological clock to cross-check??



scale parameter
 $a(t)$ [rel. units]

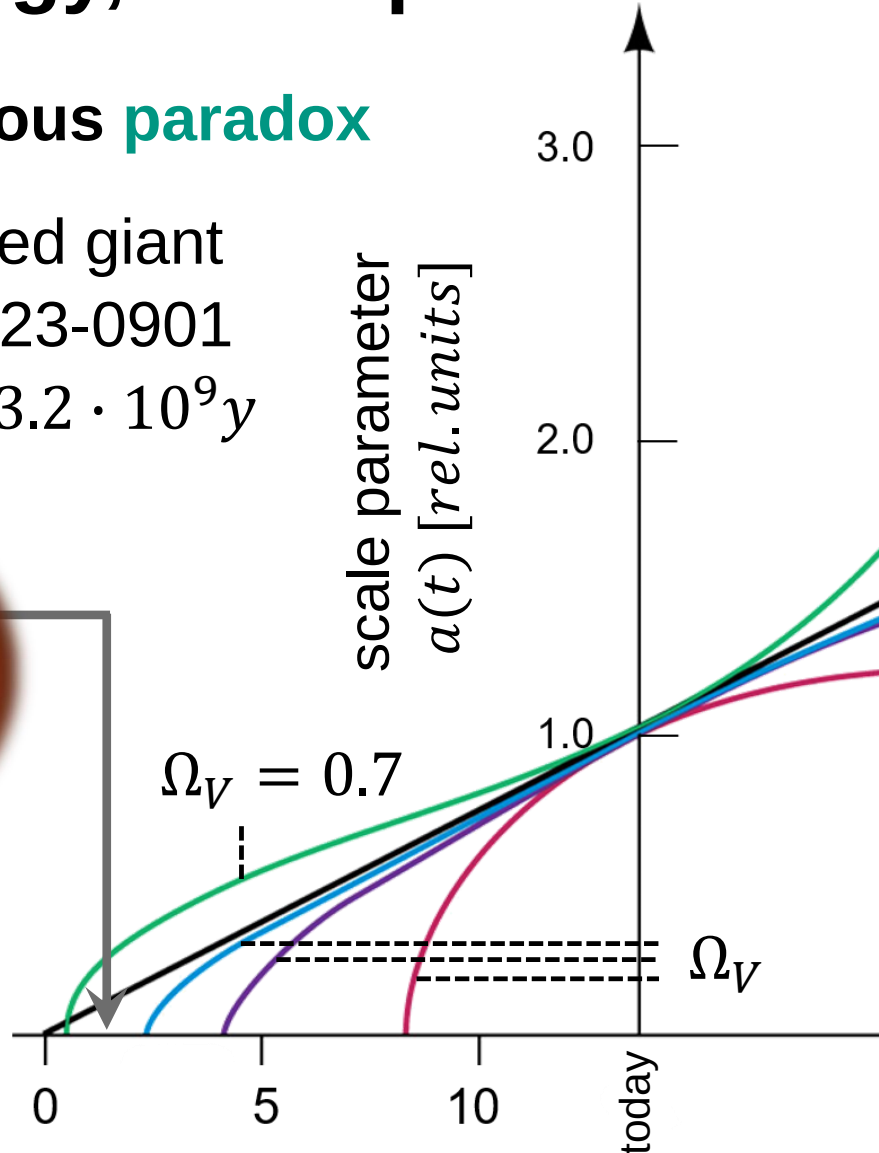
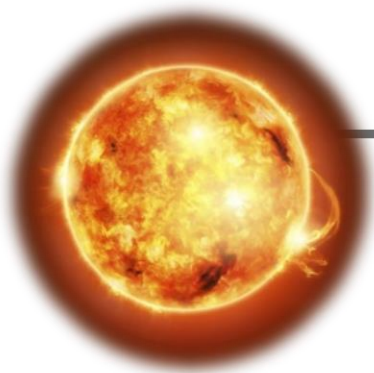


Topology, scale parameter $a(t)$ & the oldest stars

■ A famous paradox

example: red giant
star HE 1523-0901

age: $t = 13.2 \cdot 10^9 y$

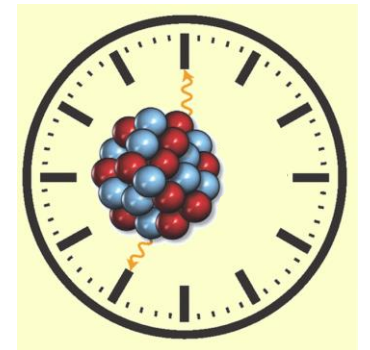


- a universe with $\Omega_m = 1$ or with $\Omega_m = 0.3$ only would be younger than the oldest stars (age by 'nuclear clocks'), apparently a paradox



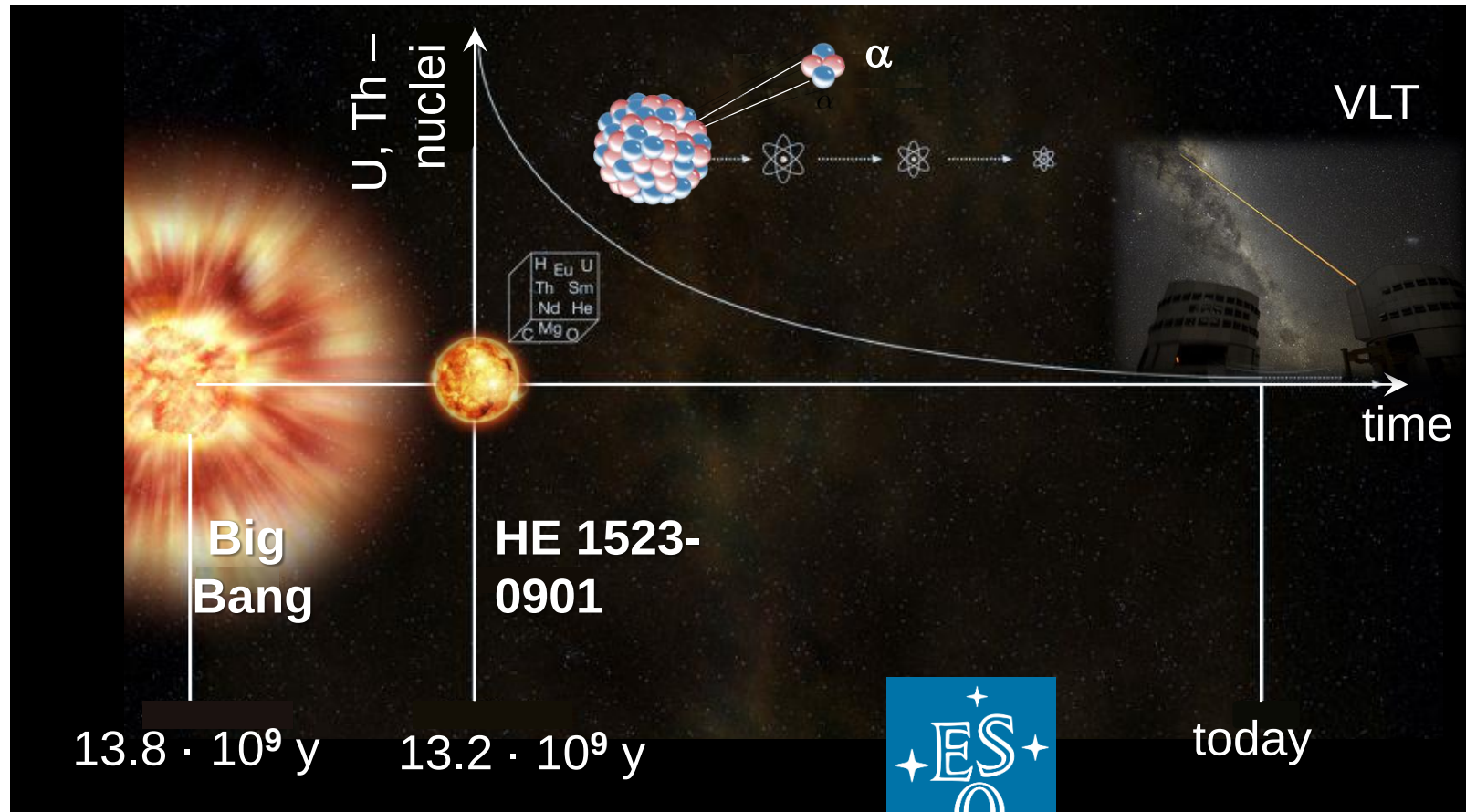
further hint for $\Lambda \neq 0$

- very long-lived nuclear decays can be used as **cosmo-chronometers**

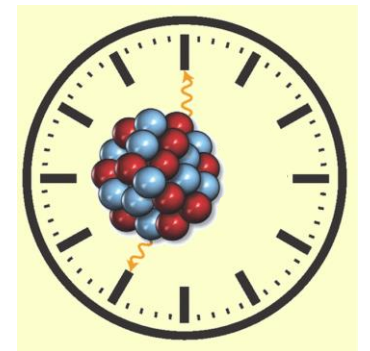


Topology, scale parameter $a(t)$ & the oldest stars

■ A famous paradox due to nuclear clocks in star HE 1523-0901

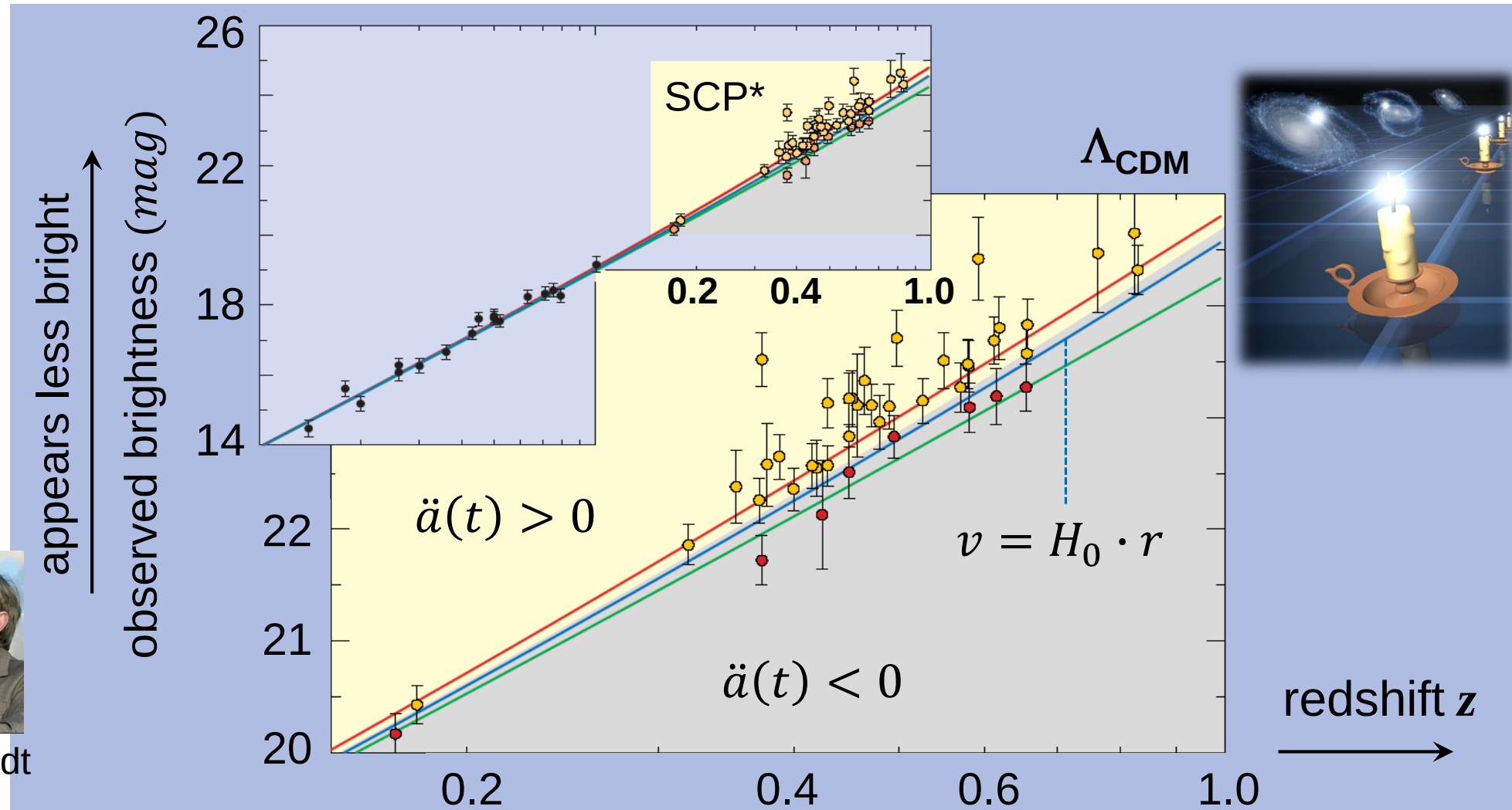


- nuclear & particle physics: very important methods for cosmology



Evidences for $\Lambda \neq 0$ & expansion with $\ddot{a}(t) > 0$

- Observed brightness of distant SNaE: **dimmer than one expects**



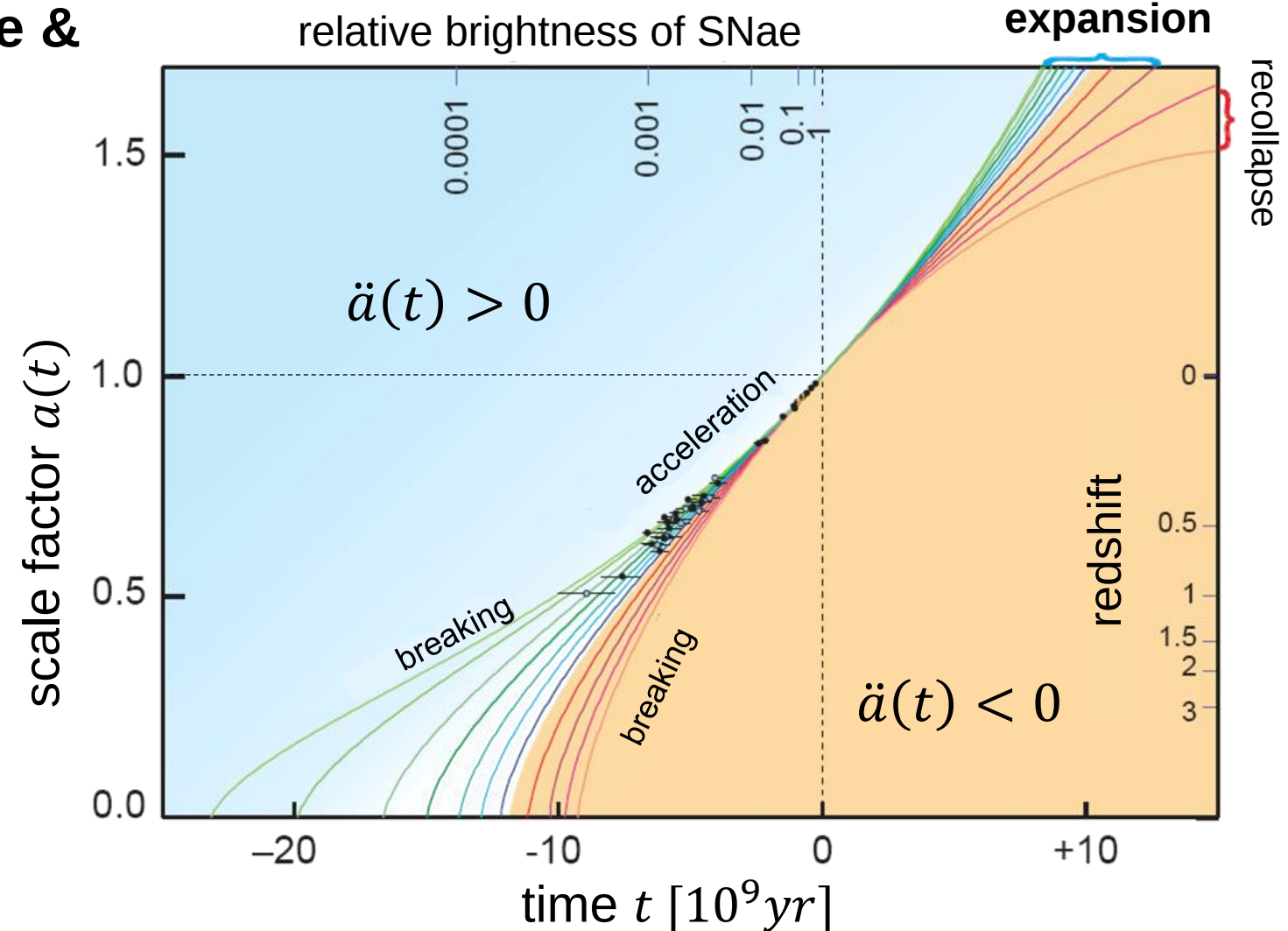
Perlmutter Riess Schmidt

Nobel prize 2011

Evidences for $\Lambda \neq 0$ & existence of dark energy

■ Observed brightness of SNaE & evolution of the scale parameter $a(t)$

- SNaE type Ia can be observed up to **redshift** $z = 1$ [$a(t) \sim 0.5$] via spectroscopy of atomic lines
- comparison of absolute SN-brightness M with apparent brightness m
 $\Rightarrow$ distance modulus $m - M$
 $\Rightarrow$ sensitivity to SN-distance r



Analysis of data from SNaE: a discrepancy?

■ Different results (2019) of data sets from SNIae: A. Riess vs. S. Sarkar



- > **6 σ** evidence for $\ddot{a}(t) > 0$
- > **4 σ** evidence against local motion!



A. Riess



S. Sarkar

The Open Supernova Catalog

perform your own analysis

- ~ **1.4 σ** 'evidence' for $\ddot{a}(t) > 0$
- ~ **3.9 σ** evidence for local motion!

DARK MATTER AND ENERGY | RESEARCH UPDATE

Dark energy debate reignited by controversial analysis of supernovae data

28 Oct 2019



Fading candle: remnant of a type Ia supernova called SNR 0509-67.5. (Courtesy: NASA/CXC/SAO/J Hughes et al/ESA/Hubble Heritage Team)

The mysterious substance known as dark energy thought to be pushing the universe apart at ever greater speeds may be nothing more than an artefact of our acceleration through a local patch of the universe. That is the controversial claim of a group of physicists who reckon they have found flaws in the evidence underpinning the Nobel-prize winning discovery of cosmic

Global data set for Ω_m & Ω_v

■ Actual best fit values

- combining data sets for:

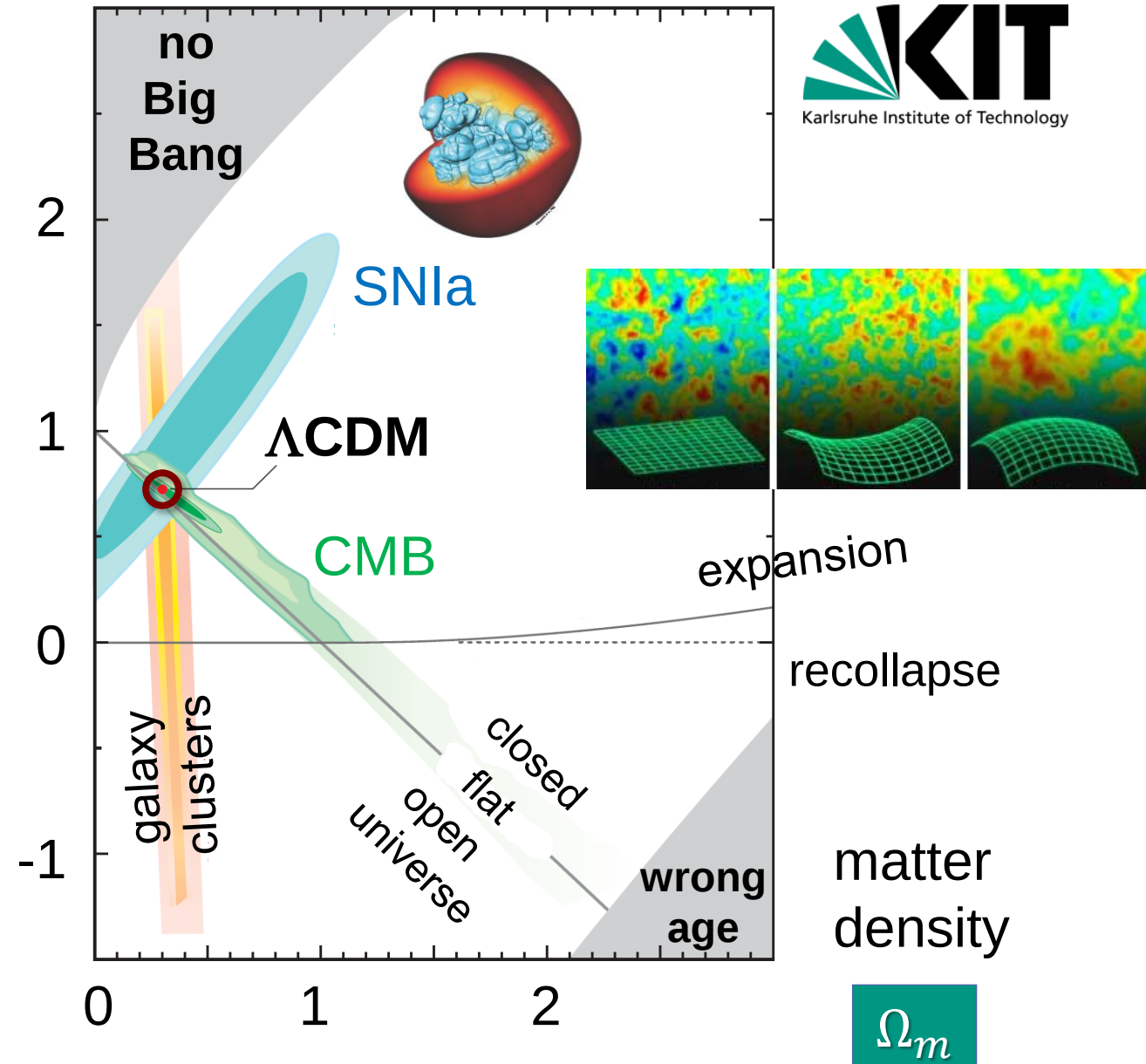
- SNIa brightness
- CMB analyses
- galaxy clusters

- 'orthogonal' methods:

- CMB analyses (3K):
sensitive to $\Omega_{tot} = \Omega_m + \Omega_v$
- SNaE-brightness:
sensitive to value $\Omega_v - \Omega_m$

