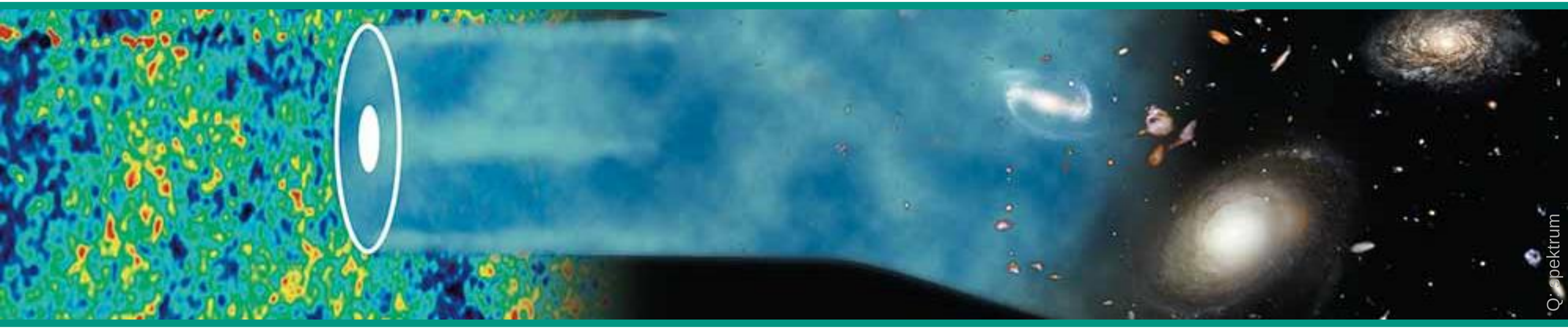
Karlsruhe Institute of Technology

Introduction to Cosmology

Winter term 23/24 Lecture 10 Jan. 9, 2024



Cosmology 2024

- **Outlook:** 2 Postdocs* at MPA riding cosmic waves of CMB in the early universe to study the parameters Ω_i

2024

BAO

2023

[Riding the Cosmic Waves - Pressepaket (mpg.de)]

Recap of Lecture 9

- $\boldsymbol{CMB}$ – **multipole expansion: a unique tool for cosmology**

  - foreground: **thermal** (dust) / **non–thermal** (synchrotron, free–free scattering)

  - $\boldsymbol{BOOMERanG}$, $\boldsymbol{WMAP}$ –**mission** to $\boldsymbol{L2}$ – point: high–resolution $\boldsymbol{CMB}$ maps

  - angular correlation function $\boldsymbol{C(\theta)}$ for $\boldsymbol{\Delta T/T}$: expansion in **multipoles** $\boldsymbol{\ell}$

  - **power spectrum**: $(\Delta \boldsymbol{T})^{\mathbf{2}} = \ell \cdot (\ell + \mathbf{1}) \cdot \boldsymbol{C_\ell}/\mathbf{2}\boldsymbol{\pi}$

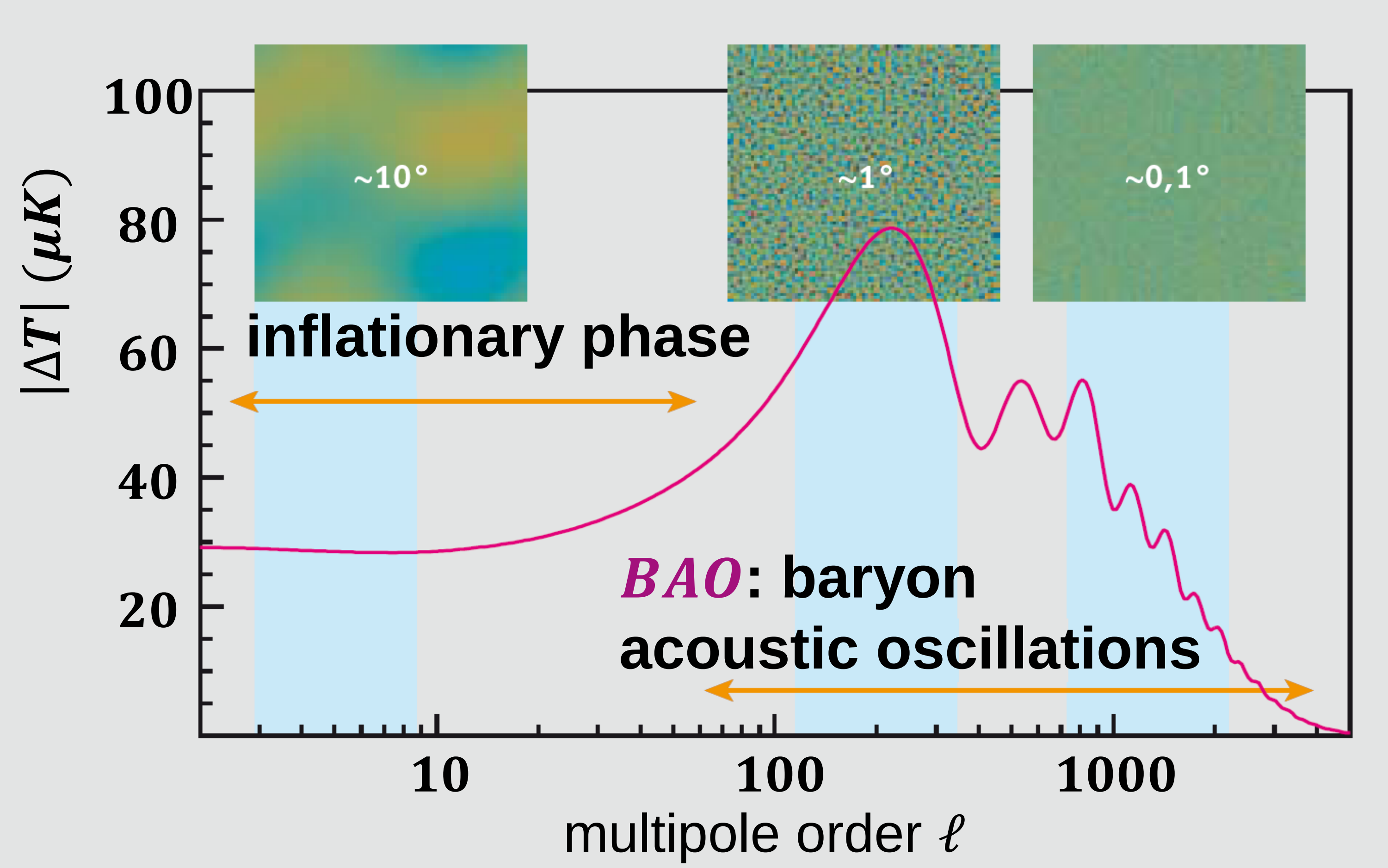
  - **large** angles: flat **Harrison Zel´dovich** spectrum (zero–point $\boldsymbol{QM}$ fluctuations)

  - **small** angles: $\boldsymbol{BAO}$ – $\boldsymbol{B}$**aryon** $\boldsymbol{A}$**coustic** $\boldsymbol{O}$**scillations** ($\boldsymbol{DM}$ signature), today

Outlook: from scale invariance to BAOs

- CMB multipole spectrum: study physics of the early universe $t < 378.000\ yr$

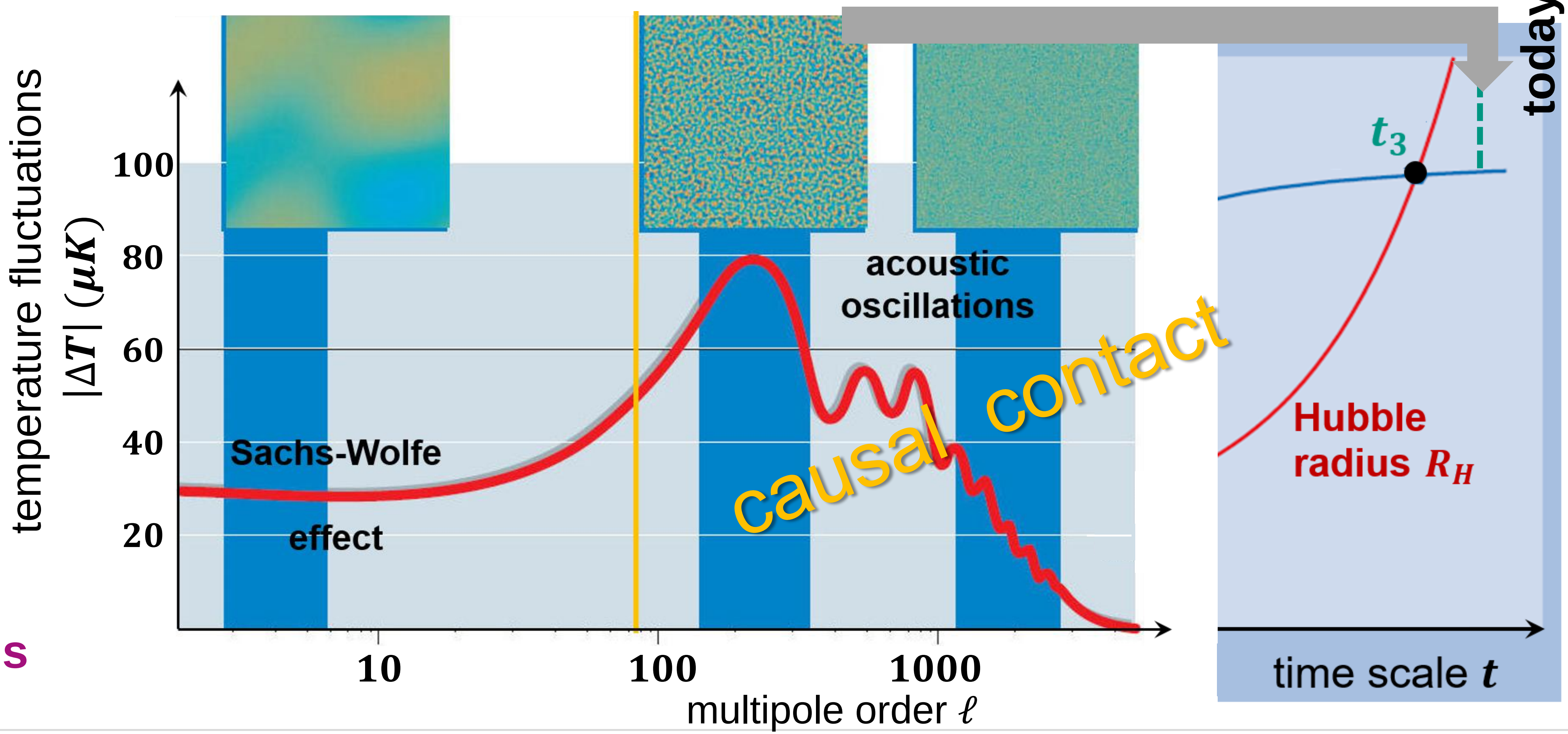
- **large scales:** fluctuations are **frozen** (´QM in the sky´)
- **small scales**: **modification** due to gravity & **acoustic sound waves** in the plasma
- BAOs: a perfect tool to measure **baryon density Ω_B** & **dark matter density Ω_{DM}**



Entire CMB spectrum up to large multipoles ℓ

- On smaller scales, the **initial density pertubations have been modified**

- **small scales**: we see CMB – **modes modified** due to different **causal interactions**



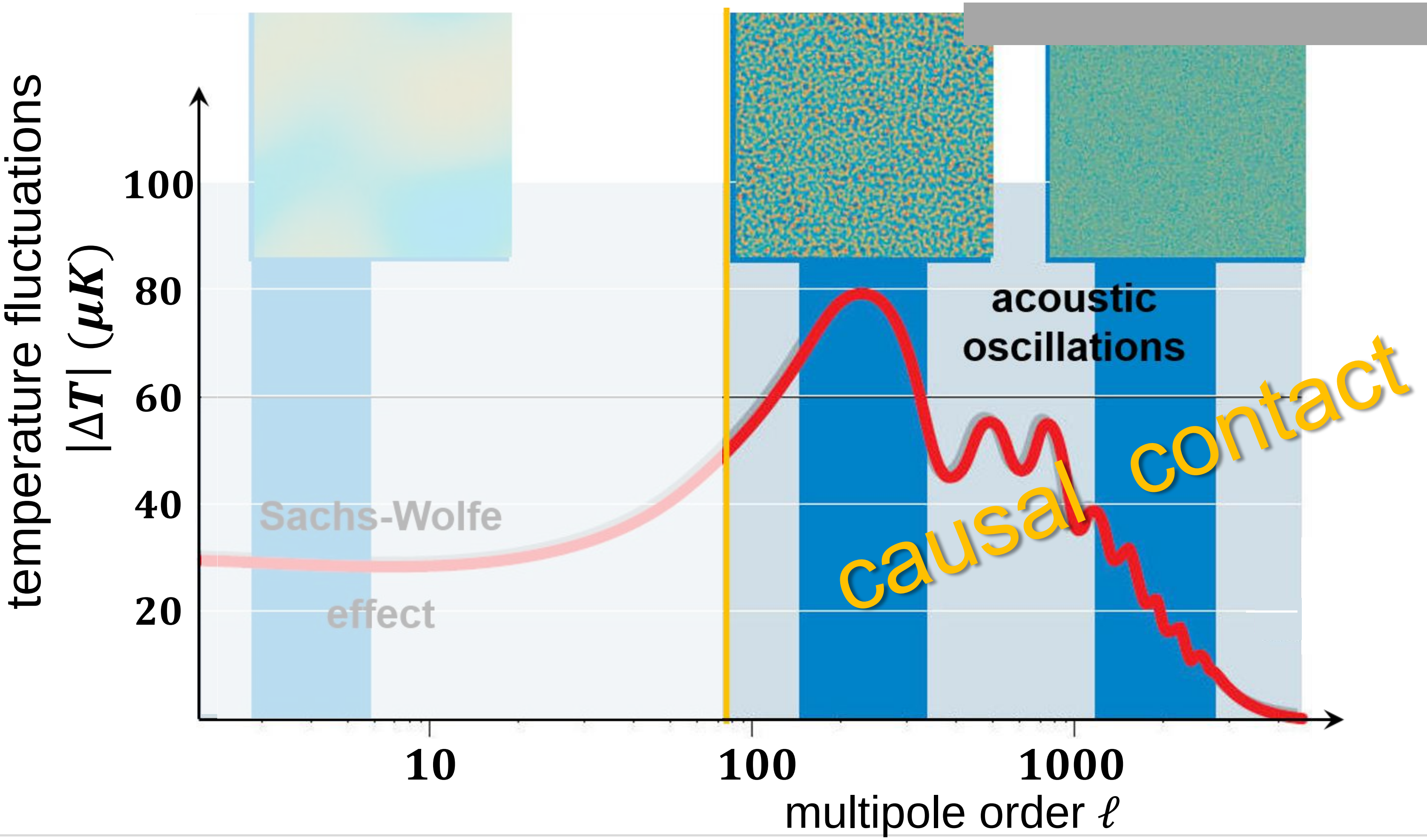
Entire CMB spectrum up to large multipoles ℓ

- On smaller scales, the **initial density pertubations have been amplified**

- a model yielding a **harmonic oscillator**:

gravity

radiation pressure



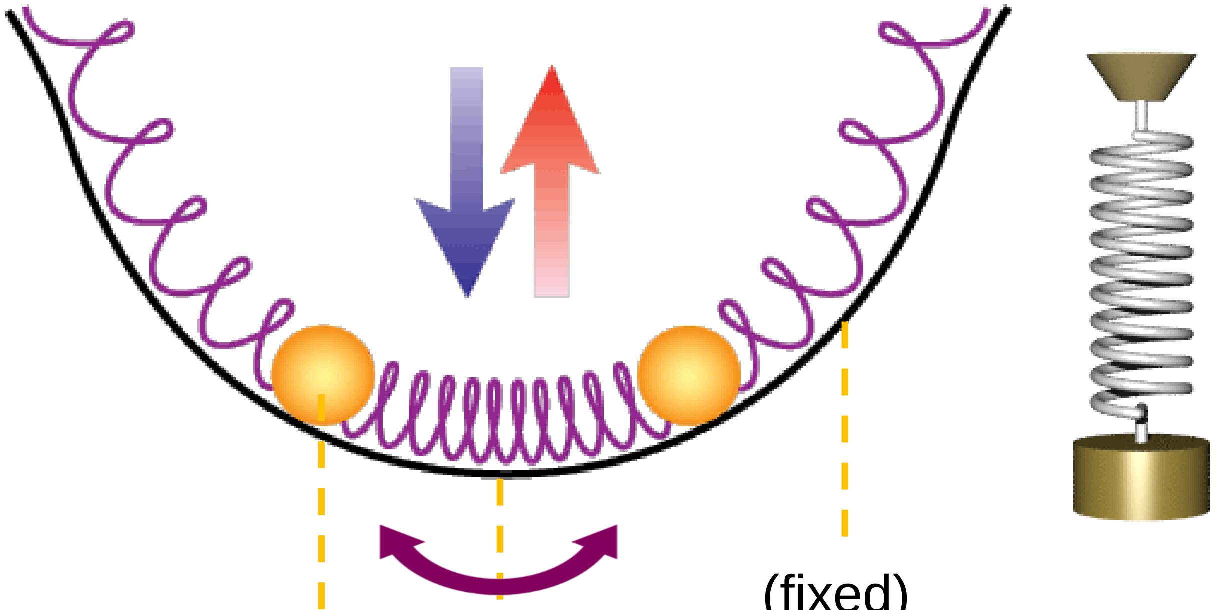
Baryon Acoustic Oscillations – BAO

- **modification is due to close coupling of baryonic matter with radiation**

- once a density perturbation is **re−entering** into causal contact, it will be shaped by **2** forces:

a) **gravitational attraction**
from dark matter, forming a **quasi−fixed gravity well** (**dark matter Ω_{DM}** is dominant & does **not** interact with photons)

