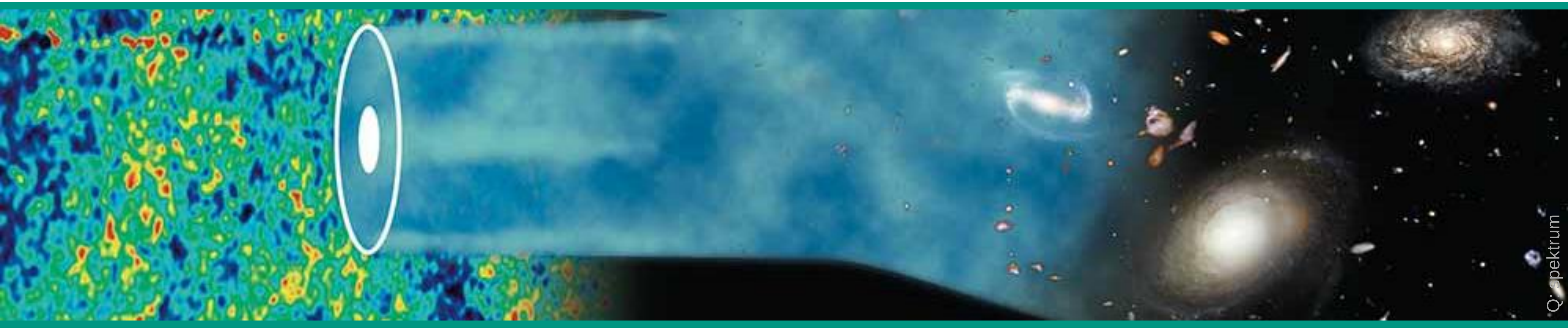
Introduction to Cosmology

Winter term 22/23 Lecture 6 Dec. 6, 2022



Recap of Lecture 5

- **Thermal universe: the radiation-dominated era with phase transitions**

  - temperature evolution: $\mathrm{T(t)} \sim \sqrt{t}$ with fixpoint $T = 1\ MeV$ at $t = 1\ s$

  - particles (energy $\boldsymbol{E_J}$) in thermodynamical equilibrium with heat bath (photons)

  - resulting Boltzmann distribution with $N_J(\mathrm{t}) \sim e^{-\boldsymbol{E_J}/k_B \boldsymbol{T(t)}}$

  - condition for **freeze-out**: $\Gamma(t) = H(t)$ ⇔ interaction rate = Hubble expansion

  - photons from matter – anti-matter annihilation, tiny **baryon asymmetry** $\boldsymbol{\eta}$

  - QCD – phase transition: quark-gluon plasma → nucleons (p, n)

RECAP: neutrino freeze-out

- **during expansion of universe: freeze-out processes of ν´s are important**

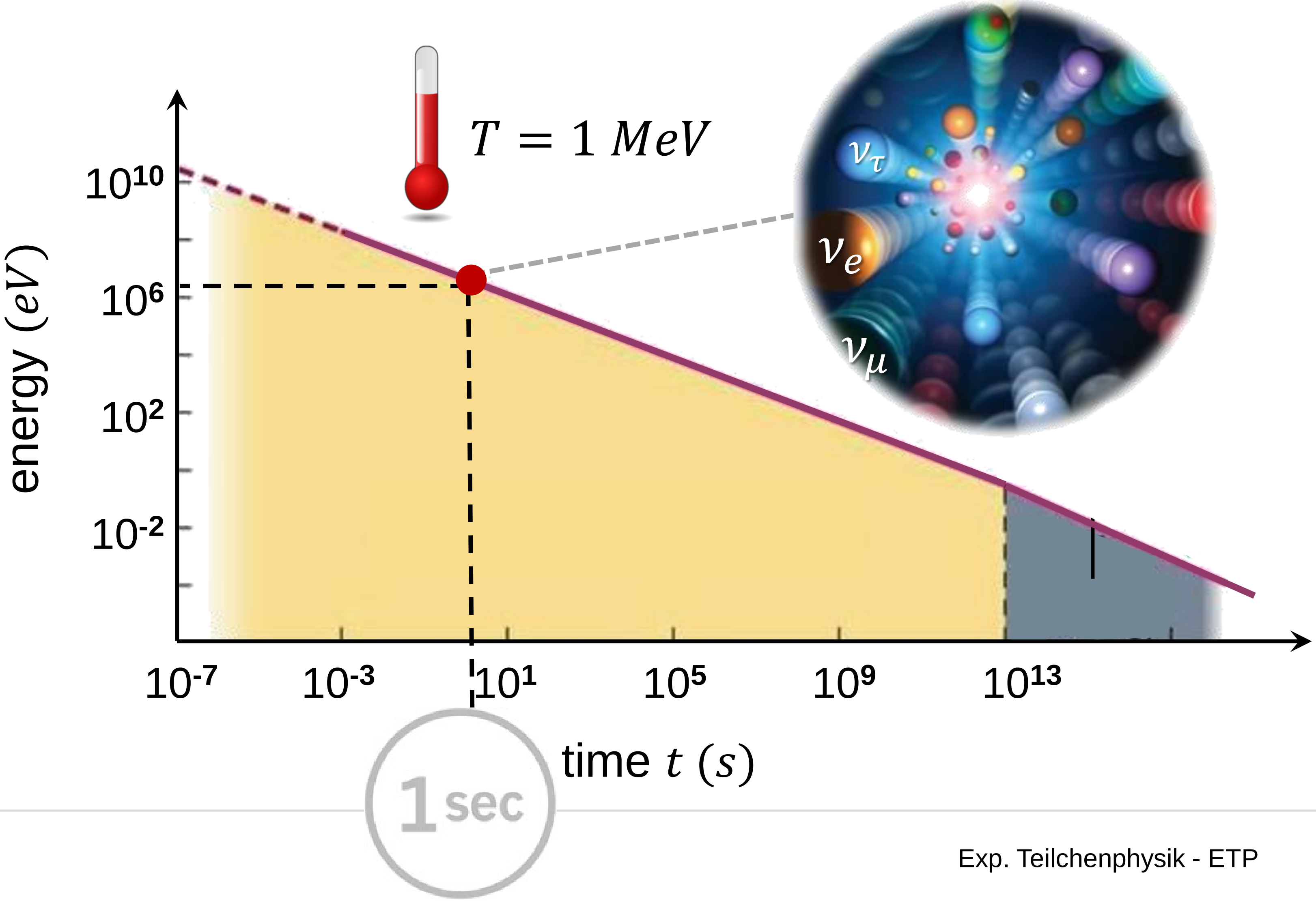
- since $t = 1\ s$ neutrinos from the Big Bang are ´free-streaming´

$T(t = 1\ s) = 1\ MeV$

$t = 1\ s$

$$\Gamma_\nu(t) = H(t)$$

weak interaction cross section



neutrino freeze-out & # of relativistic particles

- **An important time stamp: ν´s decouple at $t = 1\ s$ & at $T = 1\ MeV$**

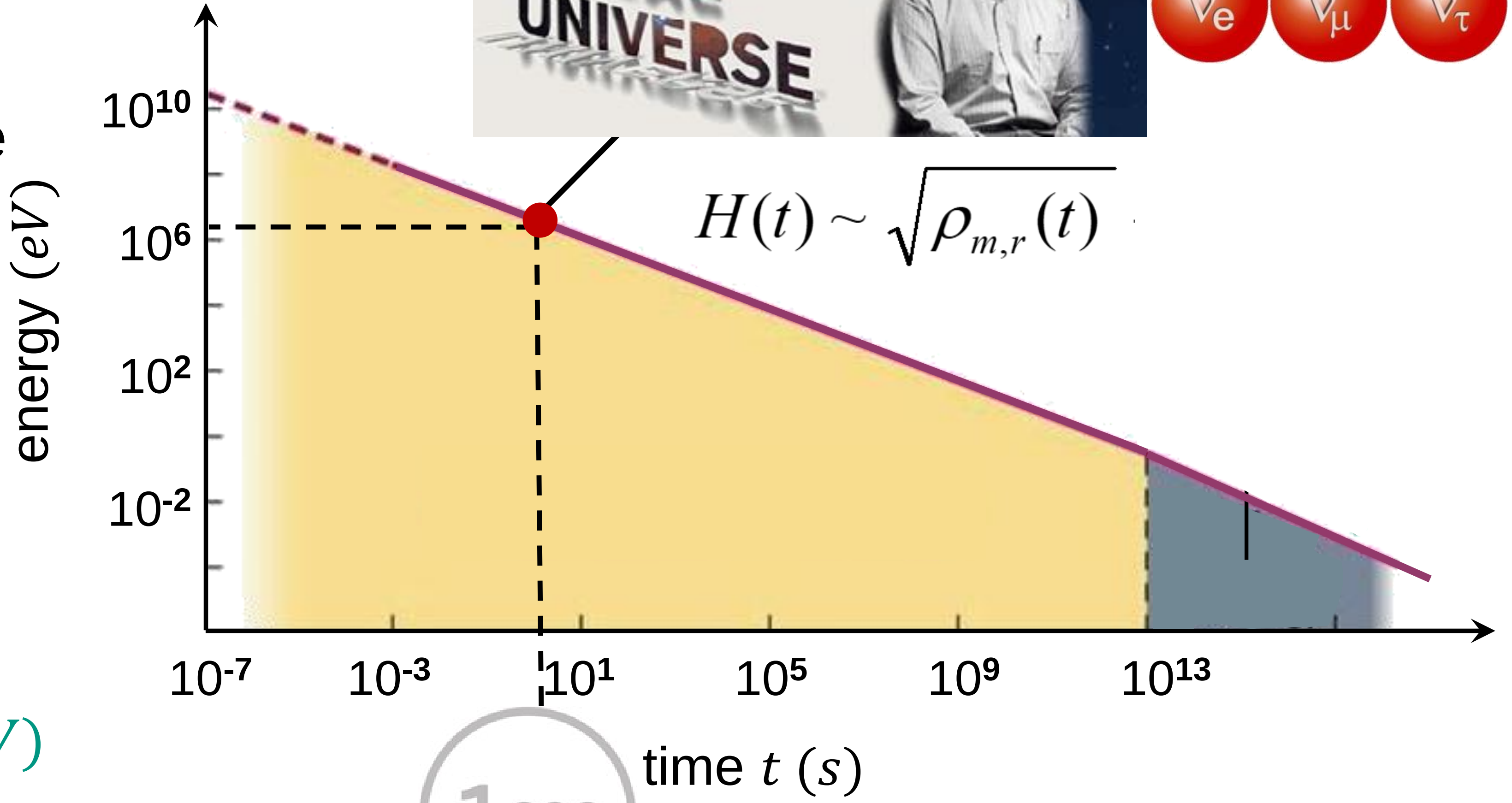
- how many relativistic ´degrees of freedom´ were present at this time?*

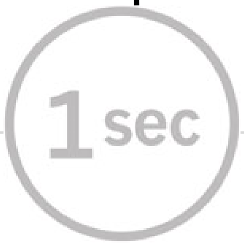
- preview: **relativistic particles** at $E = 1\ MeV$

$\boldsymbol{\gamma}$: ☑ $(m_\gamma = 0)$

$\boldsymbol{\nu_e\, \nu_\mu\, \nu_\tau}$: ☑ $(m_\nu < 0.8\ eV)$

$\boldsymbol{e^- e^+}$: ☑ $(m_e = 0.511\ MeV)$





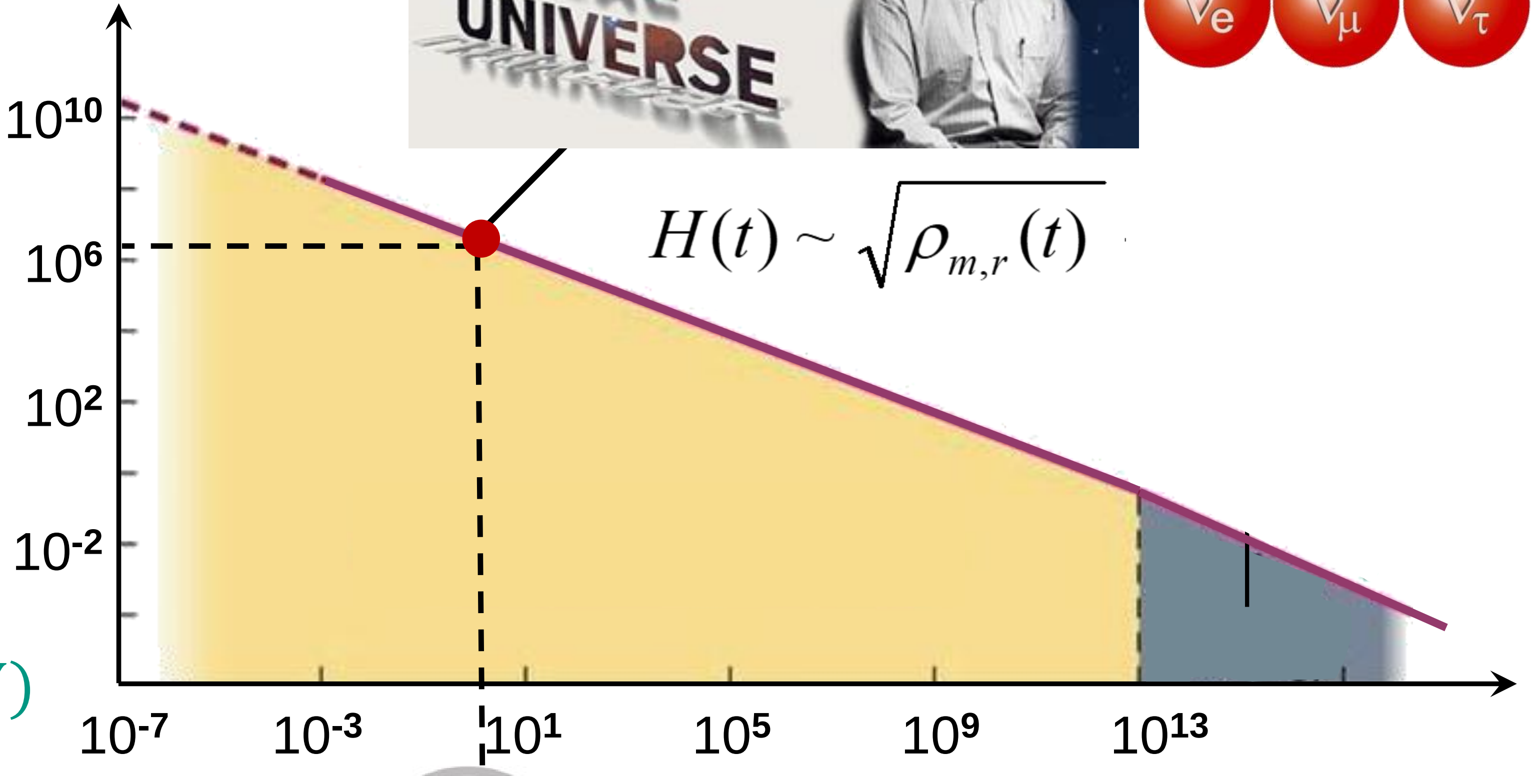
* see next lecture!

neutrino freeze-out & # of relativistic particles

- **An important time stamp: ν´s decouple at $t = 1\ s$ & $at\ T = 1\ MeV$**

- how many relativistic ´degrees of freedom´ were present at this time?*

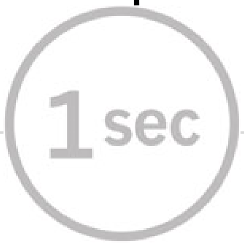
- preview: **relativistic particles** at $E = 1\ MeV$



$\boldsymbol{p, n}$: ☒ $(m_{p,n} \approx 1\ GeV)$

χ (neutralino)*: ☒ $(m_\chi \geq GeV)$

$\boldsymbol{\mu^- \mu^+}$: ☒ (decayed)



time $t\ (s)$

* Cold Dark Matter!