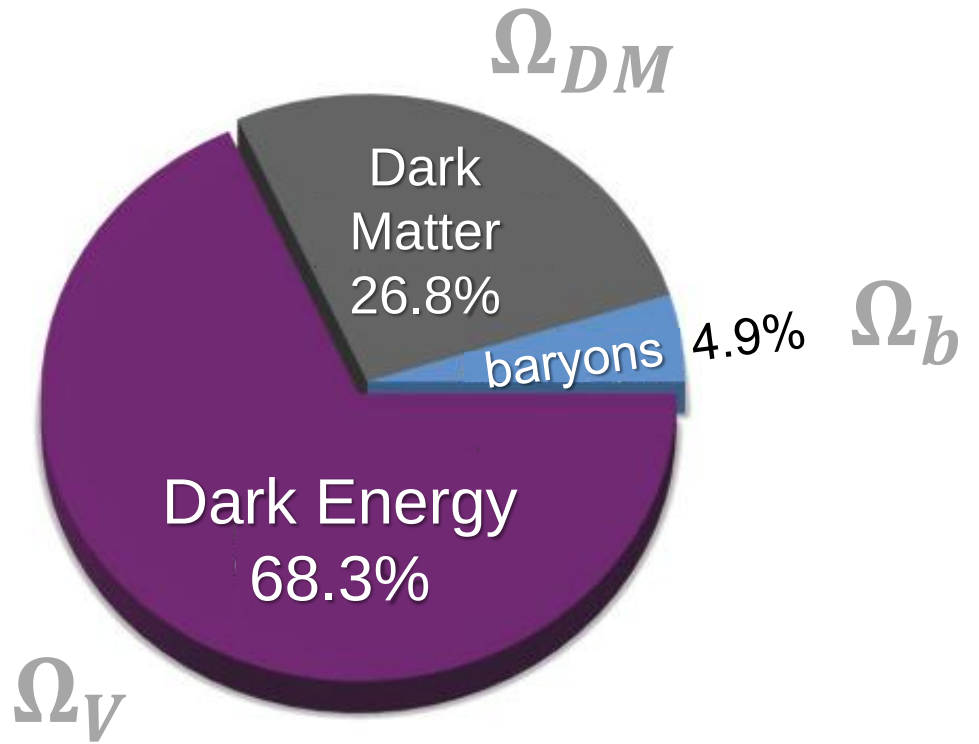
Ω_v

vacuum energy density



Global data set for Ω_m & Ω_v

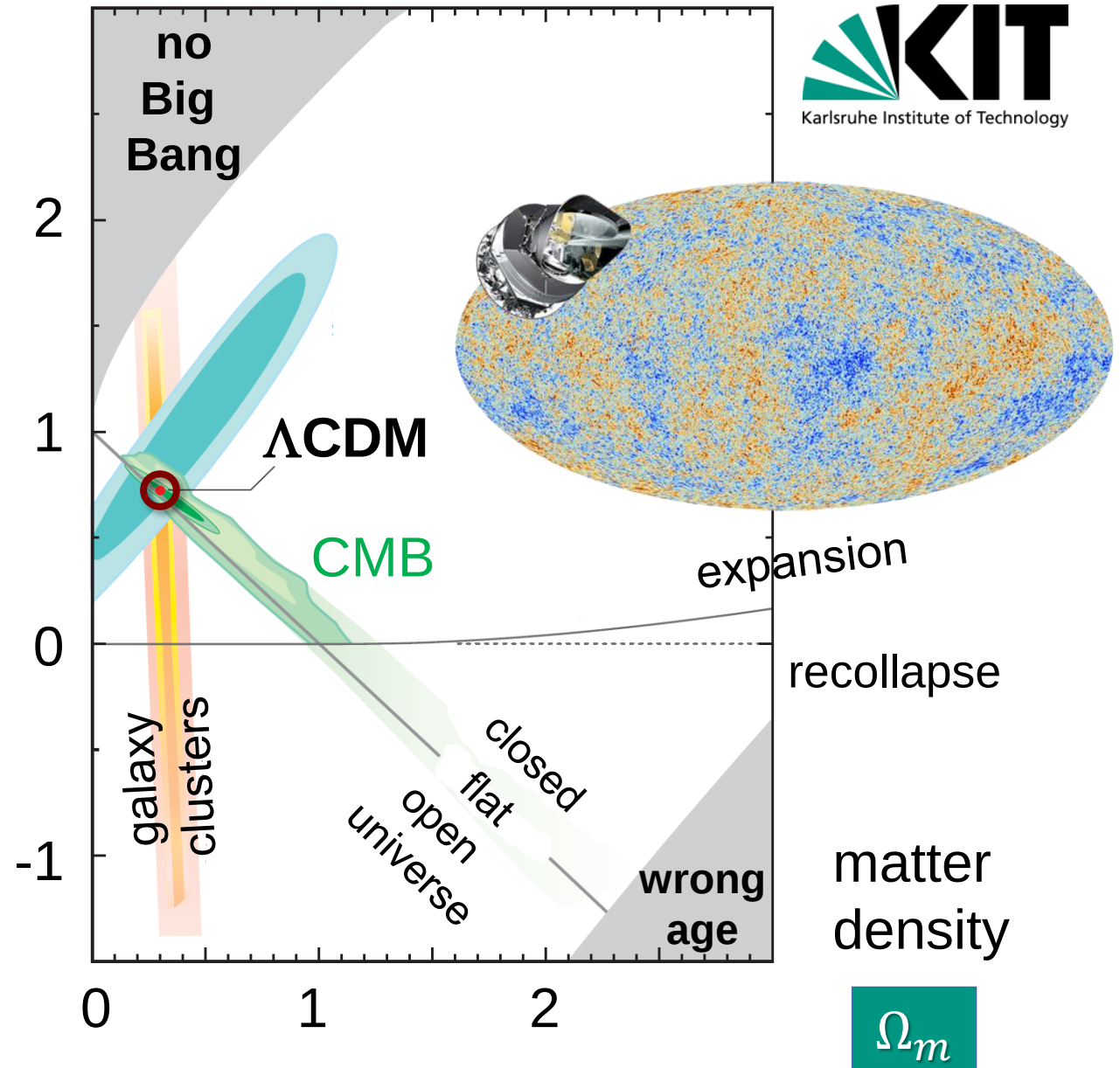
■ Λ CDM concordance model of cosmology

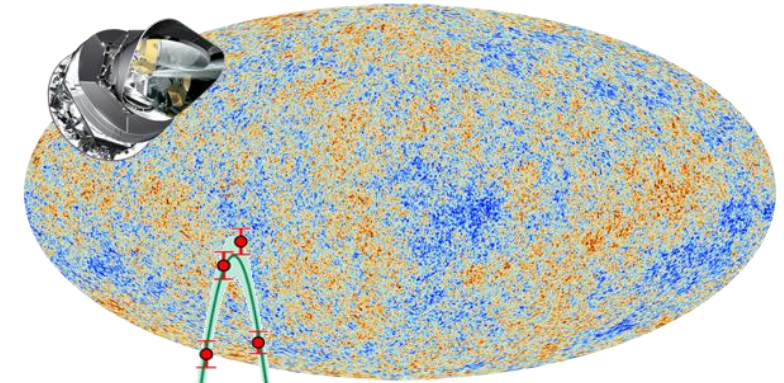
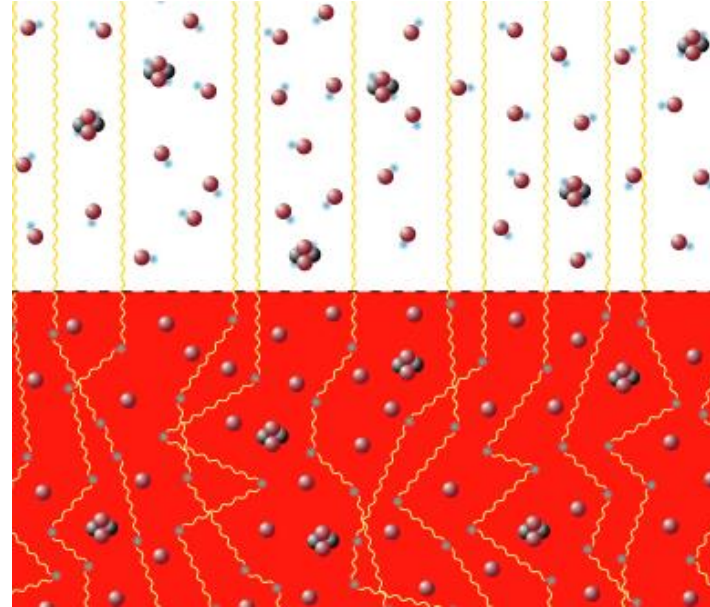


$$T_0 = 13.73 \cdot 10^9 \text{ yr}$$

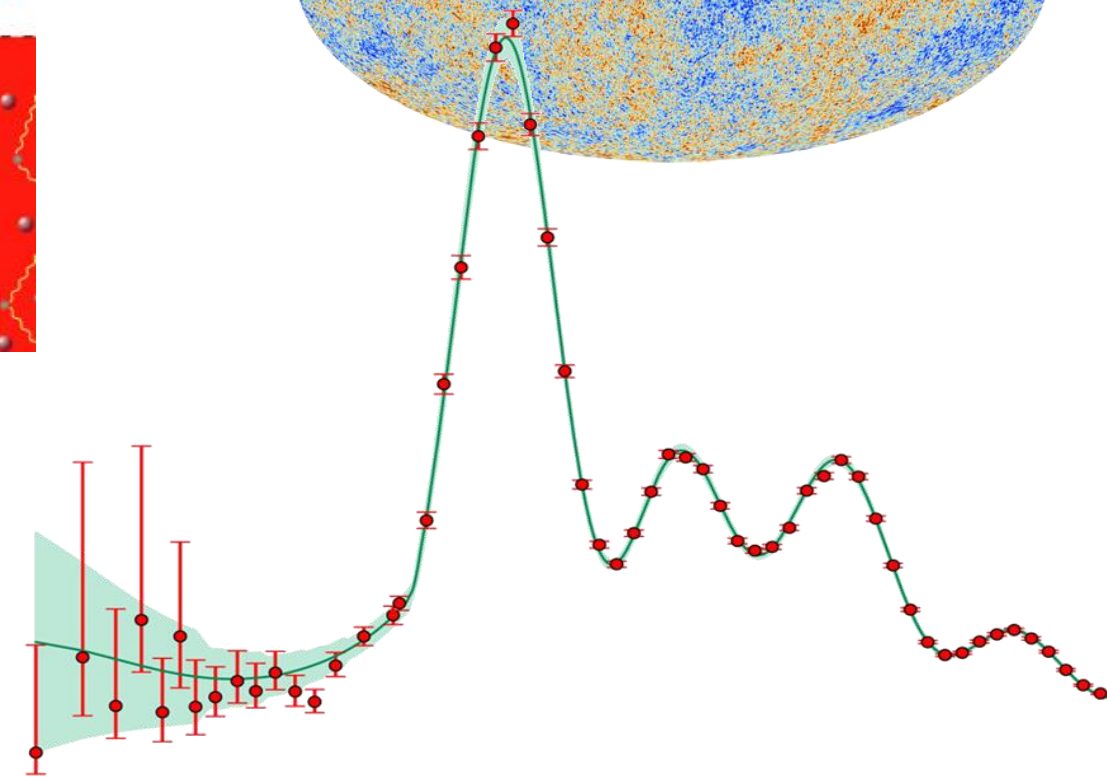
Ω_v

vacuum energy density





CHAPTER 3 – THERMAL UNIVERSE



Overview: thermal history of the universe

■ An expanding universe is **cooling down**

- shortly after Big Bang:

$$t \sim 10^{-36} \text{ s}$$

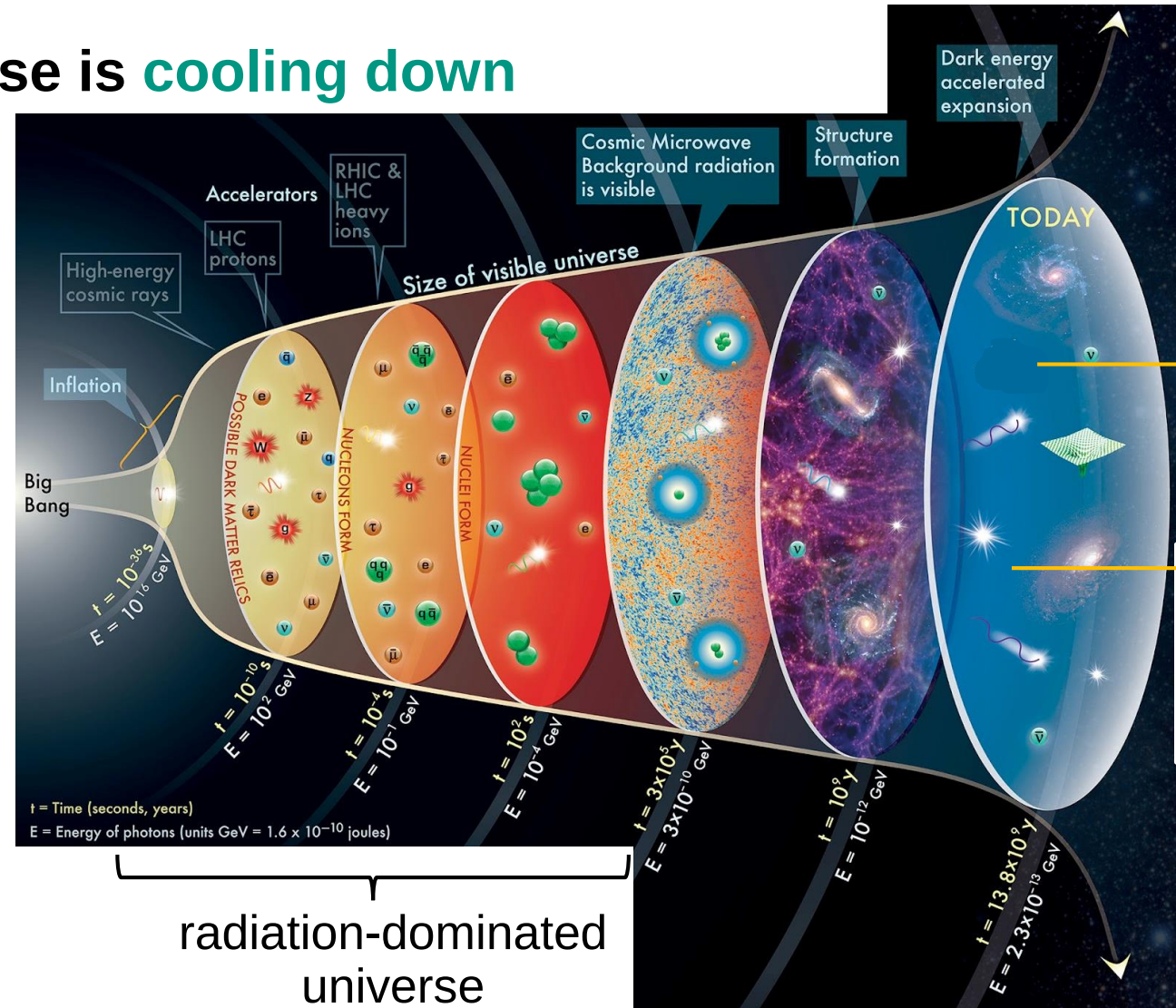
$$T \sim 10^{25} \text{ eV}$$

- present universe:

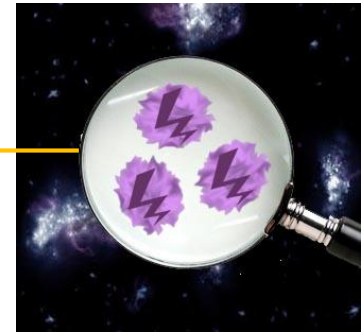
$$t \sim 13.8 \cdot 10^9 \text{ yr}$$

$$T \sim 10^{-3} \text{ eV}$$

- here: focus on **radiation-dominated universe**, up to $t \sim 47\,000 \text{ yr}$



Dark Energy



Dark Matter
CDM

Q: LBL/PDG

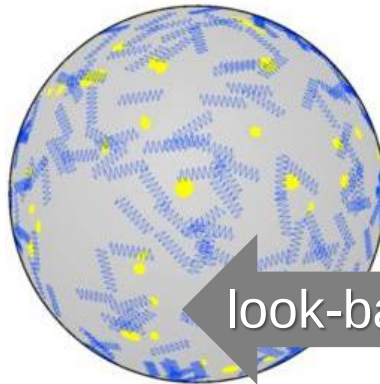
Evolution of temperature T in expanding cosmos

■ What is the relation between the **scale factor $a(t)$** & **temperature T** ?

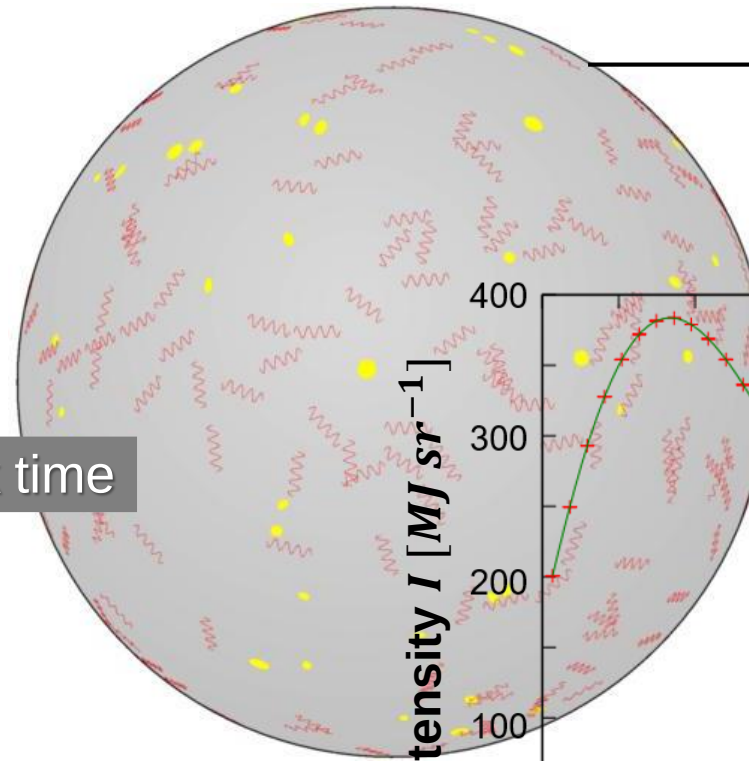
earlier (smaller & hotter) universe

$$a(t_1) = 0.5$$

$$T(t_1) = ?$$



look-back time



now: $t = t_0$

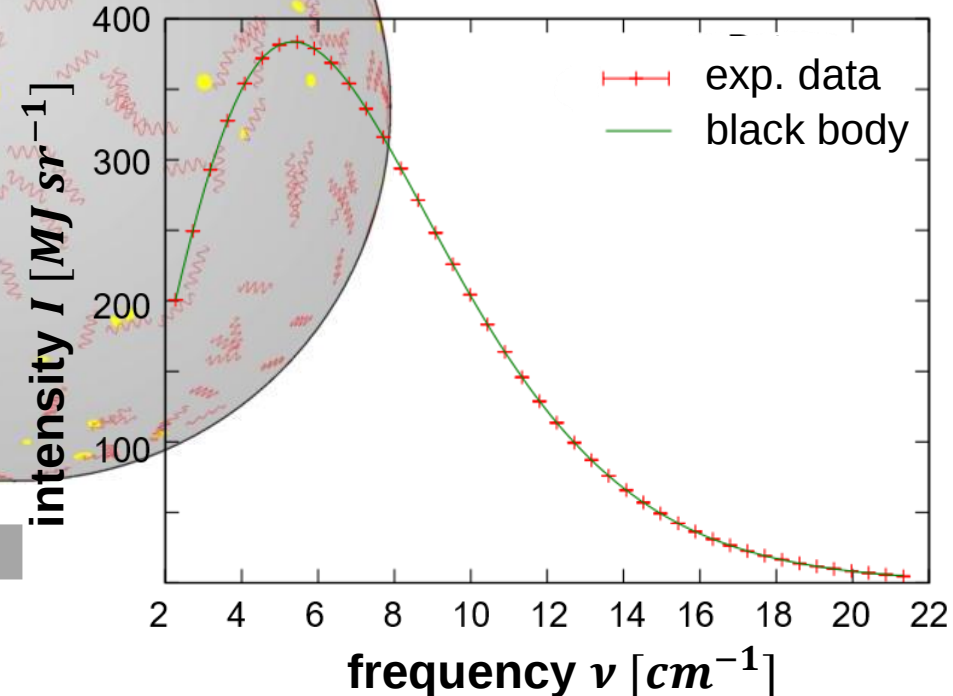
$$a(t_0) = 1.0$$

$$T(t_0) = 2.7 \text{ K}$$

- photon wavelengths are blue-shifted for $t < t_0$

- universe = perfect black body

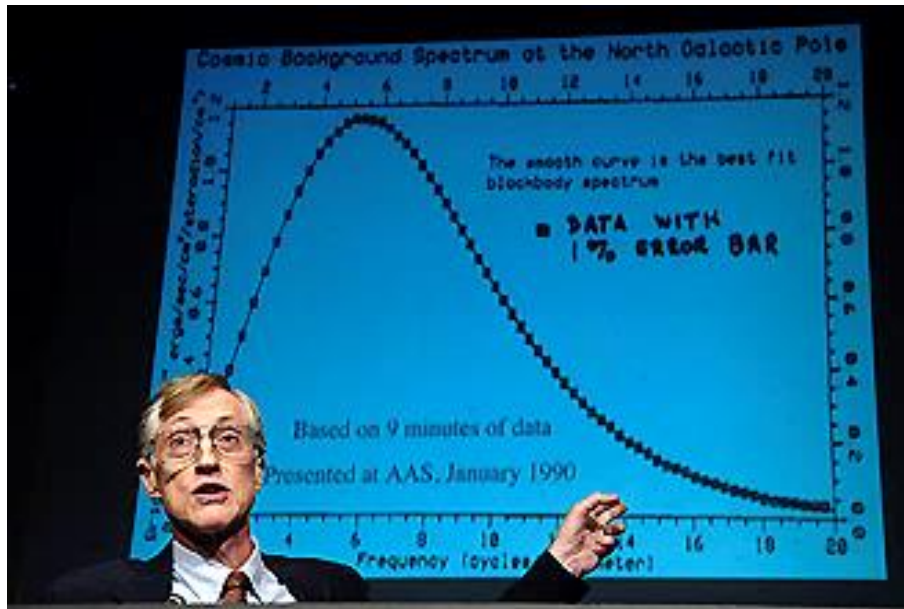
⇒ **temperature T from Planck-distribution**



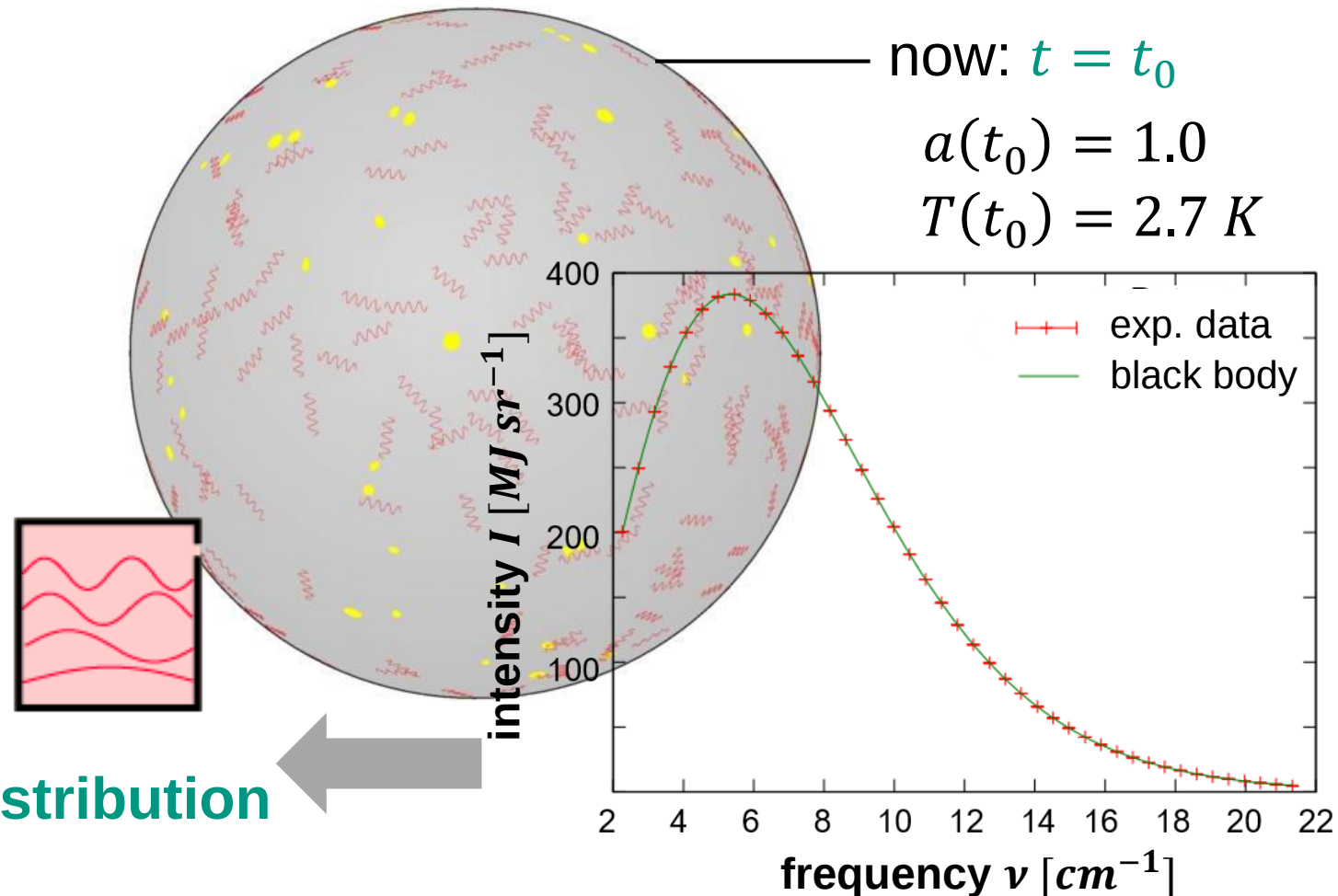
Evolution of temperature T in expanding cosmos

- What is the relation between the **scale factor $a(t)$** & **temperature T** ?