b) **radiation pressure**
from photons, restoring force on baryons



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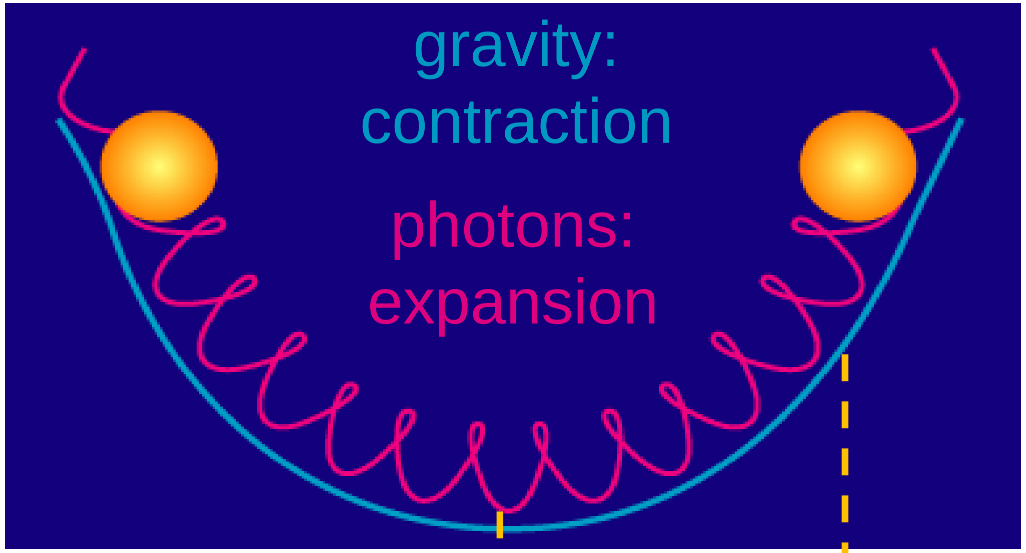

oscillating mass: **baryons**

(fixed) gravitational potential **Dark Matter**

restoring force: **photons**

Baryon Acoustic Oscillations – BAO

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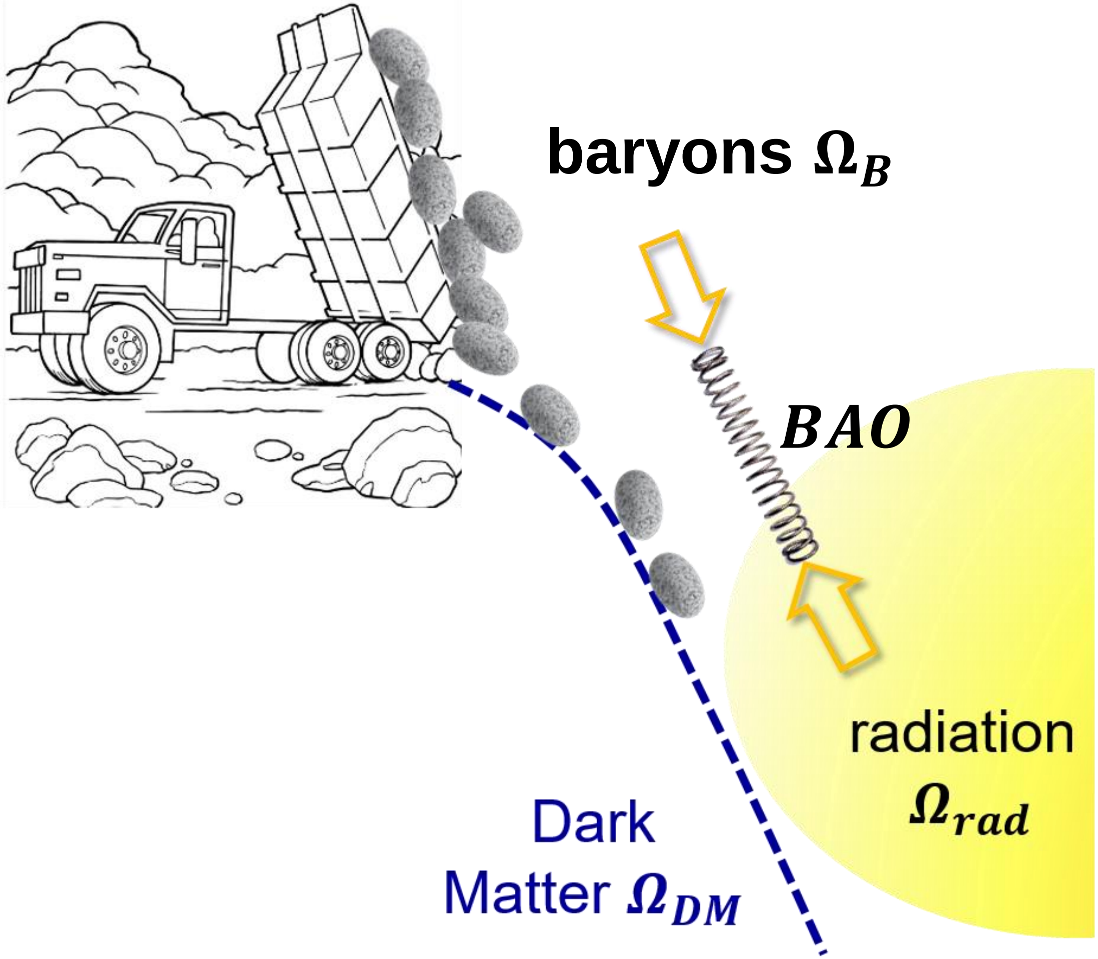
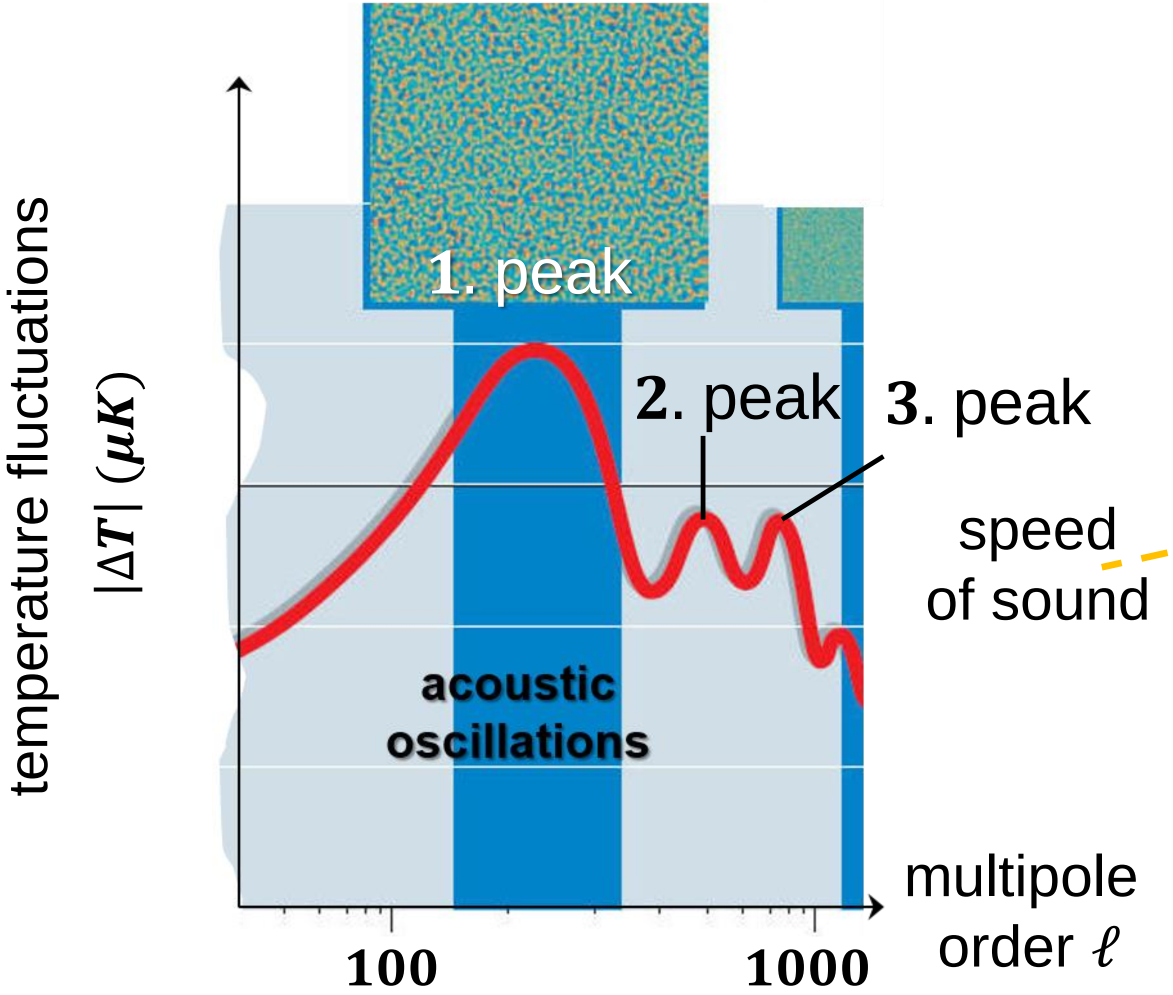
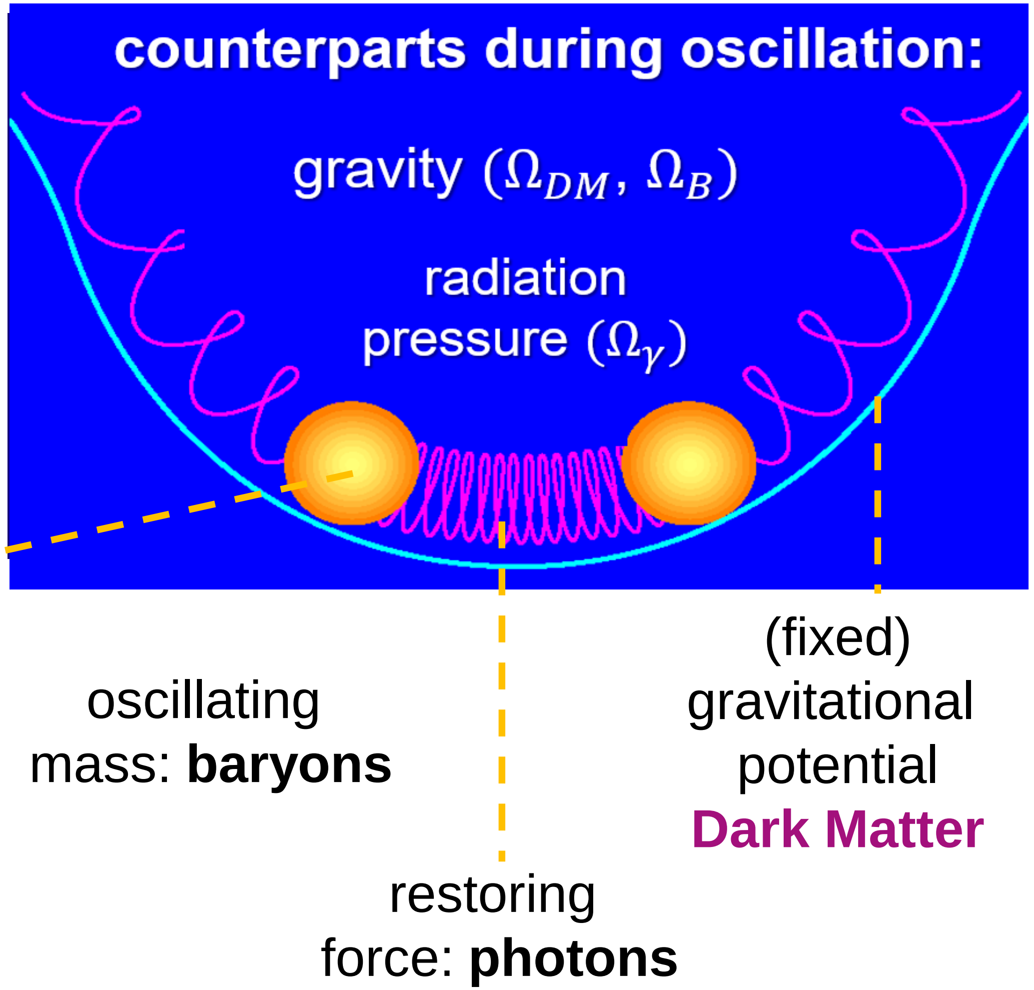


Baryon Acoustic Oscillations – *BAO*

- **modification is due to close coupling of baryonic matter with radiation**





BAO – acoustic waves in the early universe

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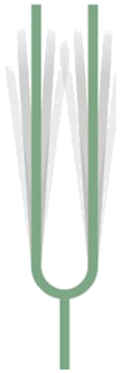
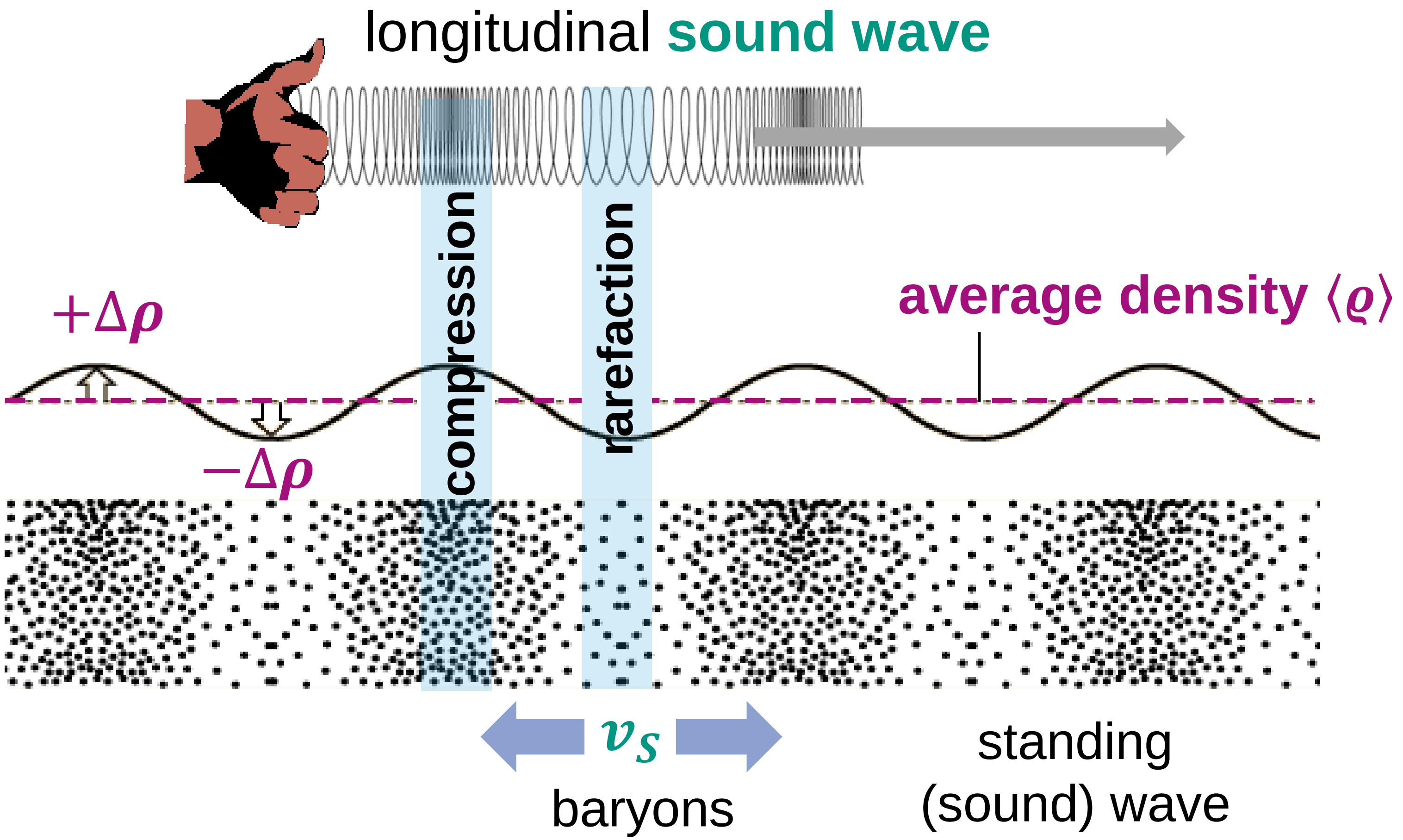
- ***John Peebles***: **standing acoustic waves in the early primordial plasma**

- $\boldsymbol{BAOs}$ are based on well–established physics: **standing acoustic waves**

- important parameter: **the speed of sound v_S**





Recap*: speed of sound v_S during phase transition

■ **Phase transition from plasma to neutral matter: change of speed of sound**

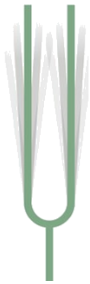
- **speed of sound v_S** before (plasma) & after (neutral atoms) recombination:

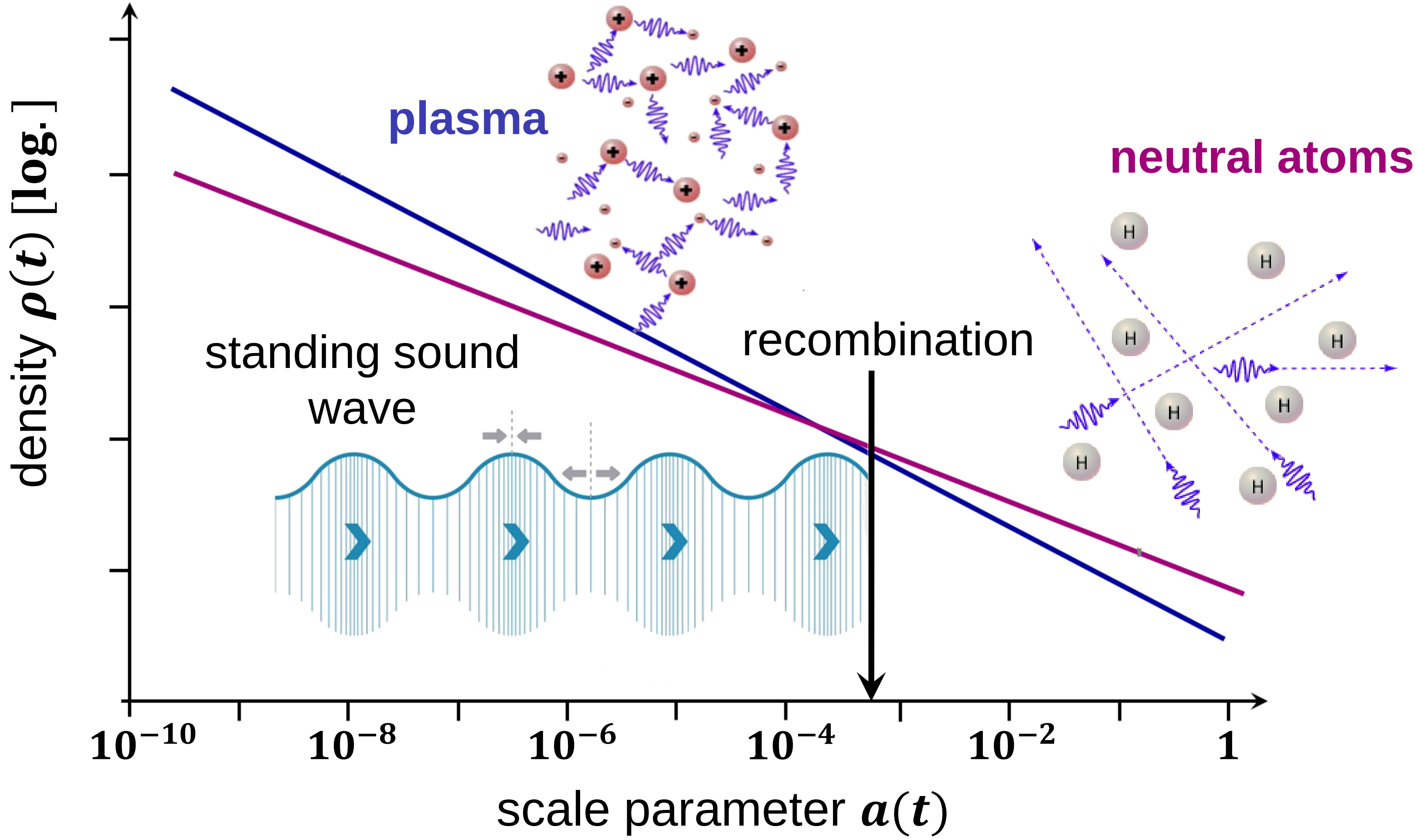
- **plasma**: very fast!

$$\boxed{v_S^2 = \frac{\partial p}{\partial \rho} = \frac{c^2}{3}}$$

- **neutral matter**:

$v_S^2 \approx 0$ ⇨ full stop!