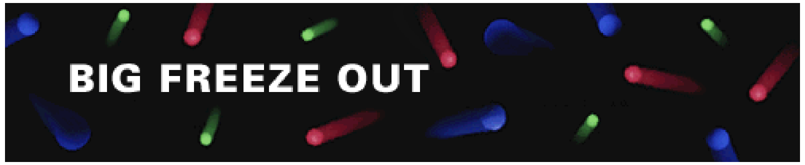
freeze-out of massless photons

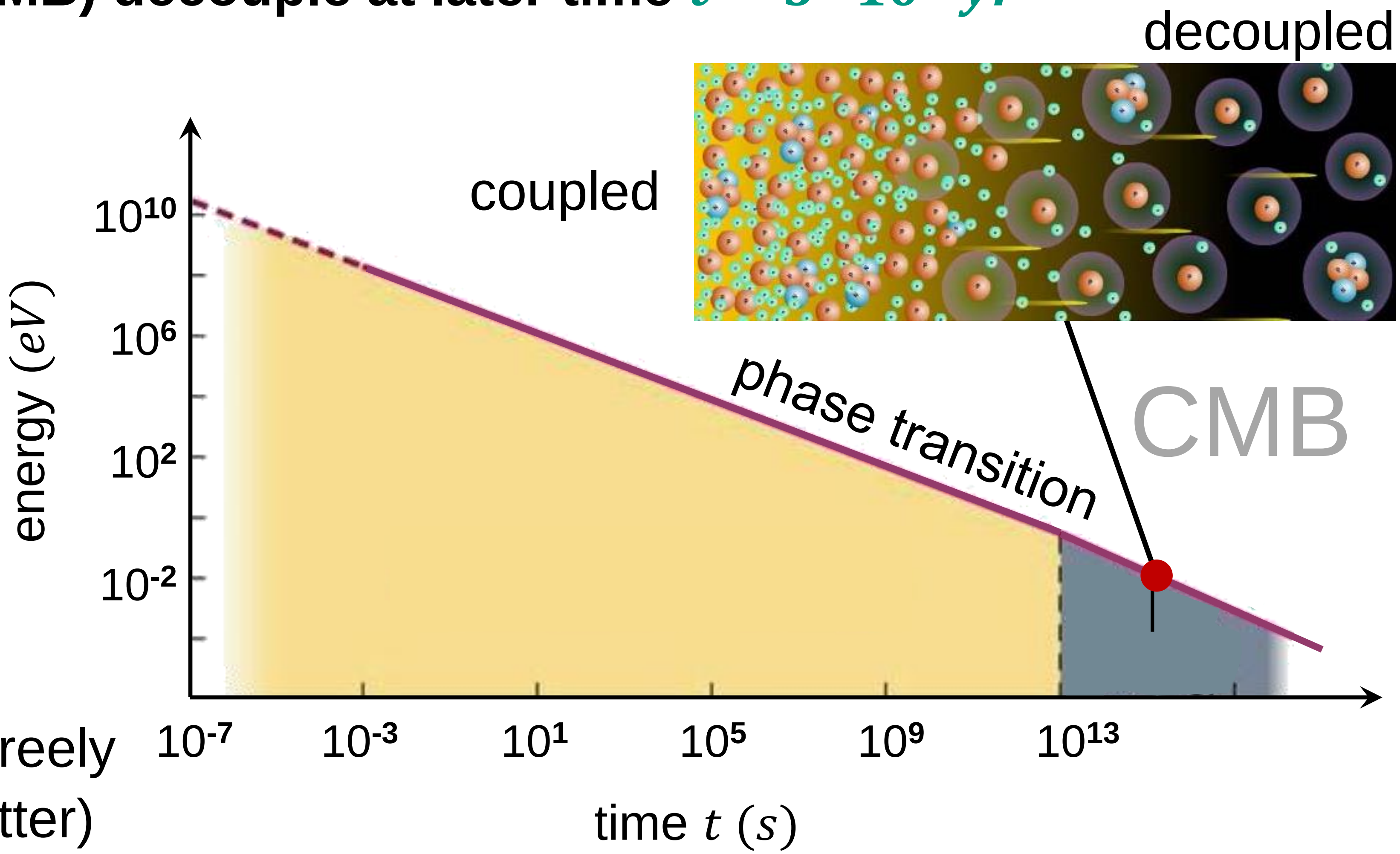
- **Matter and radiation (CMB) decouple at later time $t = 3 \cdot 10^5\ yr$**

  - freeze-out condition:

$$\Gamma(t) = H(t)$$

photon electromagnetic interaction with charges





  - photons (CMB) propagate freely (without interaction with matter)

freeze-out of massless photons

- **Matter and radiation (CMB) decouple**

  - freeze-out condition:

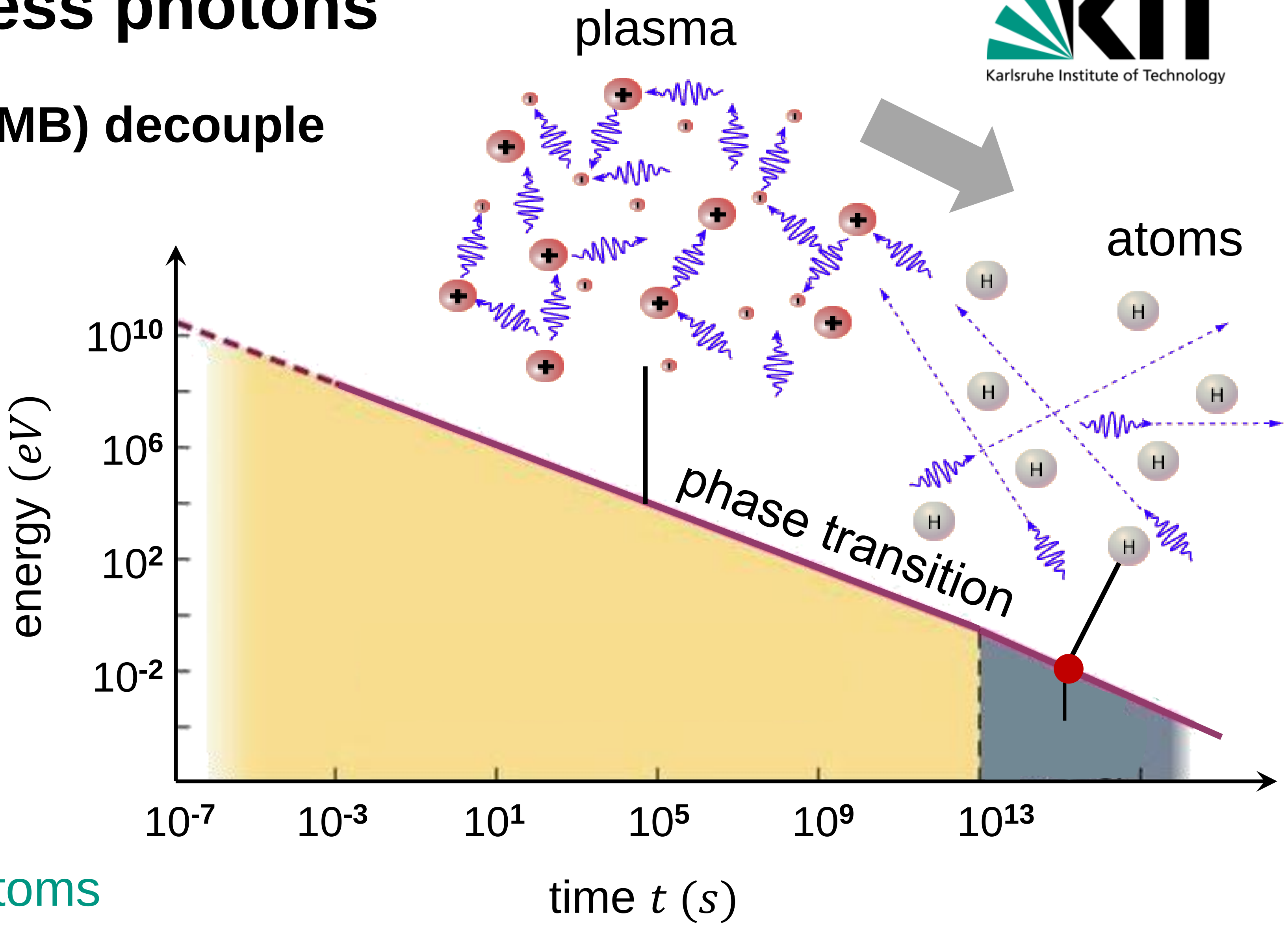
$$\Gamma(t) = H(t)$$

photon electromagnetic interaction with charges

  - phase transition:

$t < 3 \cdot 10^5 yr$: plasma

$t > 3 \cdot 10^5 yr$: neutral atoms



freeze-out of massless photons

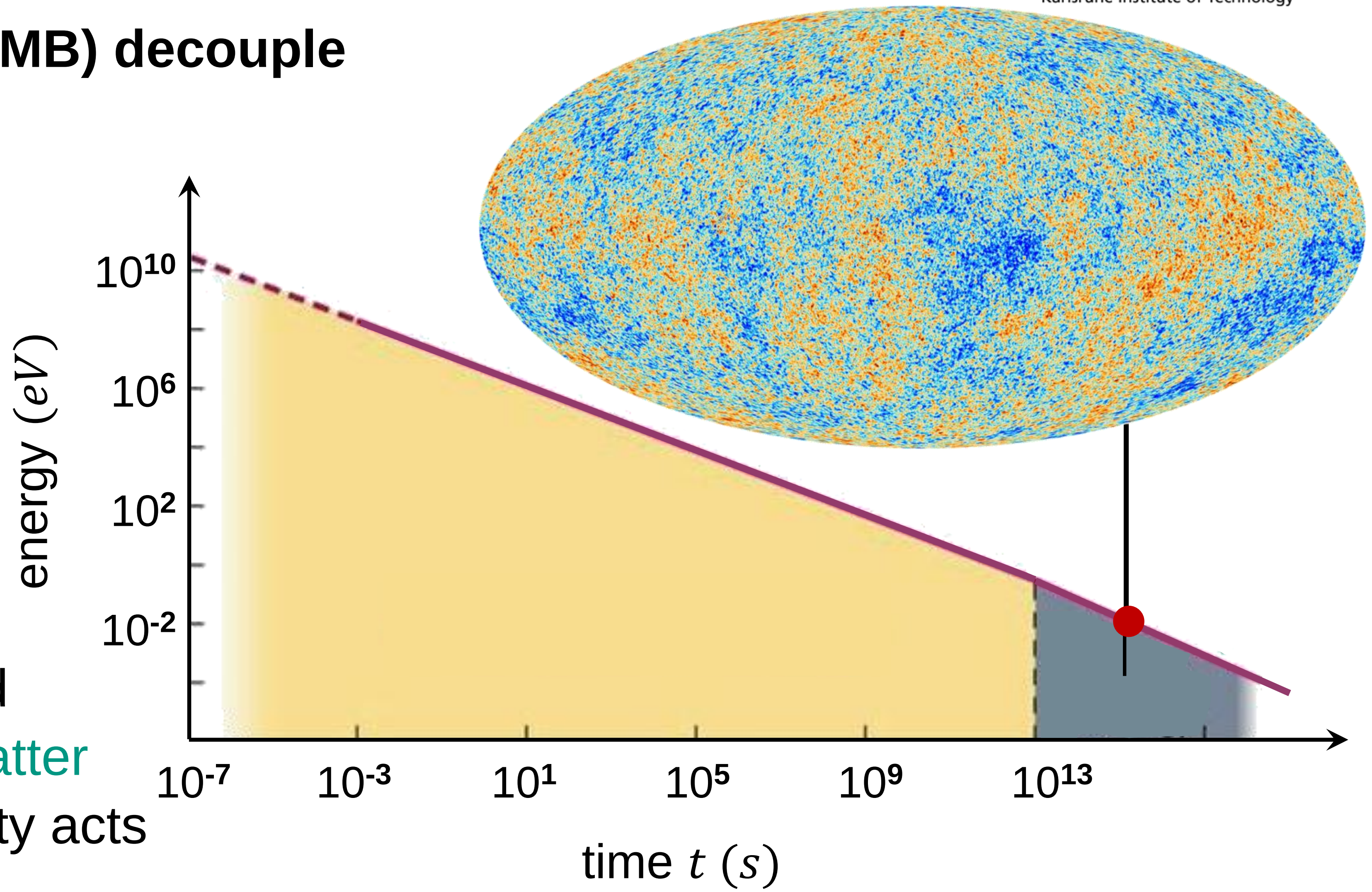
- **Matter and radiation (CMB) decouple**

  - freeze-out condition:

$$\Gamma(t) = H(t)$$

  - universe becomes transparaent for radiation

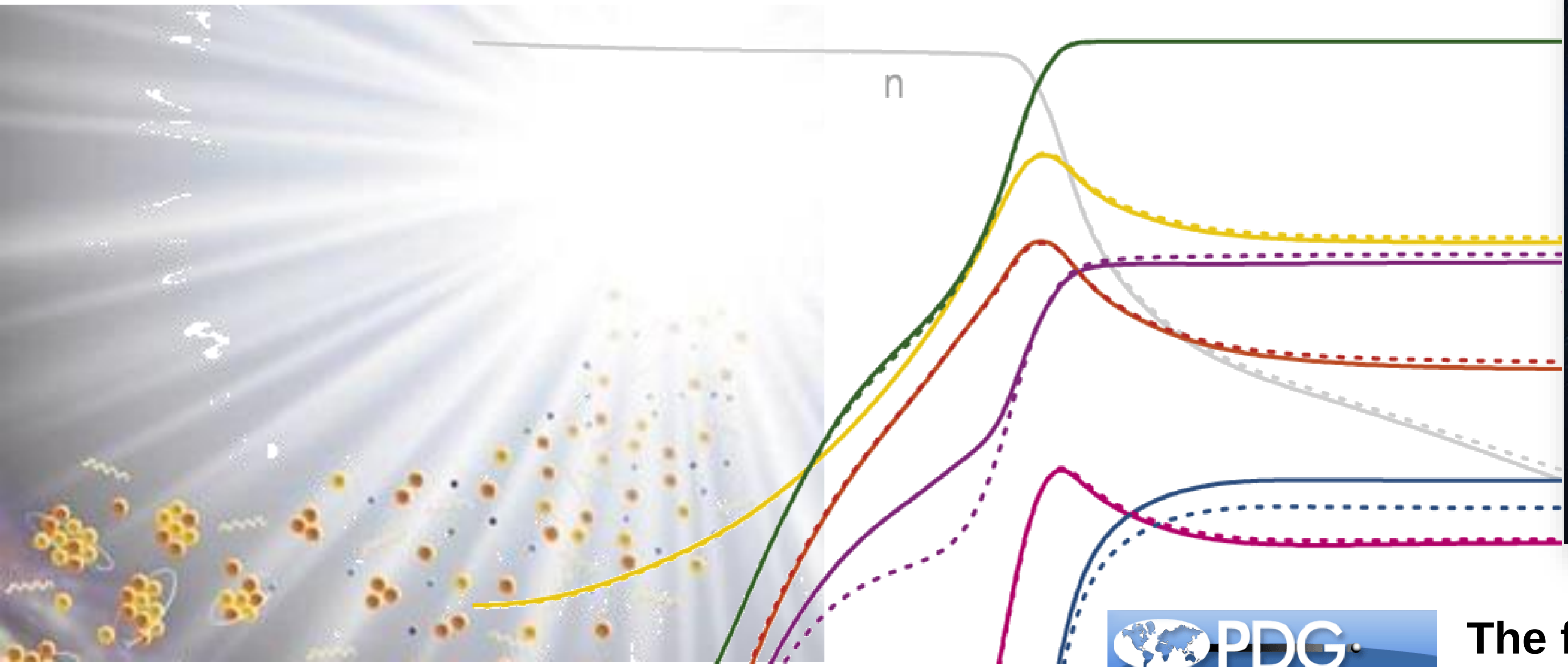
  - matter no longer coupled to radiation fluid, thus matter oscillations* stop & gravity acts