Q: NASA



- universe = perfect black body
⇒ **temperature T from Planck-distribution**



Evolution of temperature T in expanding cosmos

■ Wien's displacement law and the Cosmic Microwave Background (CMB)

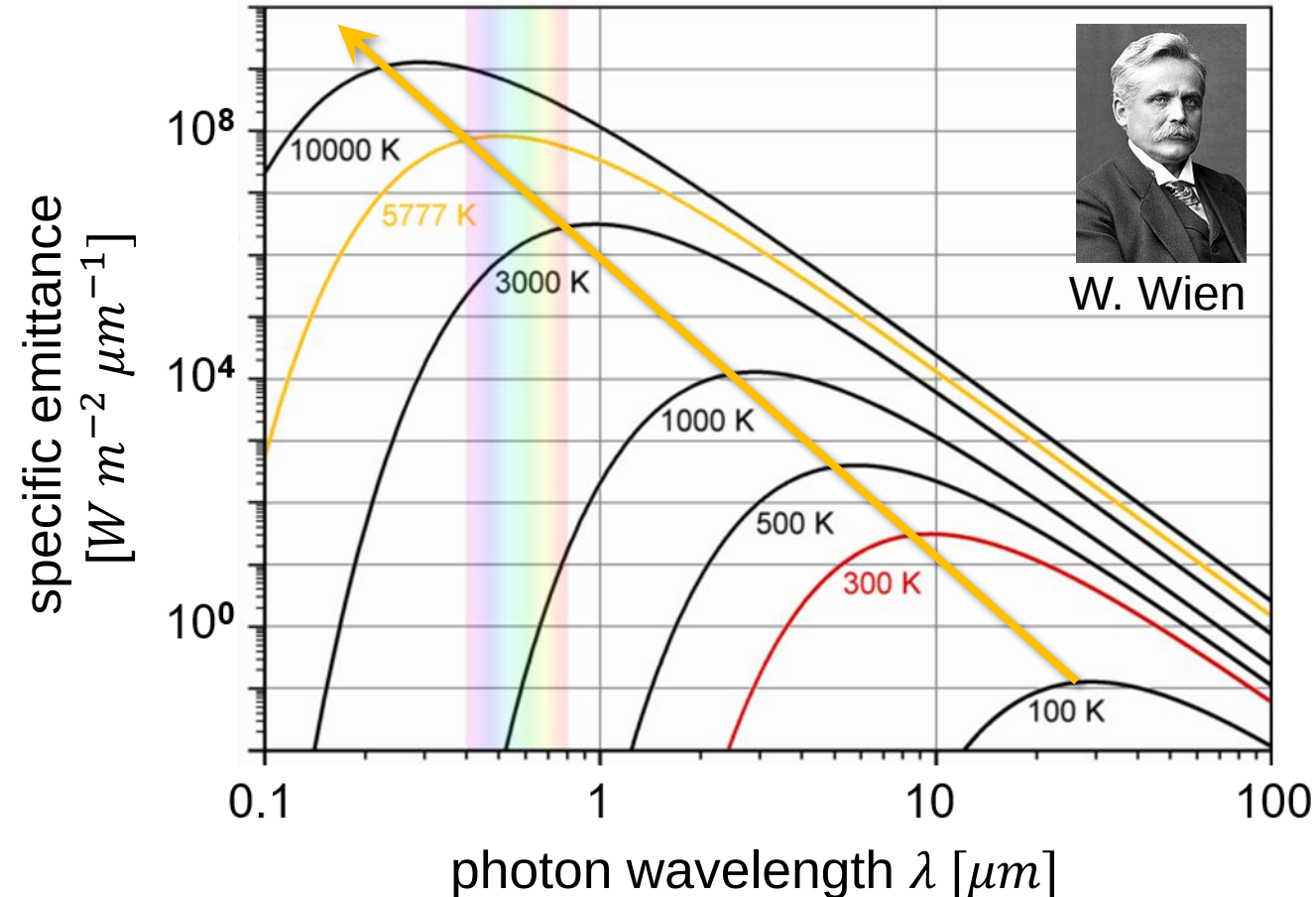
- relation between temperature T & maximum emittance λ_{max} of thermal radiation bath (Planck)

$$\lambda_{max} \cdot T = 2.8978 \text{ mm K}$$

- adiabatic expansion of cosmos (z) & CMB-photons of frequency ν

$$T_\gamma(z) = T_\gamma(z = 0) \cdot (1 + z)$$

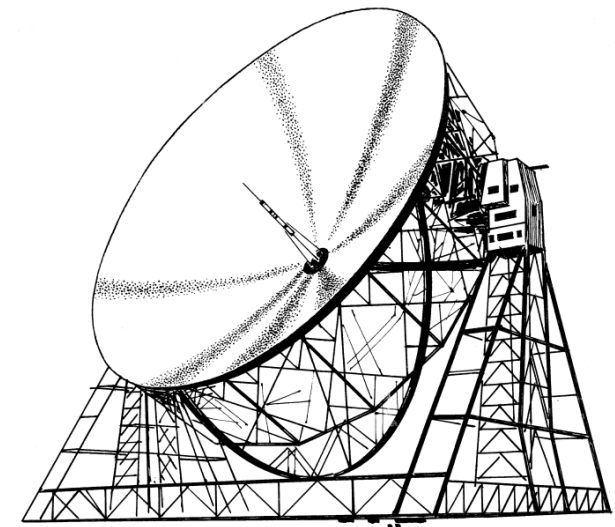
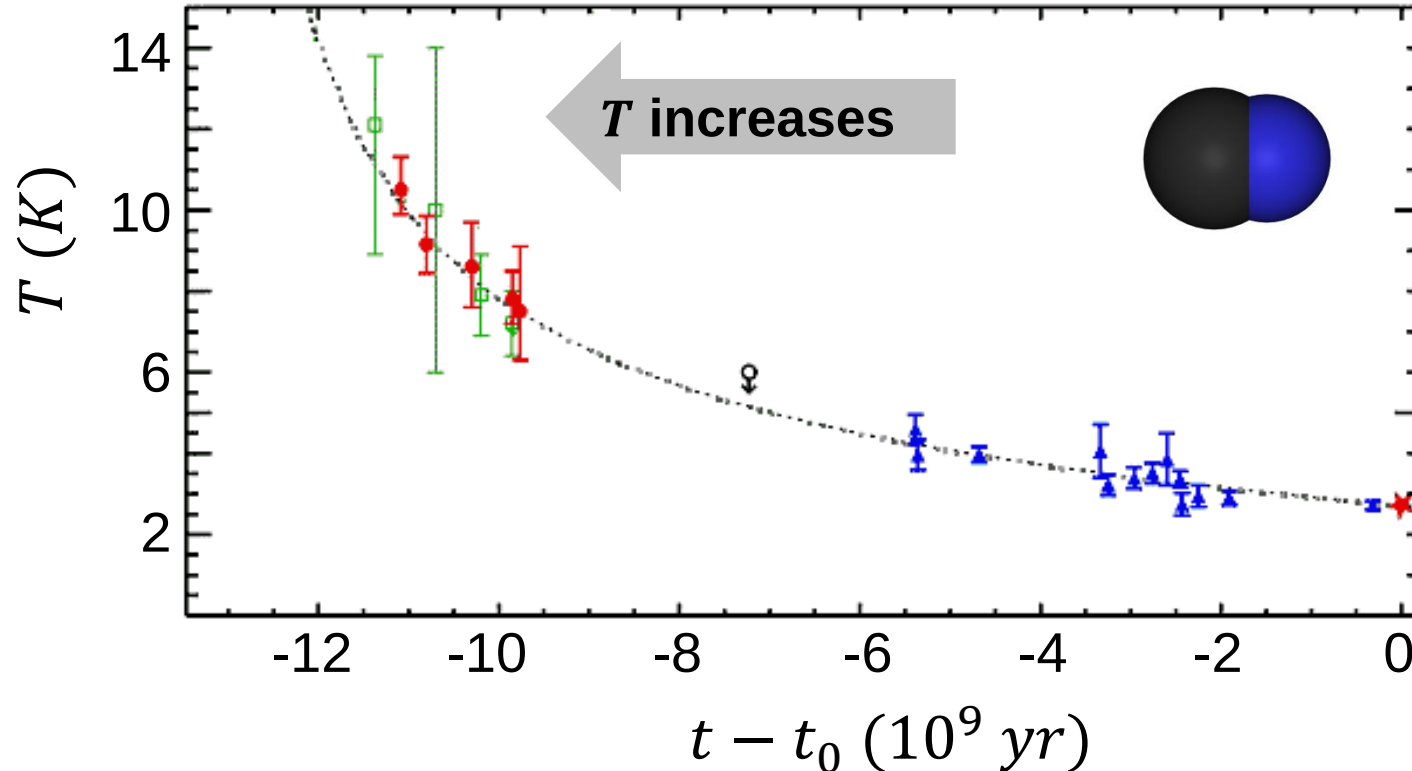
$$h\nu(z) = h\nu(z = 0) \cdot (1 + z)$$



Evolution of temperature T in expanding cosmos

■ Observational evidence for an increase in temperature T at earlier times t

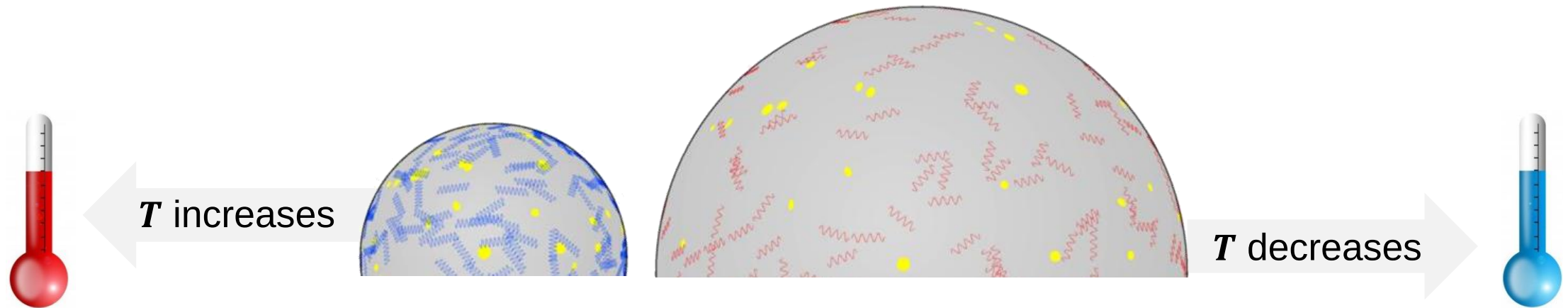
- effect: **thermal excitation** of Cyan (CN) molecules due to earlier, hotter CMB radiation ($\Rightarrow$ stronger excitation)



radio-astronomical
observations of distant clouds

Evolution of temperature T & energy conservation

■ Adiabatic expansion of the universe: a cornerstone of cosmology



A. Einstein



adiabatic cosmological expansion



J. Stefan



L. Boltzmann

Evolution of temperature T as function of t & $a(t)$

■ Connecting two descriptions: **photon bath** & **thermodynamical ensemble**

time t

$$\rho_{\gamma}(t) = N_{\gamma}(t) \cdot E_{\gamma}(t)$$

photon bath:
photons as
elementary
particles



A. Einstein

temperature T

$$\rho_{\gamma}(T) = \frac{\pi^2}{15h^3c^3} (k_B T)^4$$

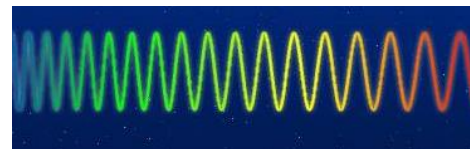
ensemble:
photons as
thermodynamical
ensemble



J. Stefan



L. Boltzmann

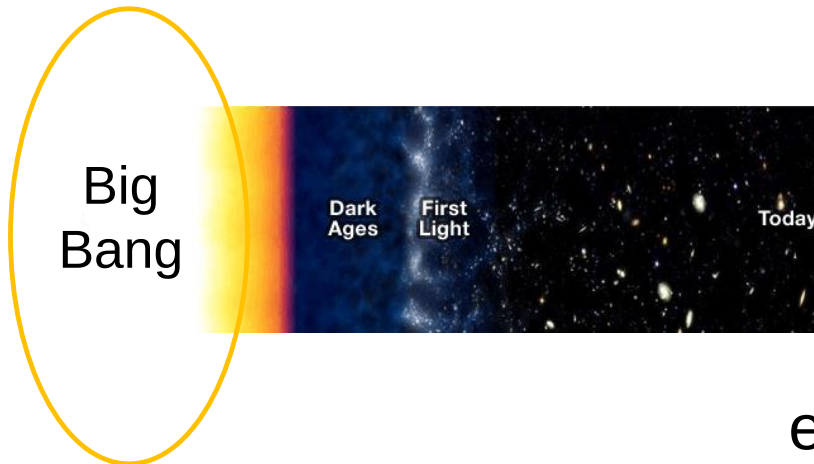


Evolution of temperature T as function of t & $a(t)$

■ Connecting two descriptions: **photon bath** & **thermodynamical ensemble**

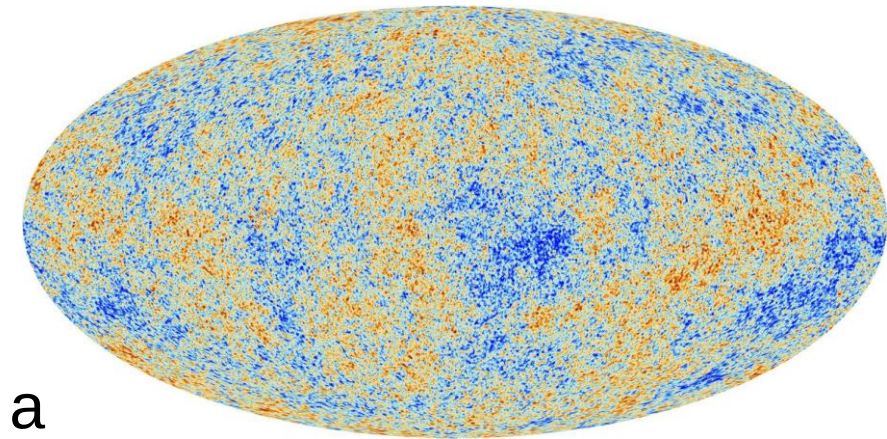
$$\rho_\gamma(t) = N_\gamma(0) \cdot a(t)^{-3} \cdot E_\gamma(0) \cdot a(t)^{-1}$$

$$\rho_\gamma(t) \sim a(t)^{-4} \quad \longleftrightarrow \quad \rho_\gamma(T) \sim T(t)^4$$



$$T(t) \sim \frac{1}{a(t)} \sim \frac{1}{\sqrt{t}}$$

evolution of temperature T in a
radiation-dominated universe



Properties of the radiation-dominated universe

■ Temperature scale $T(t)$ and energy scale $E(t)$ of CMB photons

- important conversion factor:

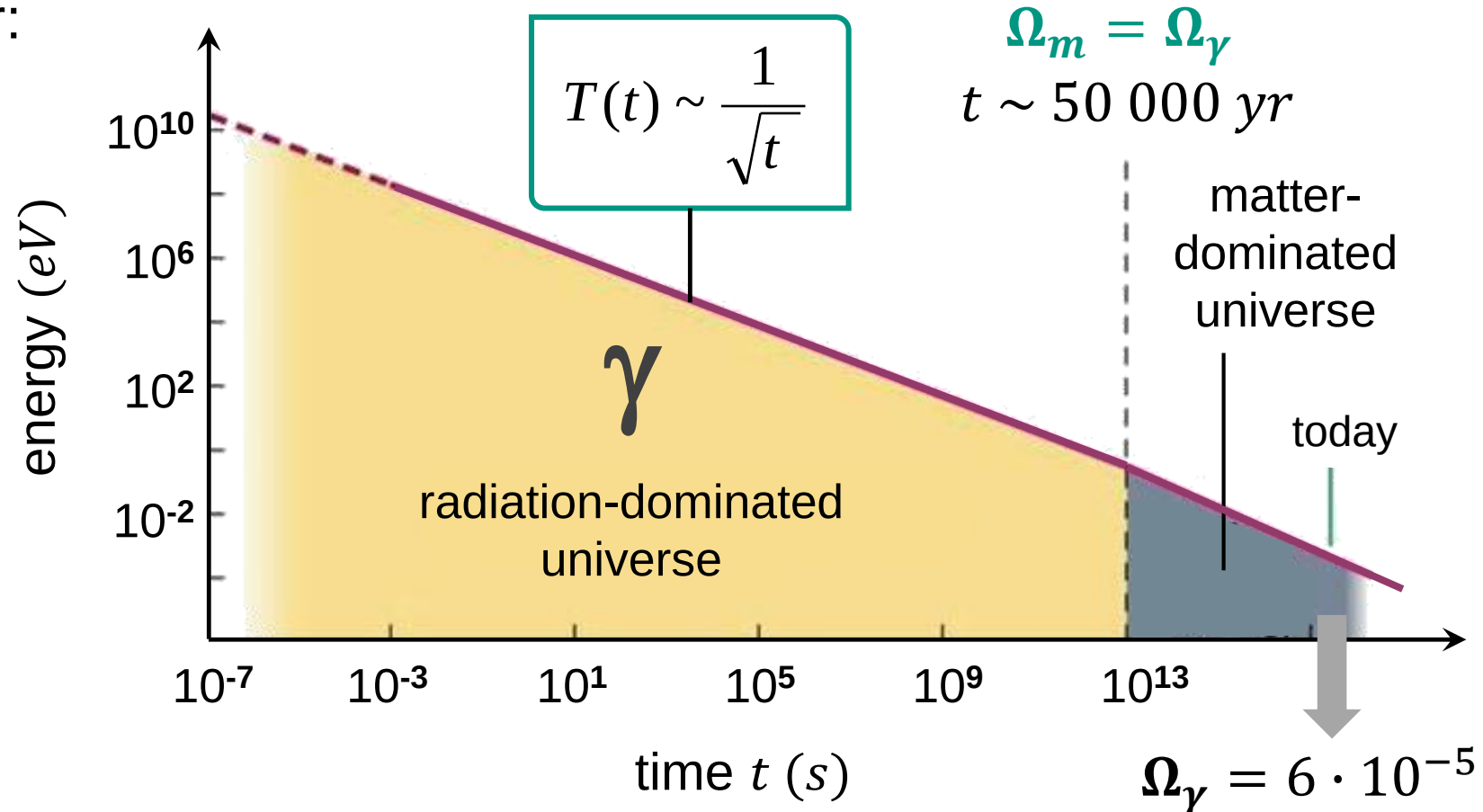
$$10^4 K \leftrightarrow 1 \text{ eV}$$

(11 605 K)

$$T(t) \cong \frac{10^{10} \text{ K}}{\sqrt{t [\text{s}]}}$$

$$\cong \frac{1 \text{ MeV}}{\sqrt{t [\text{s}]}}$$

at $t = 1 \text{ s} \leftrightarrow T = 1 \text{ MeV}$



Matter-dominated universe

■ After $t \sim 50\,000\text{ yr}$: evolution dominated by matter (dark matter, baryons)

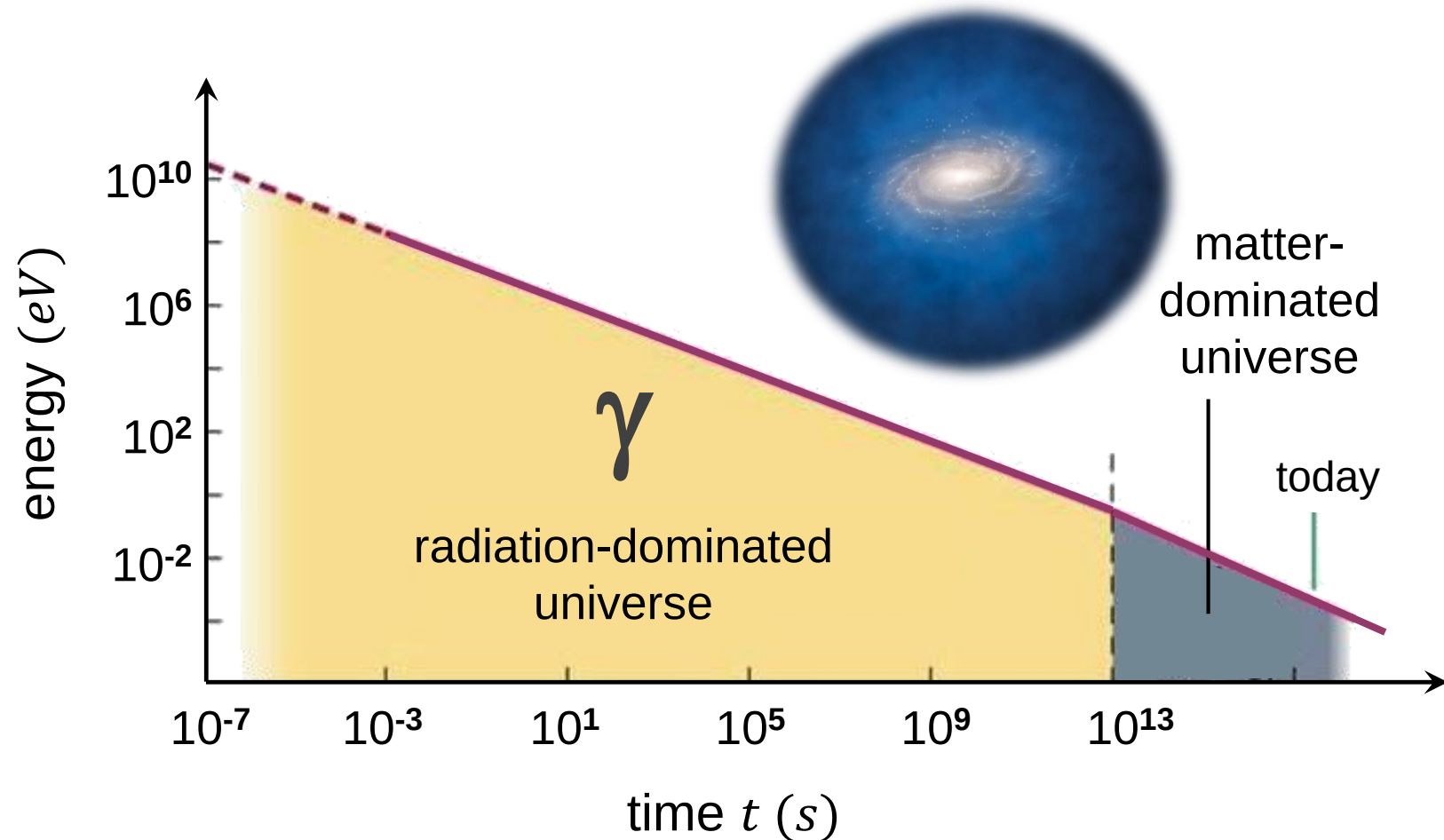
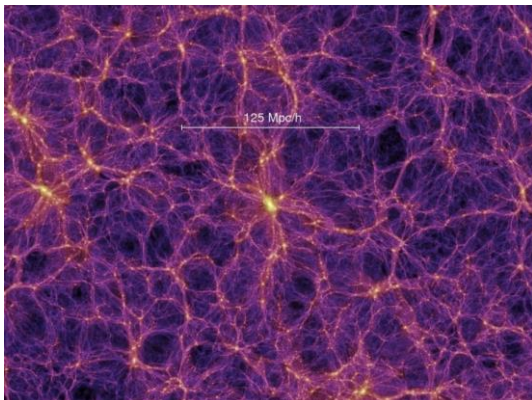
- two key players today:

Dark Matter:

gravitational attraction

Dark Energy:

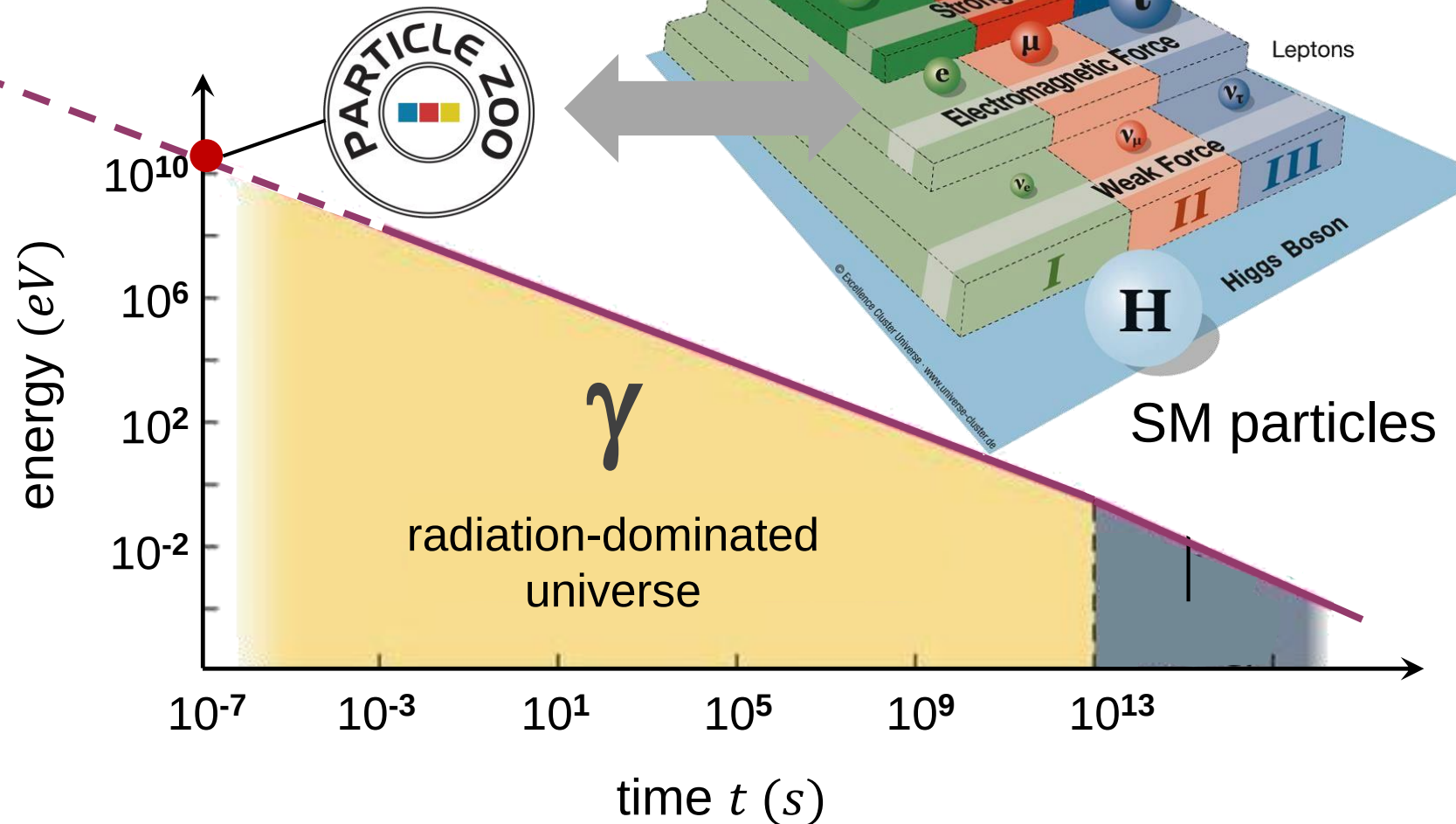
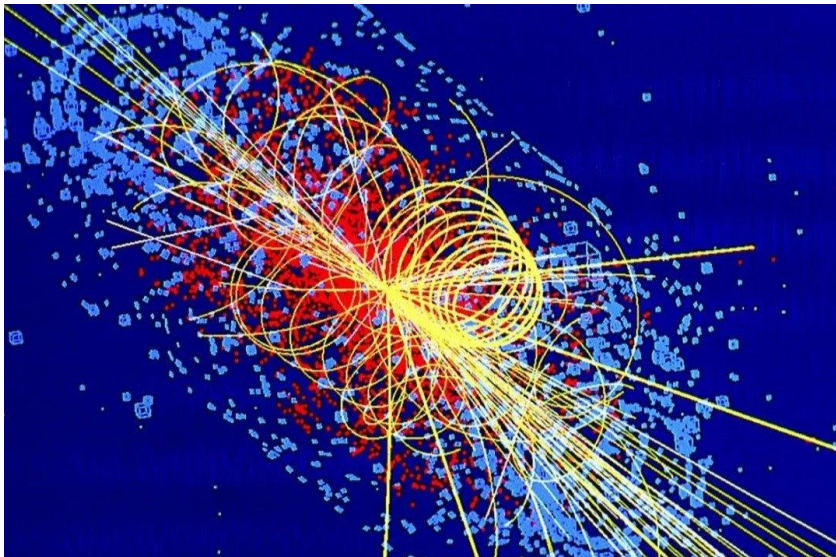
anti-gravitation