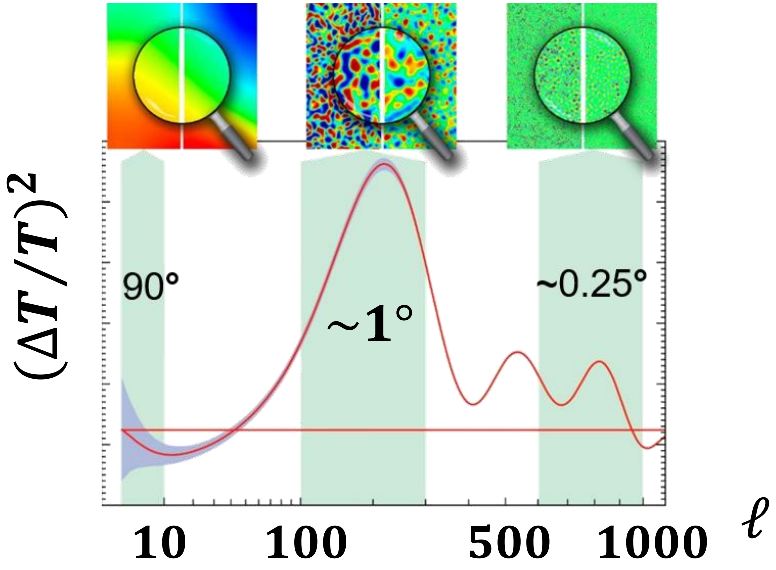
 *see lecture 8, p. 31

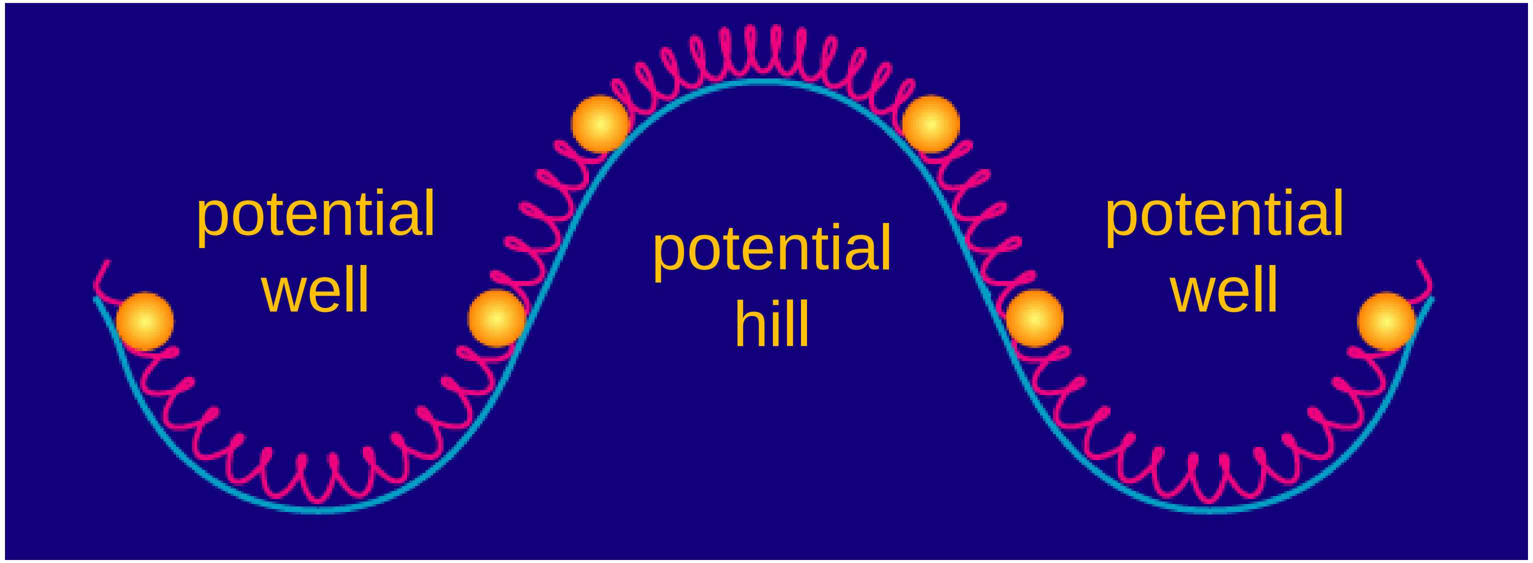
BAO – acoustic waves in the early universe

- **One more thing: acoustic oscillations are all in phase over entire universe**

- acoustic oscillations in different (non−causally linked) regions: ⇨ *<u>identical phases</u>*
  ⇨ add **coherently**: no destructive interference (as expected for random phases)

- a possible solution, again: **inflation** in the very early universe has **synchronised** all density fluctuations – thus they all start with the **same phase**

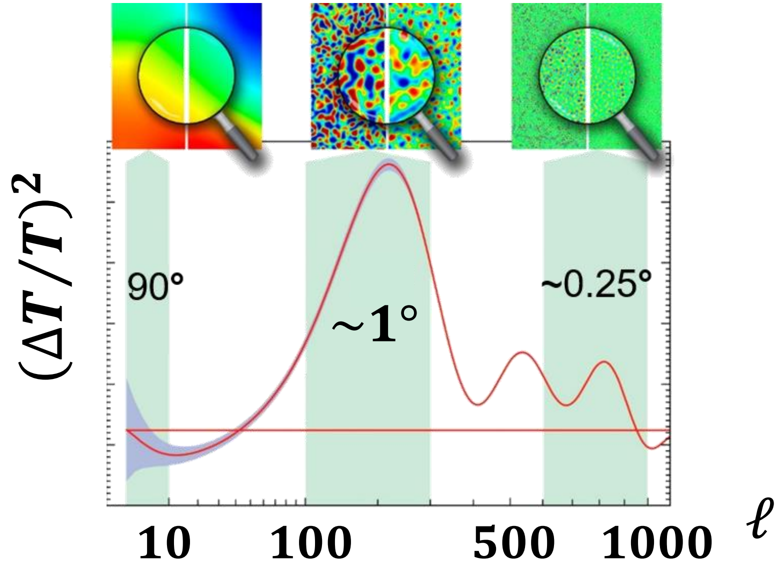


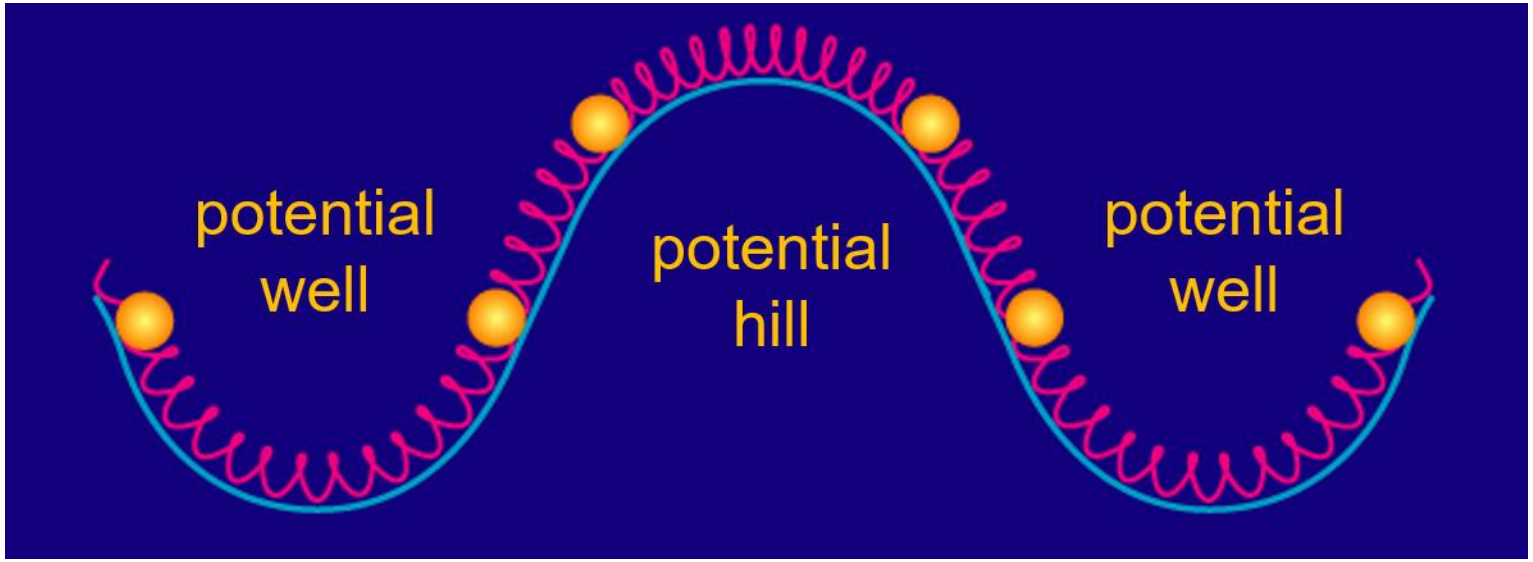


BAO – acoustic waves in the early universe

- **One more thing: acoustic oscillations only in the plasma state $t < 378000\ yr$**

- $t = 0$: causal interaction of baryons & photons in presence of DM gravity wells
  ⇨ a density perturbation starts to move with **speed of sound $v_S = c/\sqrt{3}$**
- $t = 378000\ yr$: after recombination no baryon−photon interaction, sound wave is ´**frozen**´ as $v_S \approx 0$ (⇨ sound horizon of fundamental mode from $t = 0$ to $t = t_{rec}$)



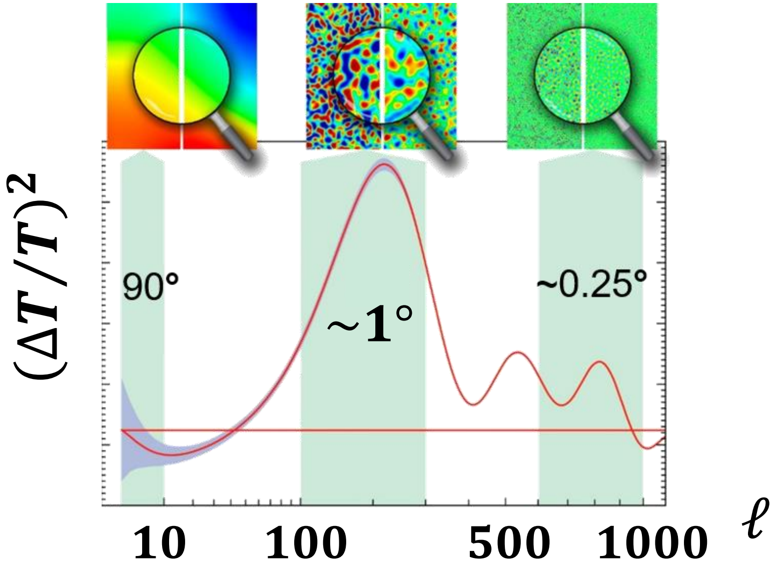


BAO – acoustic waves in the early universe

- **Analogon: fundamental mode & overtones as standing acoustic wave(s)**

wave–lengths λ_n of eigen–modes

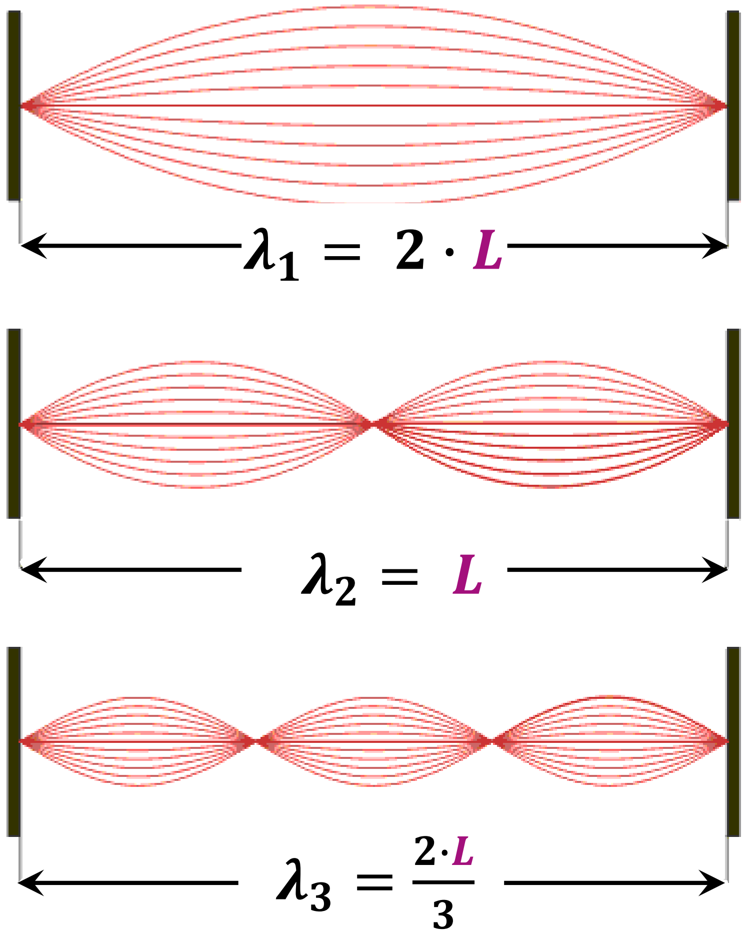
$$\boxed{\lambda_n = \frac{2L}{n}} \; *$$



fundamental mode

1. overtone

2. overtone

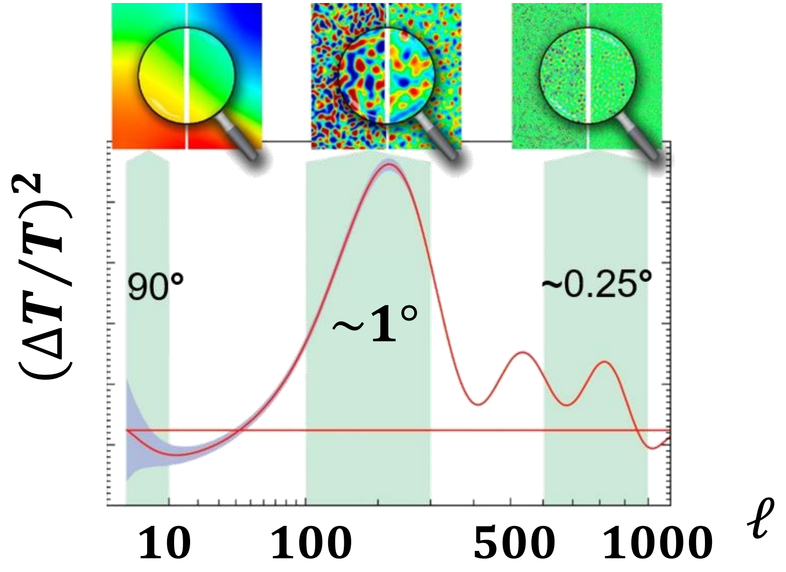


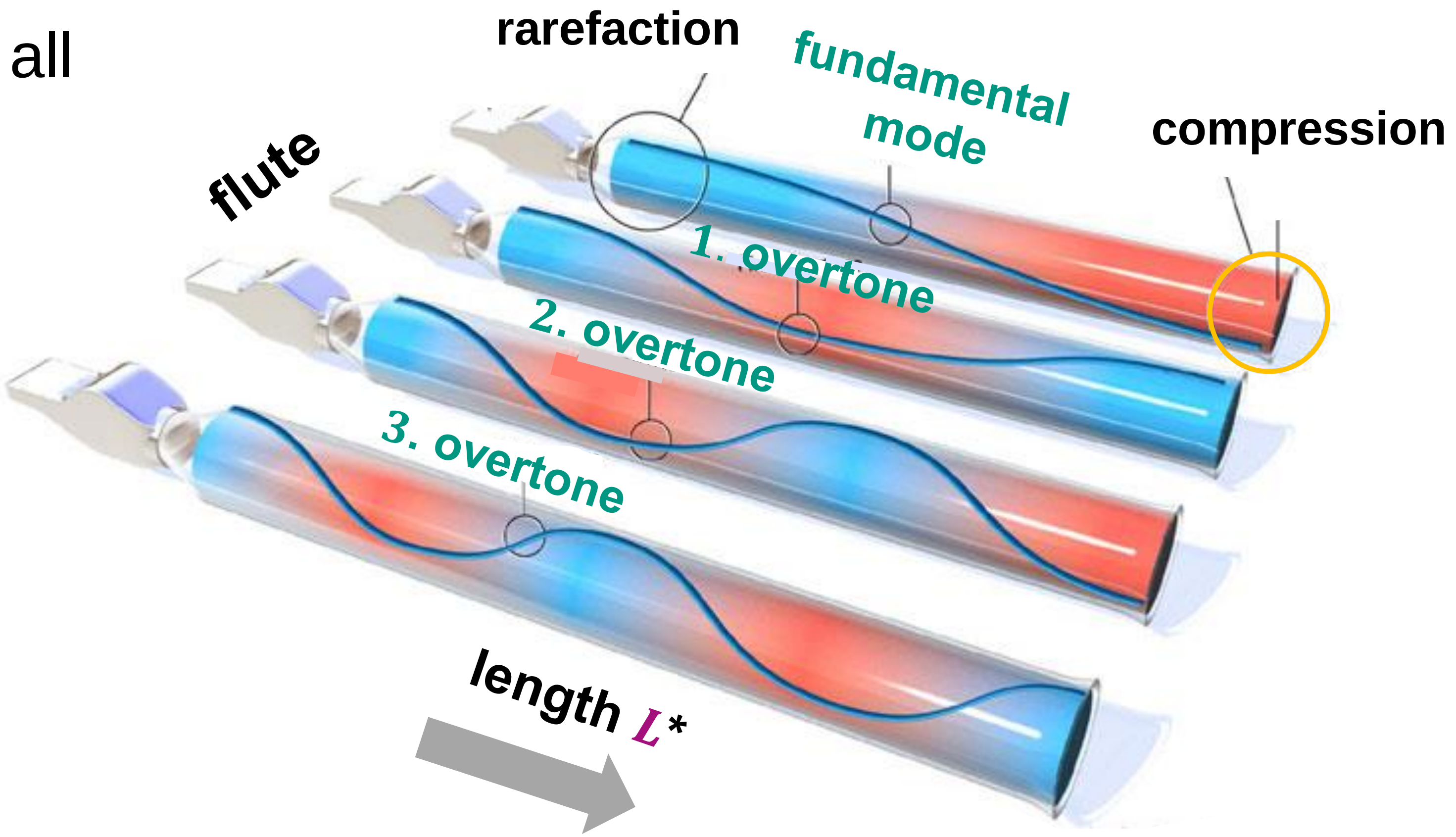
 *L = **sound horizon at** $t = t_{rec}$