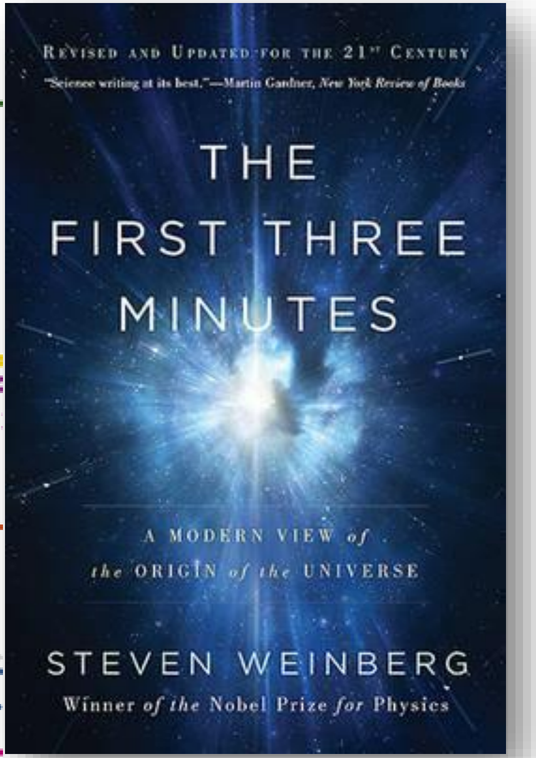


* Baryon Acoustic Oscillations, see chapter 4.2

3.1 Primordial nucleosynthesis (BBN*)

- **The origin of the light elements: a cornerstone for the hot Big Bang**





Steven Weinberg
The first three minutes
Spektrum, 2009

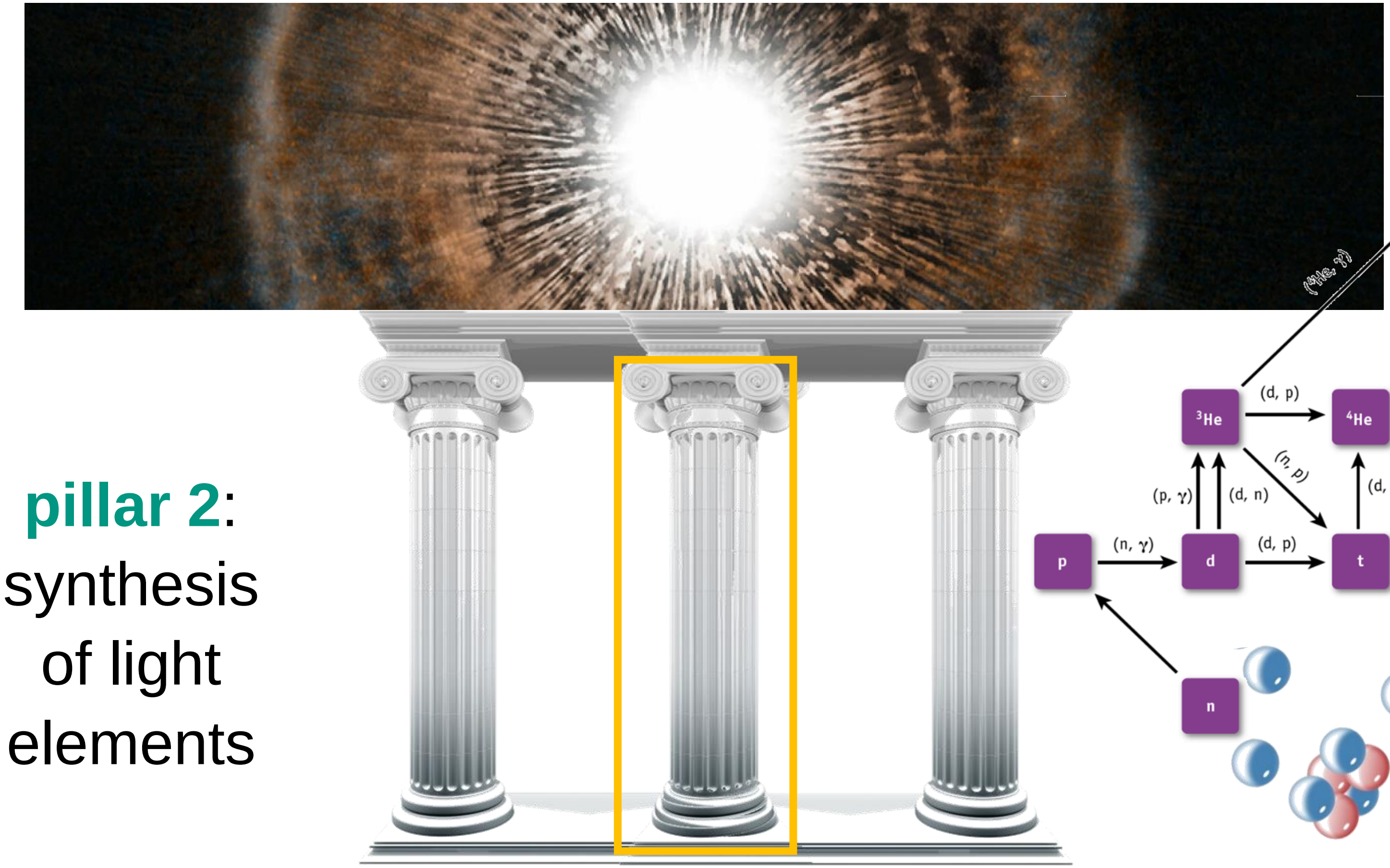
[rpp2022-rev-bbang-nucleosynthesis.pdf (lbl.gov)](rpp2022-rev-bbang-nucleosynthesis.pdf)

particle data group

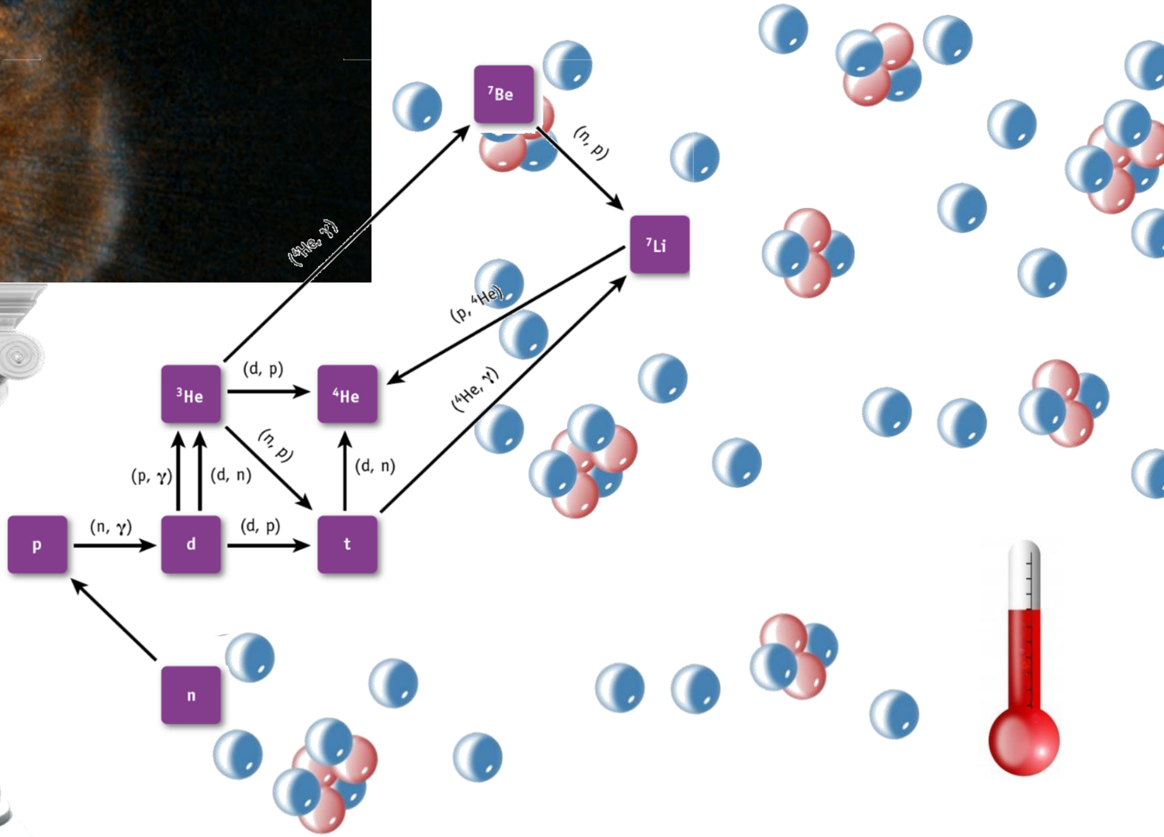
*Big Bang Nucleosynthesis

Primordial nucleosynthesis & hot Big Bang

- **Abundance of light elements as key evidence for a hot Big Bang**



pillar 2:
synthesis
of light
elements

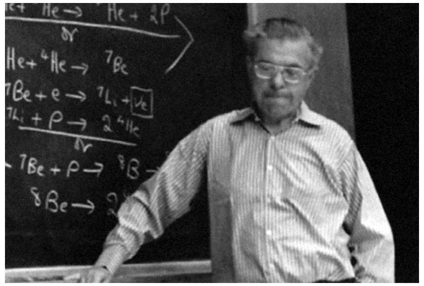


Primodial nucleosynthesis: history, breakthroughs

- **Phase-I: the ´classical´ works & discoveries from 1948 - 1977**

1948: **G. Gamov** & **R. Alpher** – all elements were forged in the early universe via neutron capture & β – decay

1957: **W. Fowler** *et al.* – **heavy elements only made in stars** (Nobel prize 1983)

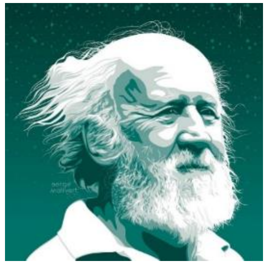


1964: **F. Hoyle** *et al.* – **He-4 production is ´primordial´,** production of heavy elements (no Nobel prize)

Primodial nucleosynthesis: history, breakthroughs

- **Phase-I: the ´classical´ works & discoveries from 1948 - 1977**

1965: **J. Peebles** – first ´modern´ calculation of BBN reaction paths (Nobel prize 2019)



1970: **H. Reeves** – deuterium spectroscopy as observational means to measure the baryon density of the universe

1977: **D. Schramm** *et al.* – **BBN limits the number of light ν-generations** (link cosmology ⇔ particle physics)

Primordial nucleosynthesis: fundamentals

Karlsruhe Institute of Technology

- **The Big Bang Nucleosynthesis…**

… allows detailed investigations of the **physics of the early universe**

… correctly predicts the **abundances of light elements** (despite a huge variation of more than 10 orders of magnitude!)

… allows an ´in-situ´ determination of the **baryon density Ω_B** as well as of the **baryon asymmetry parameter η** **(ratio baryons / photons)**

… acts as an important test bed **for novel theories**: number of light ν–generations, sterile neutrinos, novel particles (such as gravitinos,…)

… is part of **nuclear astrophysics** (element synthesis in stars & elsewhere)

Primordial nucleosynthesis: the starting point

Karlsruhe Institute of Technology

■ **The world-famous ´$\alpha - \beta - \gamma$´ paper from 1948 on nucleosnthesis**

- all elements were forged in the very early universe via neutron capture & β – decay

- based on thesis R. Alpher, supervisor: G. Gamow

- name of ´author´ H. Bethe was added ´*in absentia*´

- publication date of PRL* print: April 1, 1948

The Origin of Chemical Elements

R. A. ALPHER*
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
AND
H. BETHE
Cornell University, Ithaca, New York
AND
G. GAMOW
The George Washington University, Washington, D. C.
February 18, 1948

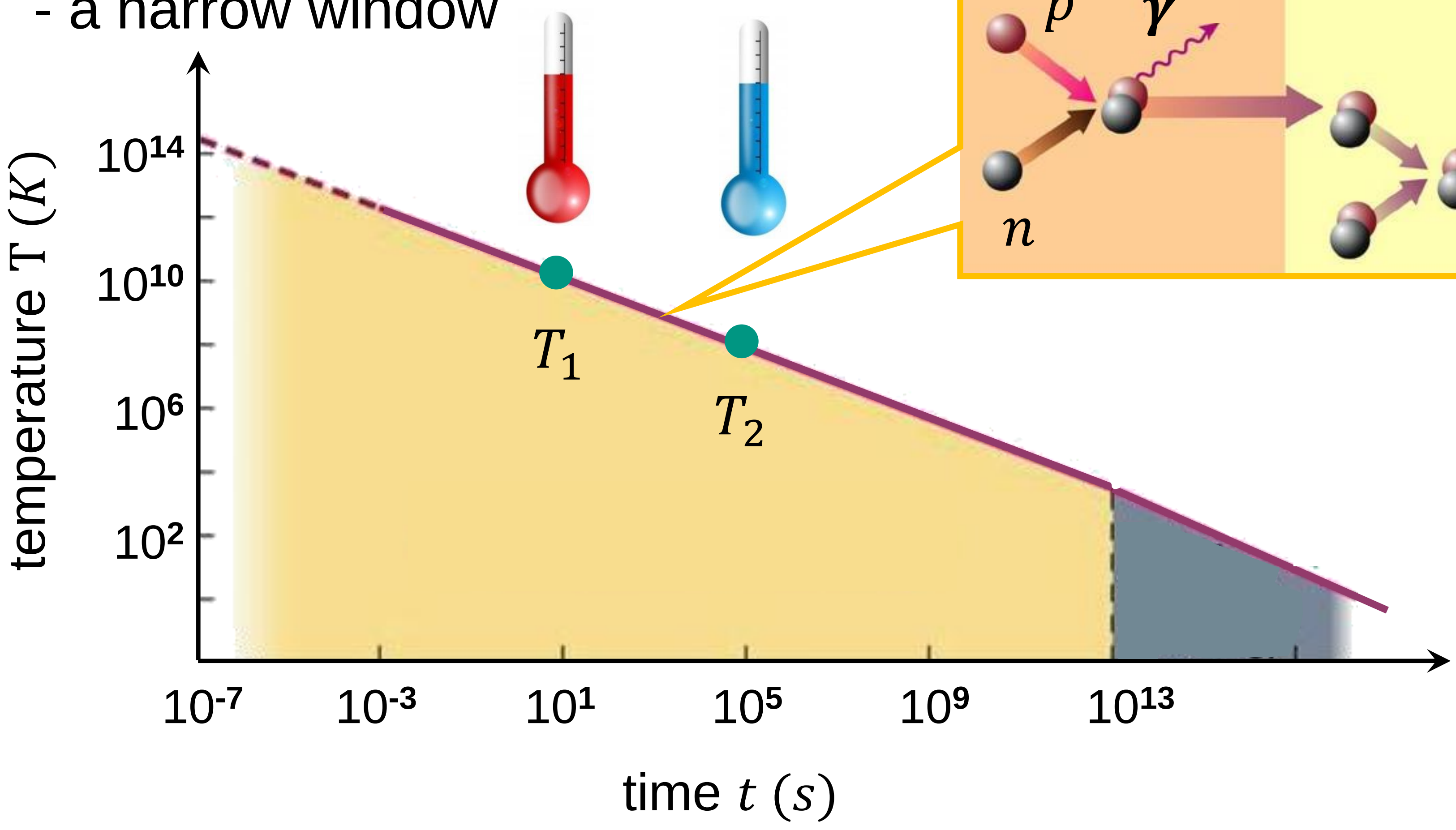
As pointed out by one of us,[1] various nuclear species must have originated not as the result of an equilibrium corresponding to a certain temperature and density, but rather as a consequence of a continuous building-up process arrested by a rapid expansion and cooling of the primordial matter. According to this picture, we must imagine the early stage of matter as a highly compressed neutron gas (overheated neutral nuclear fluid) which started decaying into protons and electrons when the gas pressure fell down as the result of universal expansion. The radiative capture of the still remaining neutrons by the newly formed protons must have led first to the formation of deuterium nuclei, and the subsequent neutron captures resulted in the building up of heavier and heavier nuclei. It