Radiation-dominated universe: Particle Zoo

■ Particle production during the Big Bang

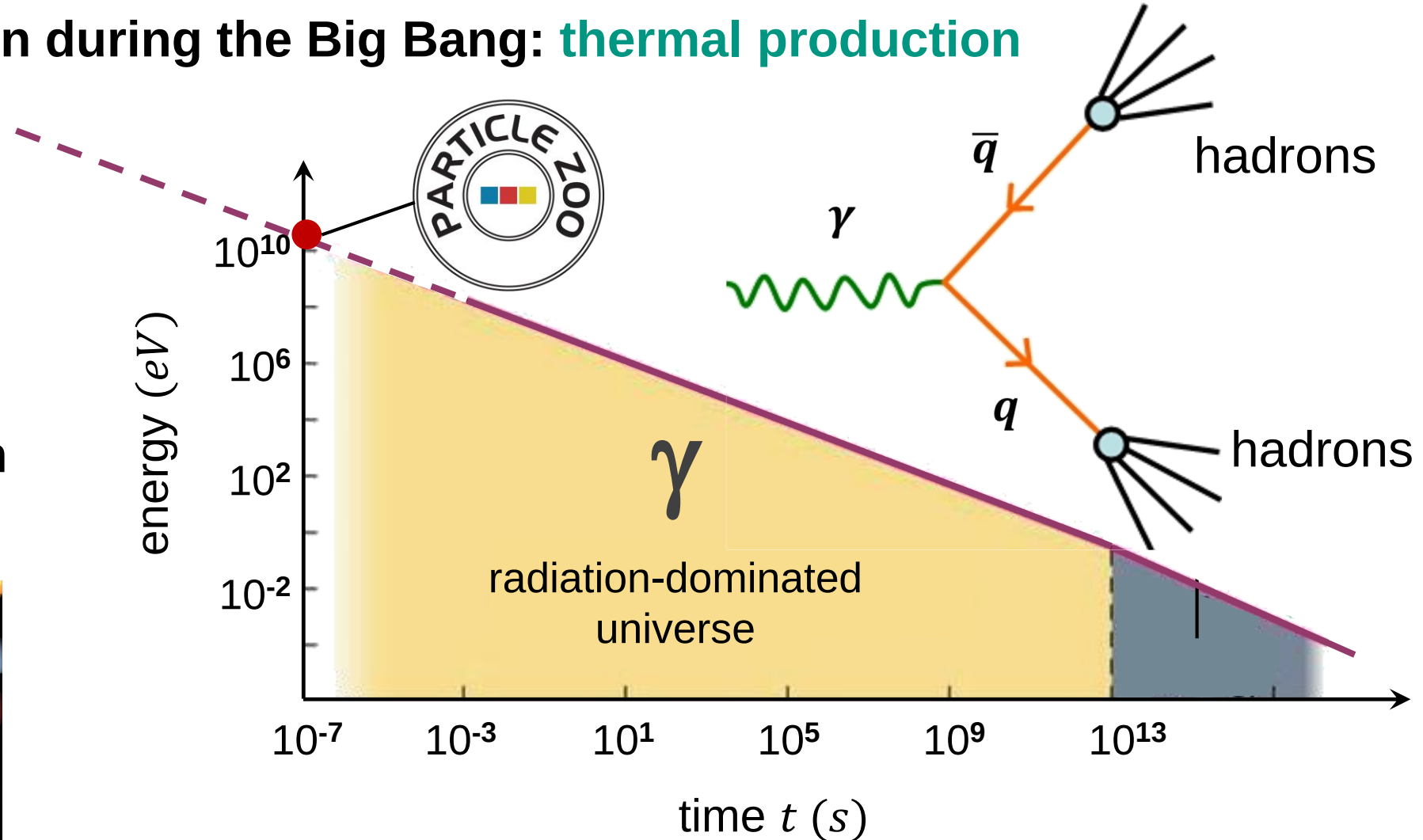
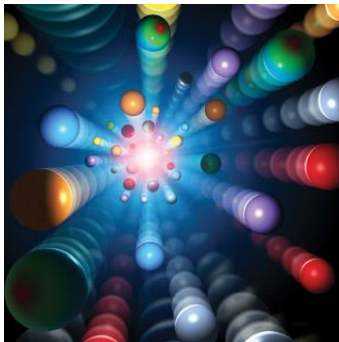
- heat bath:
particle interactions via strong & electroweak interactions



Radiation-dominated universe: Particle Zoo

■ Particle production during the Big Bang: thermal production

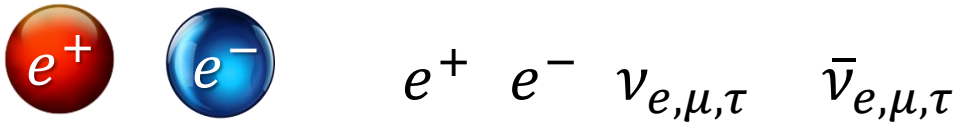
- heat bath:
particle interactions
via strong & electro-
weak interactions
- thermal production
- annihilation
- decay



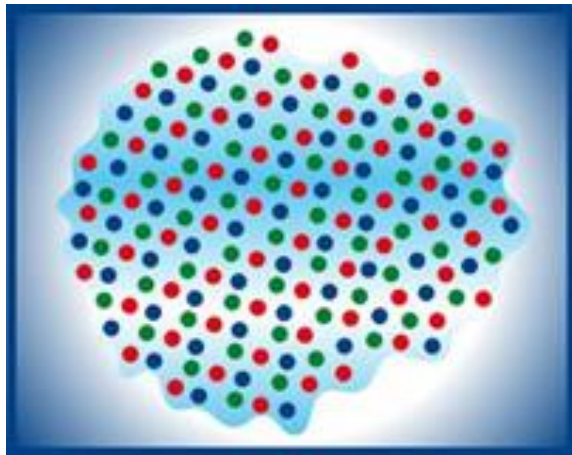
Radiation-dominated universe: Particle Zoo

Particle production during the Big Bang

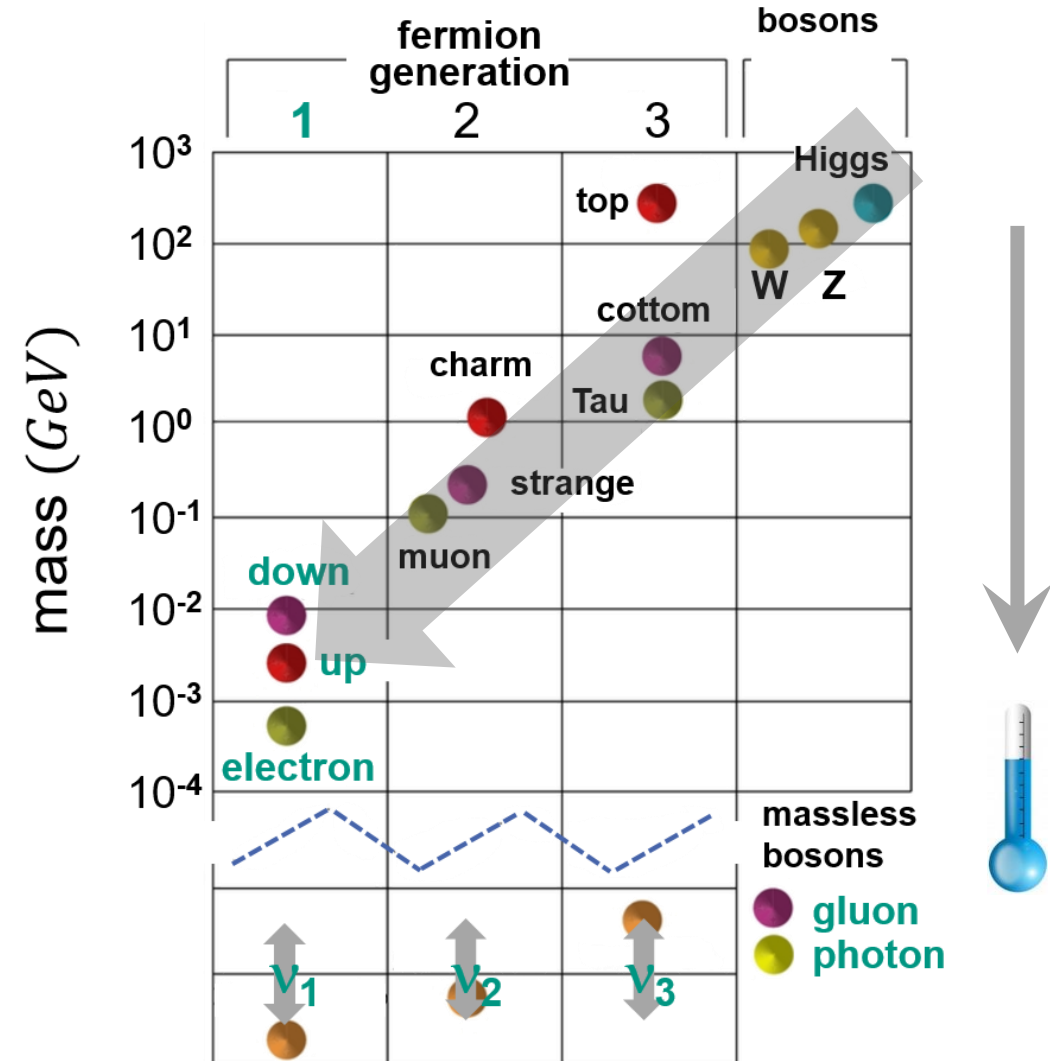
Leptons:



Quarks: u $\bar{u}$ d $\bar{d}$



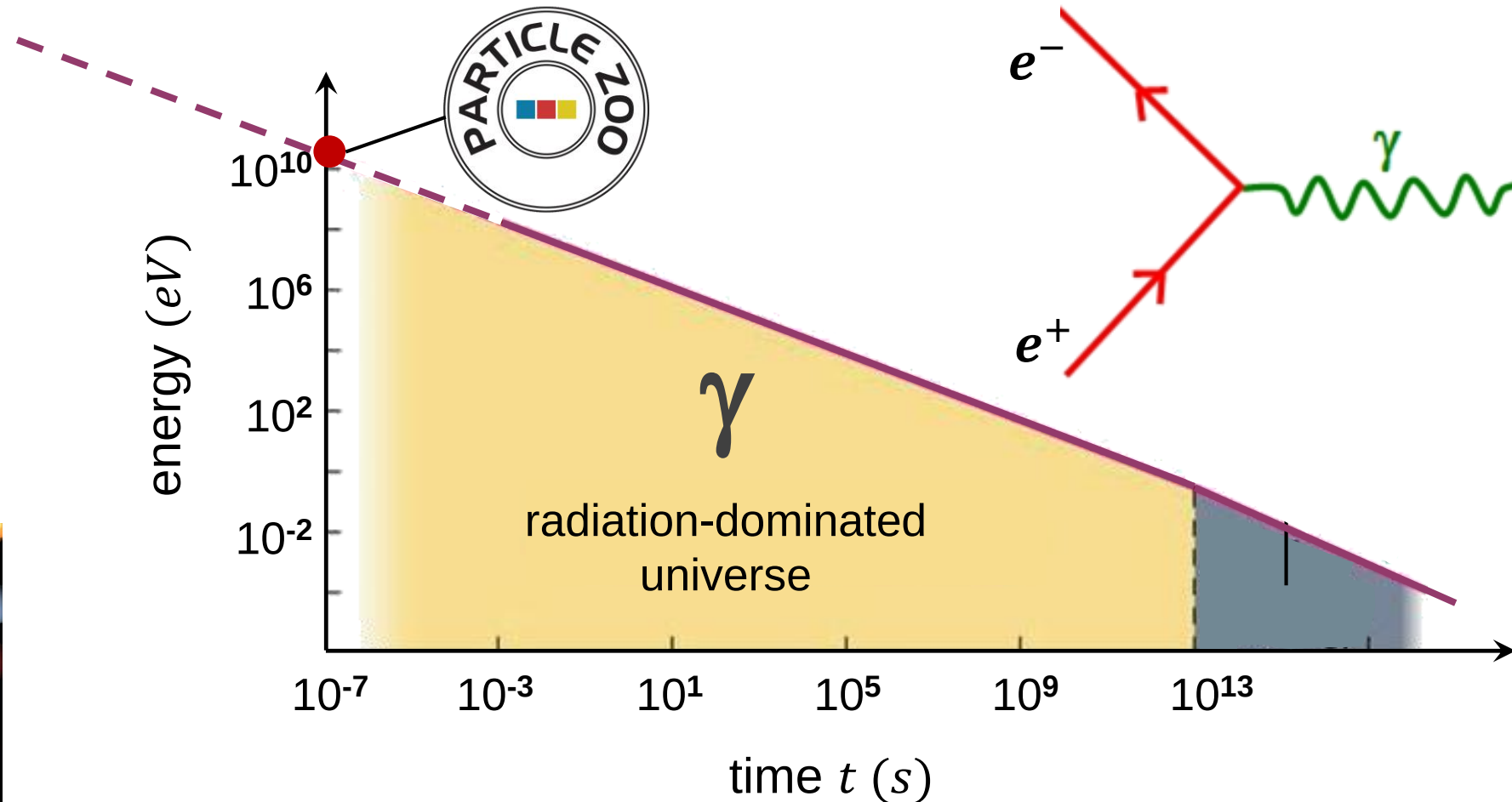
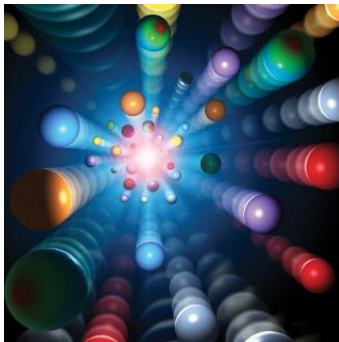
Quark-
Gluon
plasma



Radiation-dominated universe: Particle Zoo

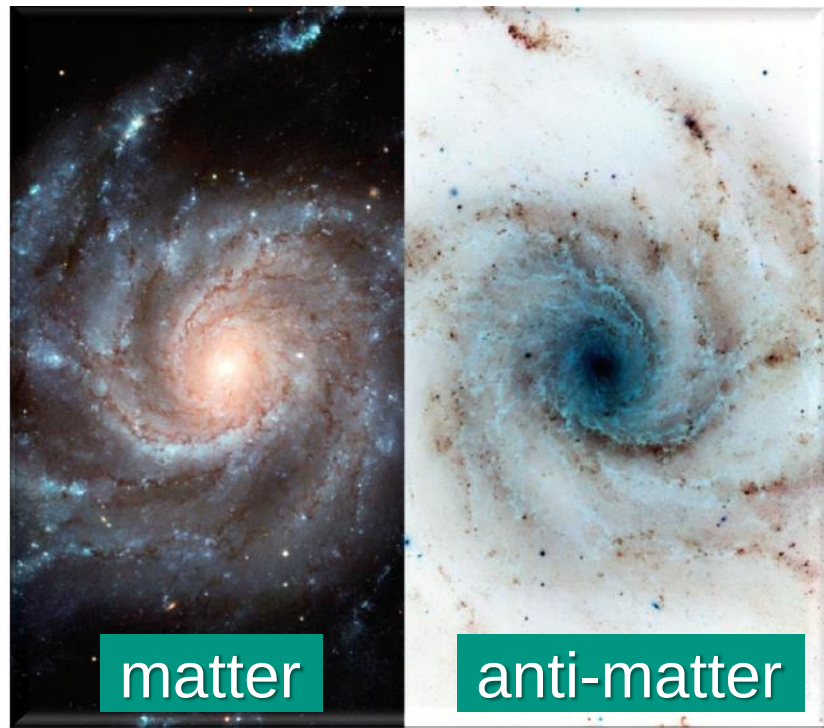
■ Particle production during the Big Bang: **annihilations**

- heat bath:
particle interactions via strong & electroweak interactions
- thermal production
- **annihilation**
- decay

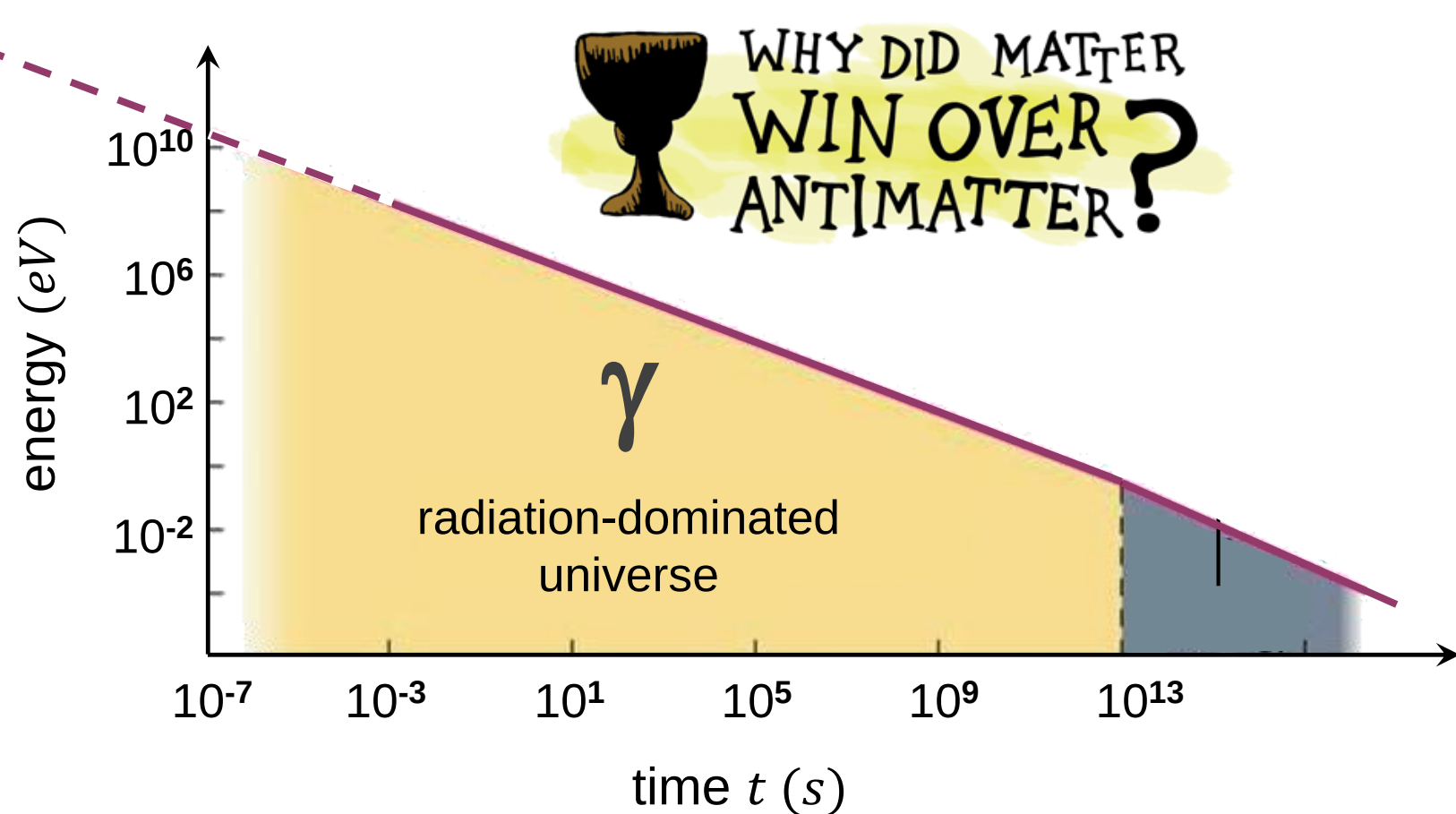


Radiation-dominated universe: Particle Zoo

■ Particle production during the Big Bang: **annihilations**



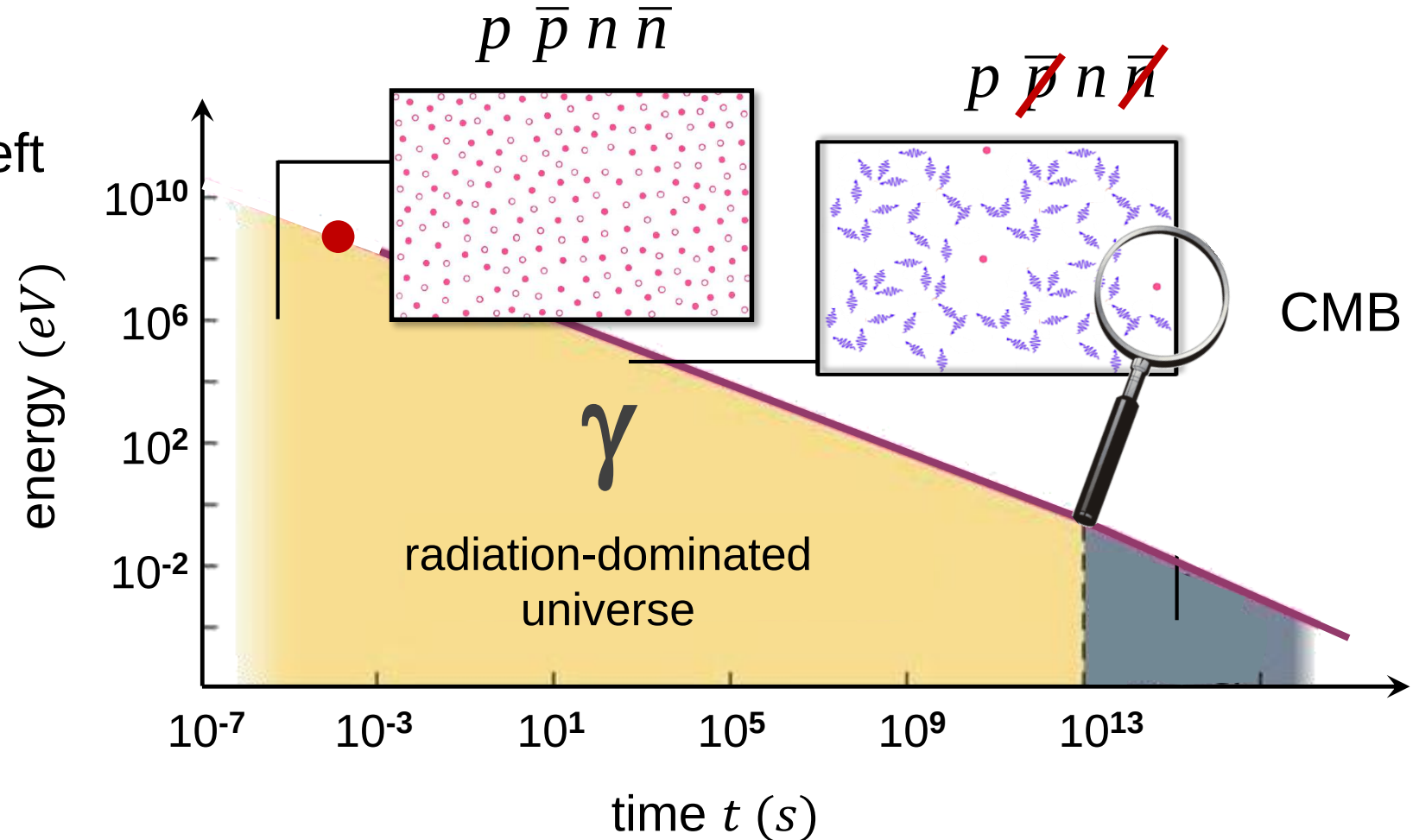
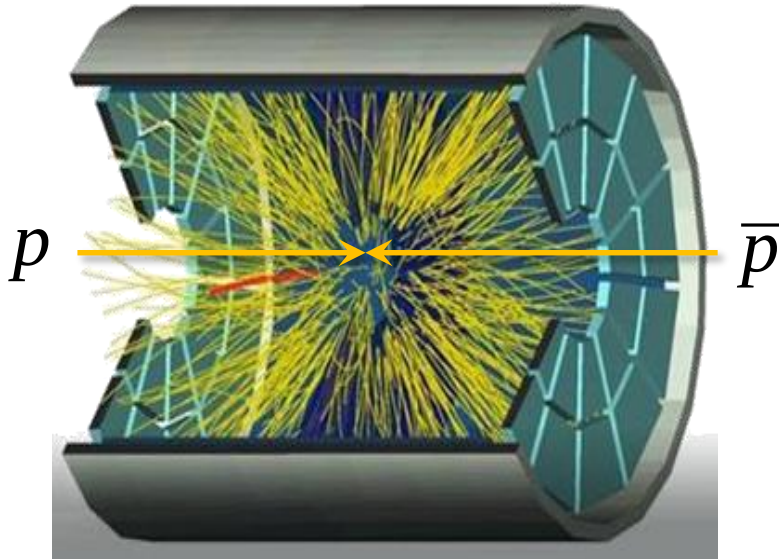
3 Sacharov-criteria