BAO – acoustic waves in the early universe

- **Analogon: fundamental mode & overtones in flute as standing acoustic wave**

- $RECAP$: only **standing acoustic waves** **will interfere constructively**, all others will be **wiped out** due to destructive interference



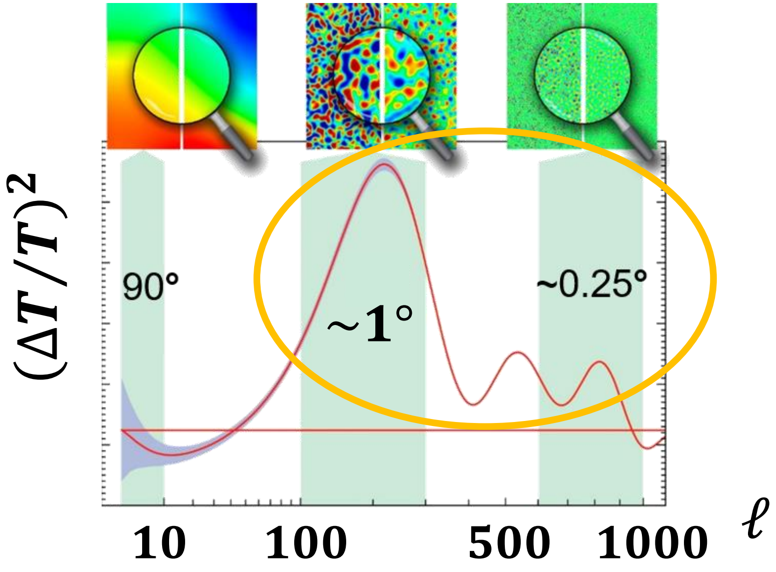


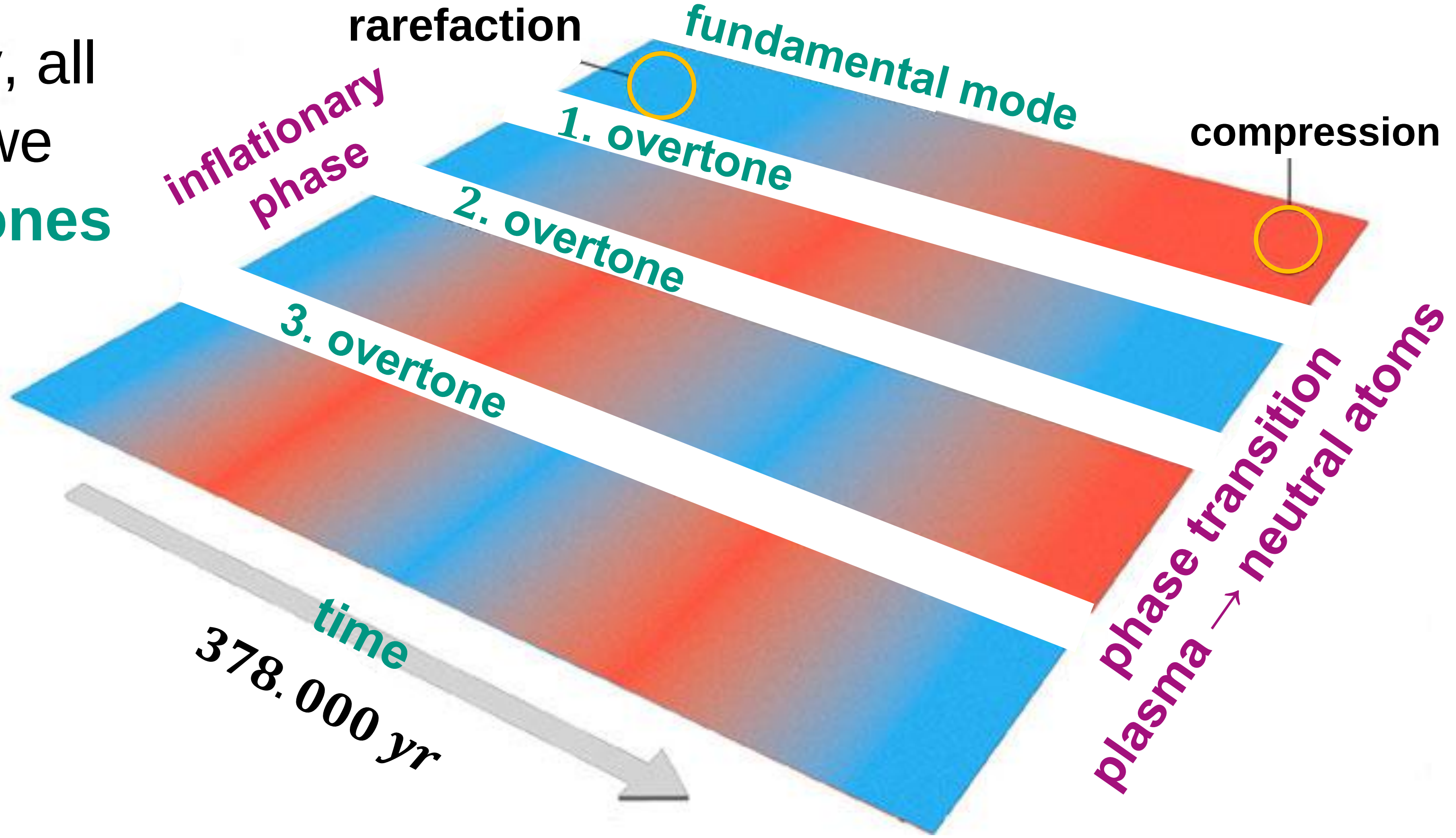
$^*L$ = sound horizon at $t = t_{rec}$

BAO – acoustic waves in the early universe

- BAO: only fundamental mode & overtones as standing acoustic waves

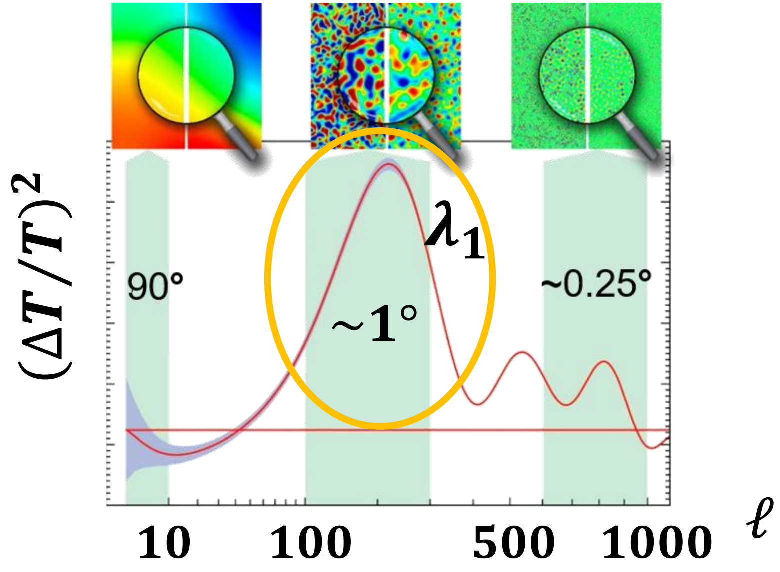
- only standing BAO waves **will interfere constructively**, all others will be **wiped out**, ⇨ we have: fundamental + overtones

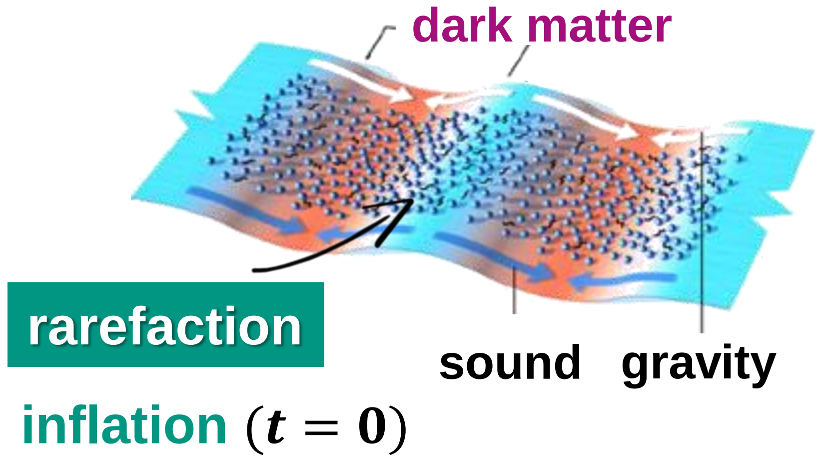


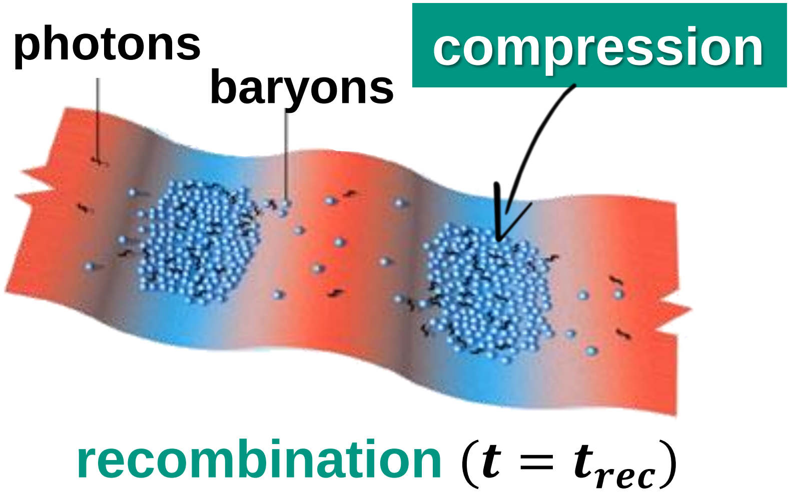


BAO – acoustic waves in the early universe

- BAO: only fundamental mode & **overtones as standing acoustic waves**
  - $t = 0$ up to $t = 378000\ yr$: fundamental mode (**largest wavelength λ_1**) with multipole $\ell \approx 200$ has just gone from **rarefaction → compression** (**one cycle**)
  - fundamental mode with $\ell \approx 200$ consists of a total of $(2\ell + 1 \approx 400)$ independent fluctuations, all adding with ***coherent phase*** to this specific mode with λ_1





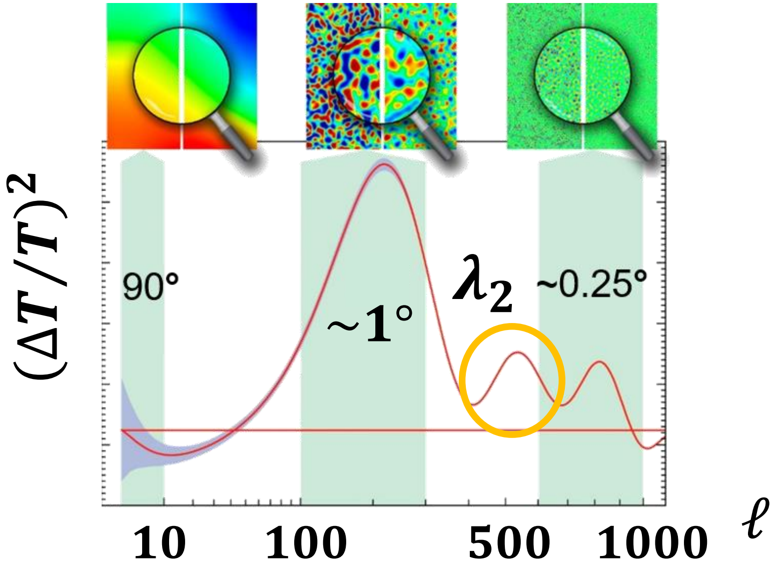


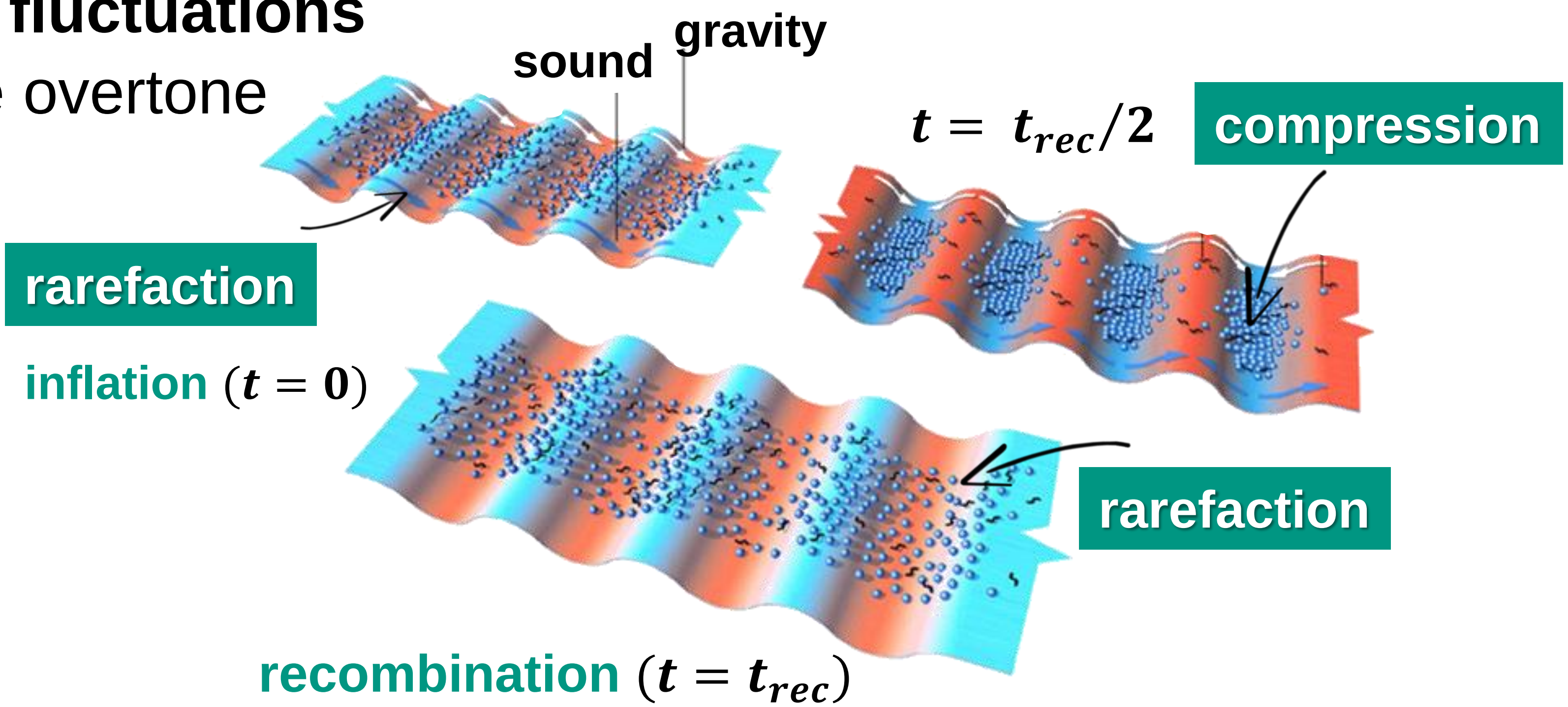
BAO – acoustic waves in the early universe

- **BAO: only fundamental mode** & **overtones as standing acoustic waves**

  - an **overtone** (here: the **first**) with shorter wavelength (here: $\lambda_2 = {}^{\lambda_1}/_2 = L$) has just gone through > **1** cycle (here: **rarefaction → compression → rarefaction**)

  - again: many **independent fluctuations** ***add coherently*** to a single overtone