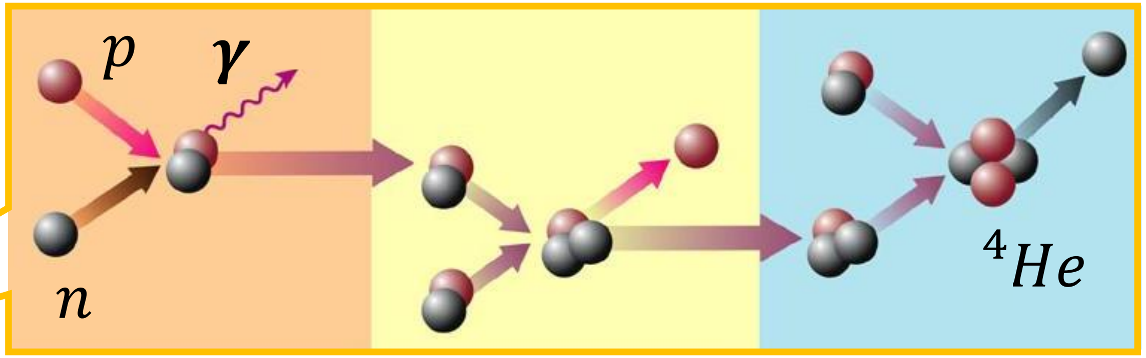
 *Phys. Rev. Letters

Primordial nucleosynthesis: fundamentals

- **Modern description of processes during light element synthesis**

- a narrow window





$T_1 = 5 \cdot 10^{10} K$

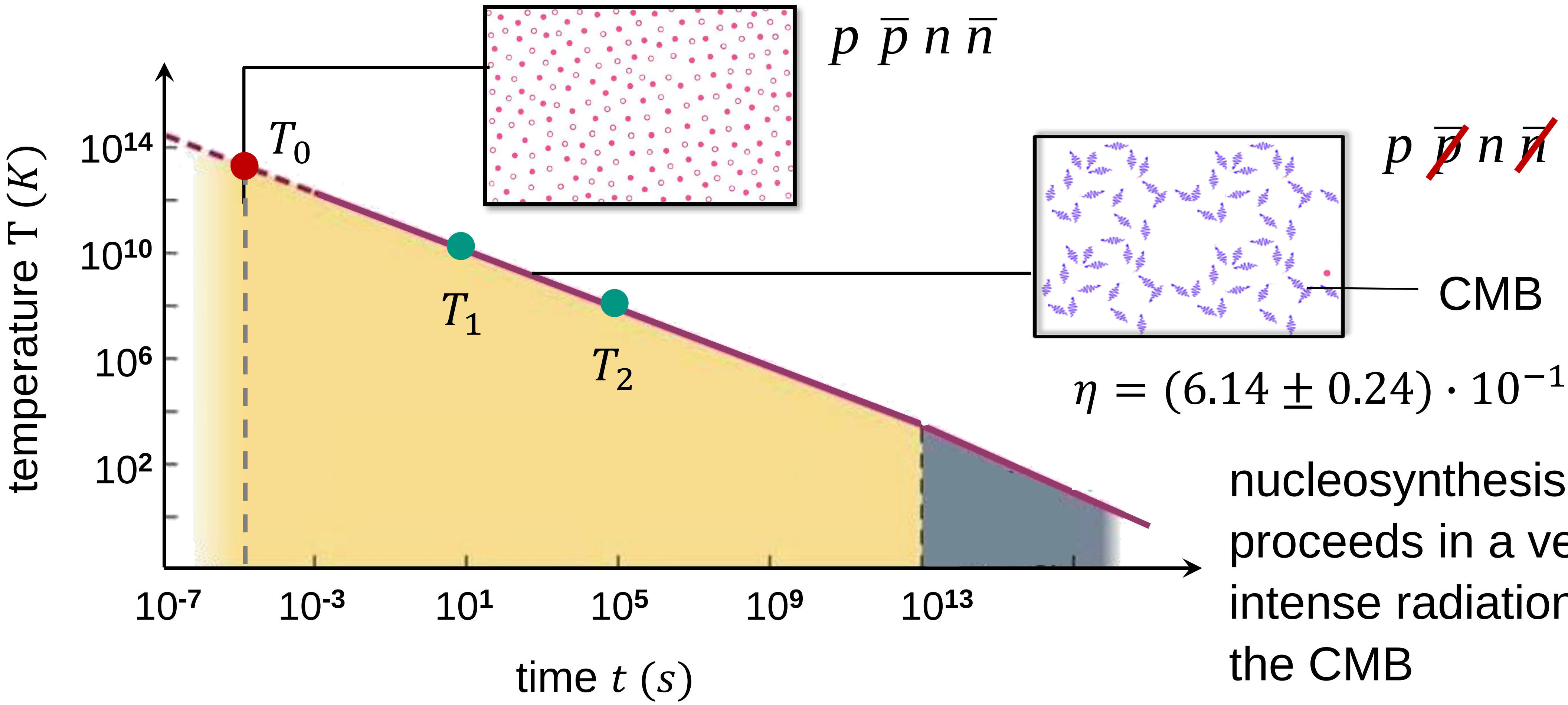
$T_2 = 5 \cdot 10^{8} K$

nucleosynthesis (BBN) ´only´ proceeds to ^4He, all heavier elements only forged later inside stars*

*see ATP-2 summer term 2023

Primordial nucleosynthesis: specific phases

- **Initial starting point $> T_0$: annihilation of matter (p, n) & anti-matter $(\bar{p}, \bar{n})$**



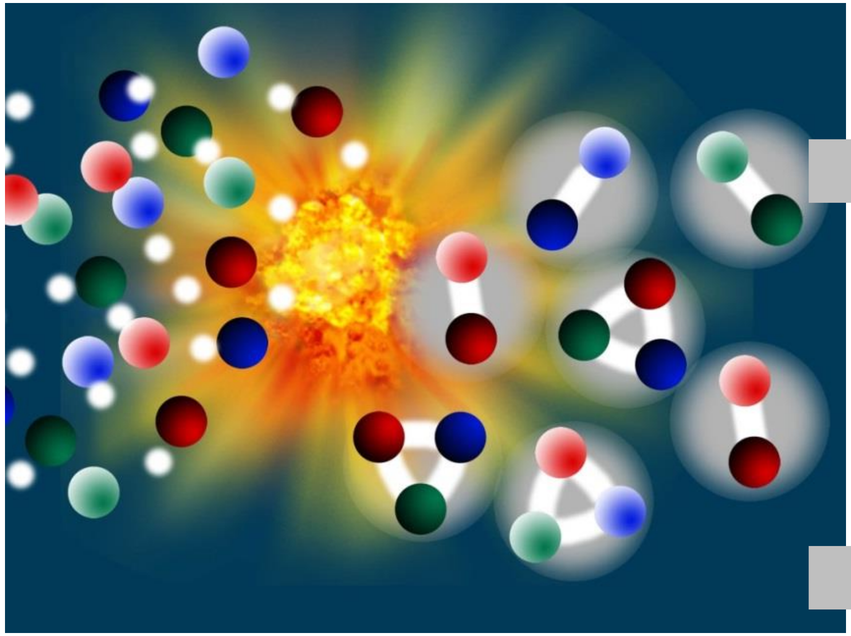
$\eta = (6.14 \pm 0.24) \cdot 10^{-10}$

nucleosynthesis (BBN) proceeds in a very intense radiation field of the CMB

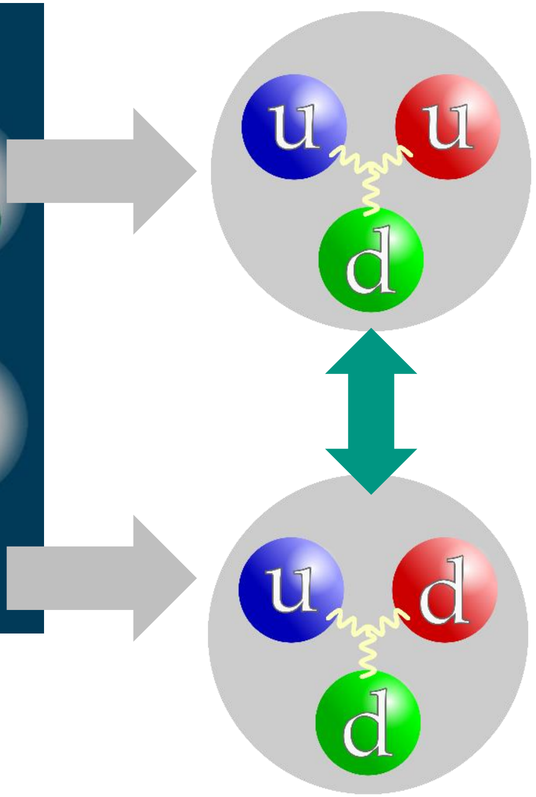
Primordial nucleosynthesis: phase 1

■ Thermodynamical equilibrium

- nucleons p, n condense out of initial quark-gluon plasma (hadronisation)



QCD phase transition
at $T \sim 150\ MeV$



$\boldsymbol{p}$ (proton)
$\boldsymbol{m = 938.3\ MeV}$

$\boldsymbol{W^{\pm}}$ **– bosons**

$\boldsymbol{n}$ (neutron)
$\boldsymbol{m = 939.6\ MeV}$

$\boldsymbol{T = 10^{11} K}$

$\boldsymbol{E = 10\ MeV}$

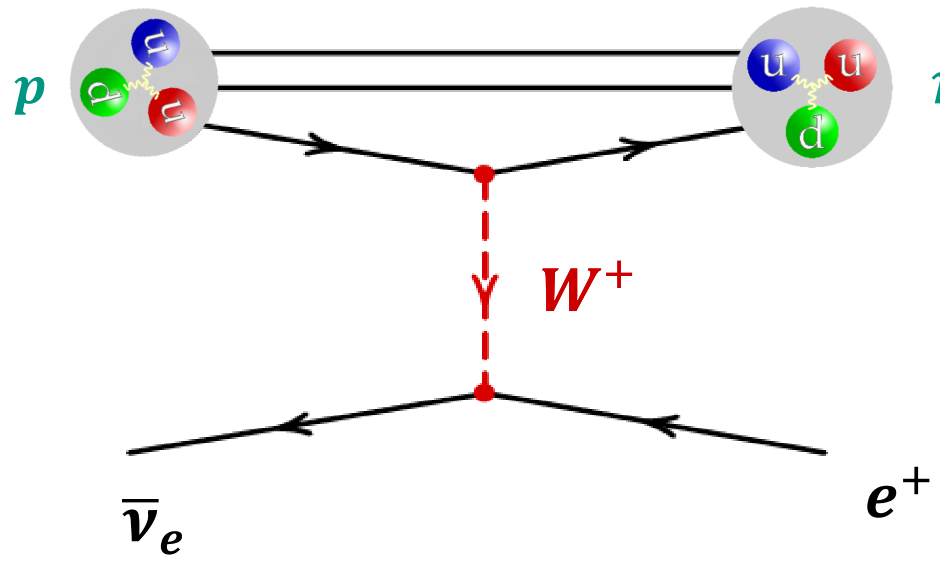
$\boldsymbol{t = 0.01\ s}$

Primordial nucleosynthesis: phase 1

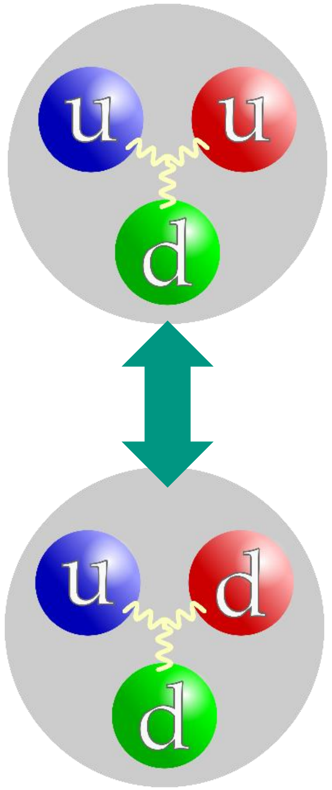
- **Thermodynamical equilibrium: example inverse β –decay**

  - nucleons p, n transform into each other due to weak interaction processes

$$p + \bar{\nu}_e \leftrightarrow n + e^+$$



inverse β – decay on $\boldsymbol{p, n}$



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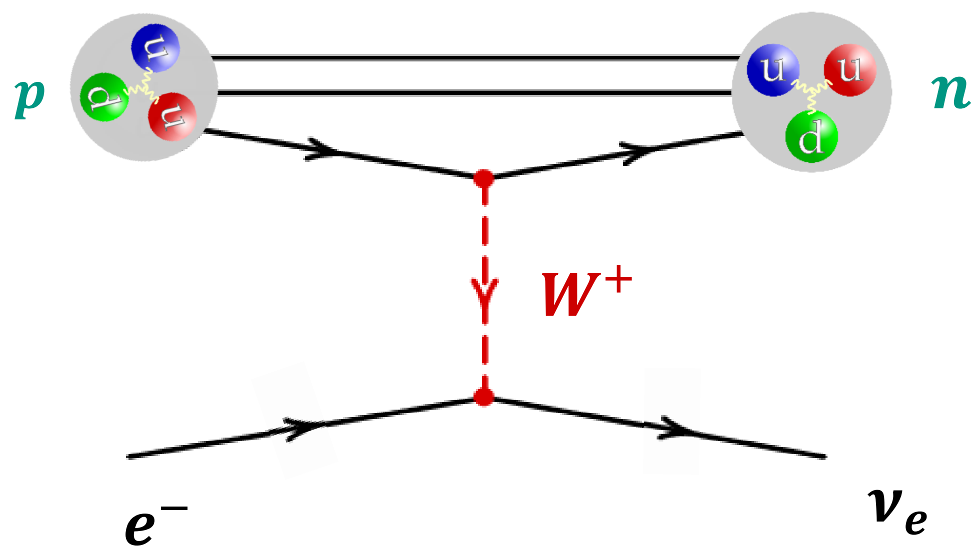
Primordial nucleosynthesis: phase 1

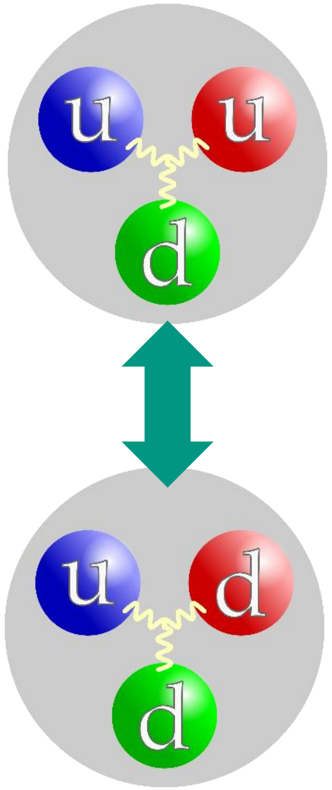
- **Thermodynamical equilibrium: example electron capture process**

  \- nucleons p, n transform into each other due to weak interaction processes

$$p + e^{-} \leftrightarrow n + \nu_e$$



electron capture on $\boldsymbol{p}$



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$\boldsymbol{n}$ (neutron)
$\boldsymbol{m = 939.6\ MeV}$

$\boldsymbol{T = 10^{11} K}$

$\boldsymbol{E = 10\ MeV}$

$\boldsymbol{t = 0.01\ s}$

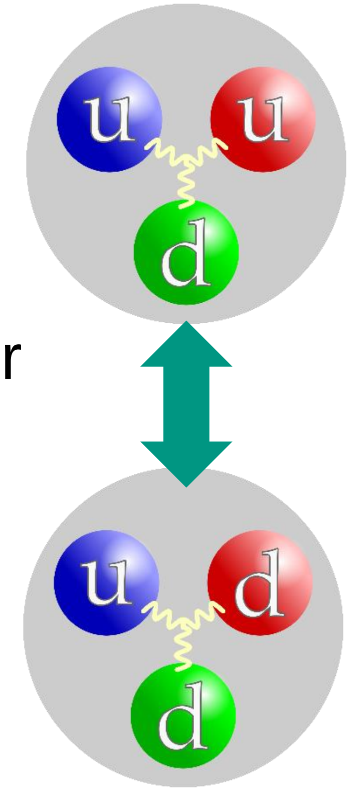
Primordial nucleosynthesis: phase 1

Thermodynamical equilibrium: example electron capture process

- nucleons p, n transform into each other due to weak interaction processes

- no change of (p, n) ratio via strong/electromagnetic interactions (strong isospin), only via weak interactions, i.e. ν –processes (before their decoupling!)

- early universe $(t < 1\ s)$: extremely high ν –densities*: $N_\nu \approx 10^{31} \ldots 10^{32}\ cm^{-3}$



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$\boldsymbol{m = 938.3\ MeV}$

$\boldsymbol{W^{\pm}}$ **– bosons**

$\boldsymbol{n}$ (neutron)
$\boldsymbol{m = 939.6\ MeV}$

$\boldsymbol{T = 10^{11} K}$

$\boldsymbol{E = 10\ MeV}$

$\boldsymbol{t = 0.01\ s}$

*presently: $N_\nu = 336\ cm^{-3}$

Primordial nucleosynthesis: phase 1

- **Thermodynamical equilibrium: calculating initial (p,n) – ratio at high T**

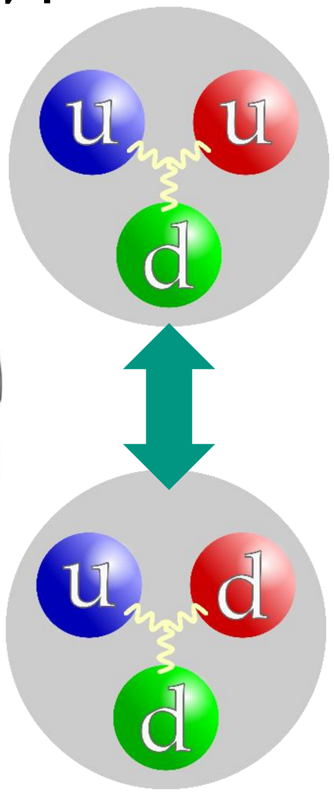
  - (p,n) – ratio follows a **Boltzmann distribution**
    different masses $\Delta m = 1.3\ MeV$!

  - for **high temperatures**
    $T(10\ MeV) \gg \Delta m\ (1.3\ MeV)$
    we have:

$$n/p = e^{-\Delta m \cdot c^2 / k_B \cdot T}$$

$$n/p = e^{-1.3/10} = 0.88$$

⇨ almost equal numbers



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$\boldsymbol{n}$ (neutron)
$\boldsymbol{m = 939.6\ MeV}$

$\boldsymbol{T = 10^{11} K}$

$\boldsymbol{E = 10\ MeV}$

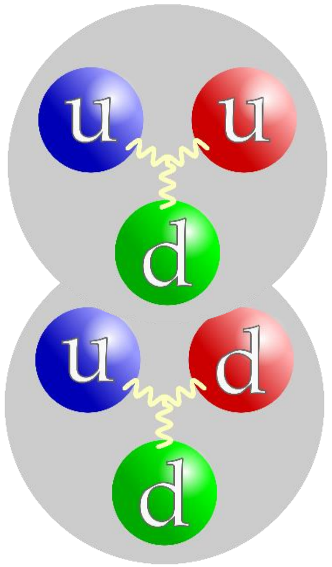
$\boldsymbol{t = 0.01\ s}$

Primordial nucleosynthesis: phase 1

- **Thermodynamical equilibrium: fusion reactions to deuterium at high T?**

  - can nucleons (p, n) already undergo fusion processes to deuterium?