Radiation-dominated universe: Particle Zoo

■ Particle production during the Big Bang: **annihilations**

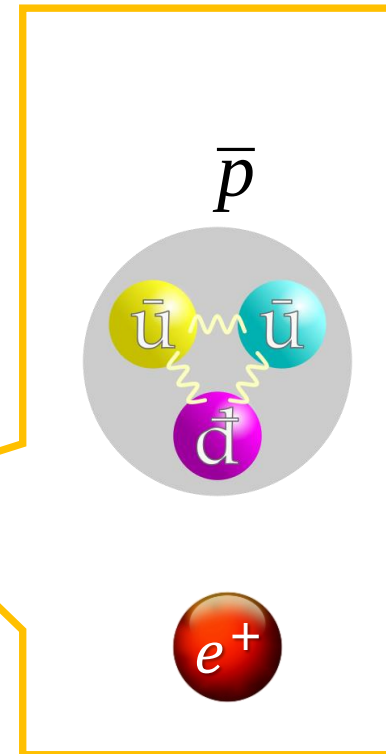
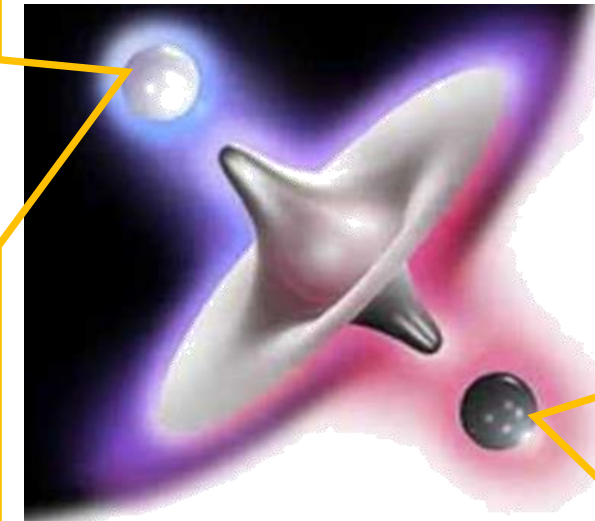
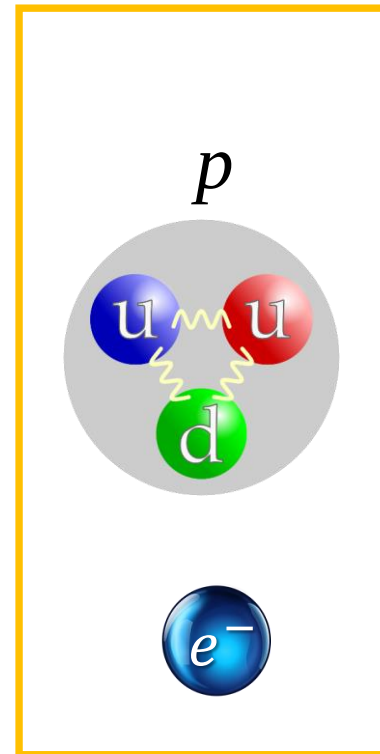
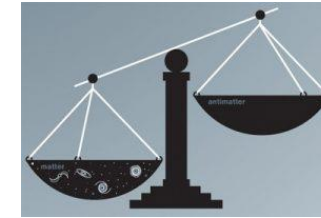
- in case of perfect CP symmetry: no matter left in universe



Radiation-dominated universe: Particle Zoo

■ Particle production during the Big Bang: **annihilations**

- in case of perfect CP symmetry: no matter left in universe



Radiation-dominated universe: Particle Zoo

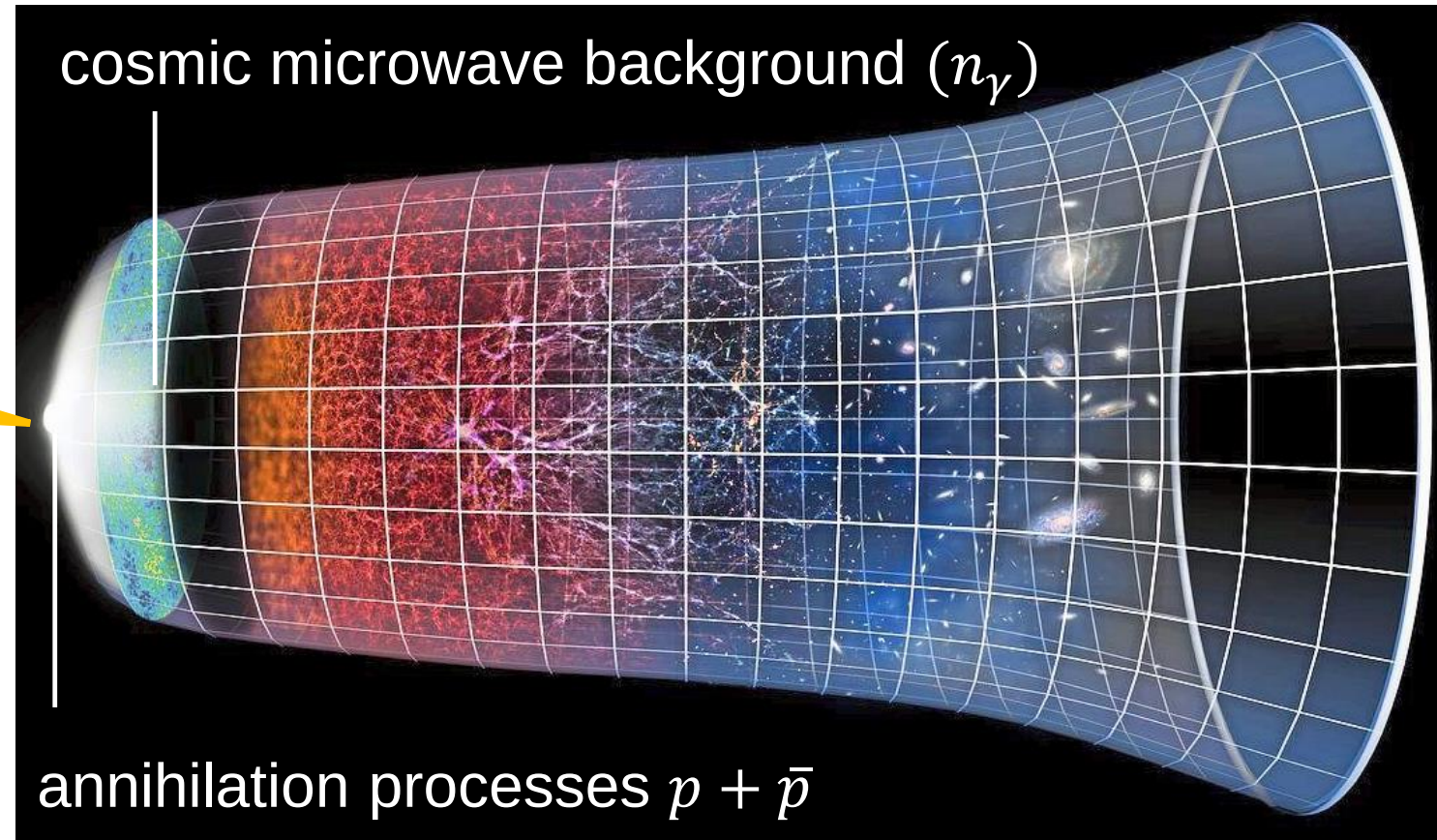
■ Particle production during the Big Bang: **origin of baryon number violation?**

- universe with a net **baryon asymmetry η**



$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$$

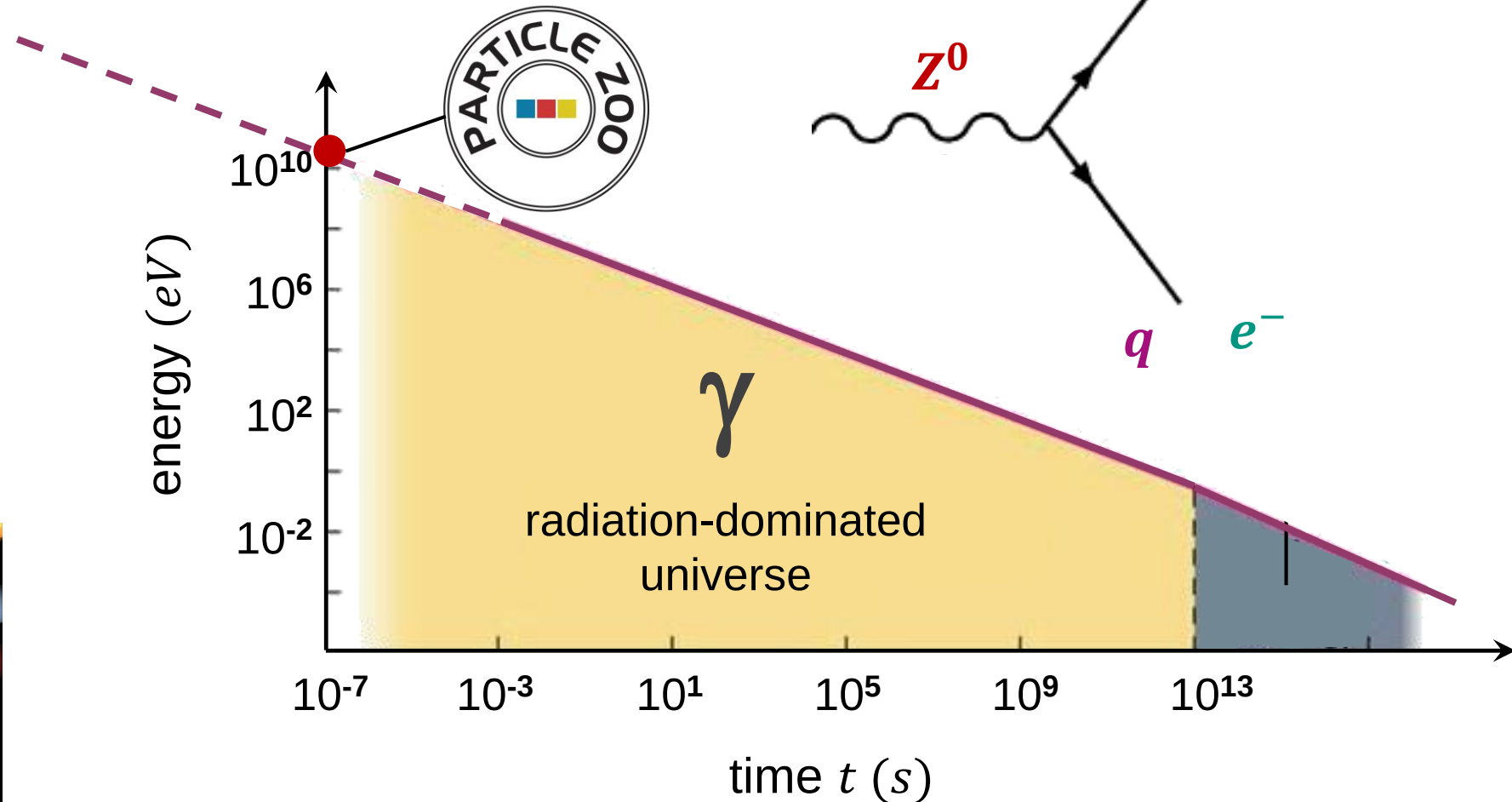
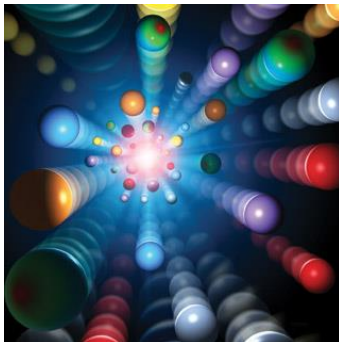
$$\eta = (6.14 \pm 0.24) \cdot 10^{-10}$$



Radiation-dominated universe: Particle Zoo

■ Particle production during the Big Bang: decays

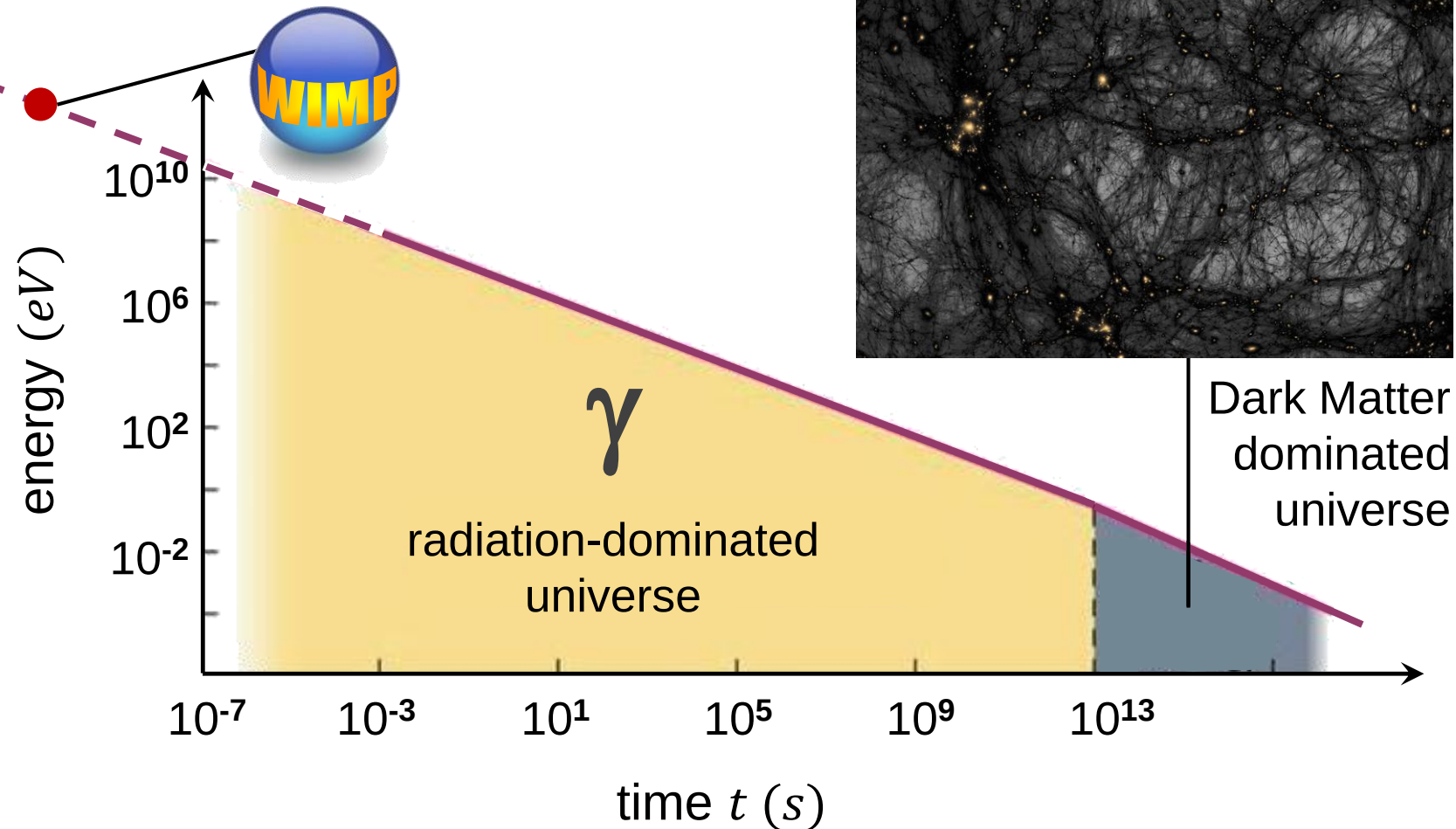
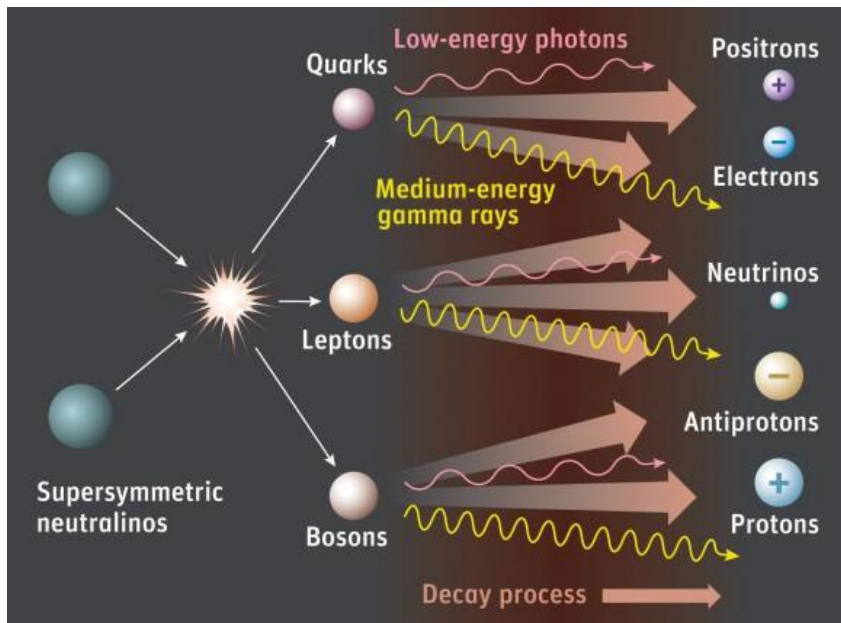
- heat bath:
particle interactions via strong & electroweak interactions
- thermal production
- annihilation
- **decay**



Radiation-dominated universe: Particle Zoo

■ Particle production during the Big Bang: **WIMPs**

- heat bath:
particle interactions
generate dark matter

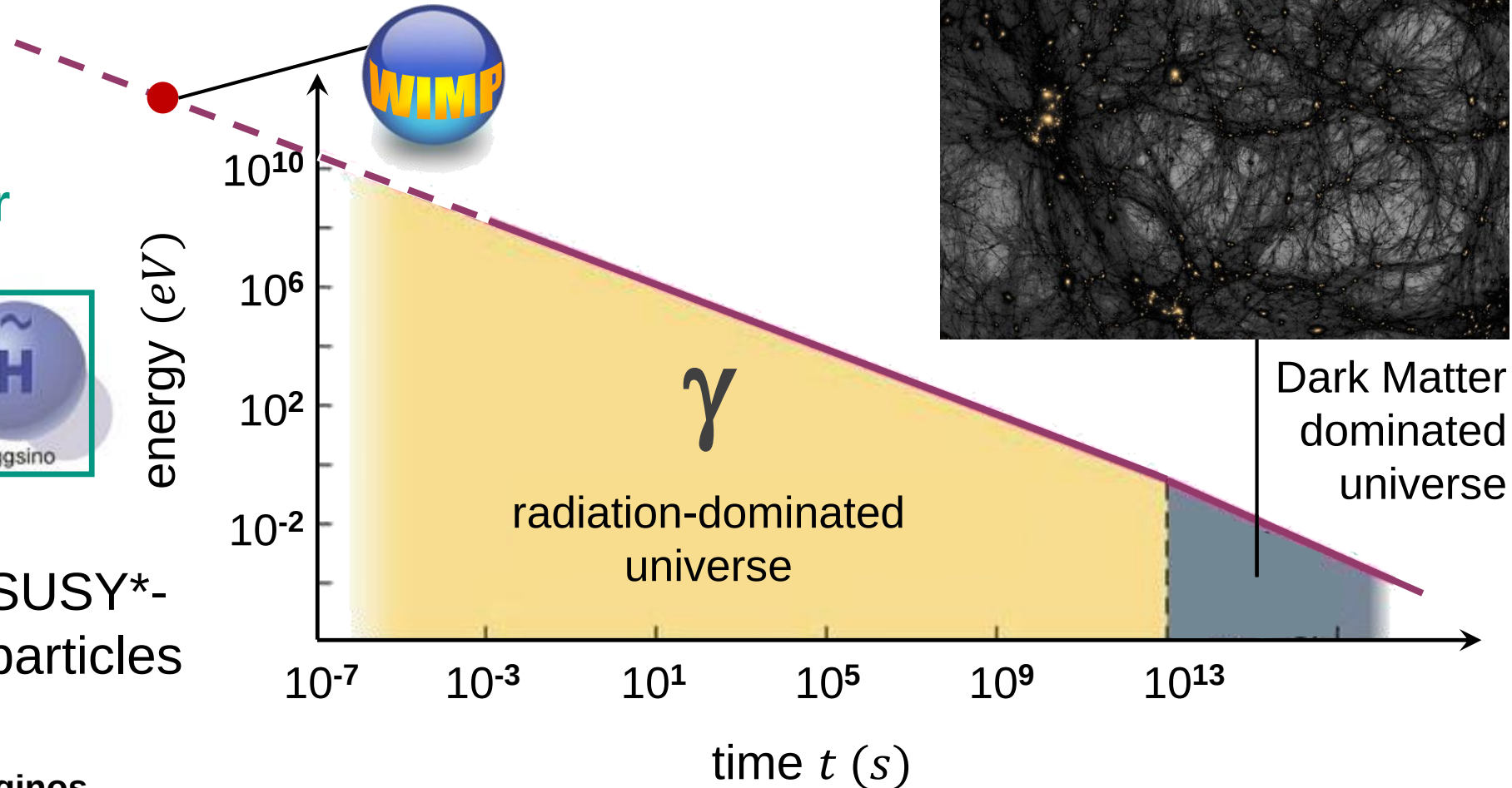
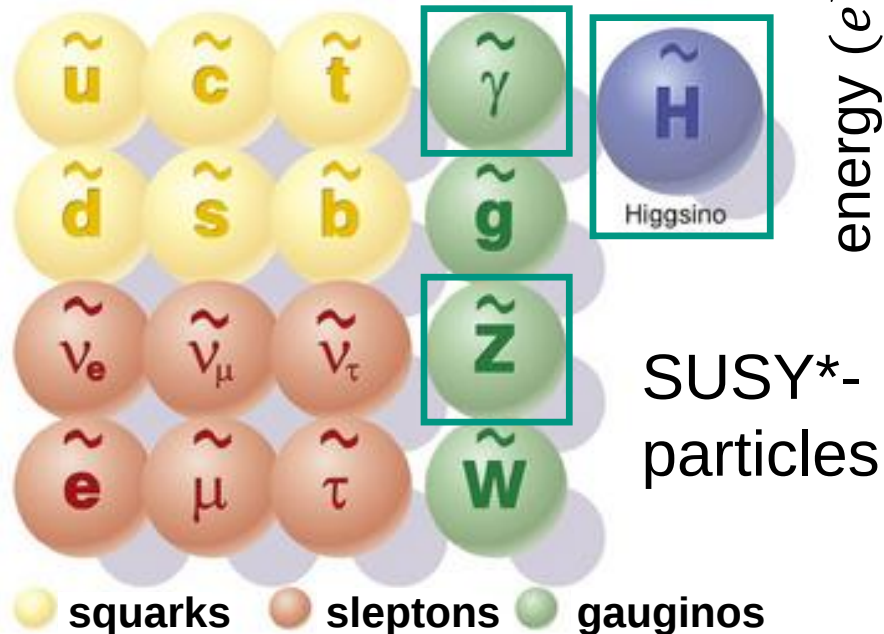


Radiation-dominated universe: Particle Zoo

■ Particle production during the Big Bang: **WIMPs**

- heat bath:

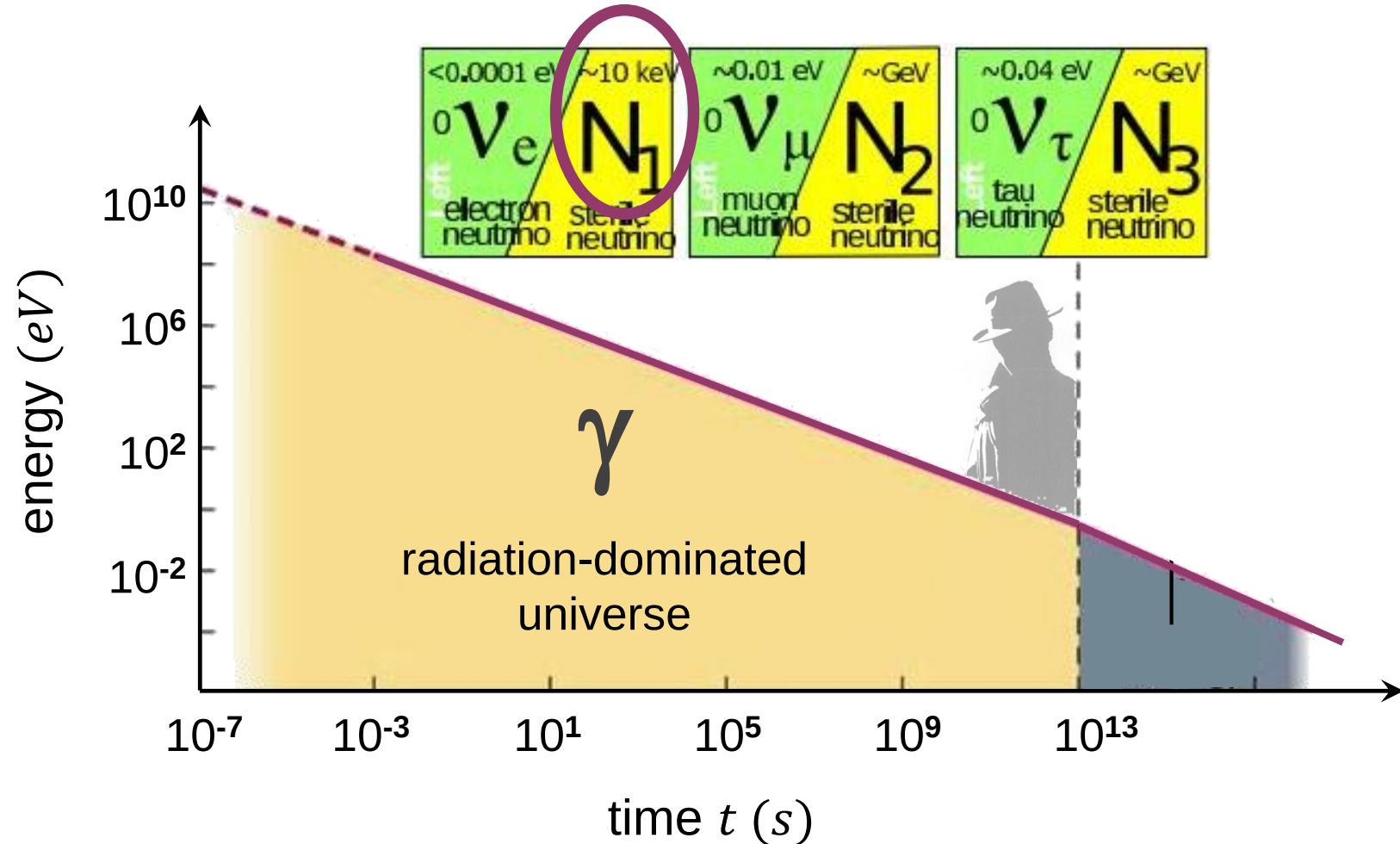
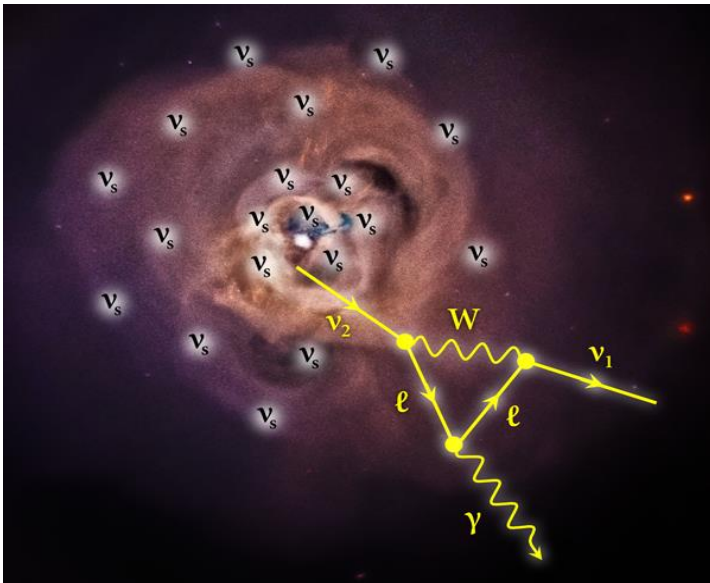
**particle interactions
generate dark matter**



Radiation-dominated universe: other processes

■ Non-thermal processes: sterile neutrinos

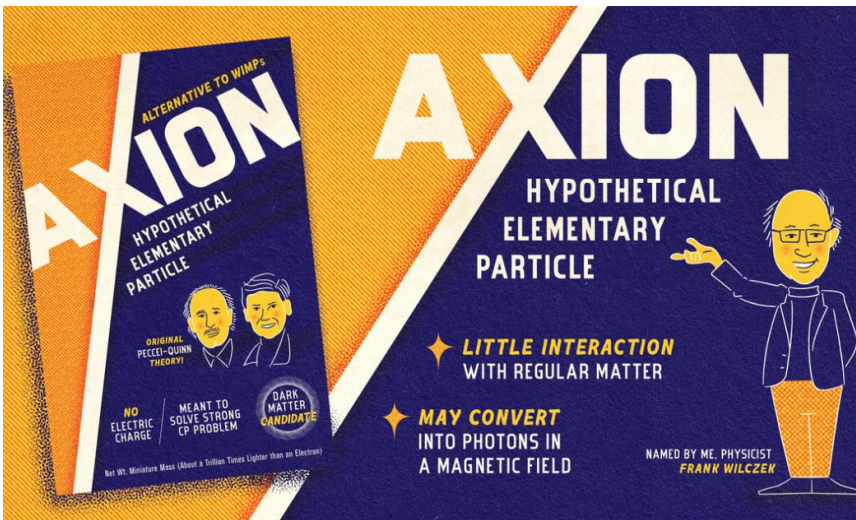
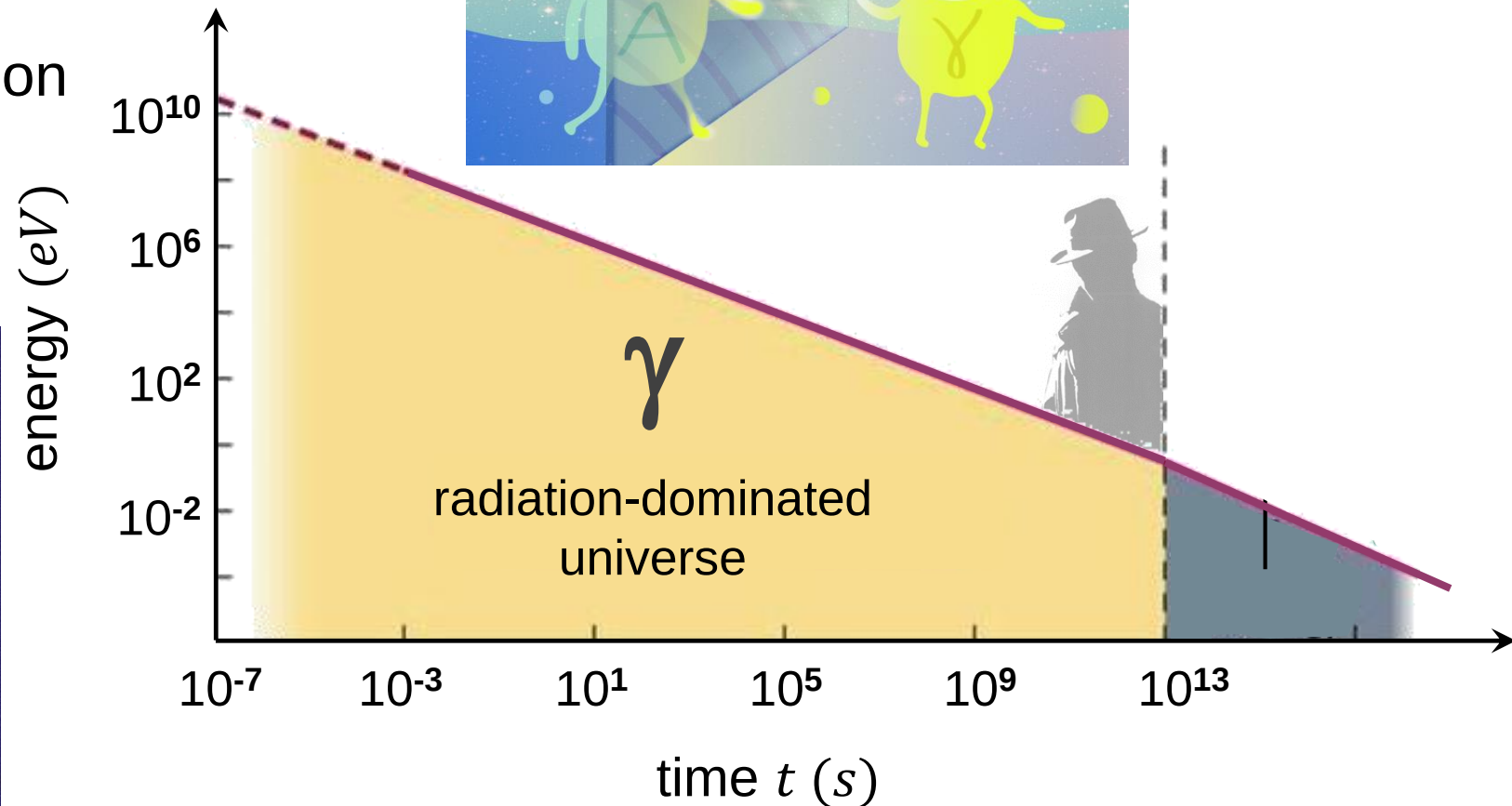
- particles with no coupling to heat bath: production via oscillations in case of sterile (RH) neutrinos



Radiation-dominated universe: other processes

■ Non-thermal processes: axions

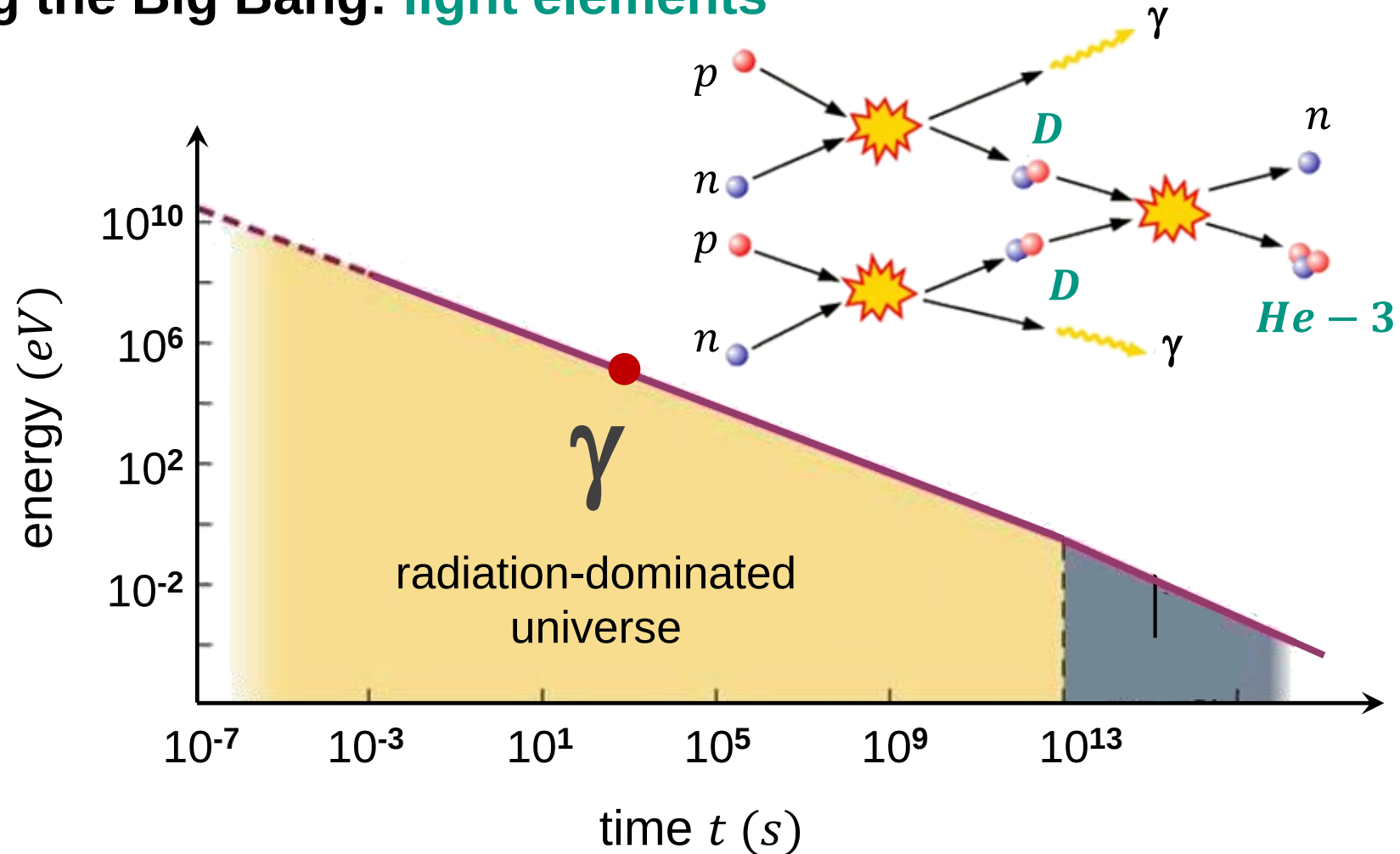
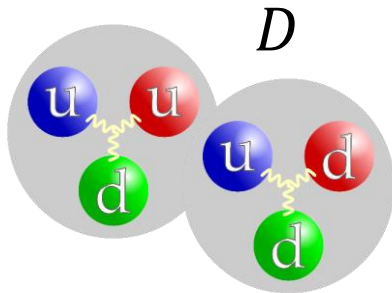
- particles with no coupling to heat bath: axion production via breaking of specific (Peccei-Quinn) symmetry



Radiation-dominated universe: Nucleosynthesis

■ Nuclear reaction during the Big Bang: light elements

- $t = 1 \dots 3 \text{ min}$
- $T \approx 10^8 \text{ K}$
- $E \approx \text{few keV}$
- fusion of light isotopes:
 $D, {}^3\text{He}, {}^4\text{He}, {}^7\text{Li}, \dots$



Radiation-dominated universe: phase transitions

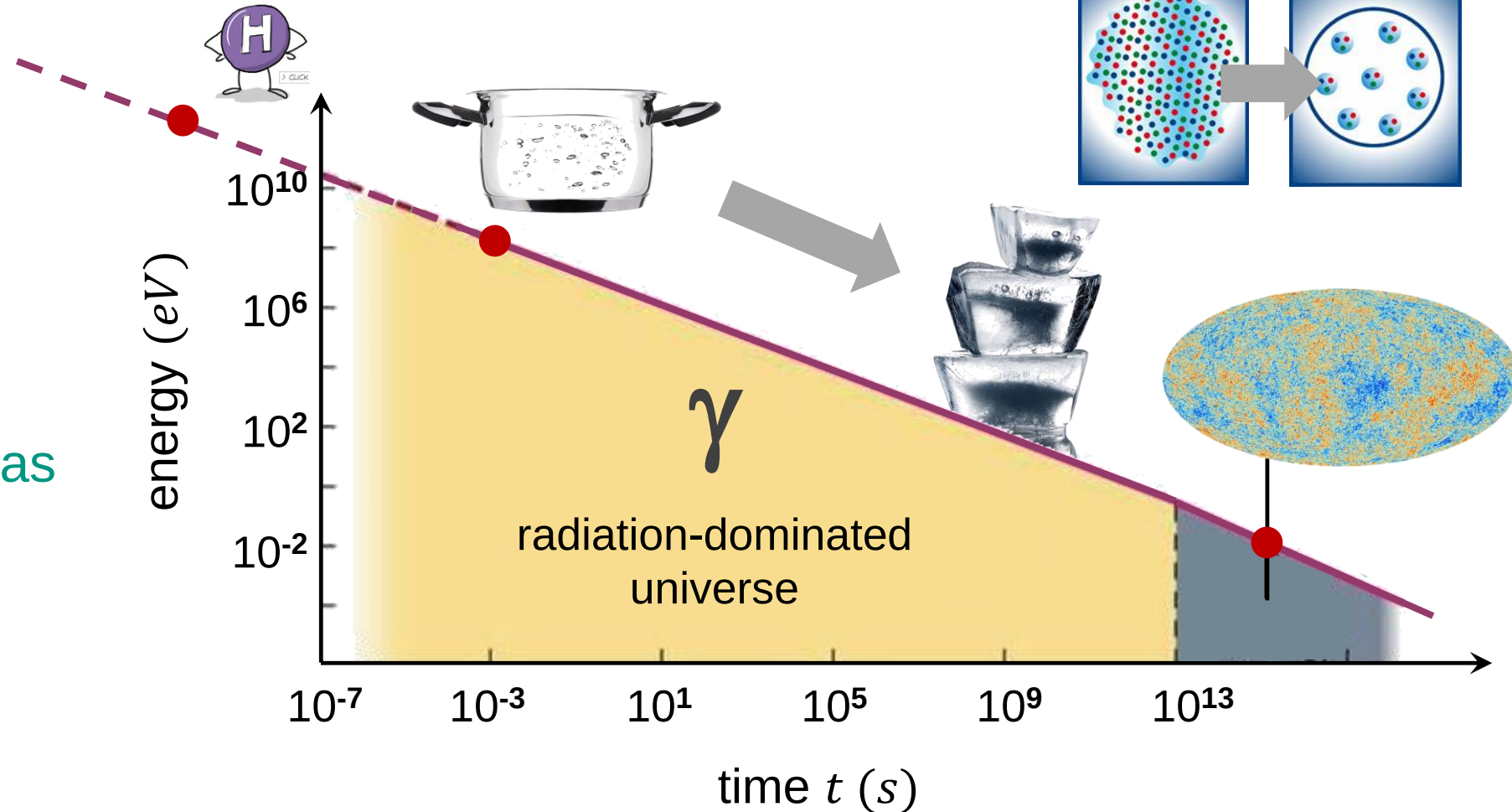
■ During expansion & cooling of universe: **phase transitions**

- many examples:

Higgs mechanism

QCD transition

plasma to atomic gas



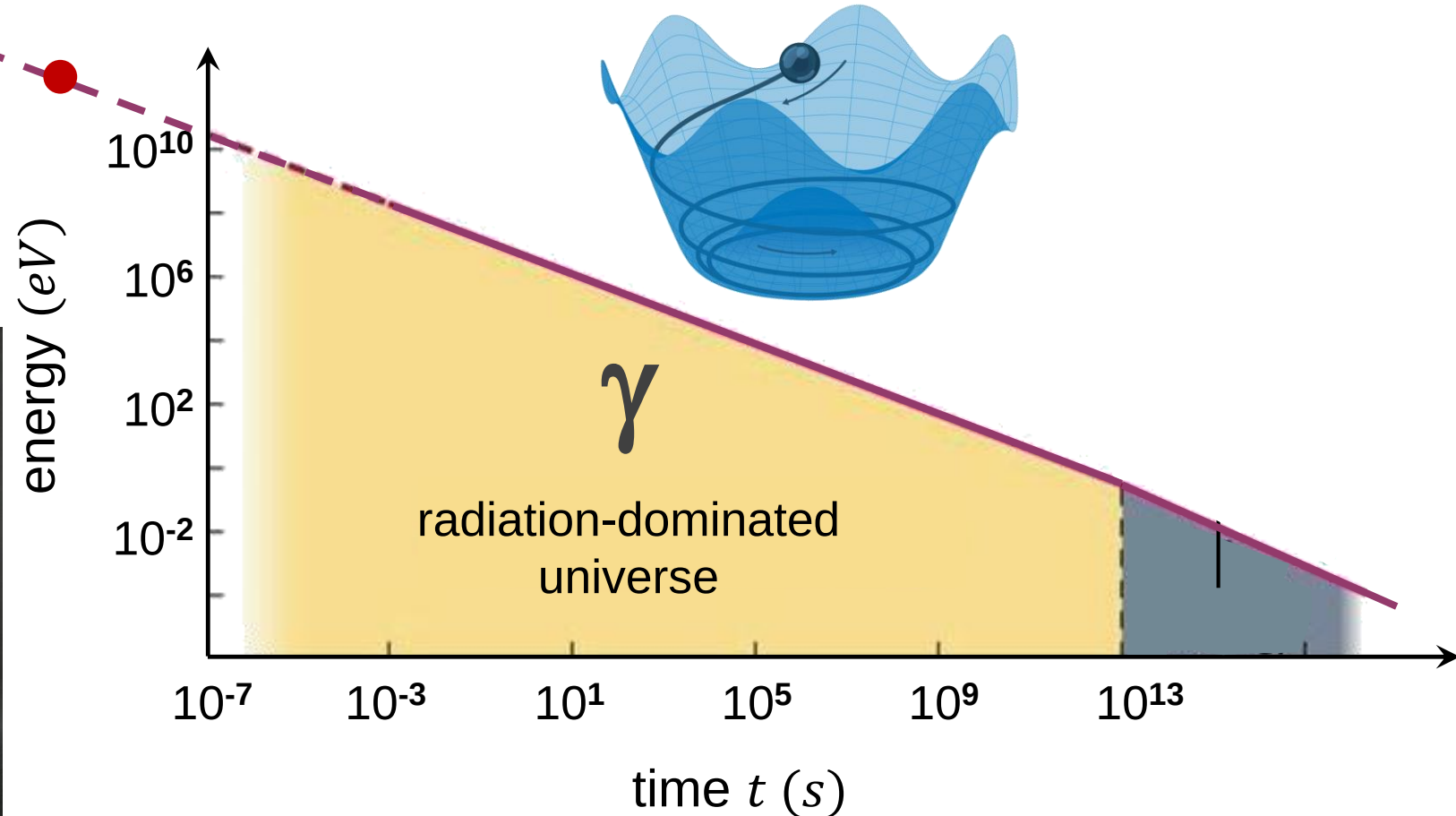
Radiation-dominated universe: phase transitions

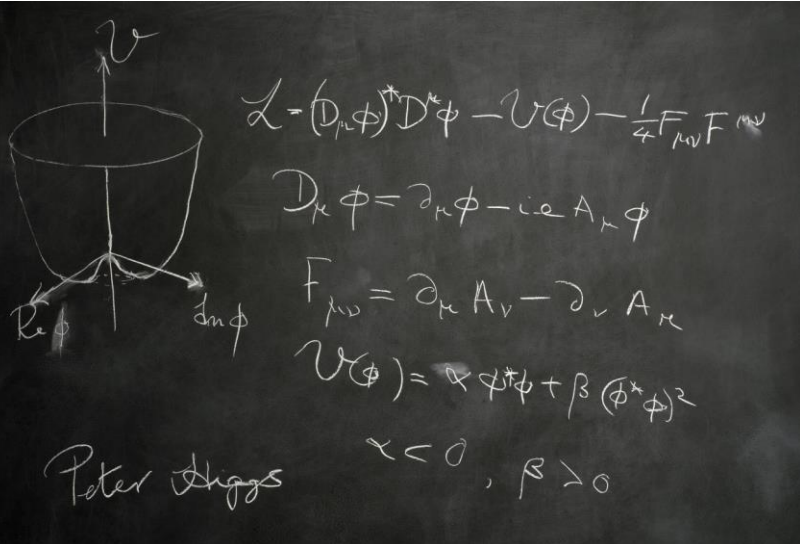
■ During expansion & cooling of universe: **electroweak phase transition**

- example:

Higgs mechanism (TeV)

⇒ particles acquire
non-zero masses





Handwritten notes on a chalkboard showing the Higgs mechanism. On the left, a diagram of a Higgs potential is shown with axes $\text{Re } \phi$ and $\text{Im } \phi$, and a vertical axis v . The potential is a paraboloid opening upwards. To the right of the diagram, the following equations are written:

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - U(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$D_\mu \phi = \partial_\mu \phi - ie A_\mu \phi$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$U(\phi) = \alpha \phi^\dagger \phi + \beta (\phi^\dagger \phi)^2$$

Below the equations, it is noted that $\alpha < 0$ and $\beta > 0$. The name 'Peter Higgs' is written at the bottom left.