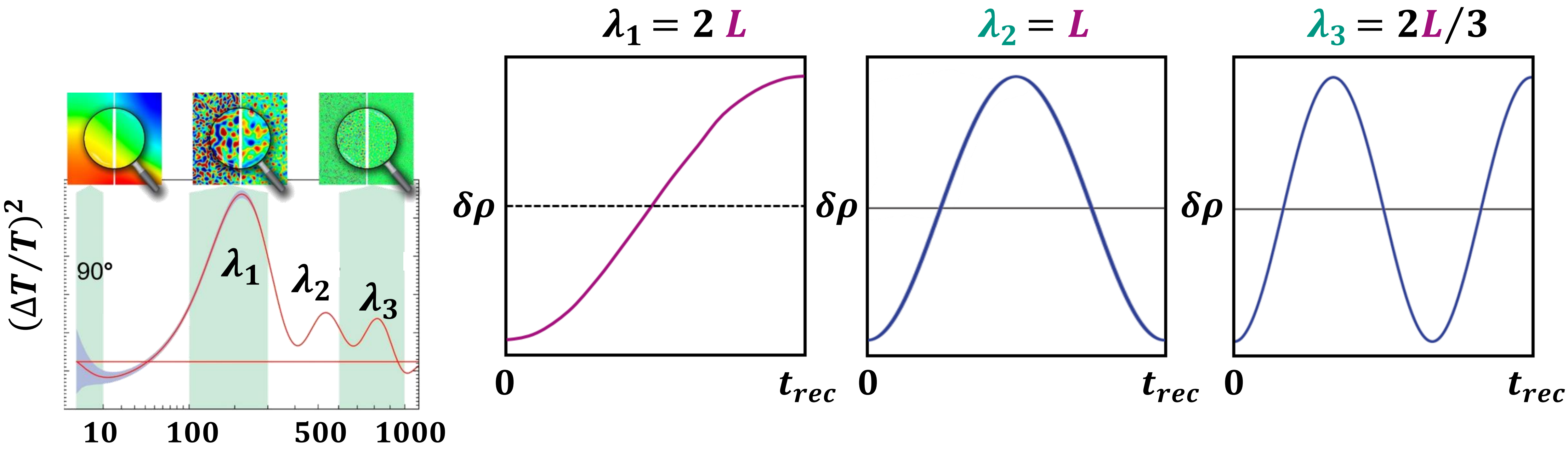




BAO – acoustic waves in the early universe

- BAO: **only fundamental mode** & **overtones as standing acoustic waves**

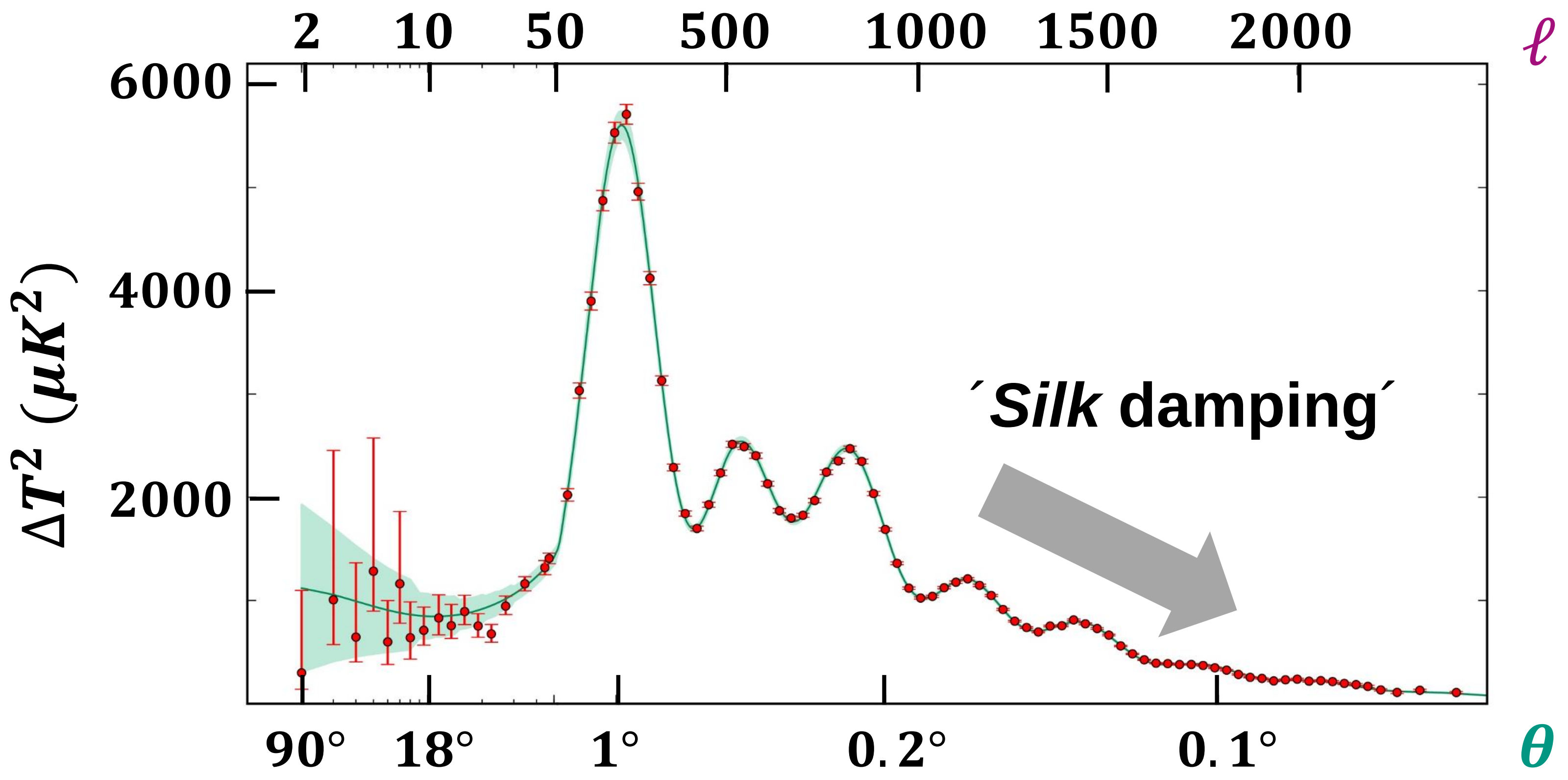
  - a ´**cosmic symphony**´ in the early universe of fundamental mode plus many overtones, each composed of $2\ell + 1$ independent (**coherent**!) fluctuations



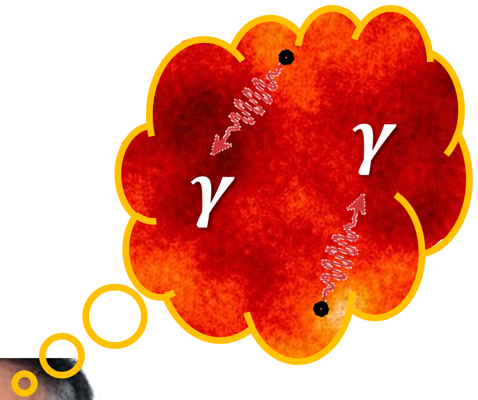
BAO – acoustic waves are damped at high ℓ

- **Why are the temperature fluctuations at large multipoles ℓ being damped?**

  - BAO take place in a strong **heat bath**: **damping** of **small angle θ amplitudes**



photons possess a finite $m$ean–$f$ree–$p$ath (mfp)

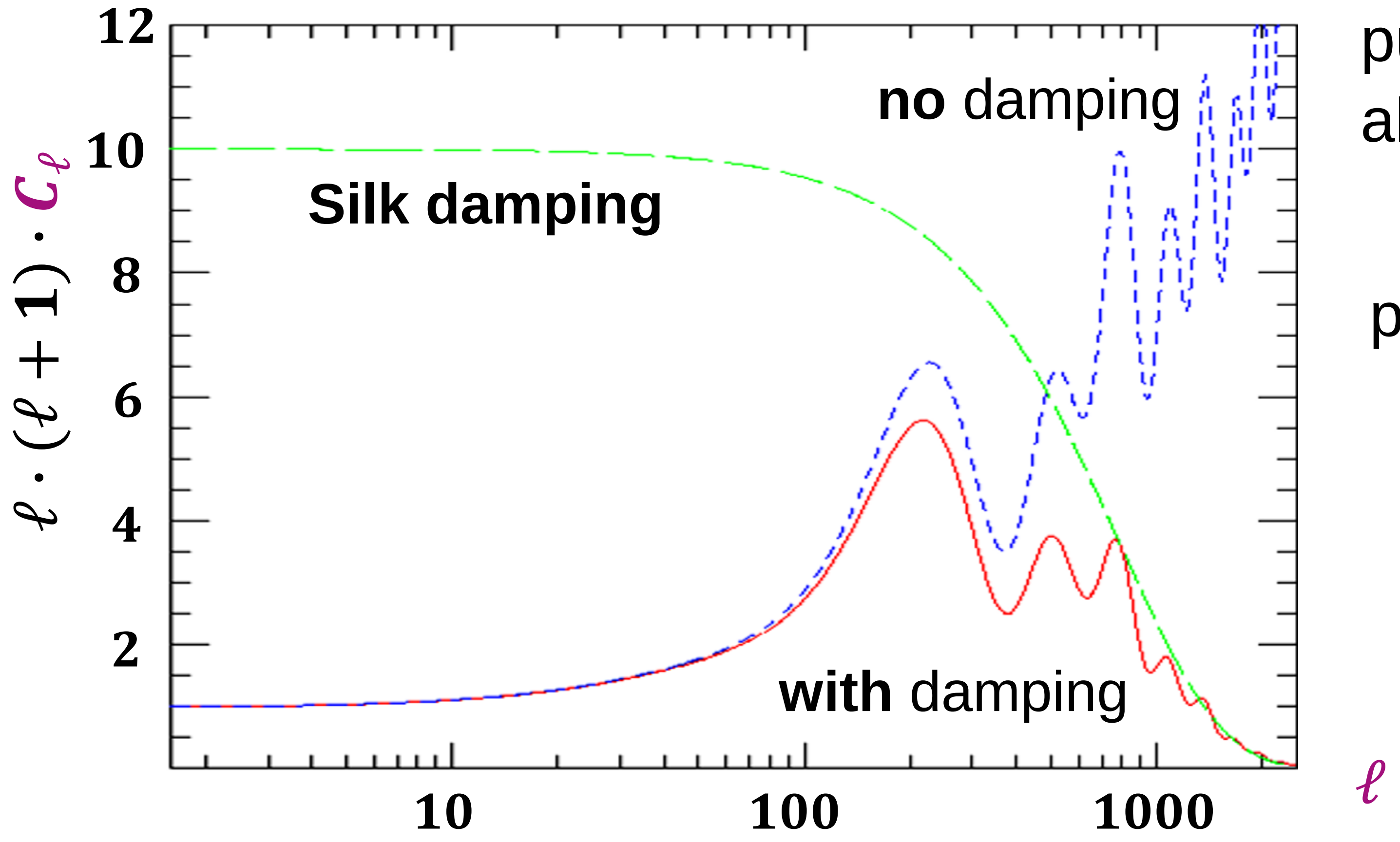


Joseph ***Silk***

BAO – acoustic waves in the early universe

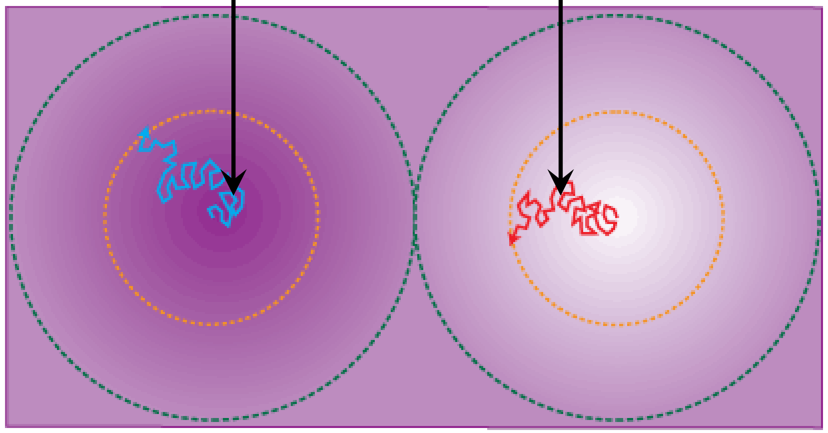
- **temperature fluctuations at large multipoles ℓ: impacted by photon diffusion**

\- photons **diffuse out** from **hotter** (overdense) to **colder** (underdense) regions, thus



pushing electrons e^- & protons p along: ⇨ **damping** of anisotropies at smaller scales (**high** ℓ)

photons **diffuse out** of perturbations

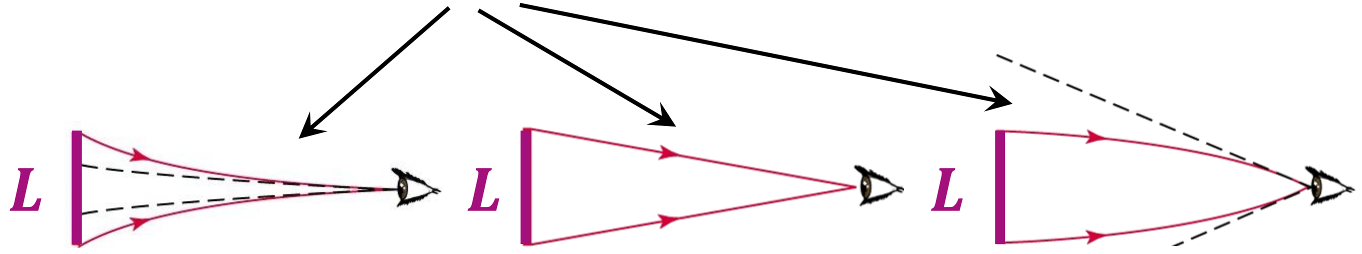


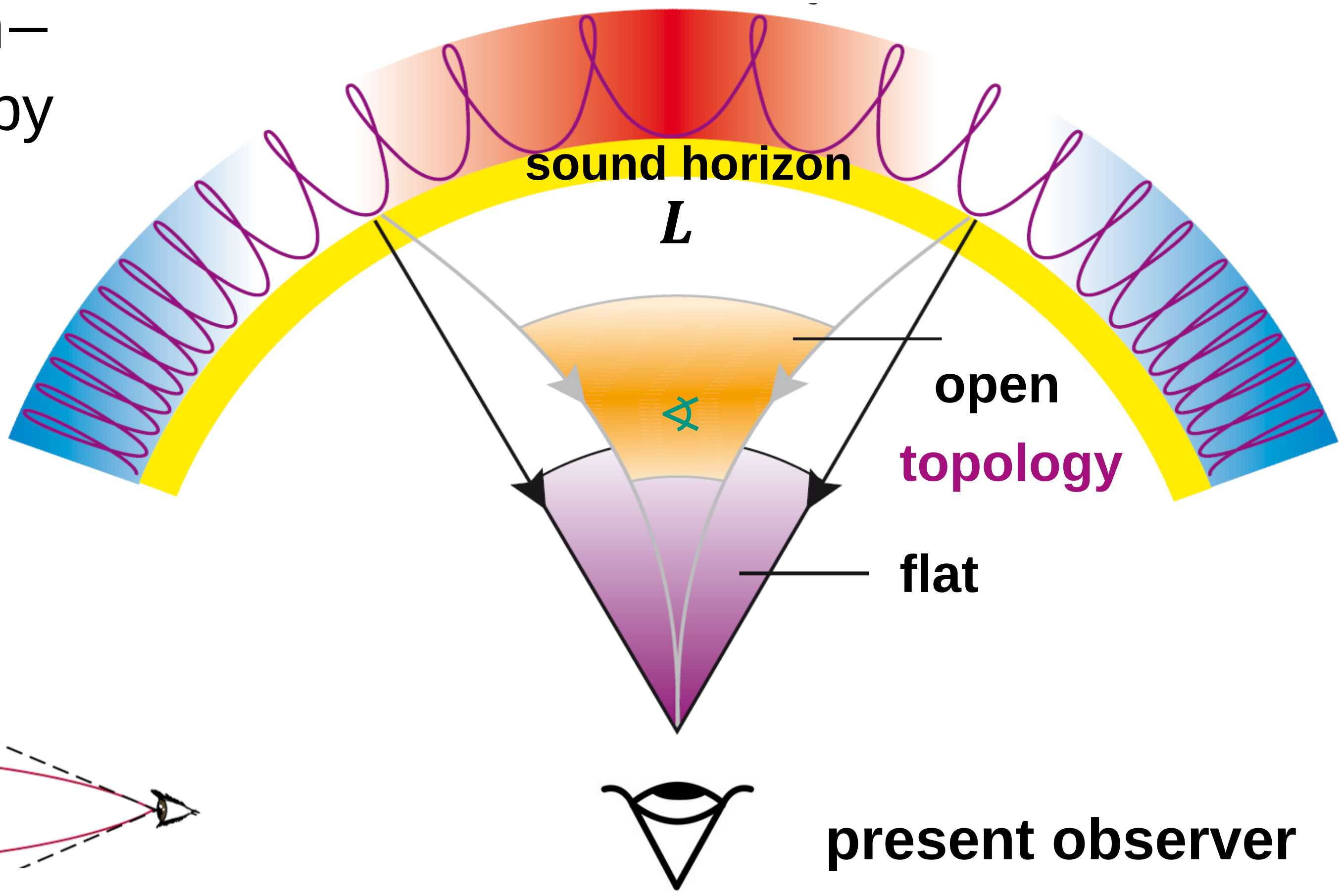
BAO – determining the topology of the universe

■ **Taking into account the expansion of the universe and its topology**

- since decoupling ($t = t_{dec}$) all length–scales (such as λ_1) have increased by factor $a = (1 + z_{dec}) \approx 1100$ due to the **cosmic expansion** since t_{dec}

- now: let´s investigate whether the corresponding **angle** $\sphericalangle$ is **modified** by the **topology** of the universe