  - for **high temperatures** $T(10\ MeV)$ we have to consider whether deuterium is stable under these conditions



d (deuterium)
$E_B = 2.2\ MeV$

strong interaction

$T = 10^{11} K$

$E = 10\ MeV$

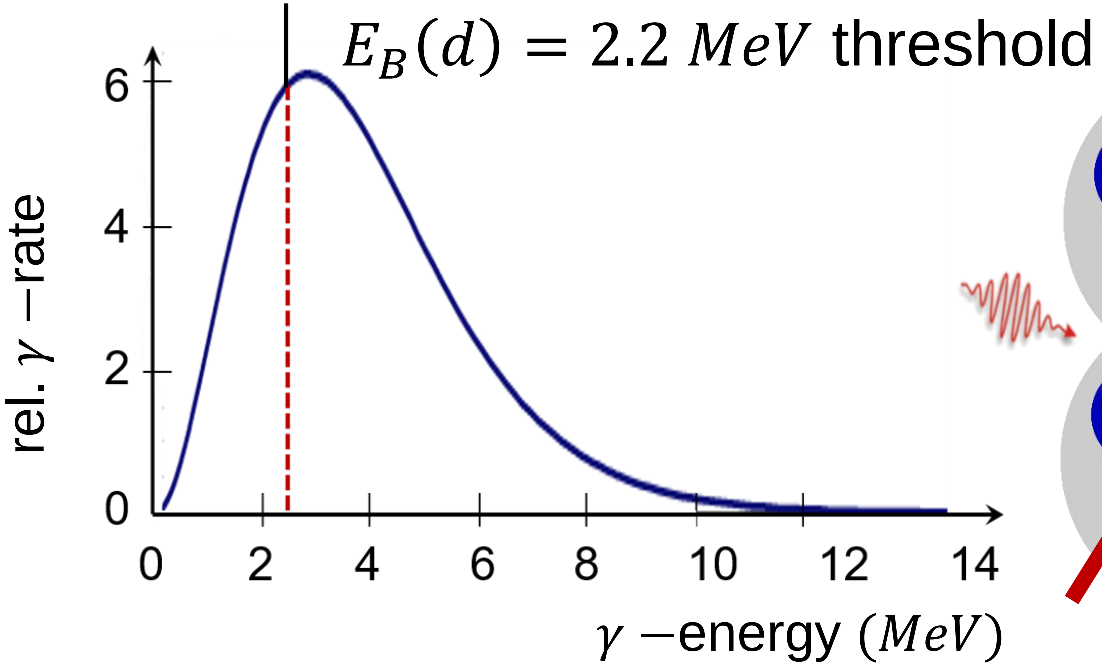
$t = 0.01\ s$

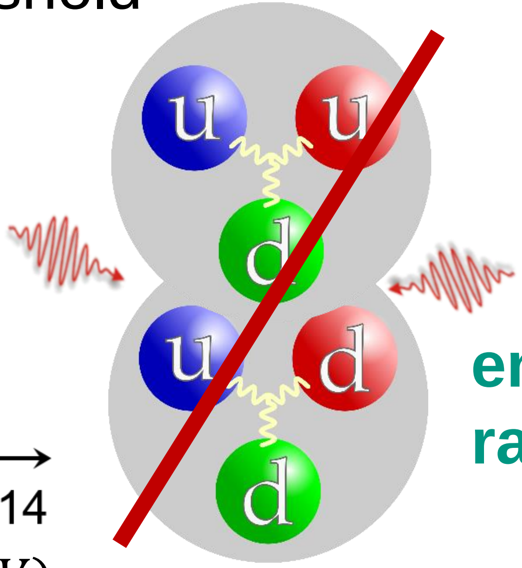
⇨ fusion reactions (?) in an **intense heat bath**

Primordial nucleosynthesis: phase 1

- **Thermodynamical equilibrium: fusion reactions to deuterium at high T?**

  - all deuterium nuclei forged at $T = MeV - scale$ are immediately destroyed





$\sim 10^{10}\ \gamma\text{´}s$ per d

energetic gamma rays break up d

$\boldsymbol{T = 10^{11} K}$

$\boldsymbol{E = 10\ MeV}$

$\boldsymbol{t = 0.01\ s}$

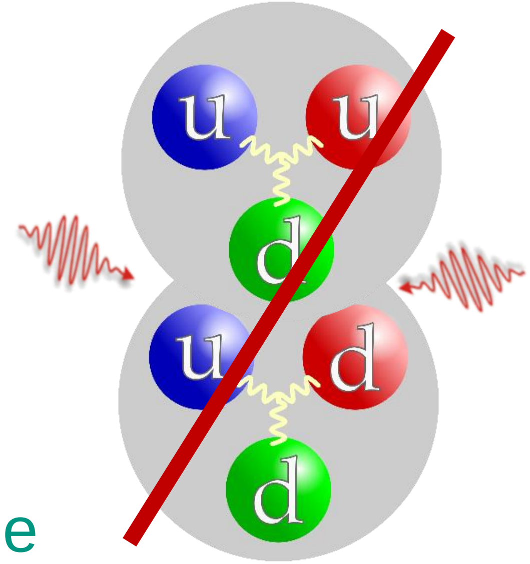
⇨ fusion reactions cannot preceed in an **intense heat bath**
nucleons (p, n) remain unpaired

Primordial nucleosynthesis: phase 2

- **Universe has to expand & cool before fusion reactions can proceed**

  - d is **bottleneck** for fusion reactions due to energtic γ´s at $MeV - scale$

  - only ´solution´:



$T \ll 10^{11} K$

$E \ll 10\ MeV$

$t \gg 0.01\ s$

  - waiting for temperature T to drop: neutrinos decouple after $t = 1\ s$ $(1\ MeV)$
    ⇨ no further (n, p) transformations ⇨ (n, p) – ratio is fixed

Primordial nucleosynthesis: phase 2

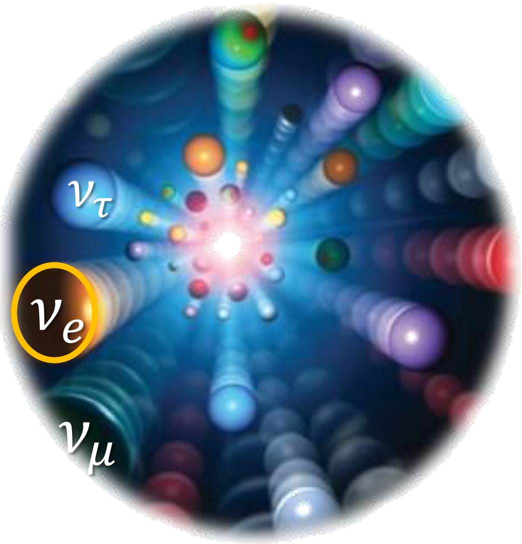
- **Freeze-out of neutrinos at $t \approx 1\ s$ of key importance for (n, p) –ratio**

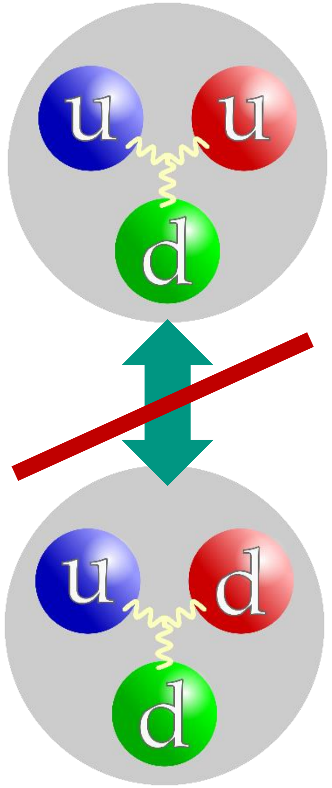
  - detailed calculations for neutrino freeze-out energy for ν_e

$$\Gamma_\nu(t) = H(t)$$

$\sim G_F \cdot T^5$ $\sim T^2$

$$T_{fr} = 0.7\ MeV$$





p (proton)
$m = 938.3\ MeV$

~~$W^\pm$ – bosons~~

n (neutron)
$m = 939.6\ MeV$

$T \approx 10^{10} K$

$E = 0.7\ MeV$

$t \approx 1\ s$

Primordial nucleosynthesis: phase 2

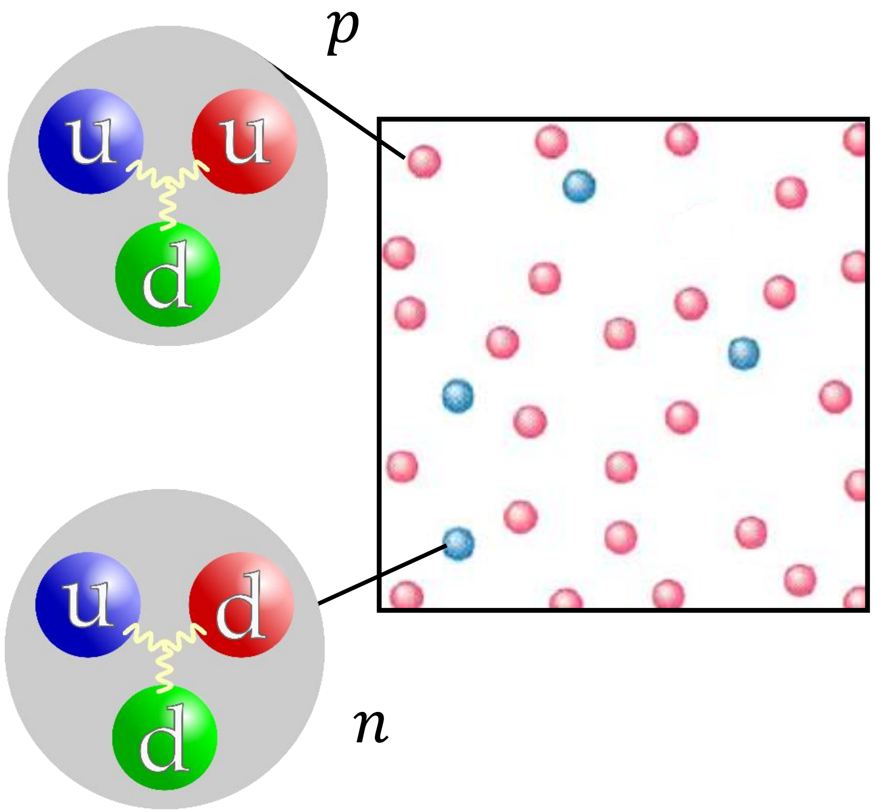
- **Freeze-out of neutrinos at $t \approx 1\ s$ of key importance for (n, p) –ratio**

  - calculated (n, p) – ratio at $T = 0.7\ MeV$

$$n/p = e^{-\Delta m \cdot c^2 / k_B \cdot T}$$

$$n/p = e^{-1.3/0.7} = 0.15 \ \approx \frac{1}{6}$$

  - (n, p) – ratio not further modified by thermodynamics



$T \approx 10^{10} K$

$E = 0.7\ MeV$

$t \approx 1\ s$

INSERTION: Primordial nucleosynthesis´

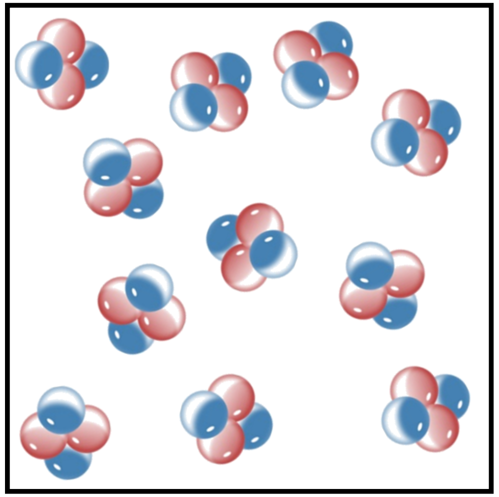
- **We consider a different universe (#42) with larger freeze-out temperature**

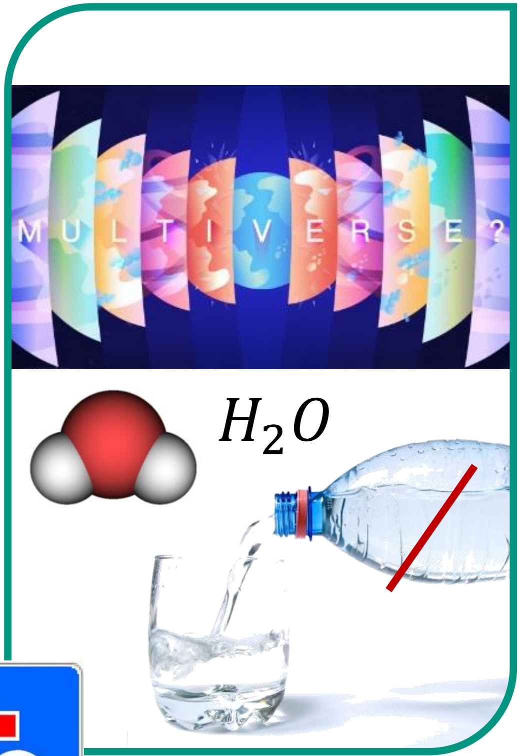
  - in case of significantly changed physical constants we have for example:

$$T_{fr} = 70\ MeV$$

$$n/p = e^{-1.3/70} = 0.981\ \approx 1$$

formation of ^4He





  - no free protons p remain to form

    molecules: H_2O and CH_2

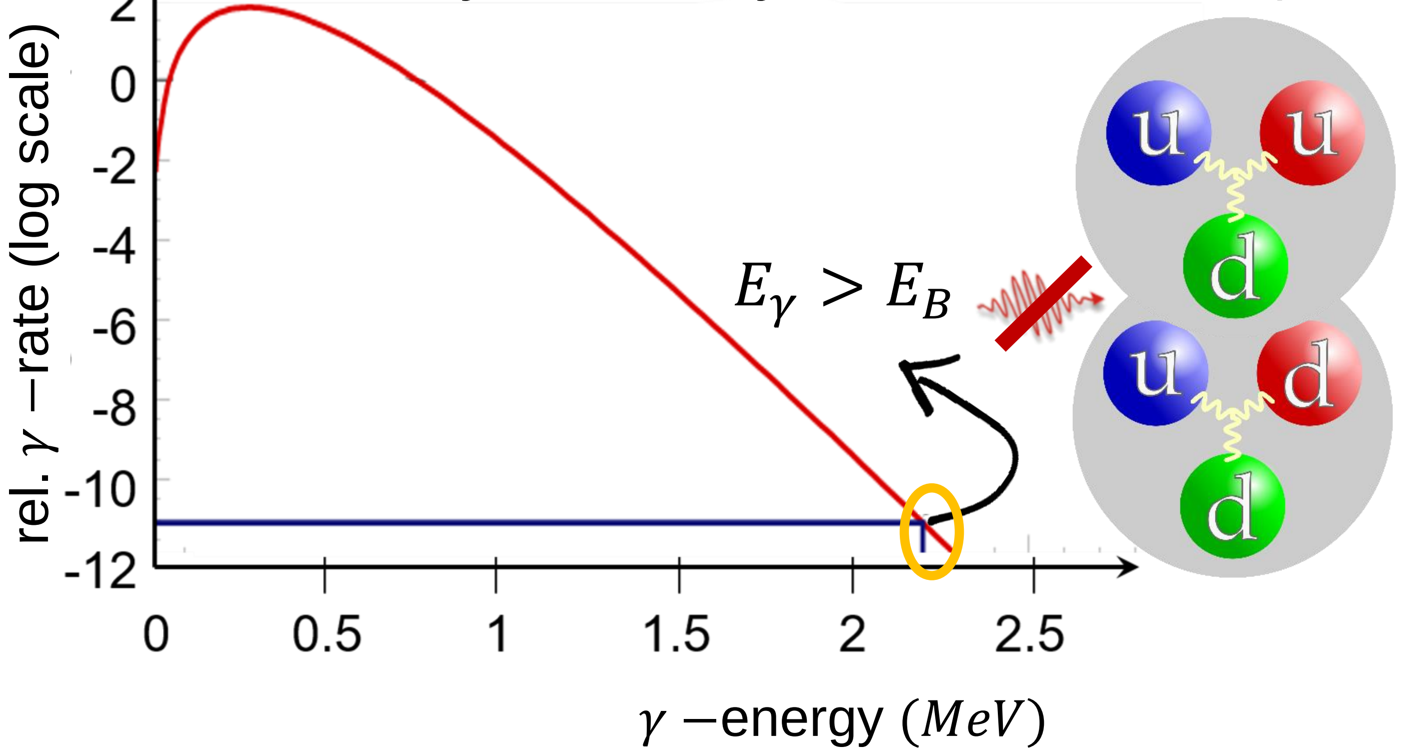
    molecules: H_2 (only fast-burning He-stars…)

Primordial nucleosynthesis: phase 2 ends

- **We need to wait for the universe to cool and expand, until d can be formed**

  - only after $t = 60\ s$ the universe is cool enough ($T < 0.1\ MeV$) so that d is stable

  - only extremely small fraction of γ´s with $E > 2.2\ MeV$!



d (deuterium)
$E_B = 2.2\ MeV$
is finally stable!