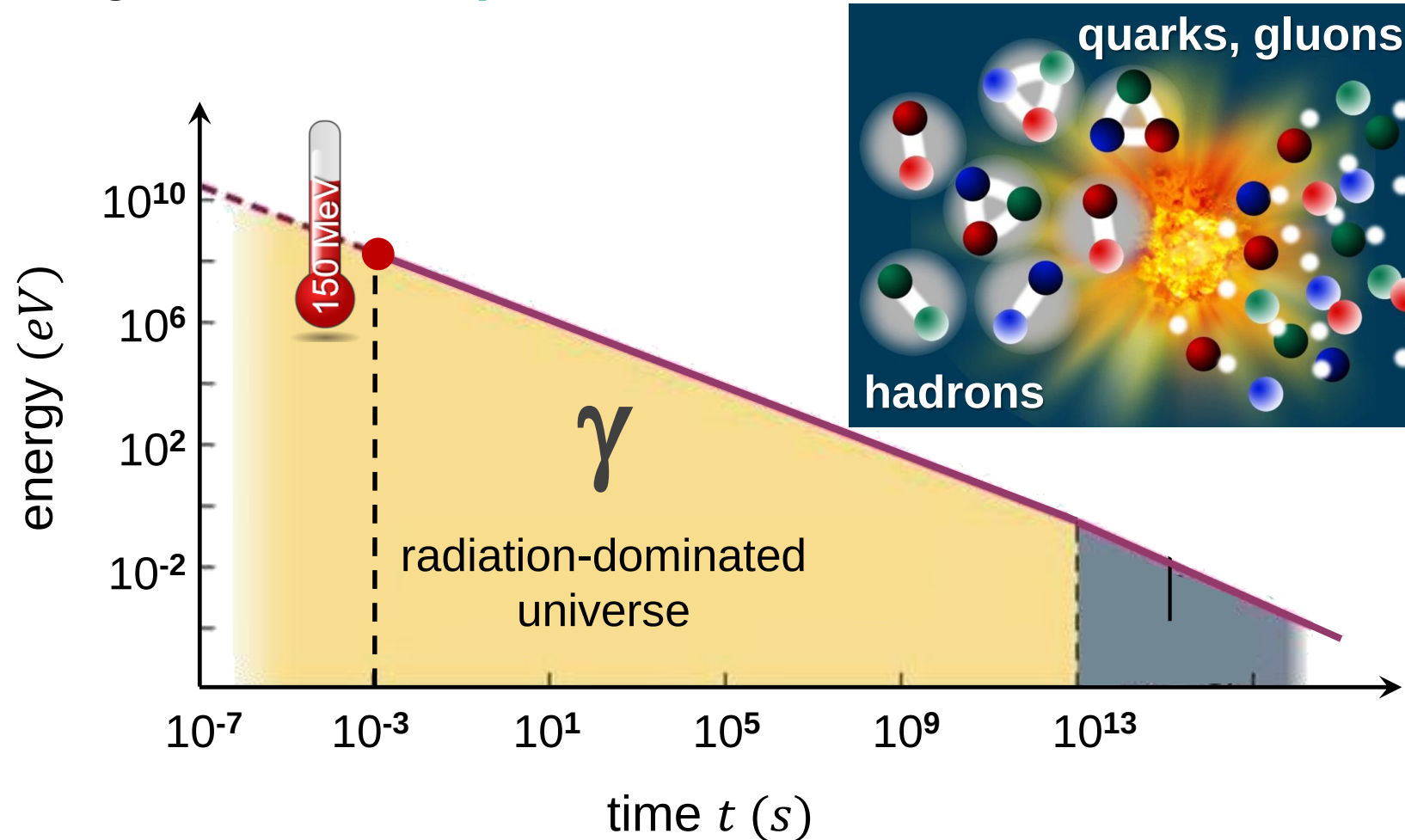
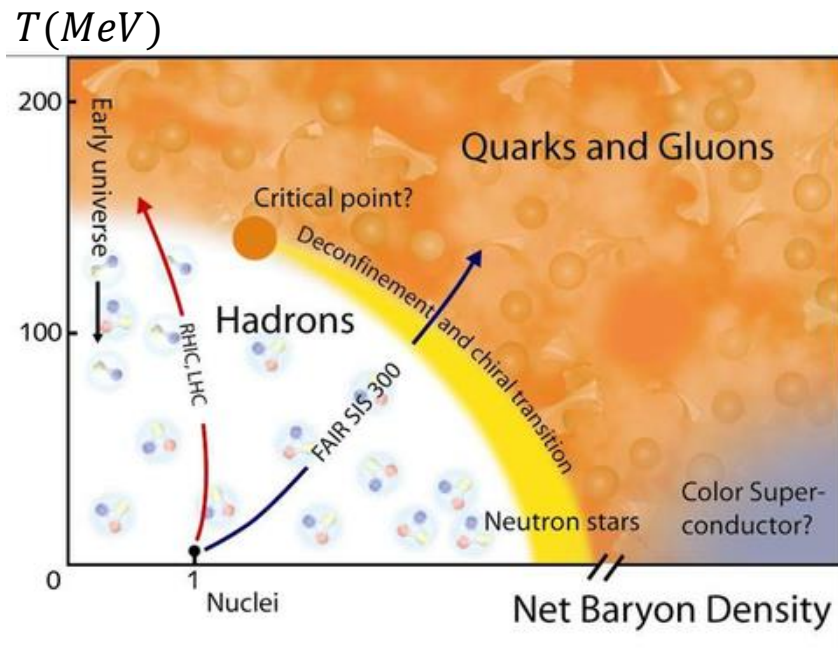
Radiation-dominated universe: phase transitions

■ During expansion & cooling of universe: phase transition of QCD

- example:

QCD at 150 MeV:

⇒ quarks form hadrons:



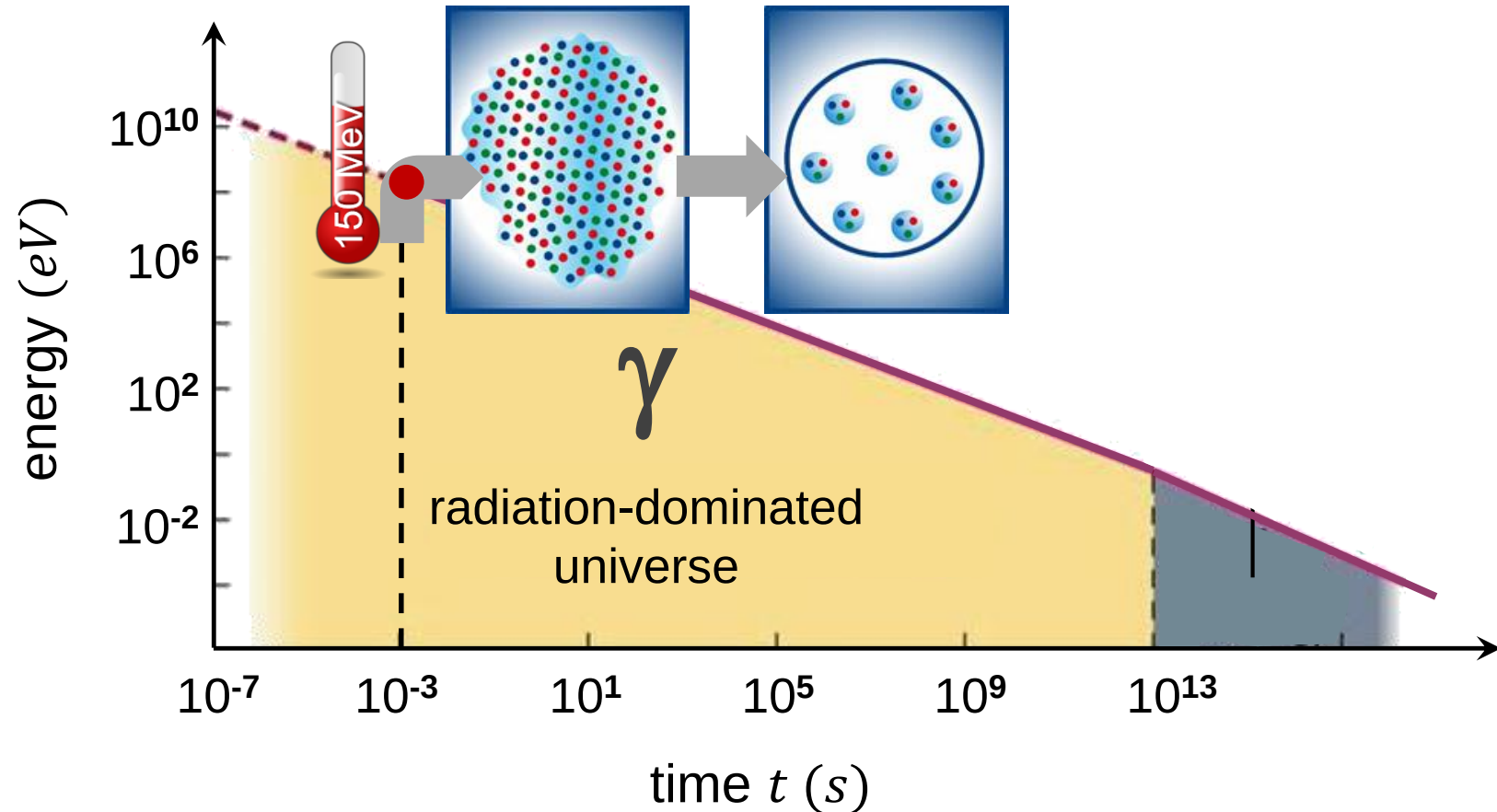
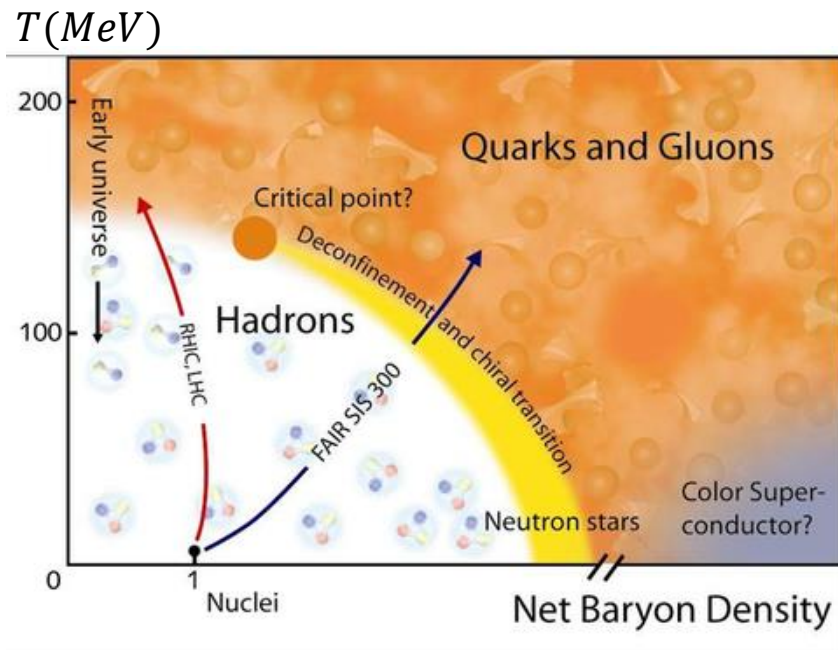
Radiation-dominated universe: phase transitions

■ During expansion & cooling of universe: **phase transition of QCD**

- example:

QCD at 150 MeV:

⇒ quarks form hadrons:



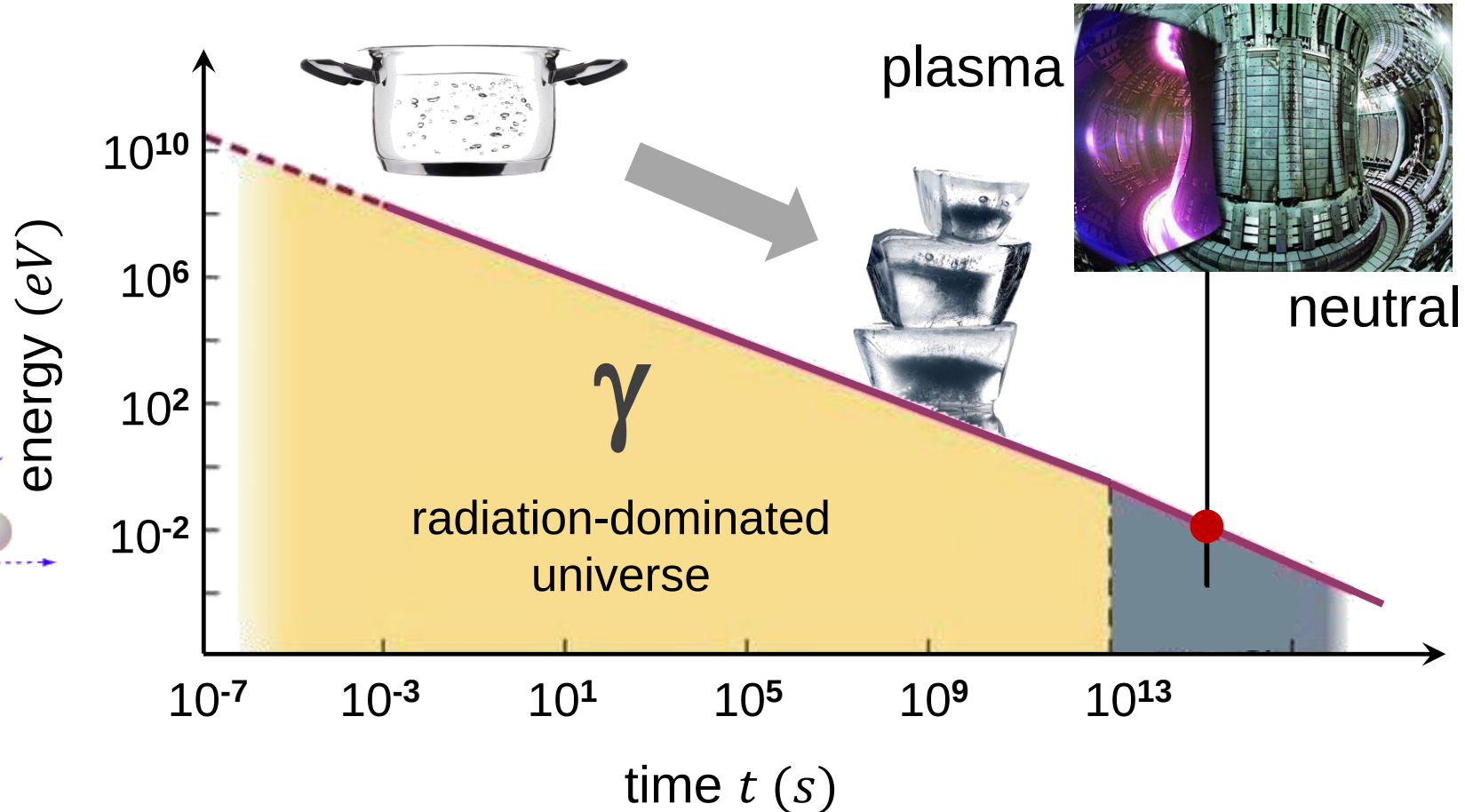
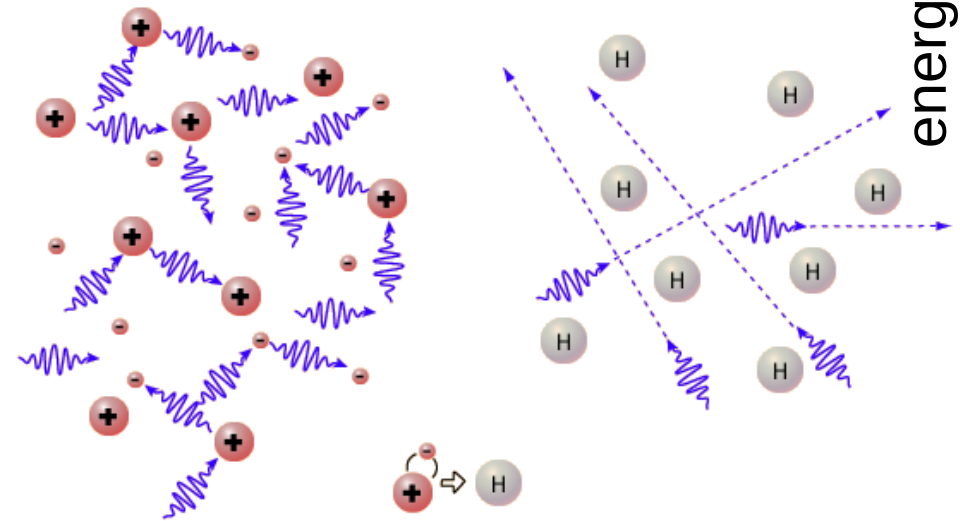
Radiation-dominated universe: phase transitions Karlsruhe Institute of Technology

■ During expansion & cooling of universe **several phase transitions** took place

- example:

plasma → **atoms**

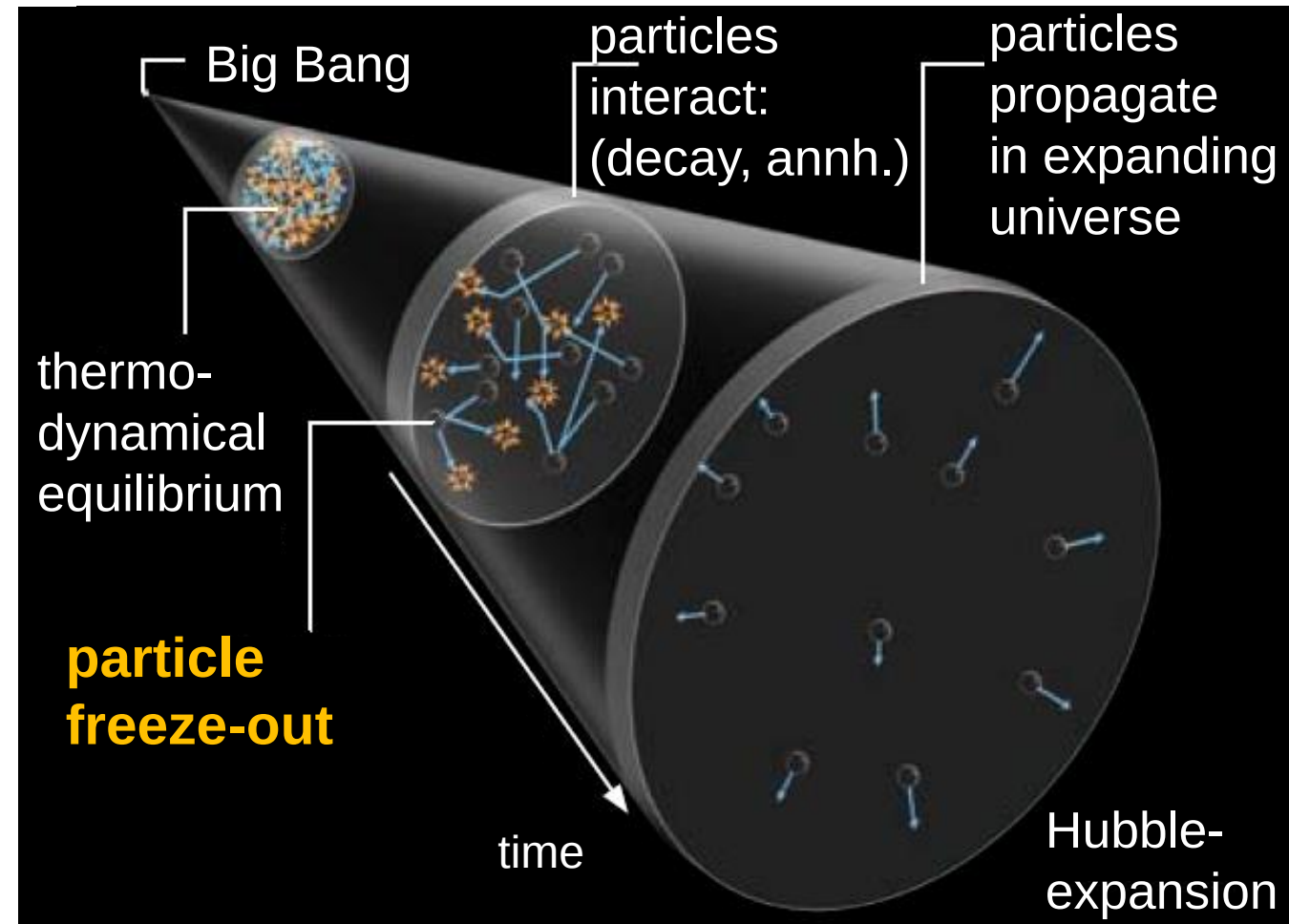
⇒ neutral atoms form,
different opacity for γ 's



Radiation-dominated universe: freeze-out

■ During expansion of universe: **freeze-out** processes are important

- **characteristic phases:**
- thermodynamical equilibrium: Boltzmann-distribution
- interactions with neighbours
- **freeze-out** & de-coupling
- free propagation

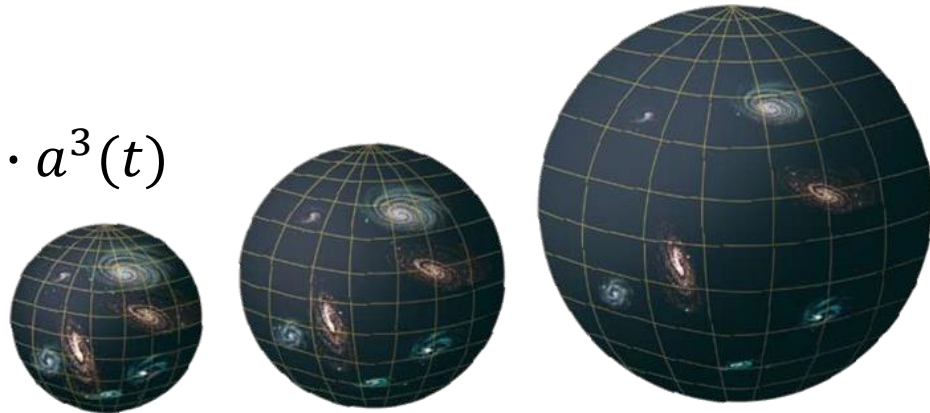


Radiation-dominated universe: equilibrium

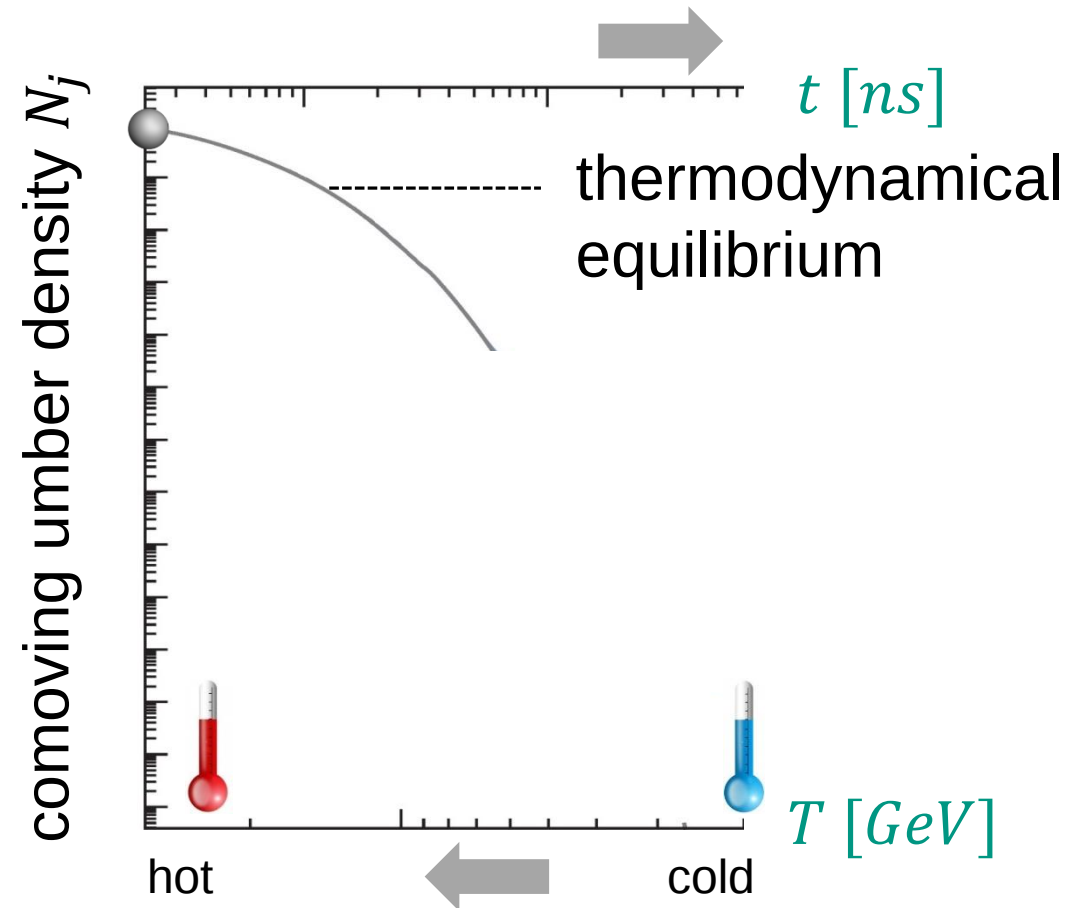
■ During expansion of universe: **phase 1 – thermal equilibrium**

- all particle number densities in **comoving coordinates** $\vec{x}$
⇒ density decrease due to the expansion of the universe taken into account

$$\rho_0 = \rho(t) \cdot a^3(t)$$



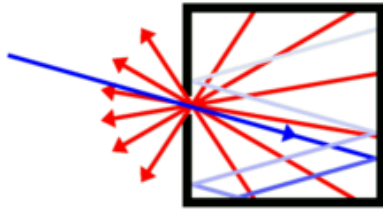
$$\vec{r}(t) = a(t) \cdot \vec{x}$$



Radiation-dominated universe: particle ensemble

■ During expansion of universe: phase 1 – Boltzmann distribution

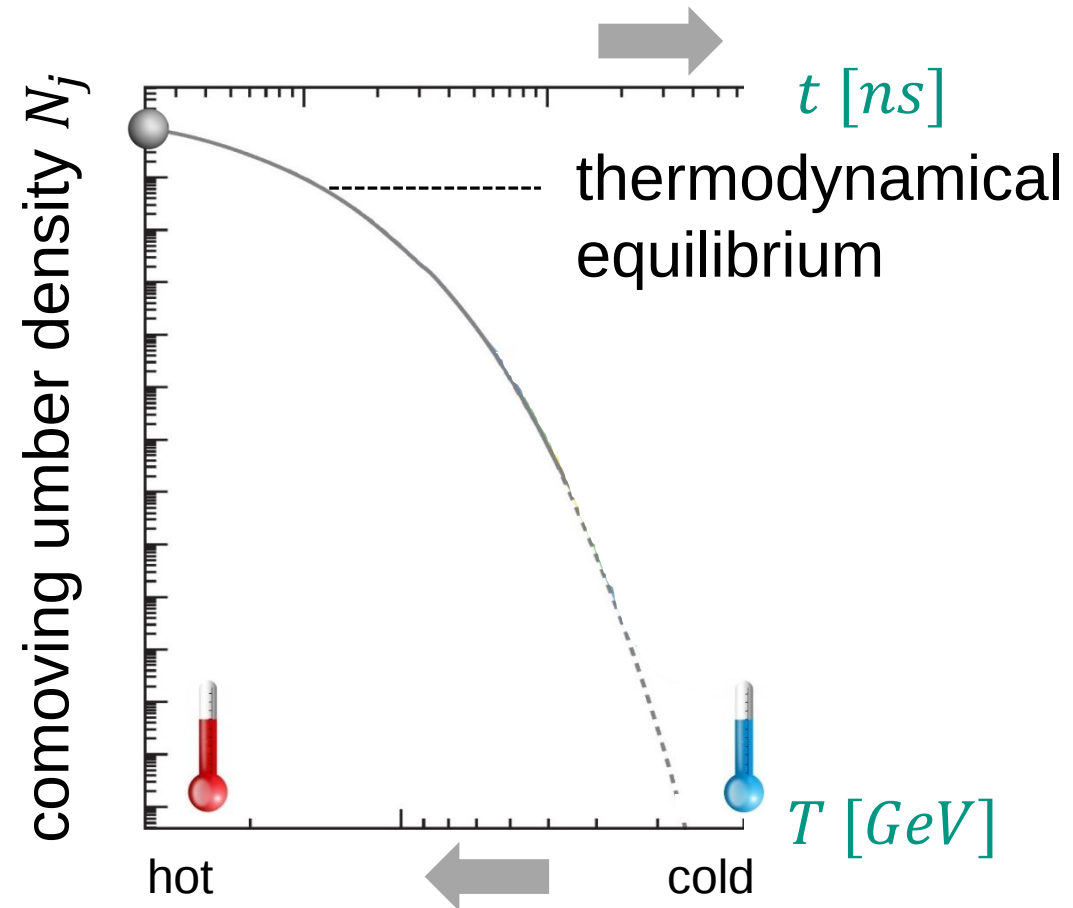
- particle species: production & annihilation (thermal heat bath)