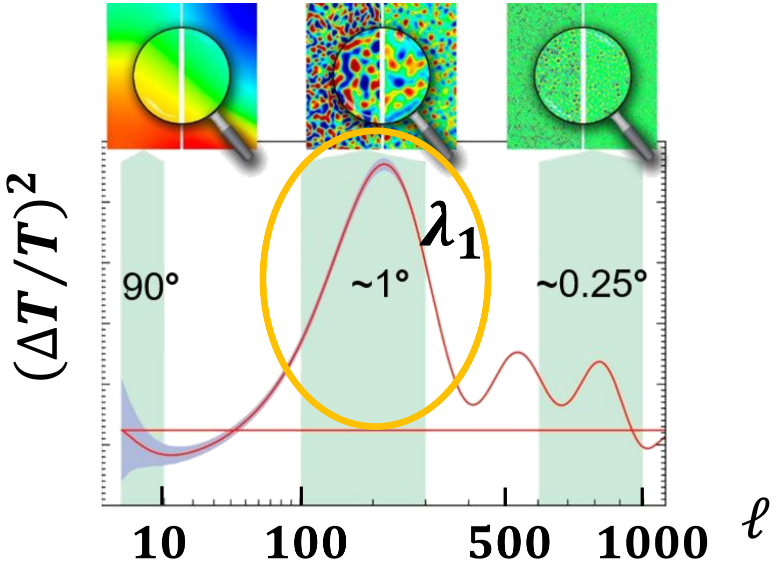


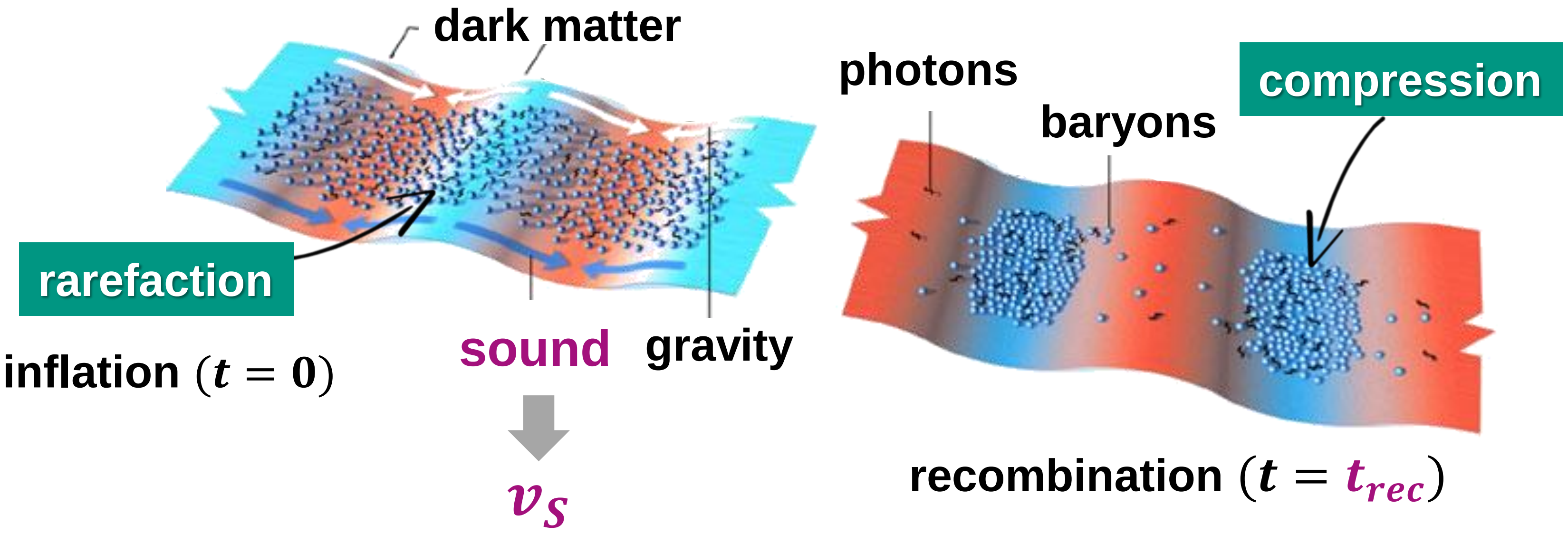
BAO – determining the topology of the universe

- **The fundamental mode revisited: the absolute sound horizon scale is known**

- the wavelength of the fundamental mode $\lambda_1 = 2\,L$:
  ⇨ measure the **absolute scale** of the **sound horizon** $L \sim v_S \cdot t_{dec} = \frac{c}{\sqrt{3}} \cdot t_{dec}$

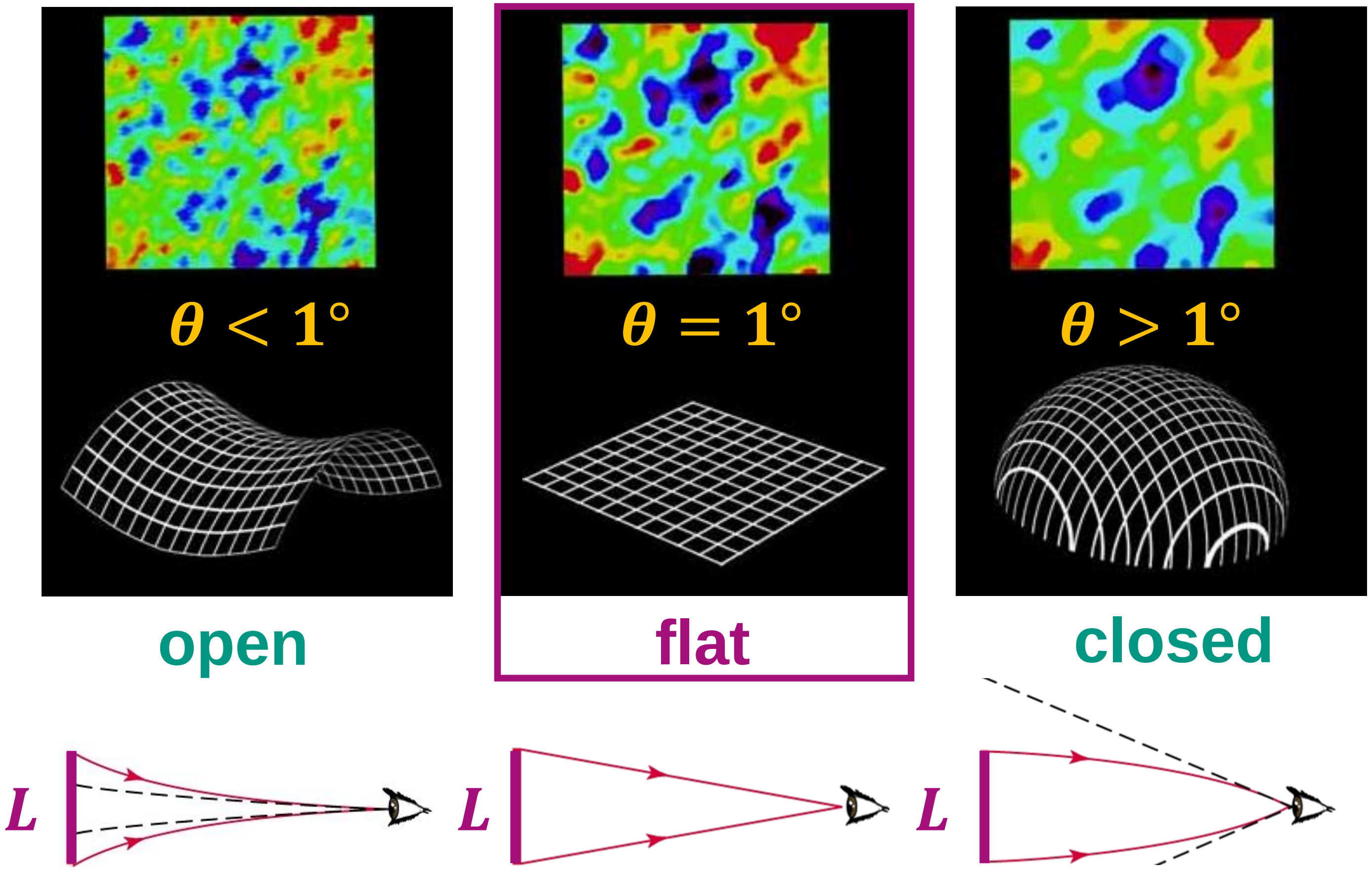
- we measure $L = \lambda_1/2$ via angular size (multipole ℓ_1) of the **fundamental mode**





BAO – determining the topology of the universe

- **Taking into account the expansion of the universe and its topology**



- for a **flat universe** we expect the fundamental mode ($1.\ CMB$ peak) to appear at **multipole** $\ell_1 \approx 200$

total energy Ω_{tot} of universe:

... via multipole order ℓ_1 of **first acoustic CMB peak**

$$\ell_1 \sim 1/\sqrt{\Omega_{tot}}$$

BAO – determining the topology of the universe

- Universe: flat topology

- position of first acoustic peak lies at $\ell_1 \approx 200$

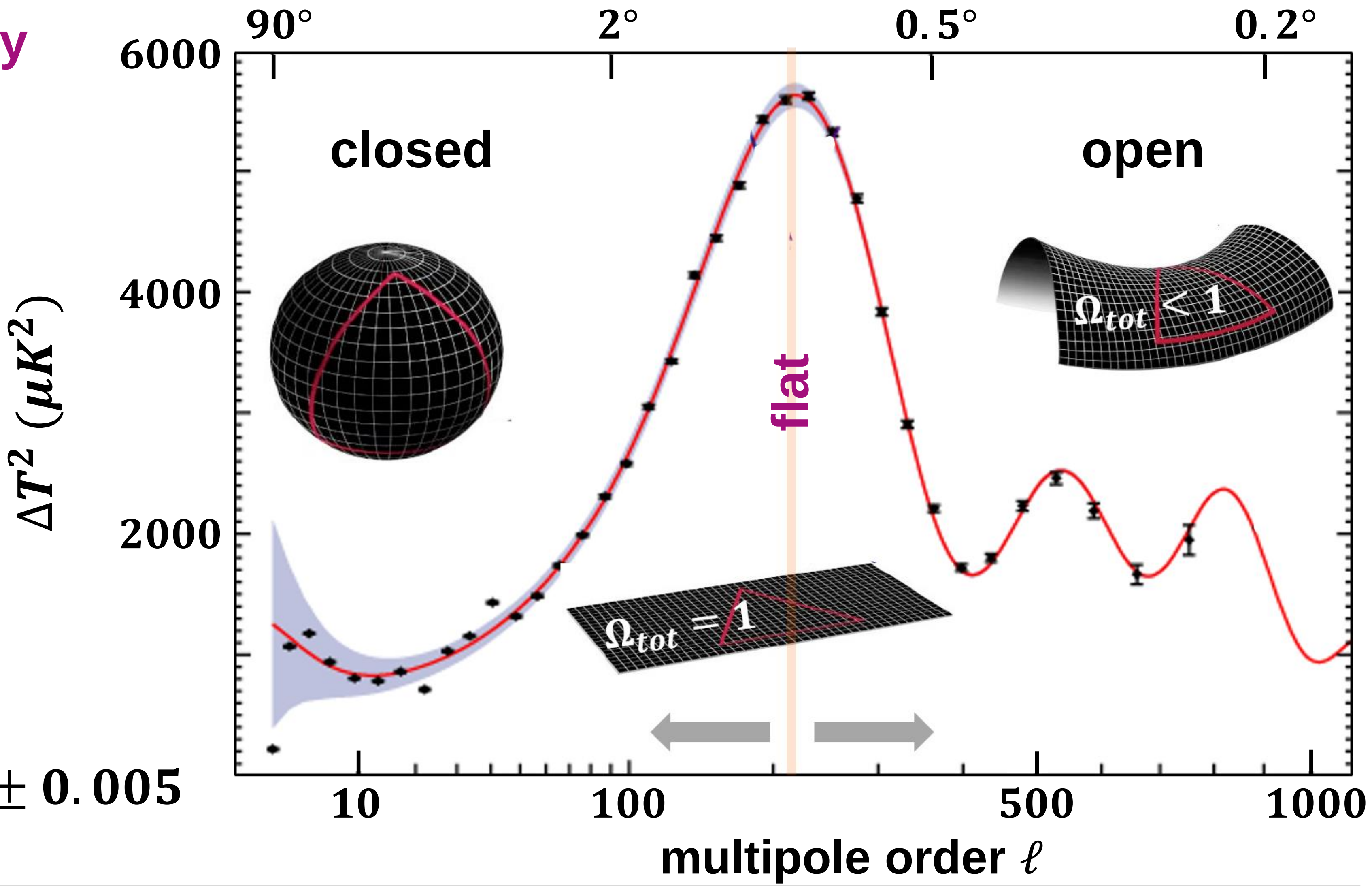
- $\Omega_k \approx 0$

Hubble constant H_0

curvature radius R_{curv}

curvature $k = +1, 0, -1$

$$\Omega_k = -\frac{kc^2}{H_0^2 \cdot R_{curv}^2} = 0.000 \pm 0.005$$



BAO – determining the baryon density Ω_B

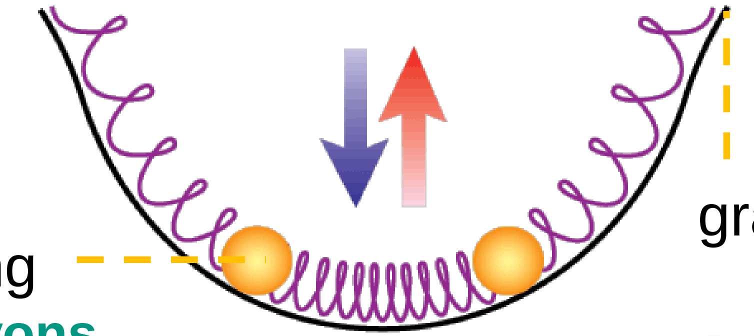
- BAO **first peak height is sensitive to Ω_B**

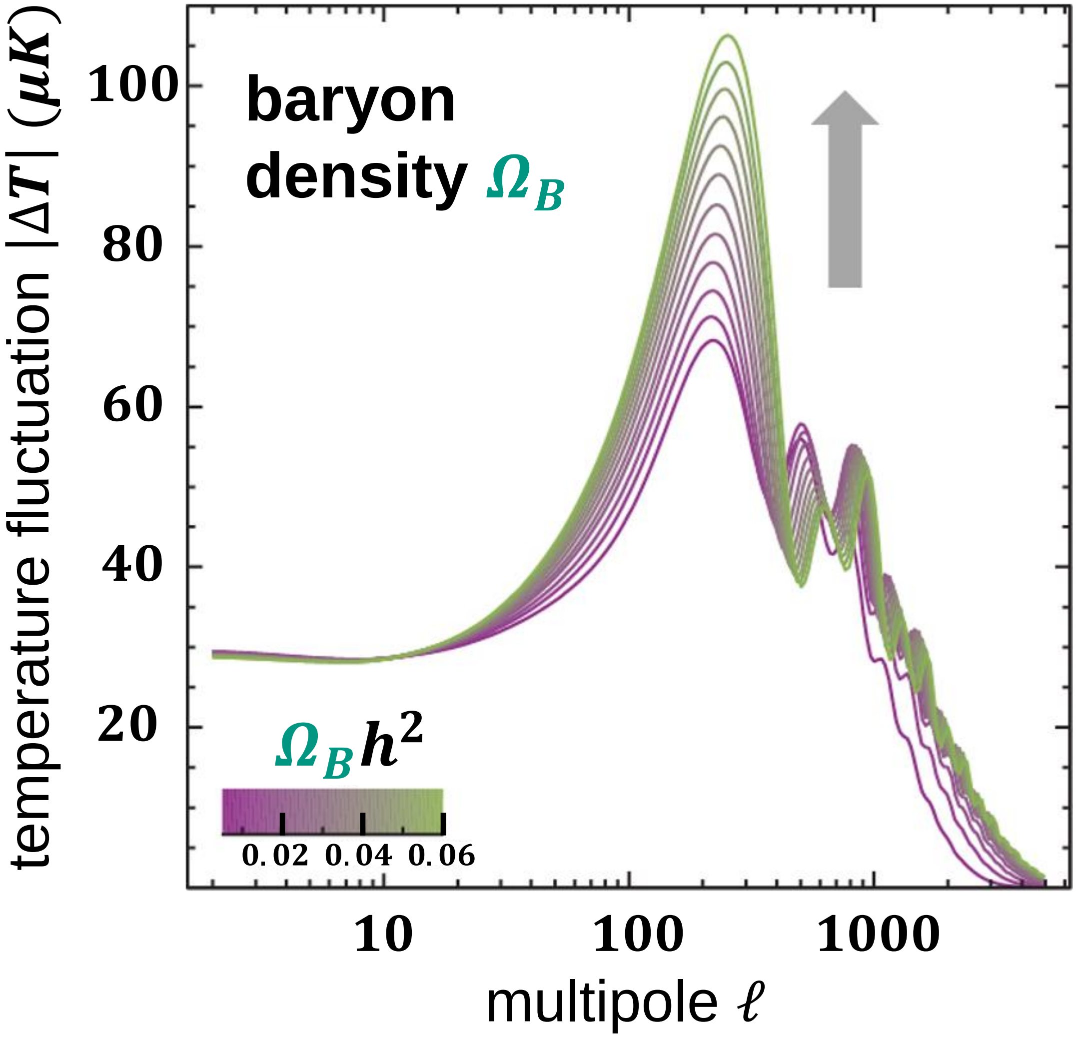
´baryon loading´:

⇨ the **height** of the first acoustic peak is sensitive to the **total # of baryons in the universe Ω_B**