$T < 10^9 K$

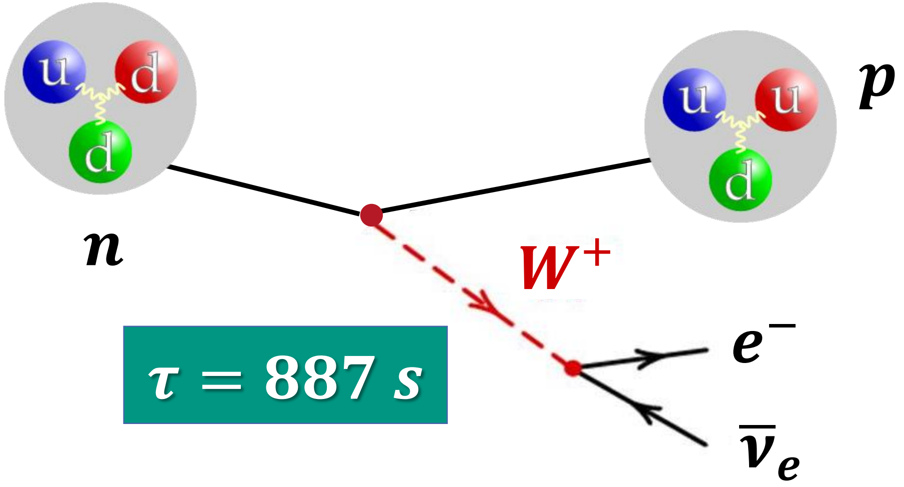
$E \approx 0.1\ MeV$

$t \approx 60\ s$

Primordial nucleosynthesis: phase 2 ends

- **We now need to take into account the finite life-time of neutrons**

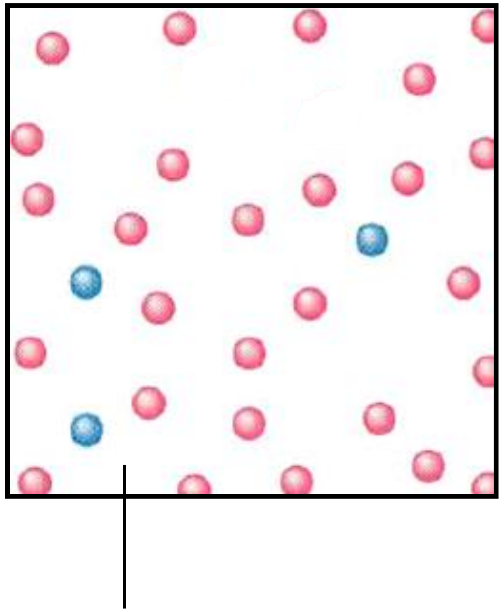
  - after $t = 60\ s$ a fraction of neutrons will have decayed ($\beta - decay$)



$\tau = 887\ s$

  - $(\mathrm{n,p}) -$ ratio after $t = 60\ s$ decreases to

$$\frac{n}{p} = \frac{1}{6} \cdot e^{-60/887} \approx \frac{1}{7}$$



$(n, p) -$ ratio now at $1/7$

$T < 10^9 K$

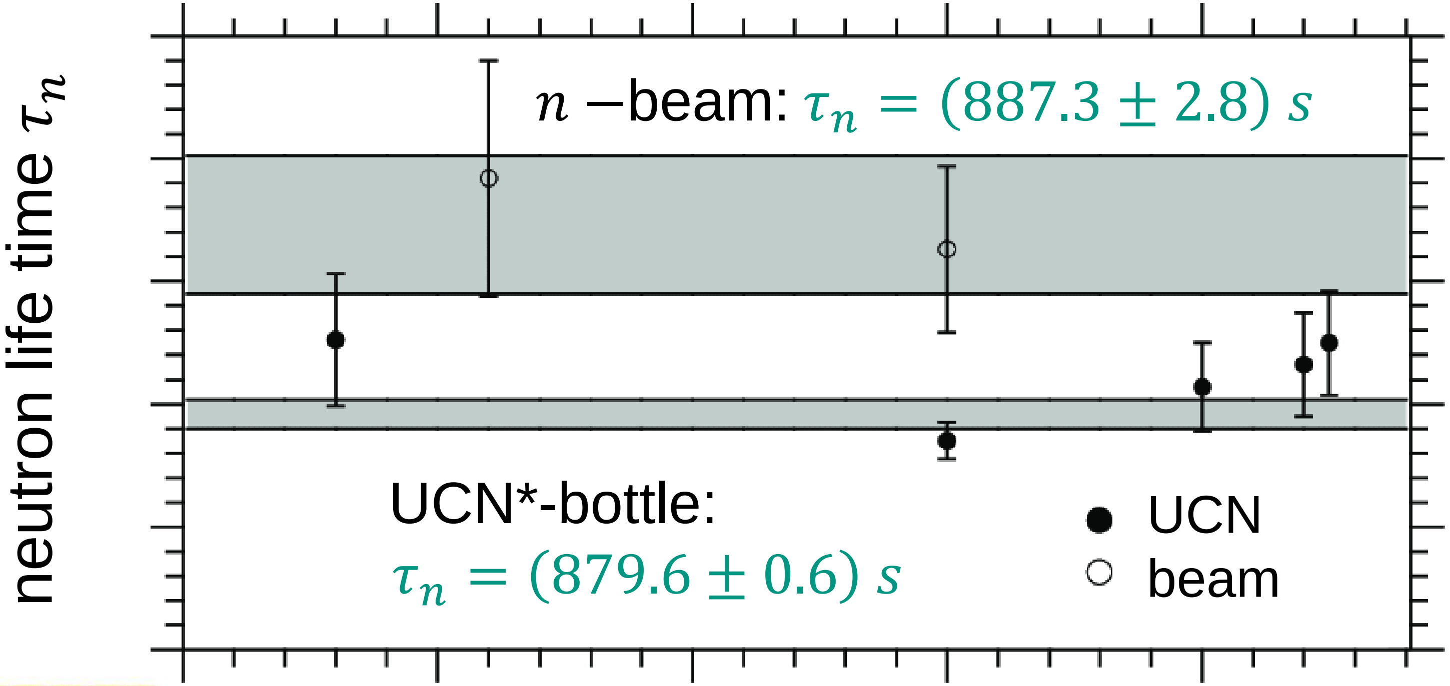
$E \approx 0.1\ MeV$

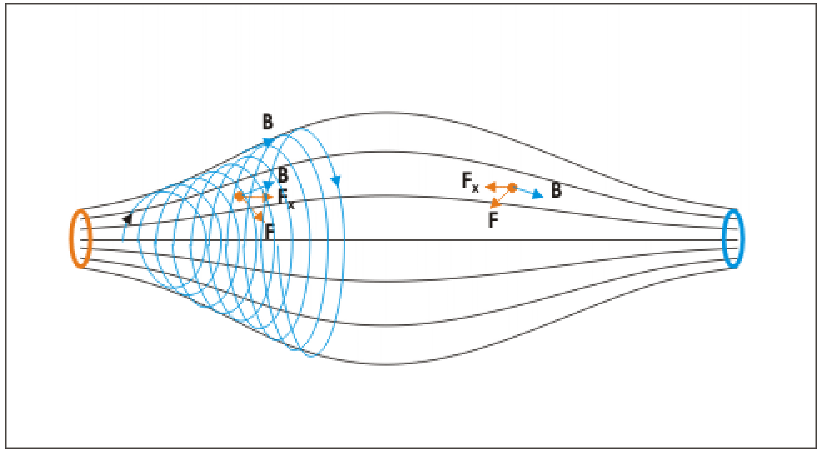
$t \approx 60\ s$

INSERTION: Primordial nucleosynthesis

- **finite life-time of neutrons: overview of exerimental results**

  - systematic effects in measurements of n – life times τ_n over past decades





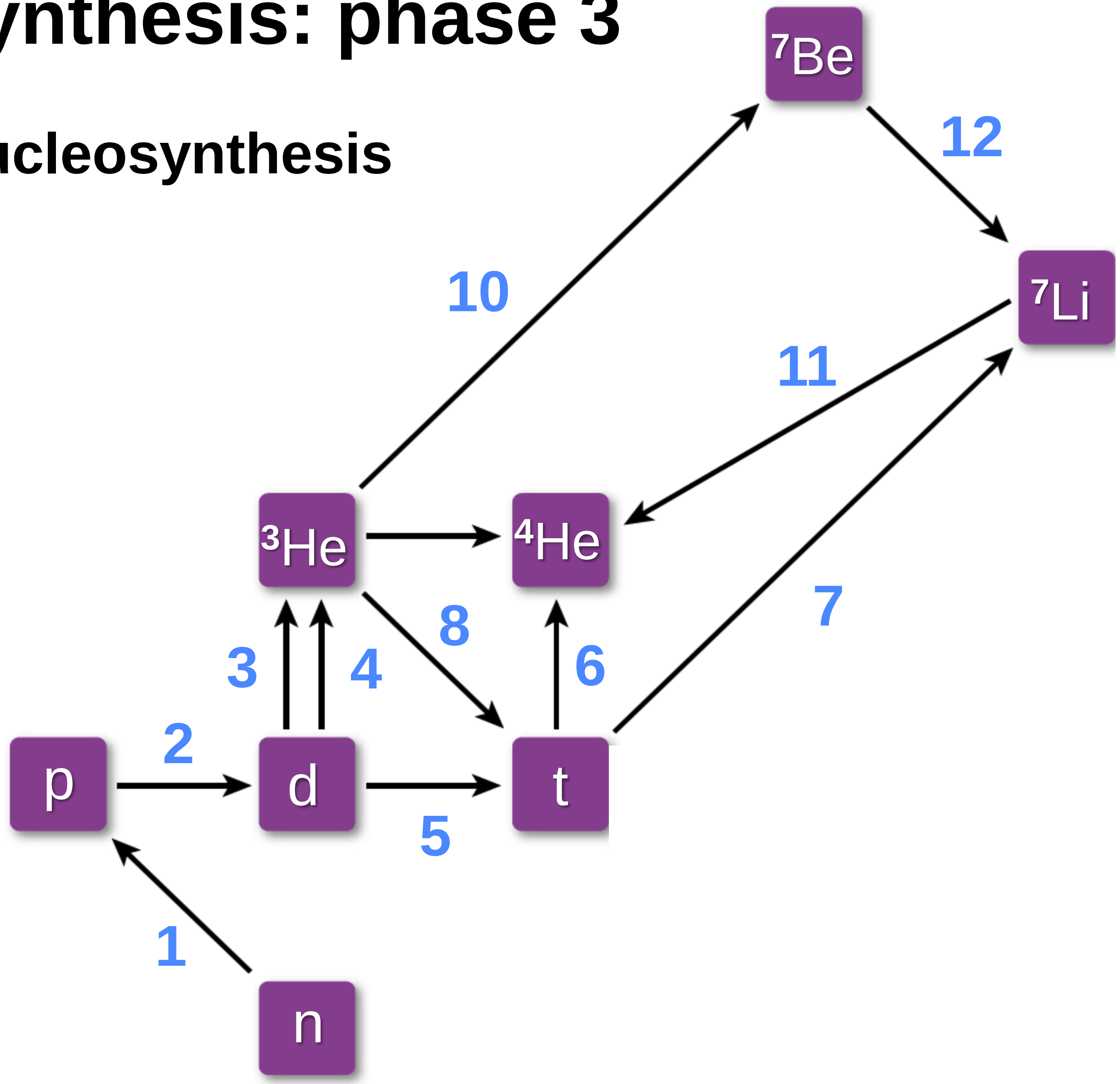
magnetic bottle

*ultra-cold neutrons

Primordial nucleosynthesis: phase 3

- **12 main pathways of nucleosynthesis**

1	$n \longrightarrow {}^{1}H + e^{-} + \bar{\nu}$
2	${}^{1}H + n \longrightarrow {}^{2}H + \gamma$
3	${}^{2}H + {}^{1}H \longrightarrow {}^{3}He + \gamma$
4	${}^{2}H + {}^{2}H \longrightarrow {}^{3}He + n$
5	${}^{2}H + {}^{2}H \longrightarrow {}^{3}H + {}^{1}H$
6	${}^{2}H + {}^{3}H \longrightarrow {}^{4}He + n$
7	${}^{3}H + {}^{4}He \longrightarrow {}^{7}Li + \gamma$
8	${}^{3}He + n \longrightarrow {}^{3}H + {}^{1}H$
9	${}^{3}He + {}^{2}H \longrightarrow {}^{4}He + {}^{1}H$
10	${}^{3}He + {}^{4}He \longrightarrow {}^{7}Be + \gamma$
11	${}^{7}Li + {}^{1}H \longrightarrow {}^{4}He + {}^{4}He$
12	${}^{7}Be + n \longrightarrow {}^{7}Li + {}^{1}H$

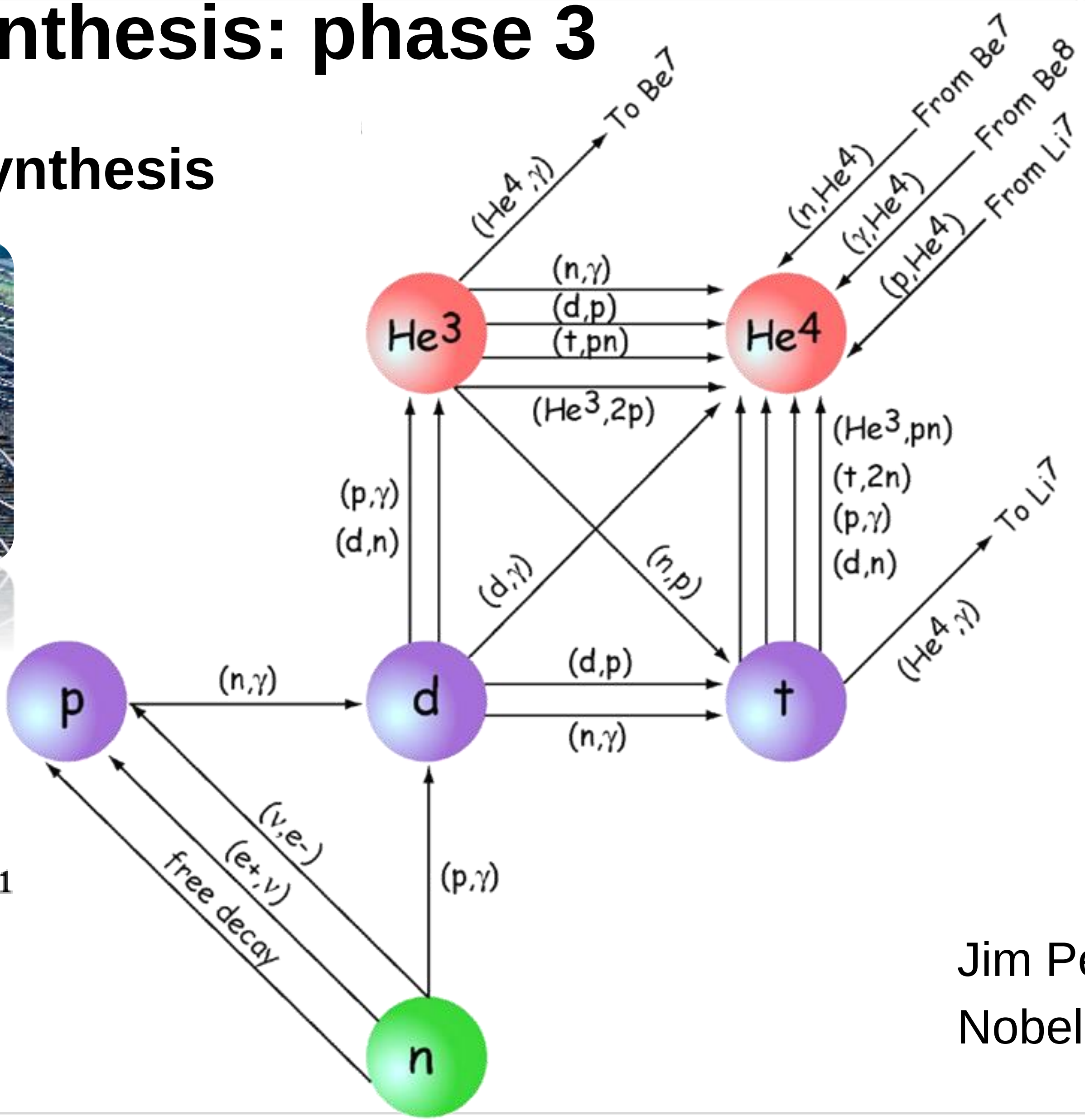


Primordial nucleosynthesis: phase 3

- all pathways of nucleosynthesis

$$N_A \sim (\sigma_A + \tau_0^{-1})^{-1}$$

$$N_{A'} \sim \left(\sigma_{A'} + \frac{\lambda_\beta}{\sigma_{A'}\Phi} + \tau_0^{-1}\right)^{-1}$$