- particle number density N_j defined by 2 free parameters:

particle energy (mass) $E_j = M_j \cdot c^2$

Temperature heat bath $E_{hb} = k_B \cdot T$



Radiation-dominated universe: particle ensemble

■ During expansion of universe: phase 1 – Boltzmann distribution

- evolution of number density

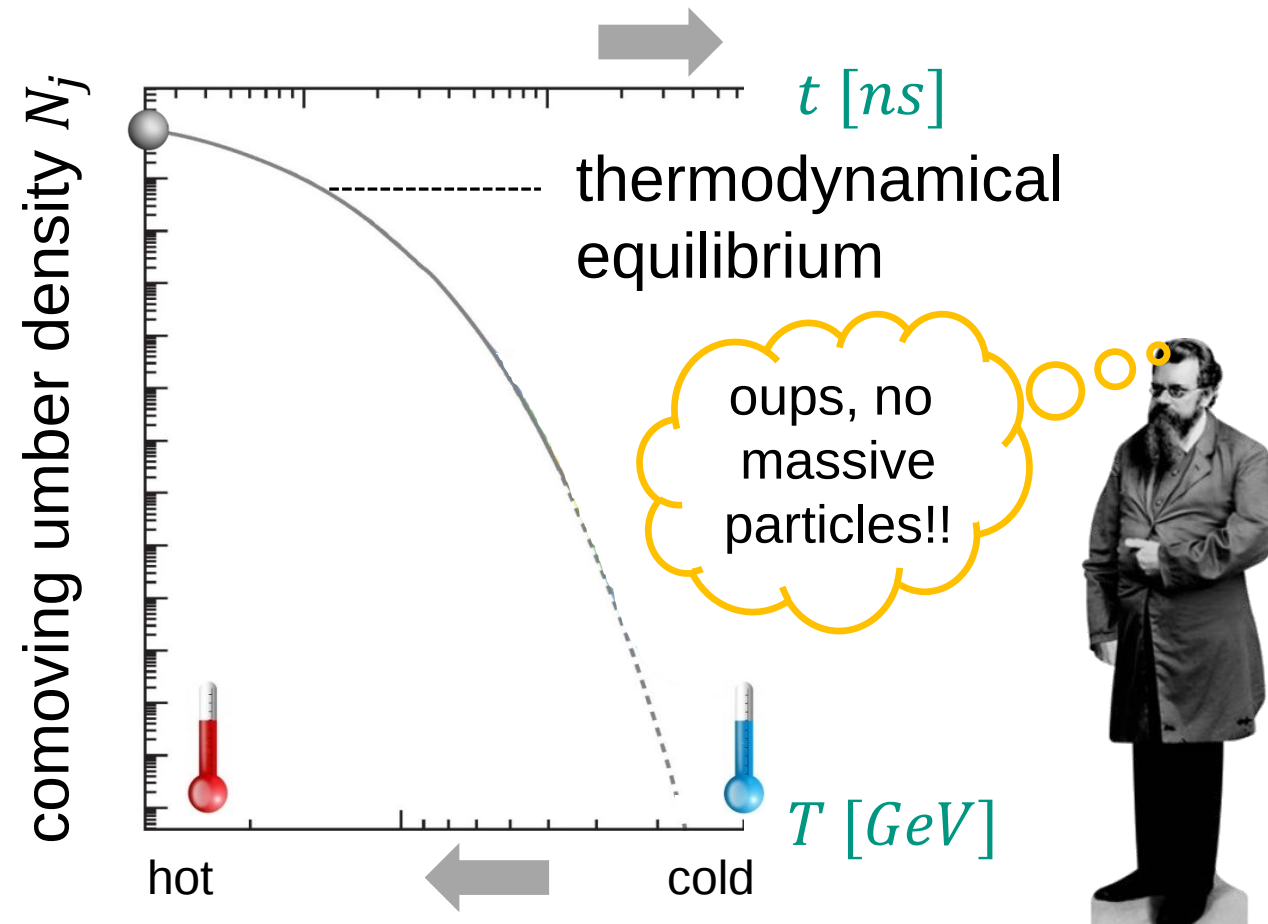
$$N_j = N_0 \cdot g_j \cdot e^{-E_j / k_B T}$$

N_0 : primary particle number density

N_j : actual particle (energy E_j) number density at temperature T

g_j : intrinsic degree of degeneracy (fermions, bosons)

⇒ for $T \rightarrow 0$ we have $N_j \rightarrow 0$



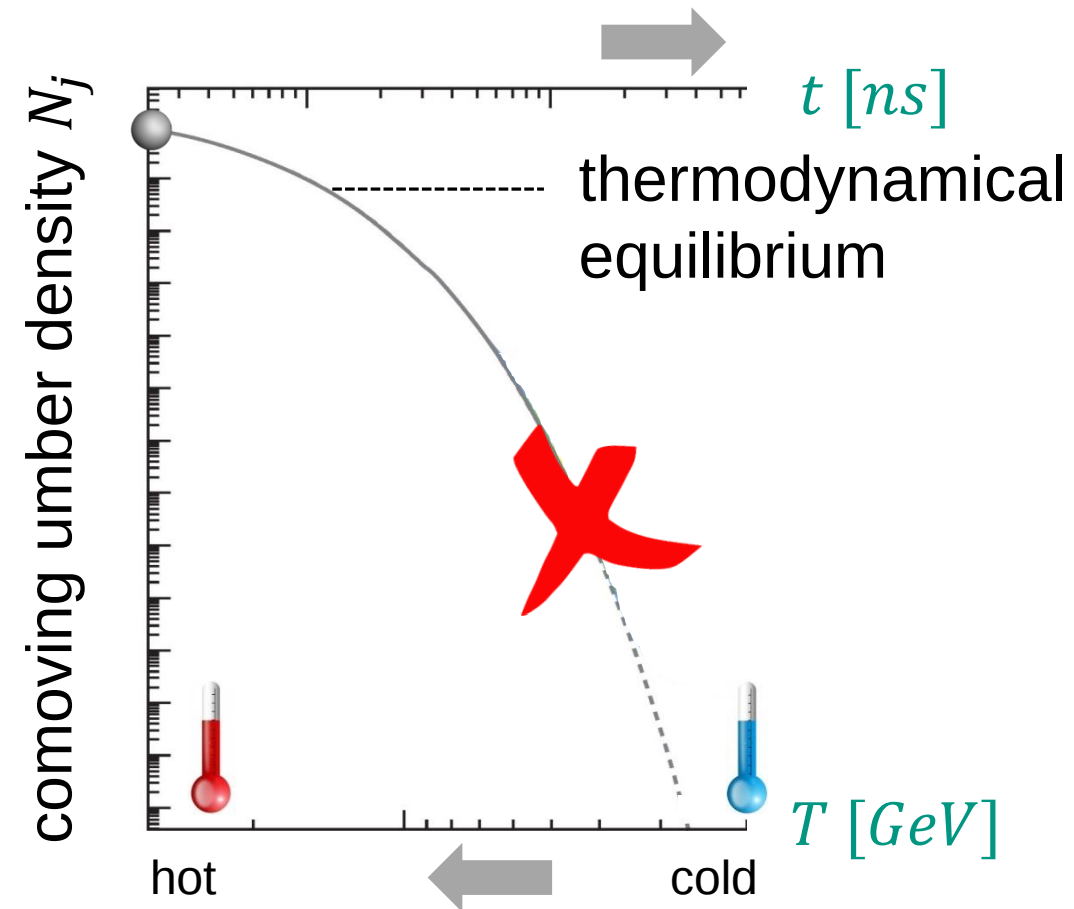
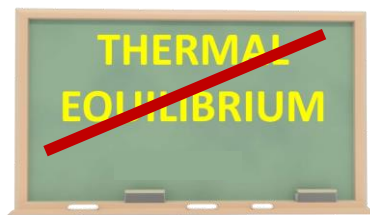
Radiation-dominated universe: particle ensemble

■ During expansion of universe: phase 2 – breaking of thermal equilibrium

- evolution of number density

$$N_j = N_0 \cdot g_j \cdot e^{-E_j/k_B T}$$

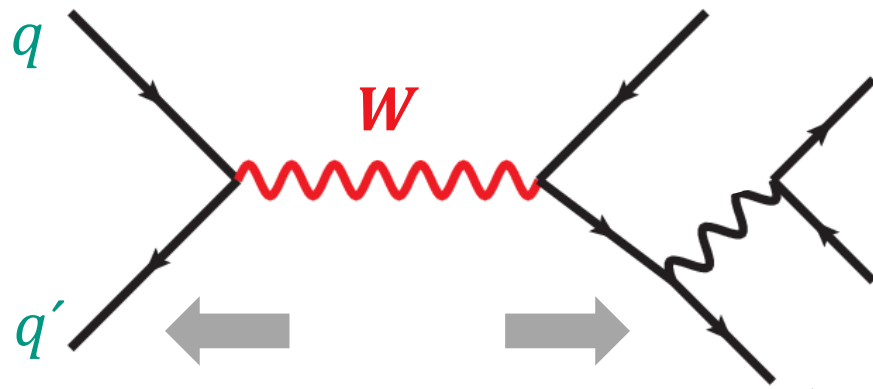
- finite number density $N_j \neq 0$
requires breaking of thermal equilibrium
(“decoupling from heat bath”)



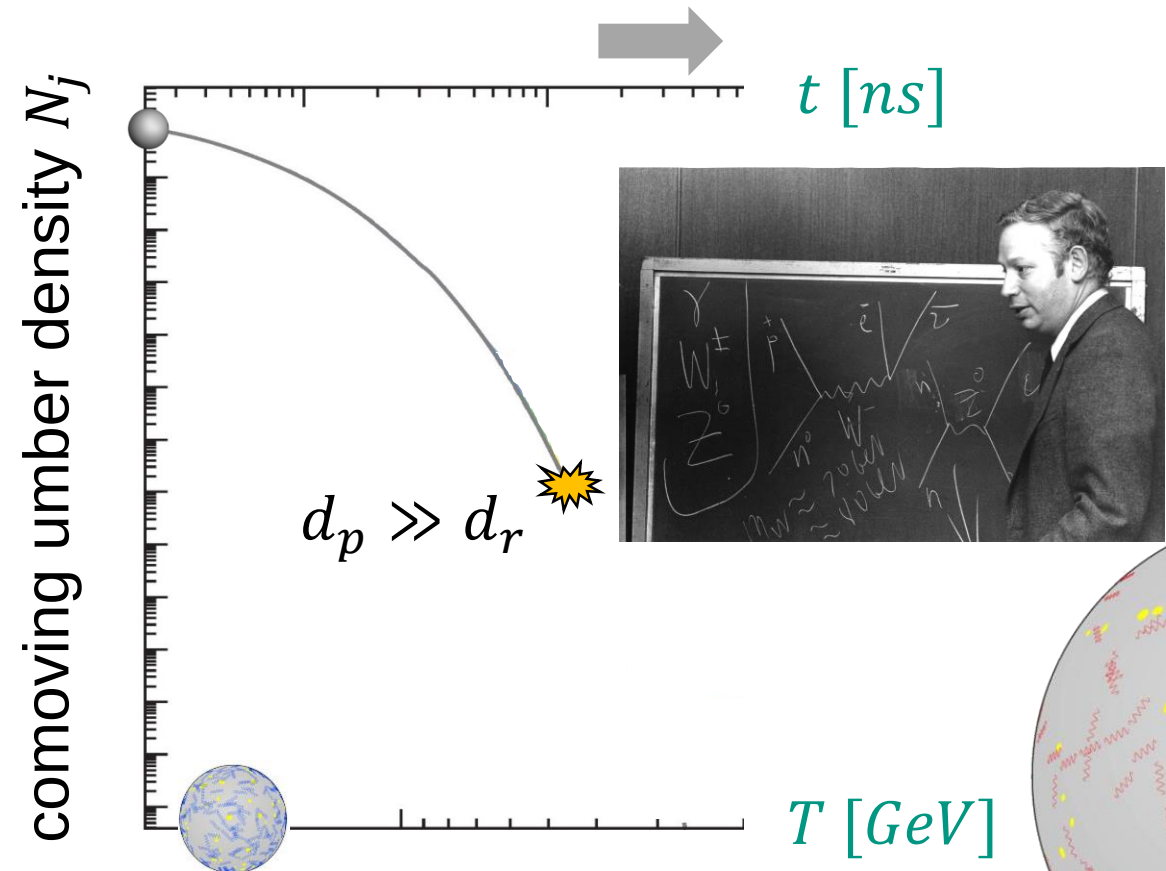
Radiation-dominated universe: particle ensemble

■ During expansion of universe: phase 2 – breaking of thermal equilibrium

- expansion of the universe takes place in 'real' coordinates $\vec{r}$



- exchange particles W, Z, g with finite range d_r (or lifetime)
- increasing distance d_p of particles



Radiation-dominated universe: particle ensemble

■ During expansion of universe: phase 3 – freeze-out of particles

- decoupling from heat bath at specific time t / temperature T :

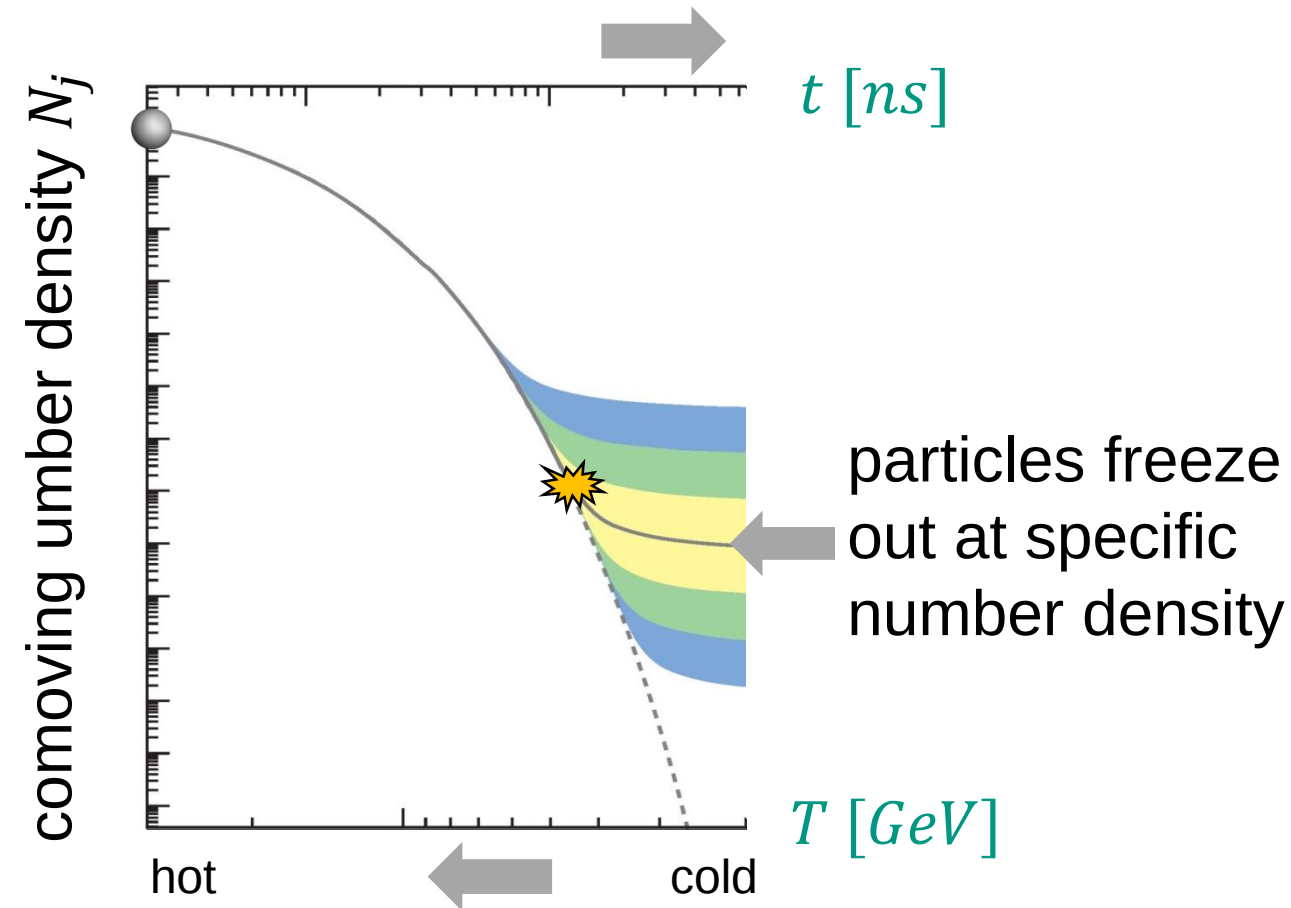


$$\Gamma(t) = H(t)$$



Hubble expansion rate $H(t)$

particle interaction rate $\Gamma(t)$



Radiation-dominated universe: neutrino freeze-out

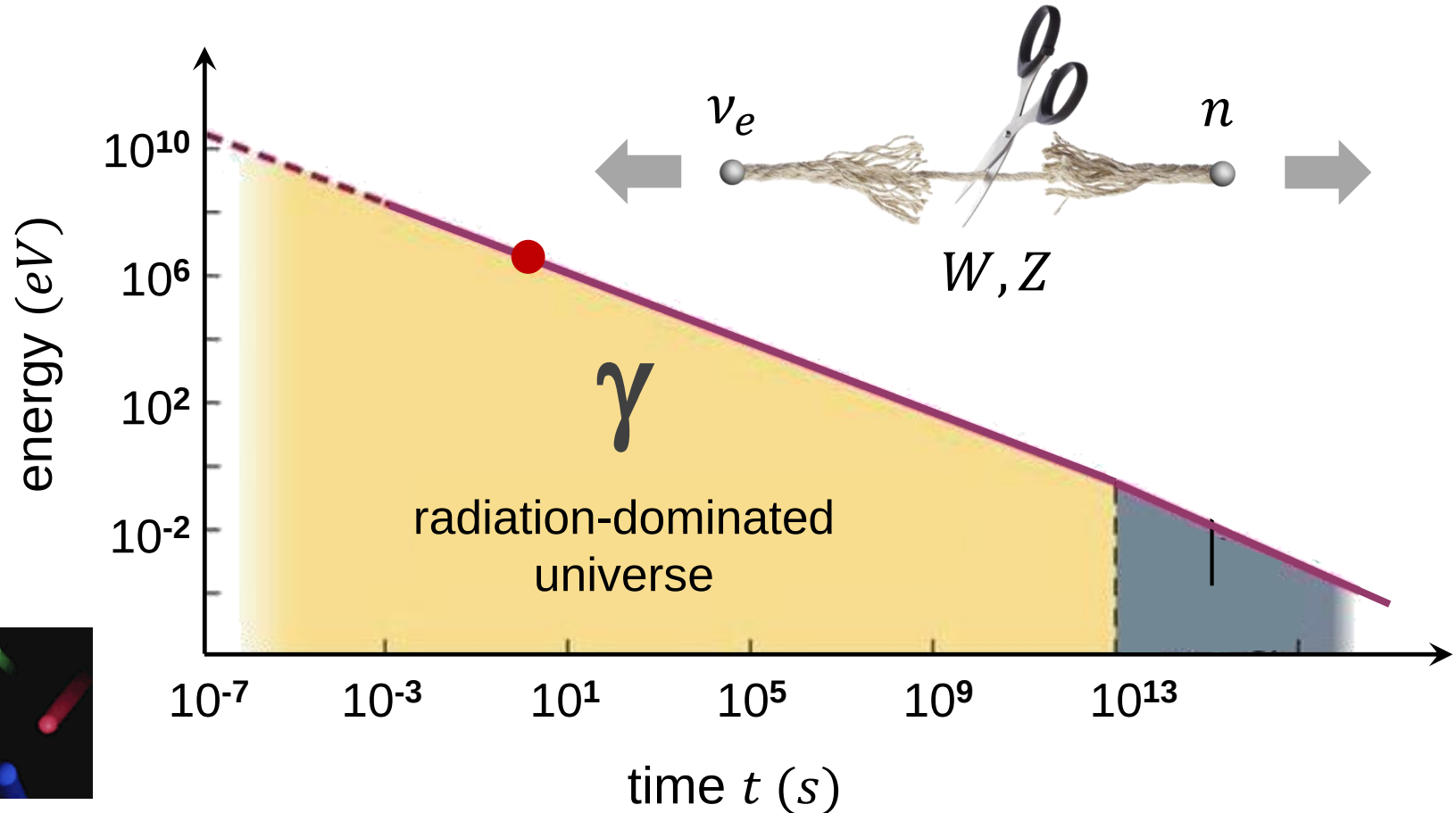
■ During expansion of universe: **freeze-out** processes of ν 's are important

- example:
neutrinos from Big Bang
decouple if

$$\Gamma_{\nu}(t) = H(t)$$

weak interaction
cross section

BIG FREEZE OUT



Radiation-dominated universe: neutrino freeze-out

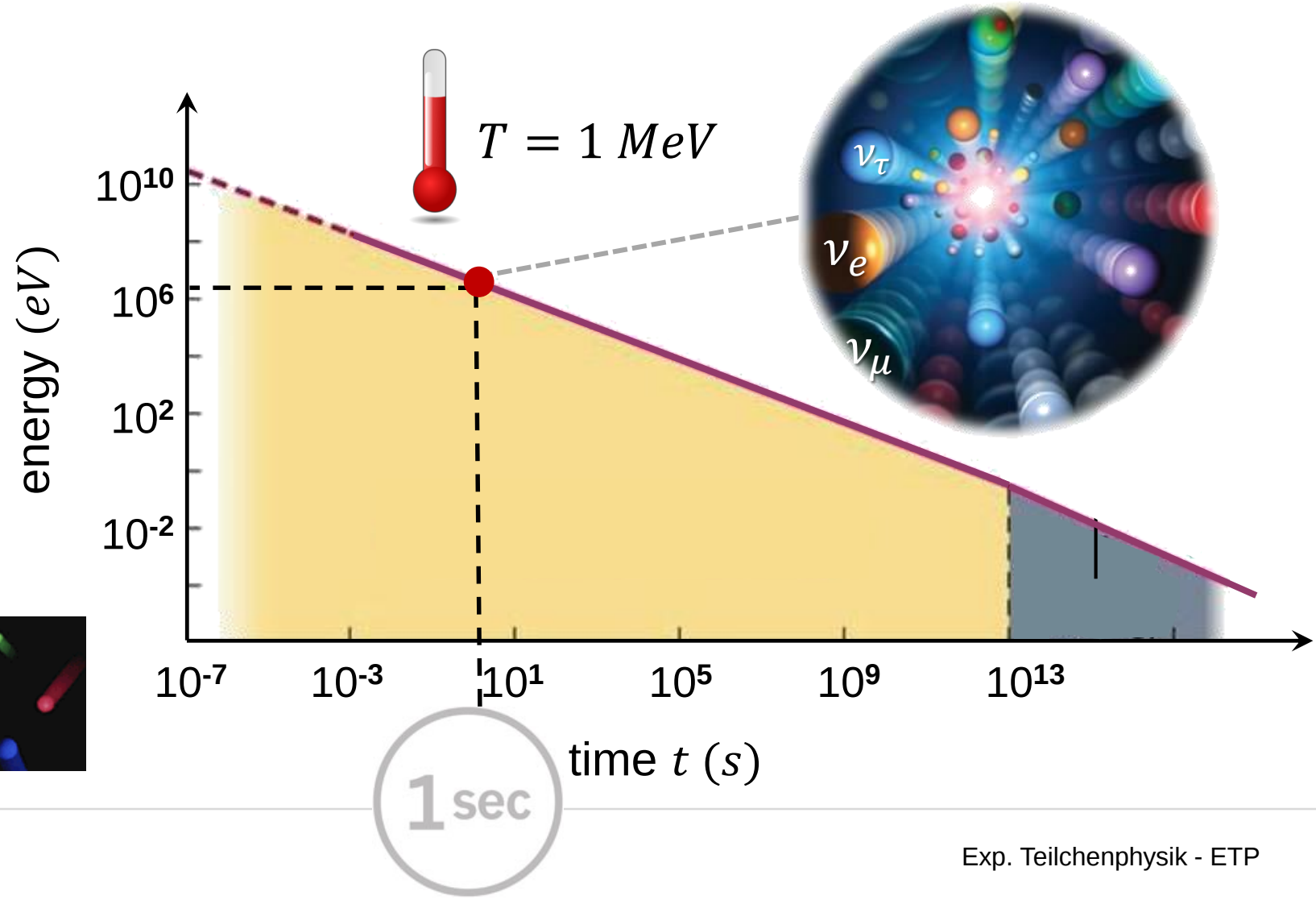
- During expansion of universe: **freeze-out** processes of ν 's are important

$$T(t = 1\text{ s}) = 1\text{ MeV}$$

$$t = 1\text{ s}$$

$$\Gamma_\nu(t) = H(t)$$

weak interaction
cross section



BIG FREEZE OUT

Radiation-dominated universe: neutrino freeze-out

KIT
Karlsruhe Institute of Technology

■ During expansion of universe: **freeze-out** processes of ν 's are important

- since $t = 1\text{ s}$ neutrinos from the Big Bang are 'free-streaming'
- detection of 'relic ν 's' allows to take a **picture of the universe at $t = 1\text{ s}$**
- goal of project KATRIN++