- more baryons Ω_B

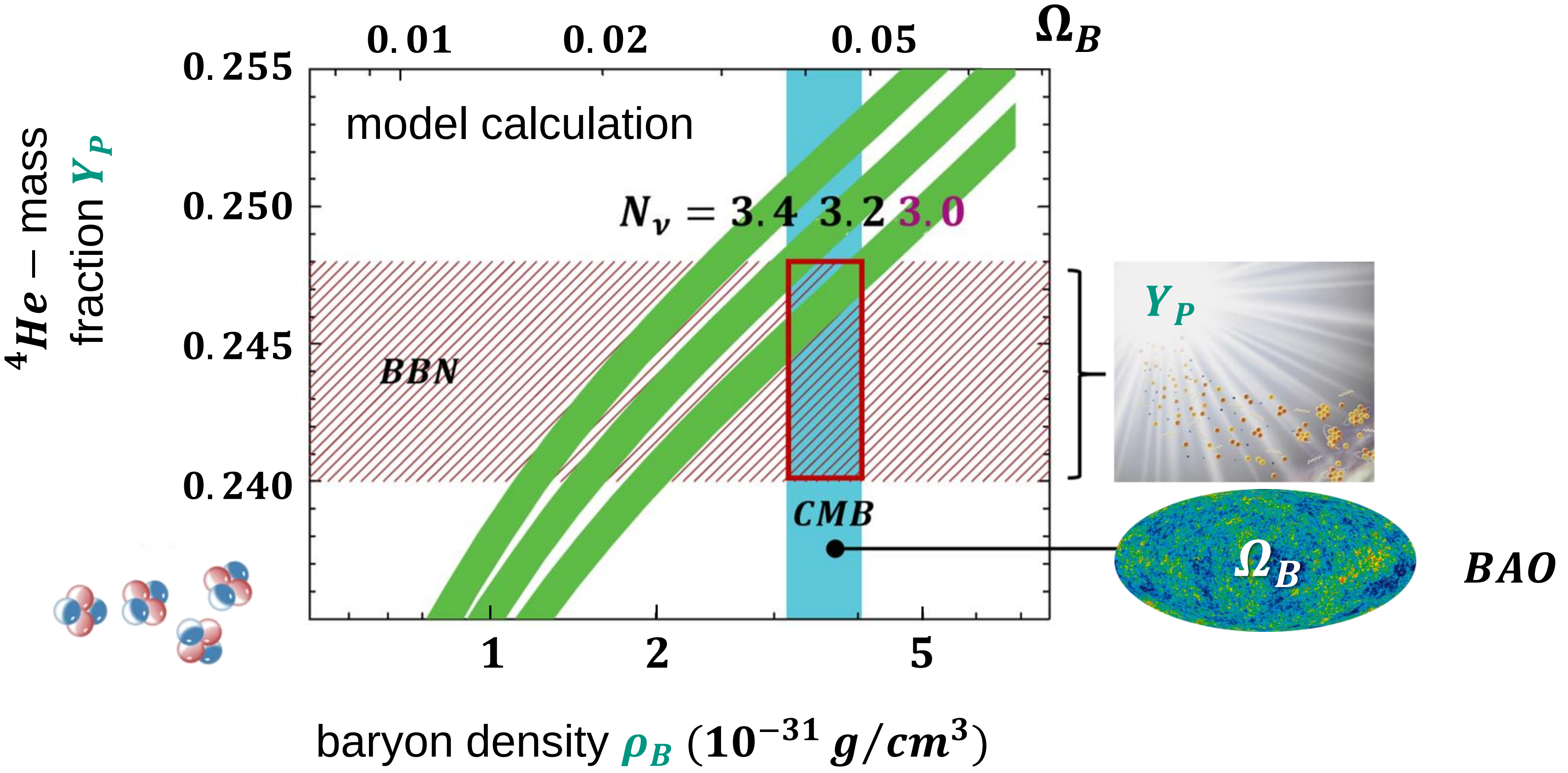
⇨ height of **1.** acoustic peak increases





BAO – determining the baryon density Ω_B

- BAO (´baryon loading´) measures Ω_B ⇔ compare to BBN* light element yields



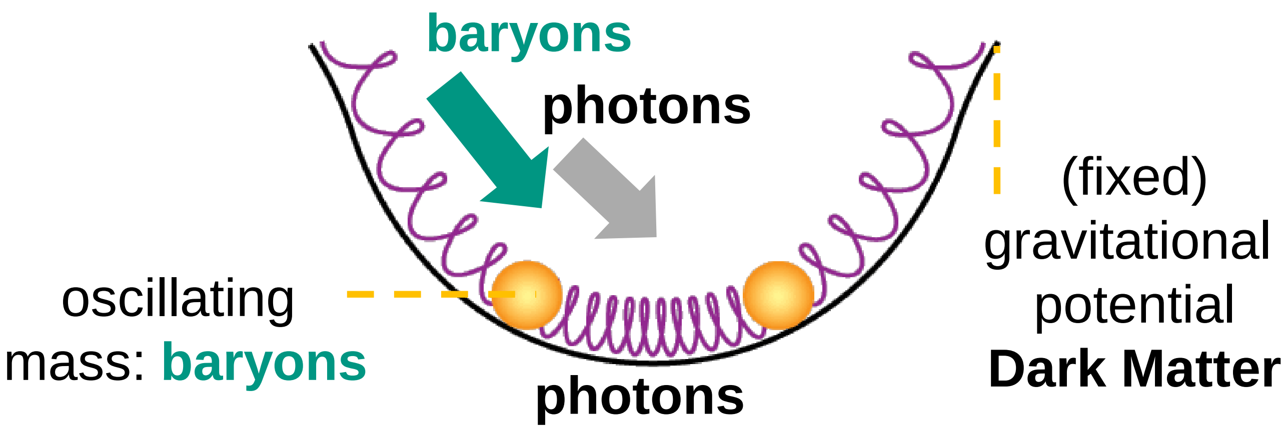
*see lecture #7

BAO – determining the baryon density Ω_B

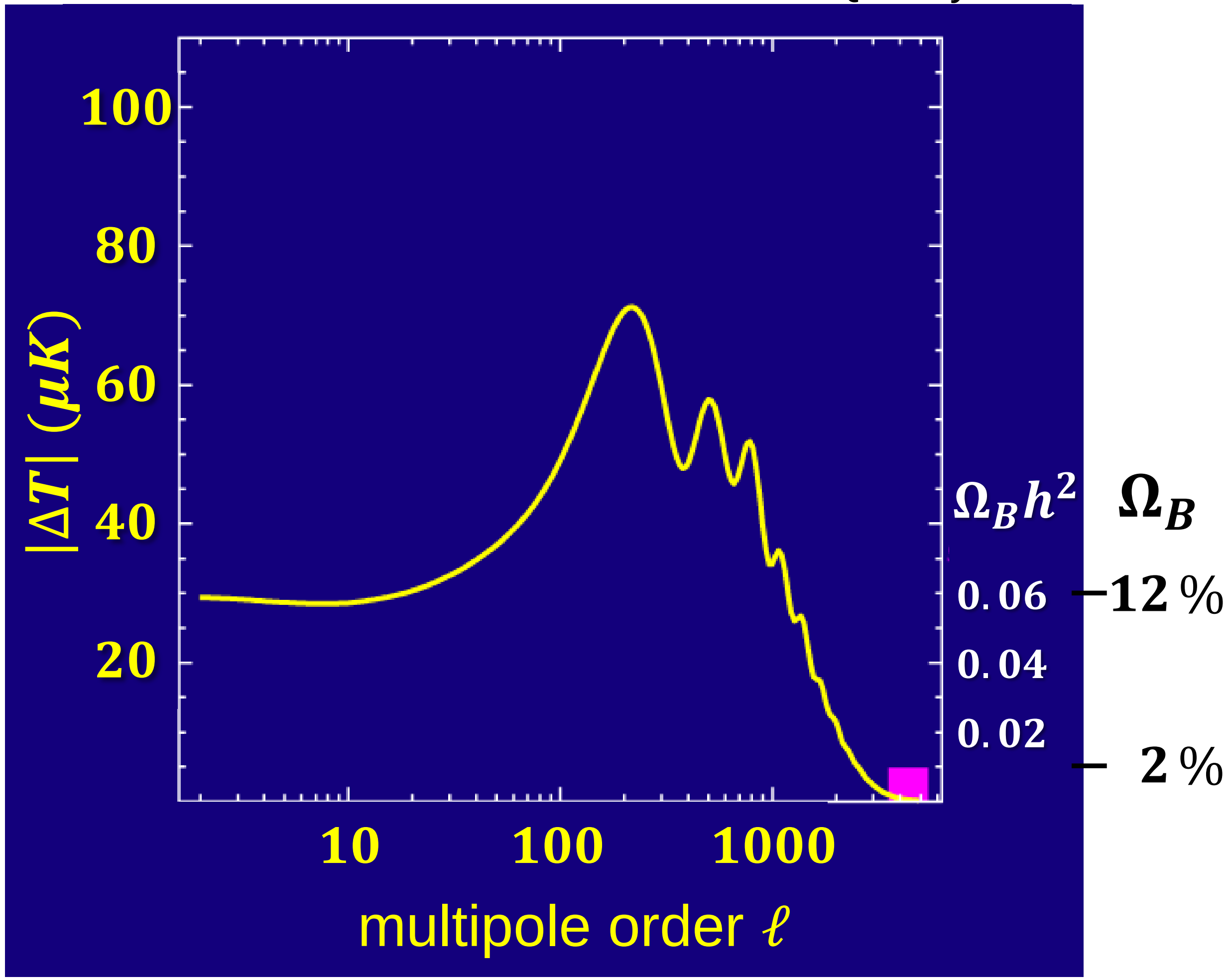
- BAO: **baryon loading** & Ω_B

- ´**baryon loading**´: baryons (matter) & radiation are ´**in phase**´ – from rarefaction to compression (**first peak**)

- thus: each 2^{nd} peak (odd numbers):
  ⇨ matter & photons are ´**in phase**´



Q: Wayne Hu

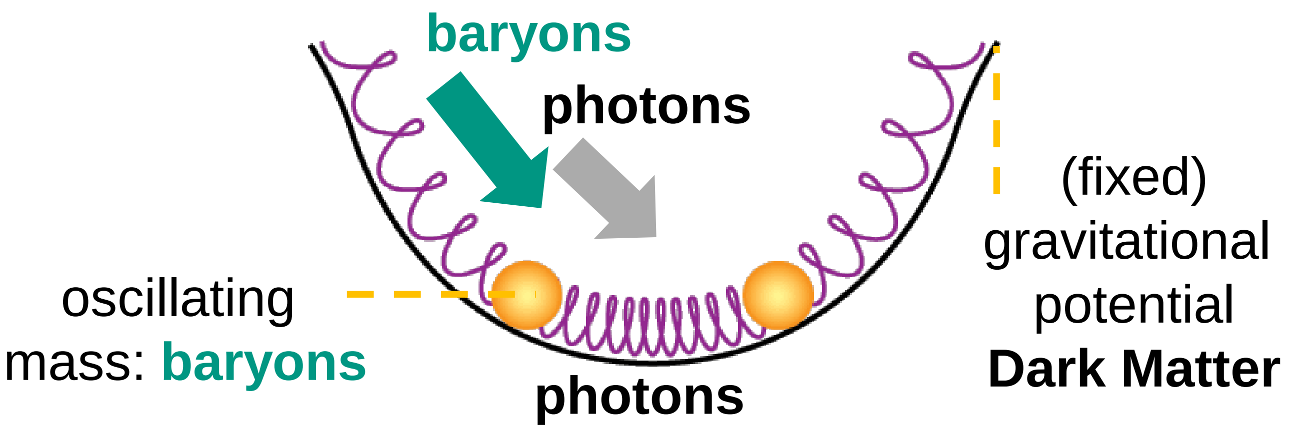


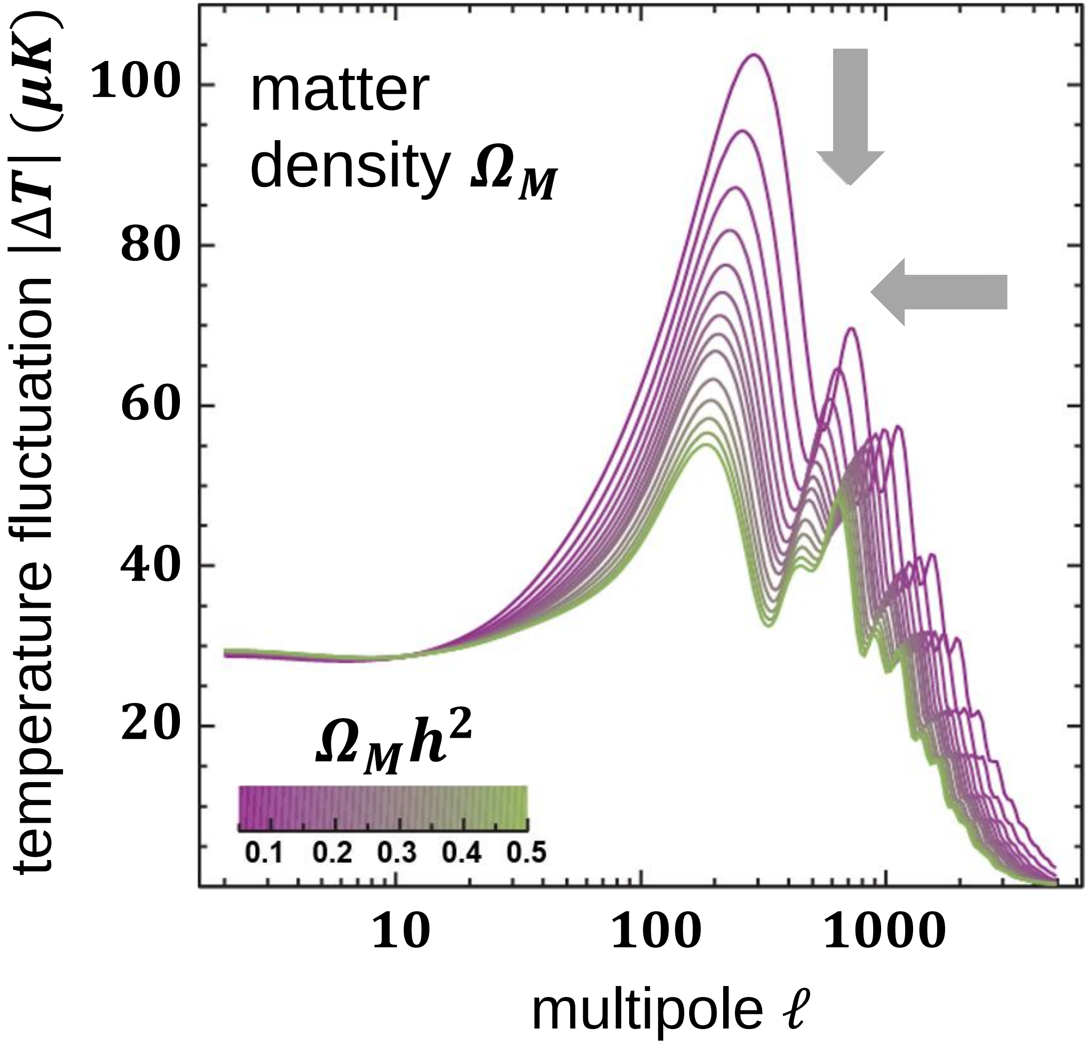
BAO – determining the dark matter density Ω_{DM}

- BAO: clear CMB evidence for $\Omega_{DM} \neq 0$

- matter density Ω_M with two contributions: dominant **Dark Matter Ω_{DM}** & **baryons Ω_B**

- **increasing** Ω_M: ⇨ scale of ΔT – fluctuations will **decrease**, need to consider both $even$ & odd peaks





BAO – determining the dark matter density Ω_{DM}

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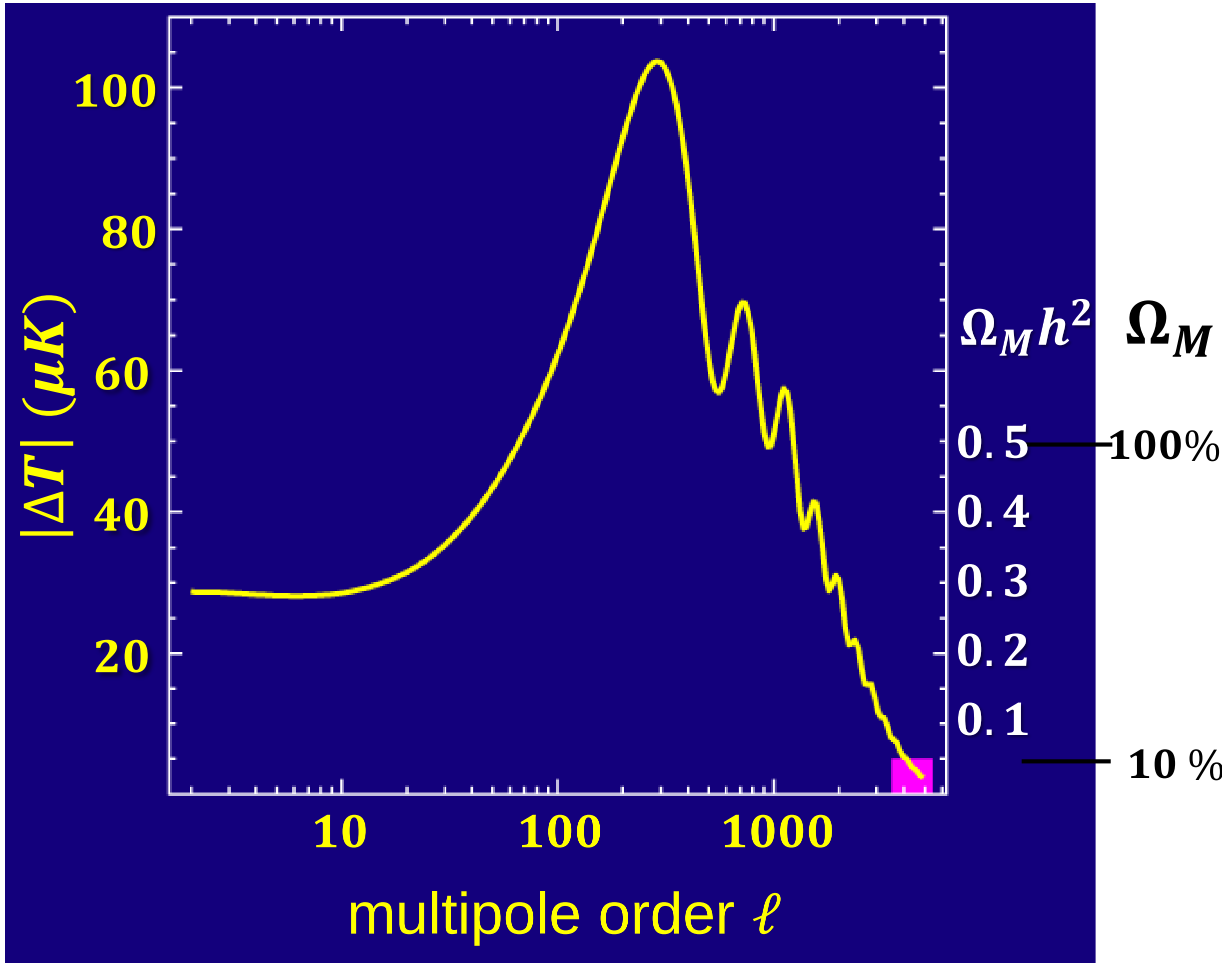
- BAO: a clear CMB evidence for $\Omega_{DM} \neq 0$

- as $\Omega_{DM} \gg \Omega_B$:
  this allows us to draw direct
  conclusions for Dark Matter density

- **best indicator** for Ω_{DM}:
  relative **heights** & **positions** of
  2. and 3. **acoustic peaks**

modern analysis technique:
perform a **global fit** to the entire
spectrum of all multipole orders!