system of coupled differential equations

Jim Peebles

Nobel prize 2019

Primordial nucleosynthesis: phase 3 channels

- **Grouping the different reaction pathways into channels (reaction ´ladder´)**

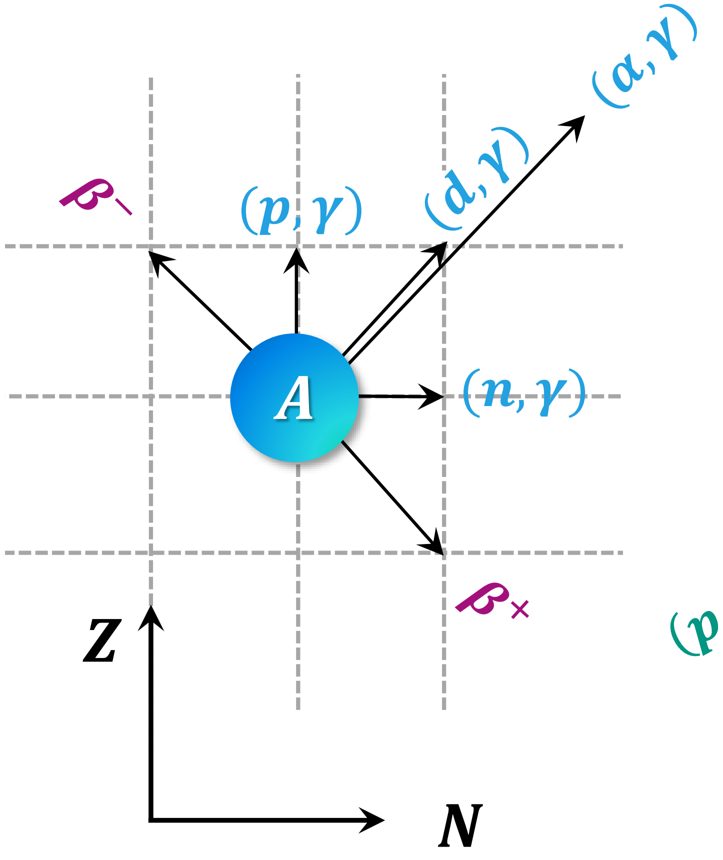
$\beta^{\pm}$ –decays

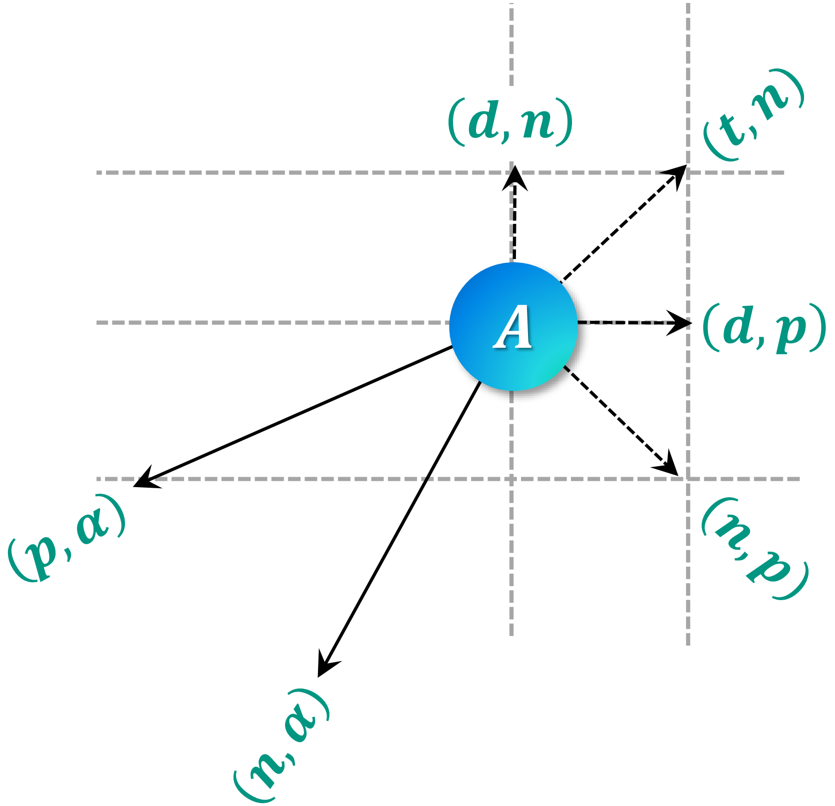
capture reactions:

$(p,\gamma), (n,\gamma),$
$(d,\gamma), (\alpha,\gamma)$

transfer processes:

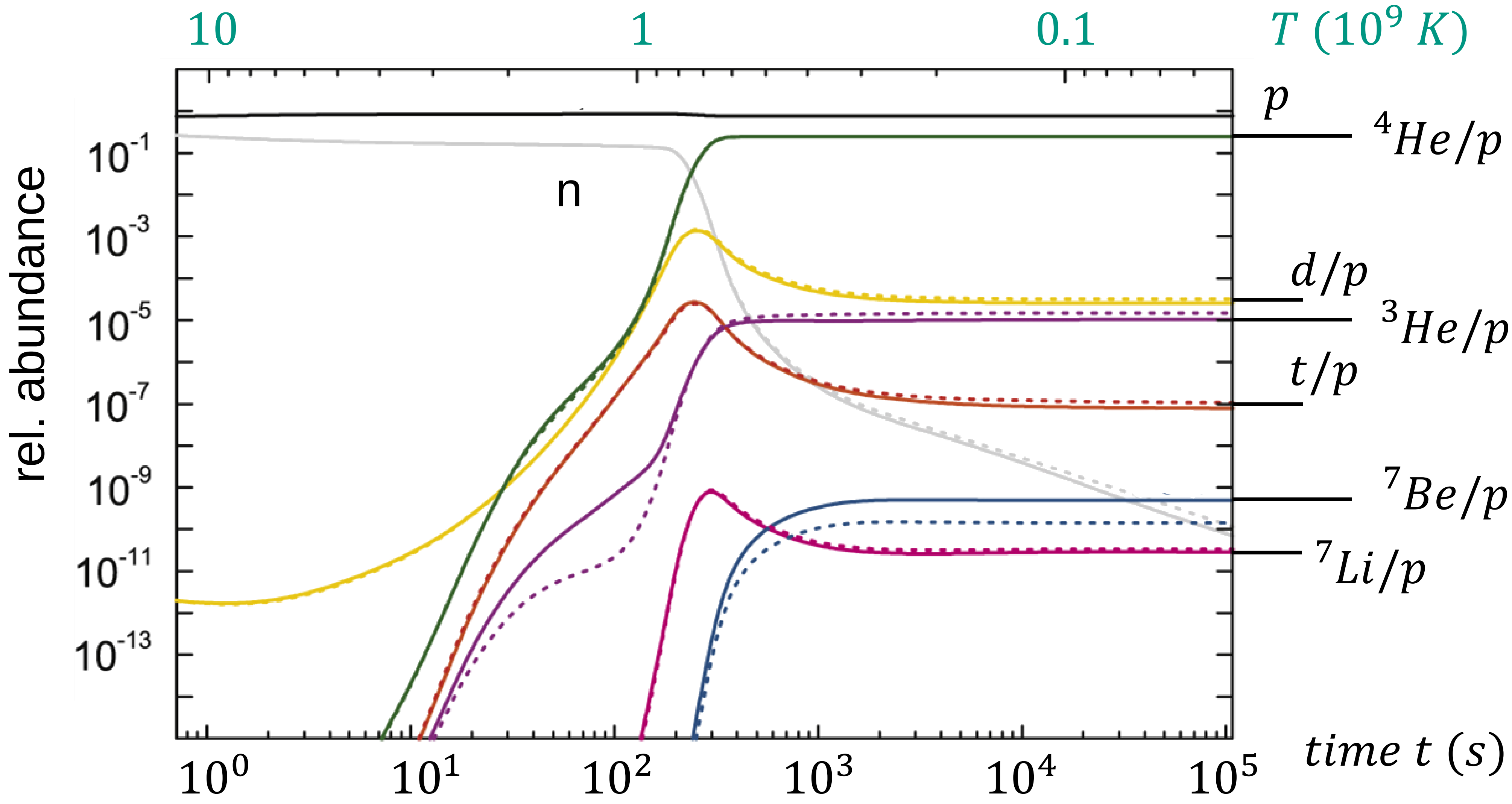
$(n,p), (d,p),$
$(d,n), (t,n), \ldots$





Primordial nucleosynthesis: fusion yields

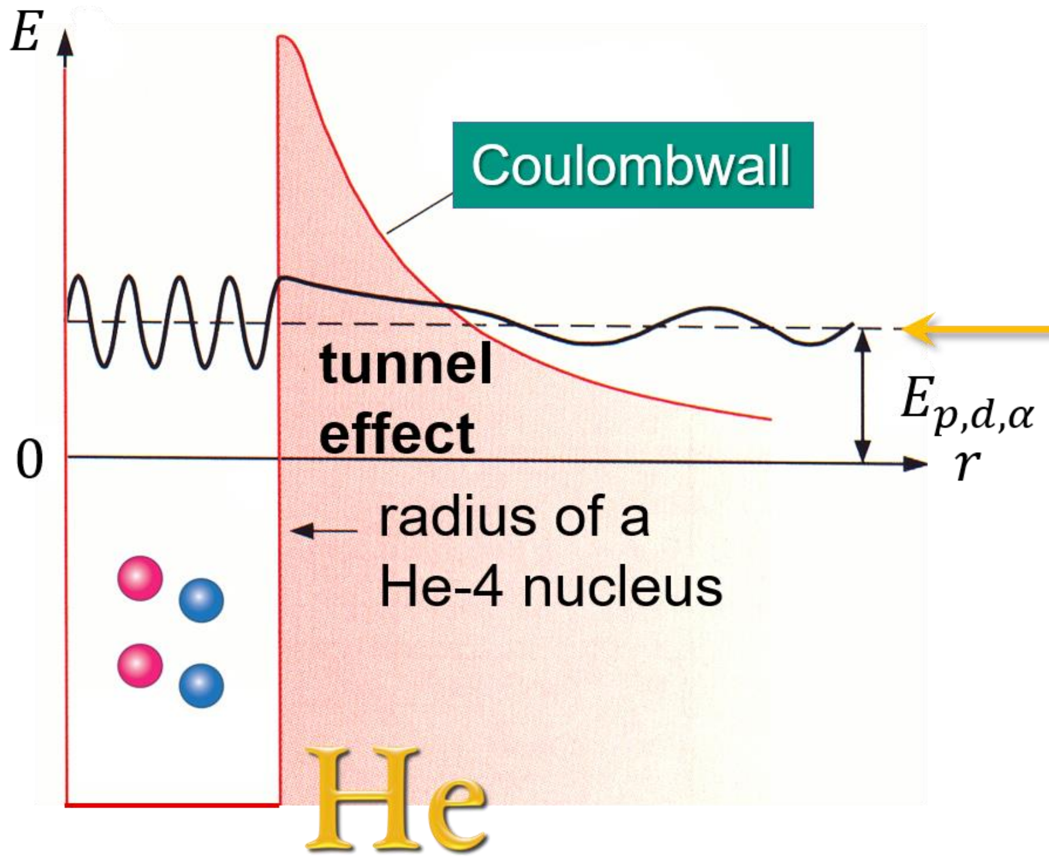
- **Results**

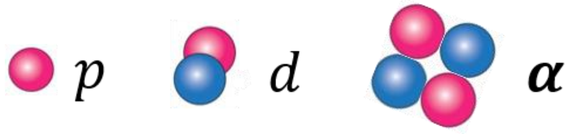


Primordial nucleosynthesis: fusion stops

- **Synthesis of light elements at low temperature ends due to Coulomb barrier**

  - energy $E_{p,d,\alpha}$ of charged particles no longer sufficient to sustain fusion reactions





$T < 5 \cdot 10^8 K$

$E \ll 0.1\, MeV$

$t \approx 300\, s$

Primoridial nucleosynthesis: all steps

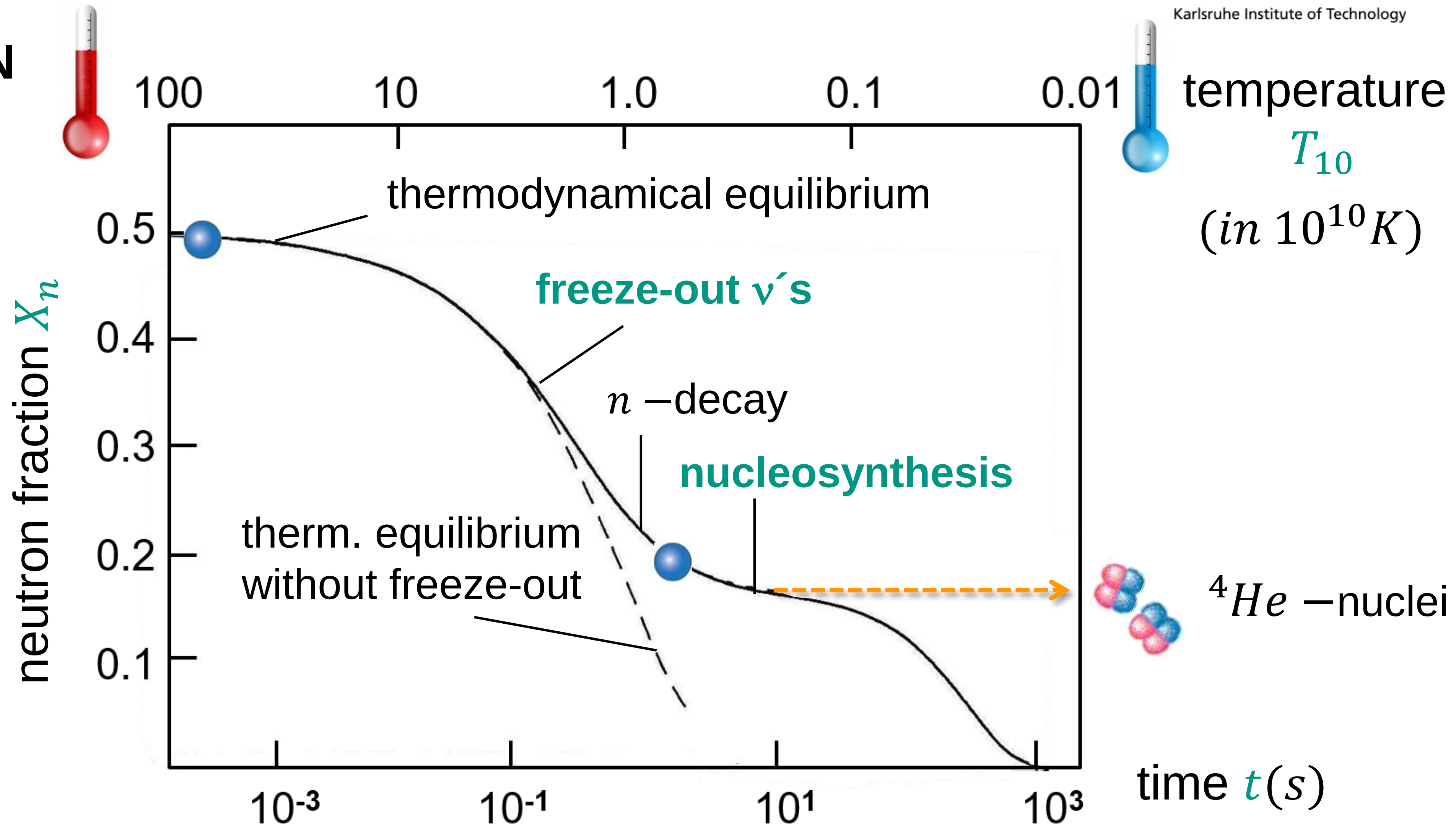
Karlsruhe Institute of Technology

Overview BBN

- fraction X_n of free n

$$X_n = \frac{N(n)}{N(p)+N(n)}$$

- **all neutrons after $t > 60\ s$ end up in ^4He**



Primordial Nucleosynthesis & parameter η

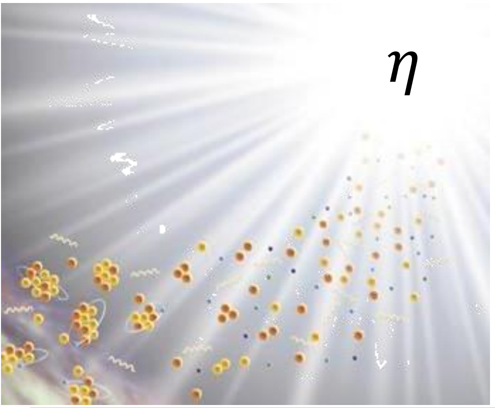
Karlsruhe Institute of Technology

- **Fusion yield of light elements depends on the size of baryon asymmetry η**

  - theoretical estimates of the light element fusion yields have been performed for different values of η:

$$\eta = \frac{n_B - n_{\overline{B}}}{n_\gamma}$$

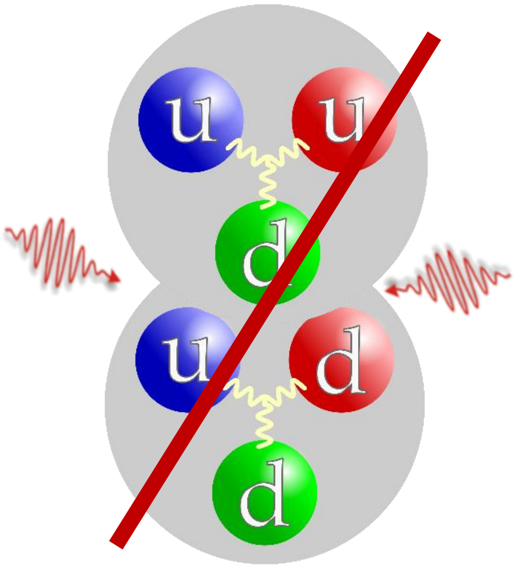
$$\eta = 10^{-6} \ldots 10^{-12}$$



η

↓

different fusion yields



observed precision fusion yields of light elements in BBN

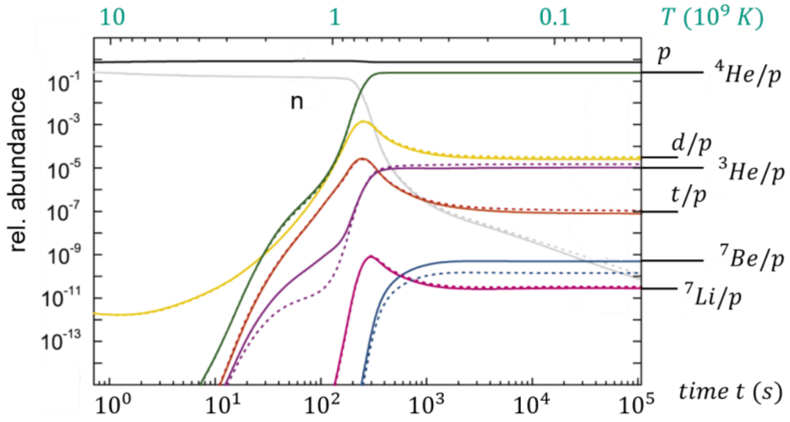
↓

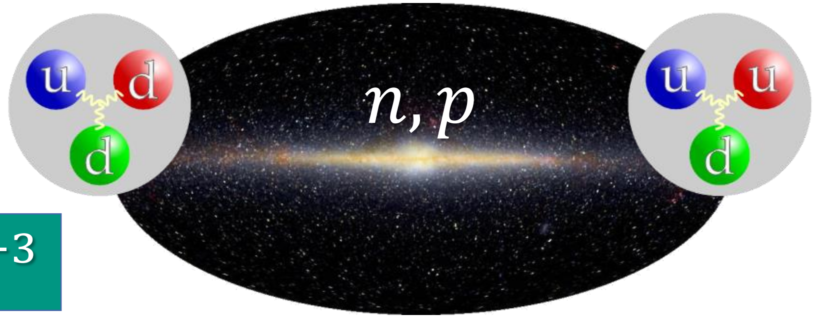
estimates of baryon asymmetry paramter η

Primordial Nucleosynthesis &baryon density ρ_B

- **Fusion yield of light elements depends on the value of the baryon density ρ_B**

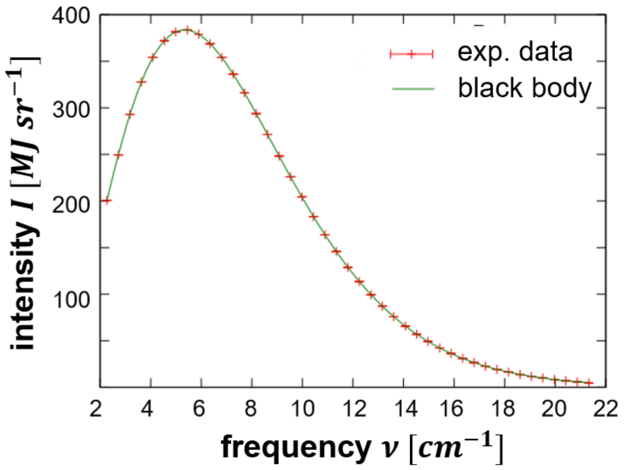
BBN calculation

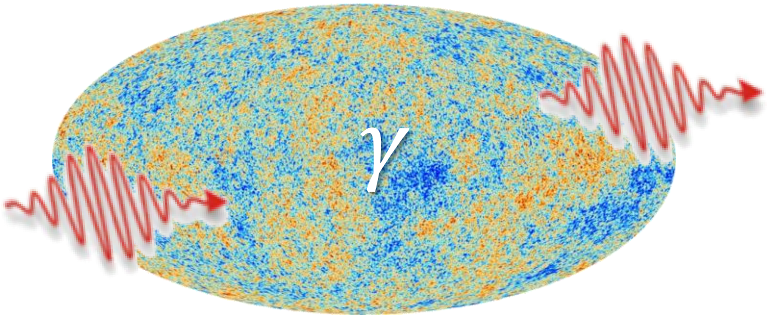




$\rho_B = ?\ cm^{-3}$

Planck distribution



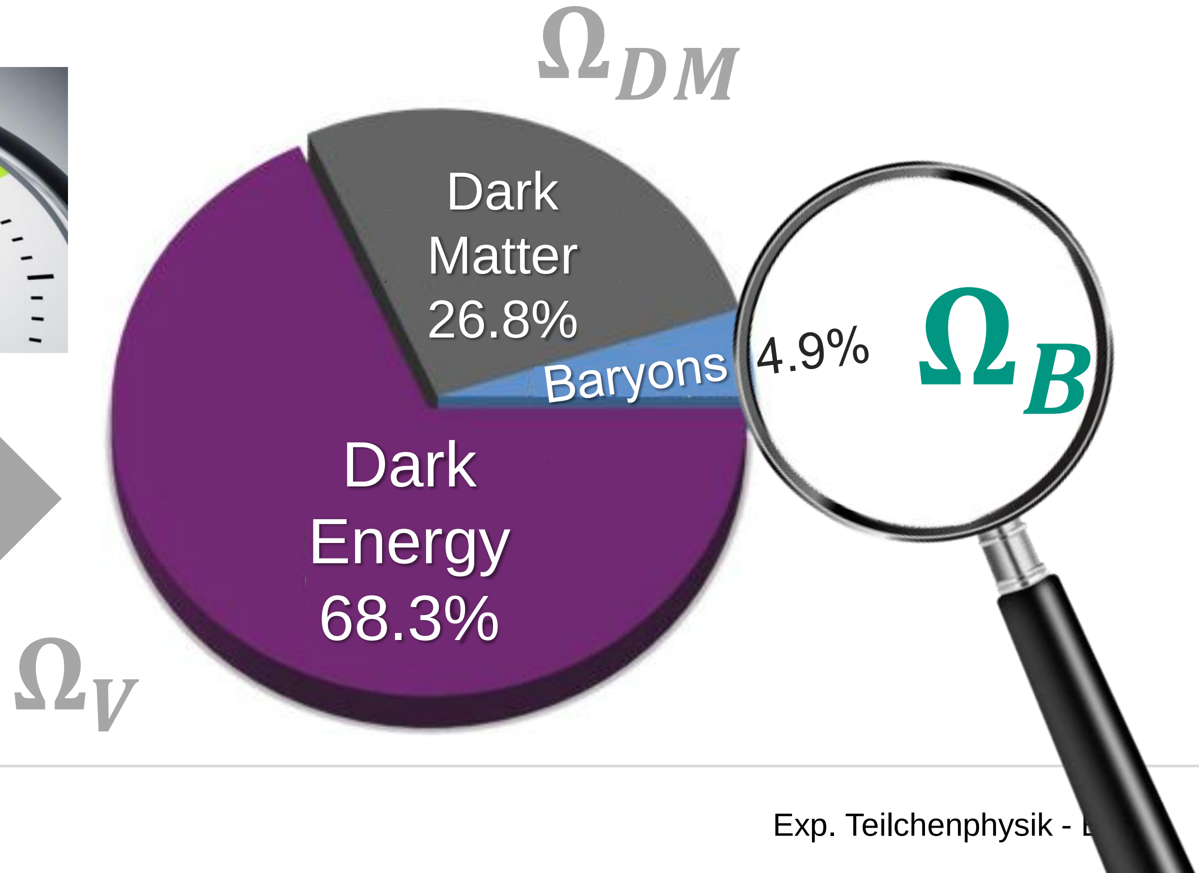


$n_{photons} = (412 \pm 2)\ cm^{-3}$

Primordial Nucleosynthesis & baryon density Ω_B

- **Extracting the baryon density parameter Ω_B from BBN synthesis of elements**

  - a spectroscopic determination of light elements d, ${}^3He$, ${}^{6,7}Li$ determines Ω_B



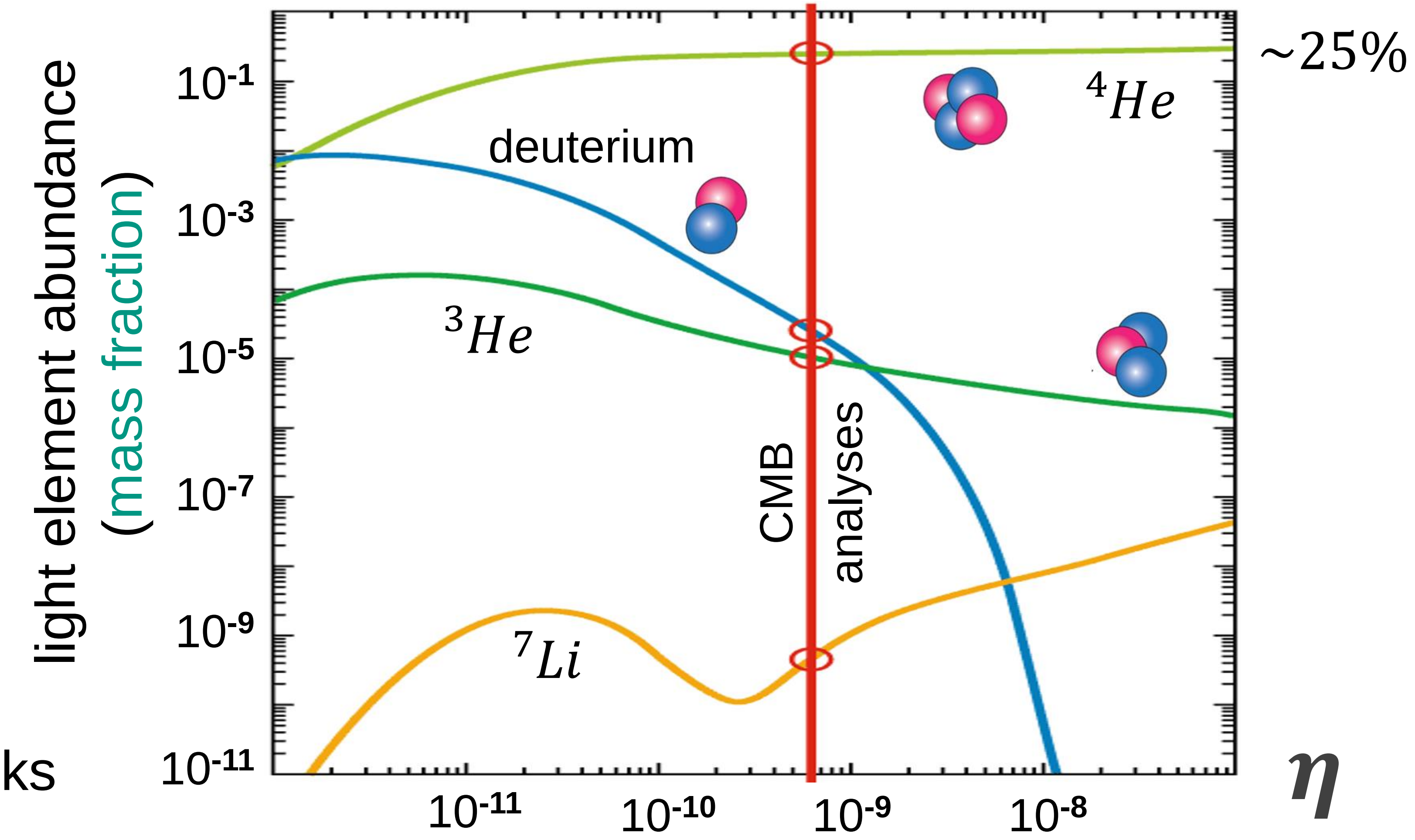
Primoridal nucleosyntheseis - calculations

- **Results of fusion yields for light elements as function of paramter η**

- fusion yield of ^4He is dominating & increases for a larger baryon asymmetry η

- yield of other isotopes is much smaller & covers many orders of magnitude

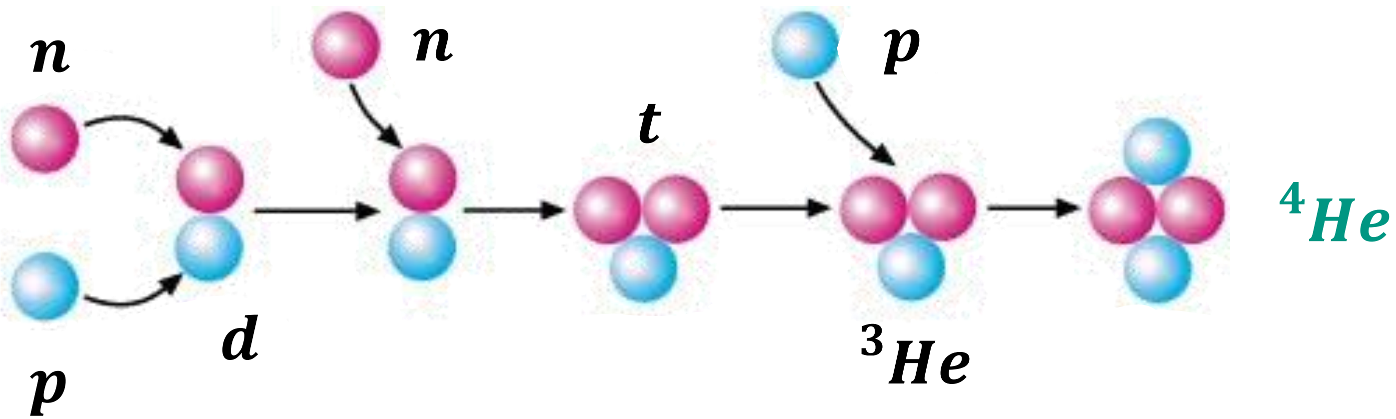
- yield of each isotope as $f(\eta)$ is unique and thus allows important cross-checks

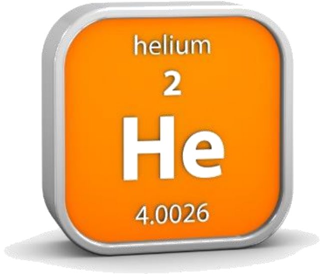


Primoridal nucleosyntheseis & yield of ^{4}He

- **Synthesis of light elements essentially stops at ^{4}He**
  - fusion beyond ^{4}He difficult, as this is a **very stable** (doubly magical) **nucleus**
  - ^{4}He is difficult to destroy, also tiny cross-sections for capture of p, n, d
  - almost all neutrons are incorporated into ^{4}He (n limit its formation)
  - in literature: primordial ^{4}He – **mass** fraction Y_p

one of many BBN reaction pathways to ^{4}He-formation





Primoridal nucleosyntheseis & yield of ^4He

- **A quick estimate on the ^4He –mass fraction Y_p as function of (n, p) ratio**

- we start with the calculated (n, p) –ratio of $1 : 7$ at the onset of BBN

- we then form ^4He via the initial ratio out of $\underbrace{(2n + 2p) + 12\,p}_{2\,n\,:\,14\,p} = {}^4He + 12\,p$

- we now calculate the **^4He mass fraction Y_p** $= {}^4He/(2\,n + 14\,p)$

  with $m_n = m_p = m_{nucl}$ we have $= 4\,m_{nucl}/16\,m_{nucl} = 0.25$

- for an (n, p) –ratio of $1 : 7$ we expect a **^4He mass fraction $Y_p = 0.25$**