BAO – a ´powerhouse´ for cosmology

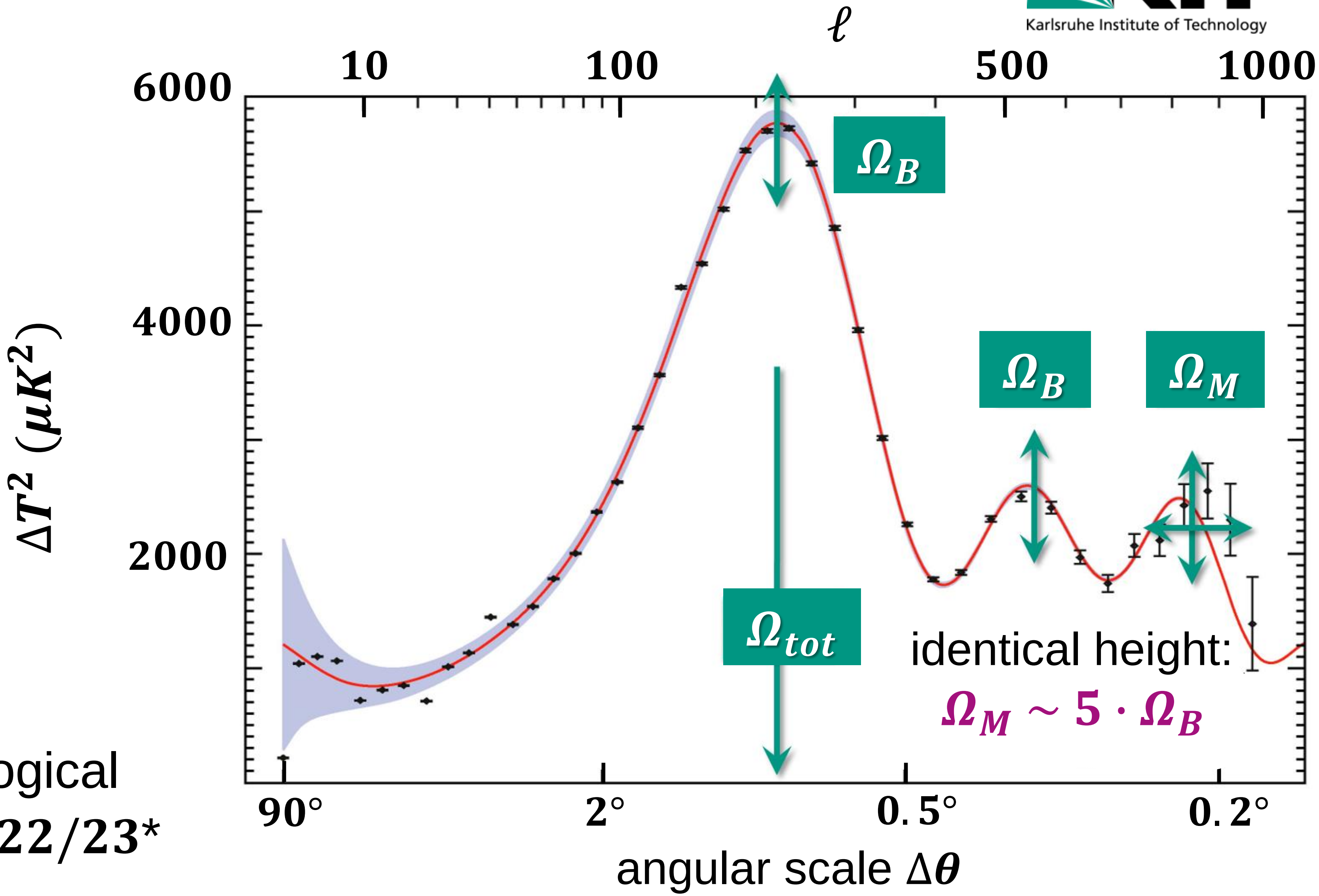
- BAO: global analysis

- **flat**, Euclidian universe
  with $\Omega_{tot} = 1$

- **baryon** density
  $\Omega_B h^2 = 0.02237$
  ± 0.00015

- **matter** density
  $\Omega_M = 0.315 \pm 0.007$

- $8/2023$: present cosmological parameters, see $PDG\ 2022/23$*



* rpp2022-rev-cosmological-parameters.pdf (lbl.gov)

$ESA´s\ Planck$ mission $2009 - 13$

primary mirror
aperture $1.75\ m$

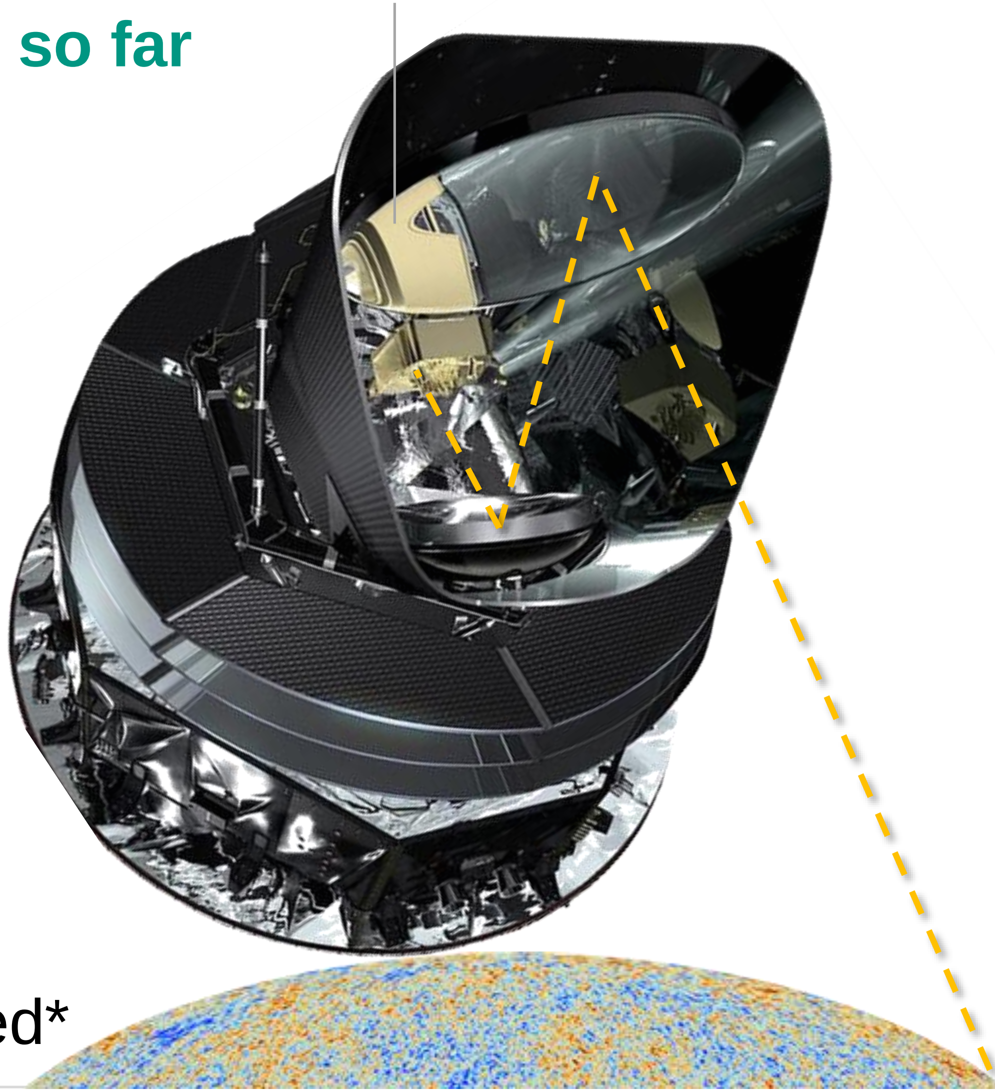
- CMB measurements at the highest resolution so far

  - nominal resolutions of $Planck$:

    - $\Delta T/T \sim 2 \cdot 10^{-6}$, $\Delta\theta = 4´ \ldots 33´$

    - frequency range: $30 \ldots 857\ GHz$

  - May $14, 2009$: start with Ariane 5
  - Aug $08, 2009$: begin of data taking
  - Jan $16, 2012$: LHe – reservoir empty
    (5 *full sky surveys*)
  - Mar $21, 2013$: publication of first results
  - Oct $23, 2013$: $Planck$ deactivated
  - Aug $10, 2021$: final 2018 results $V4$ published*



* 1807.06209.pdf (arxiv.org)

$\boldsymbol{Planck}$ – instrumentation of the focal plane

- $\boldsymbol{LFI}$ and $\boldsymbol{HFI}$ cover 9 frequency bands

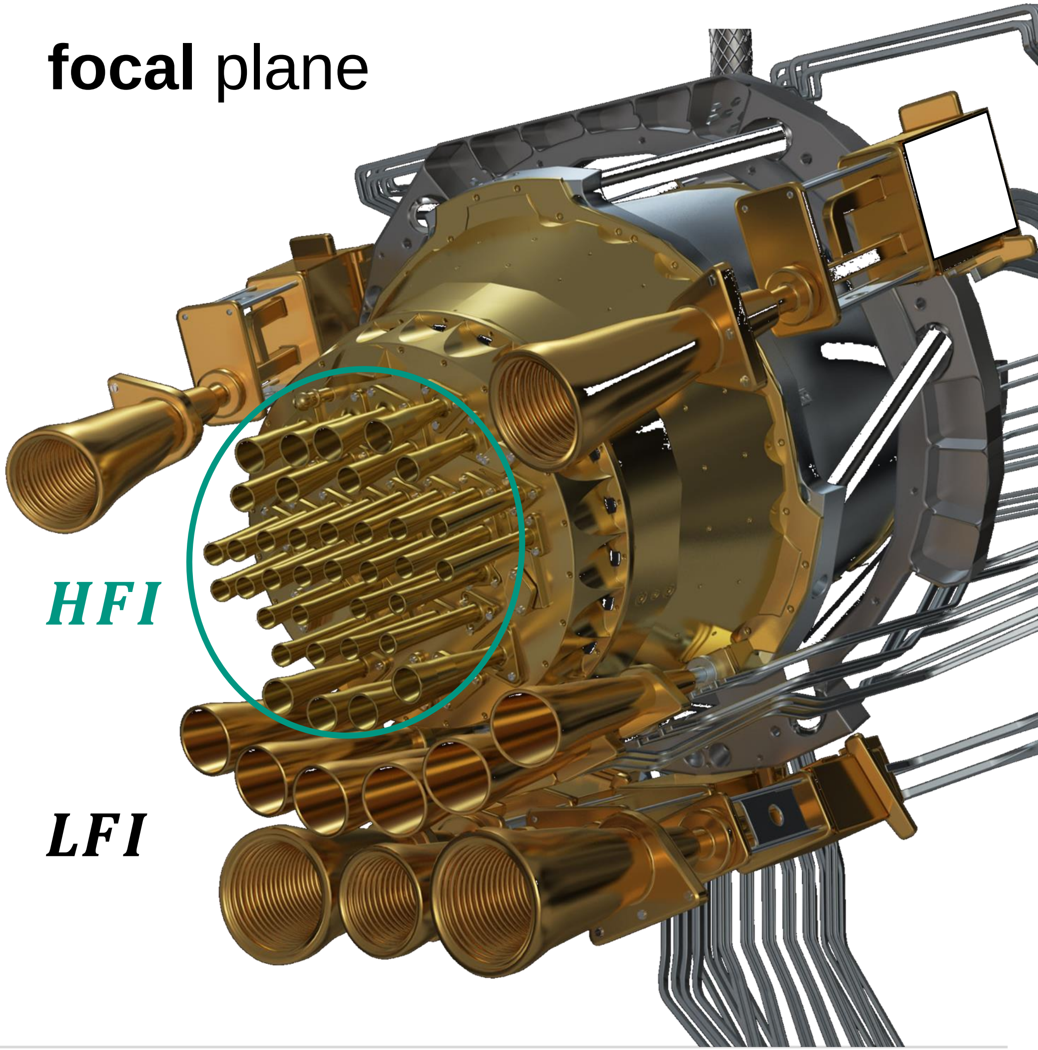
$\boldsymbol{LFI}$ – $\boldsymbol{L}$ow $\boldsymbol{F}$requency $\boldsymbol{I}$nstrument
$\boldsymbol{f = 27 \ldots 77\ GHz\ \ T = 20\ K}$

$\boldsymbol{HFI}$ – $\boldsymbol{H}$igh $\boldsymbol{F}$requency $\boldsymbol{I}$nstrument
$\boldsymbol{f = 100 \ldots 857\ GHz\ \ T = 0.1\ K}$

- $\boldsymbol{CMB}$ is guided via the **primary** & via **secondary** mirrors onto the ´**focal plane**´ with two instruments* $\boldsymbol{LFI}$ and $\boldsymbol{HFI}$

- $\boldsymbol{LFI}$ & $\boldsymbol{HFI}$ are cooled via $\boldsymbol{LHe}$ reservoir

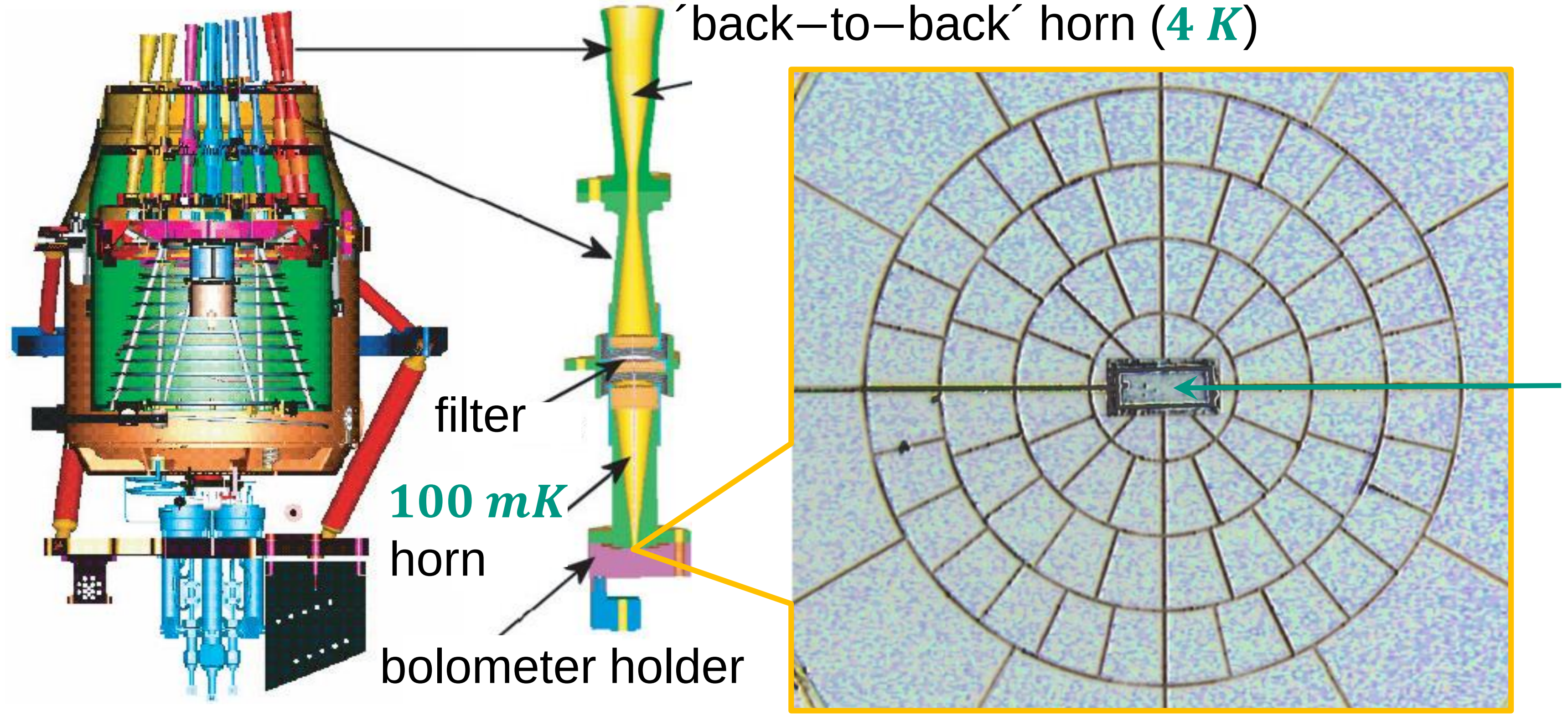
focal plane



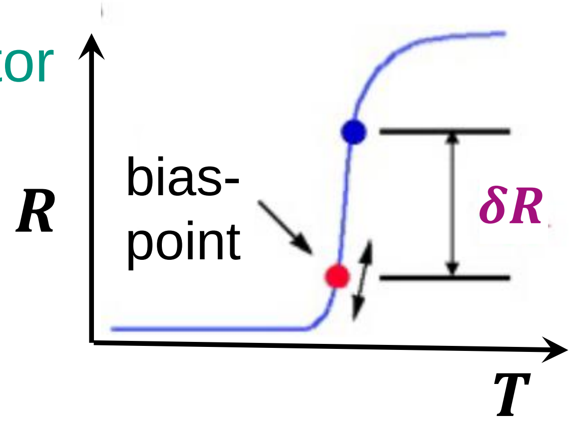
*$\boldsymbol{Q}$: The $\boldsymbol{LFI}$ horns are larger than the $\boldsymbol{HFI}$ ones, why?

$Planck$ – HFI: a ´spider–web´ bolometer

- **microwaves are absorbed by a super–conducting bolometer (´spider–web´)**
  - **absorption** of microwaves: **tiny increase** of bolometer temperature T, which then is read–out by a **thermistor**

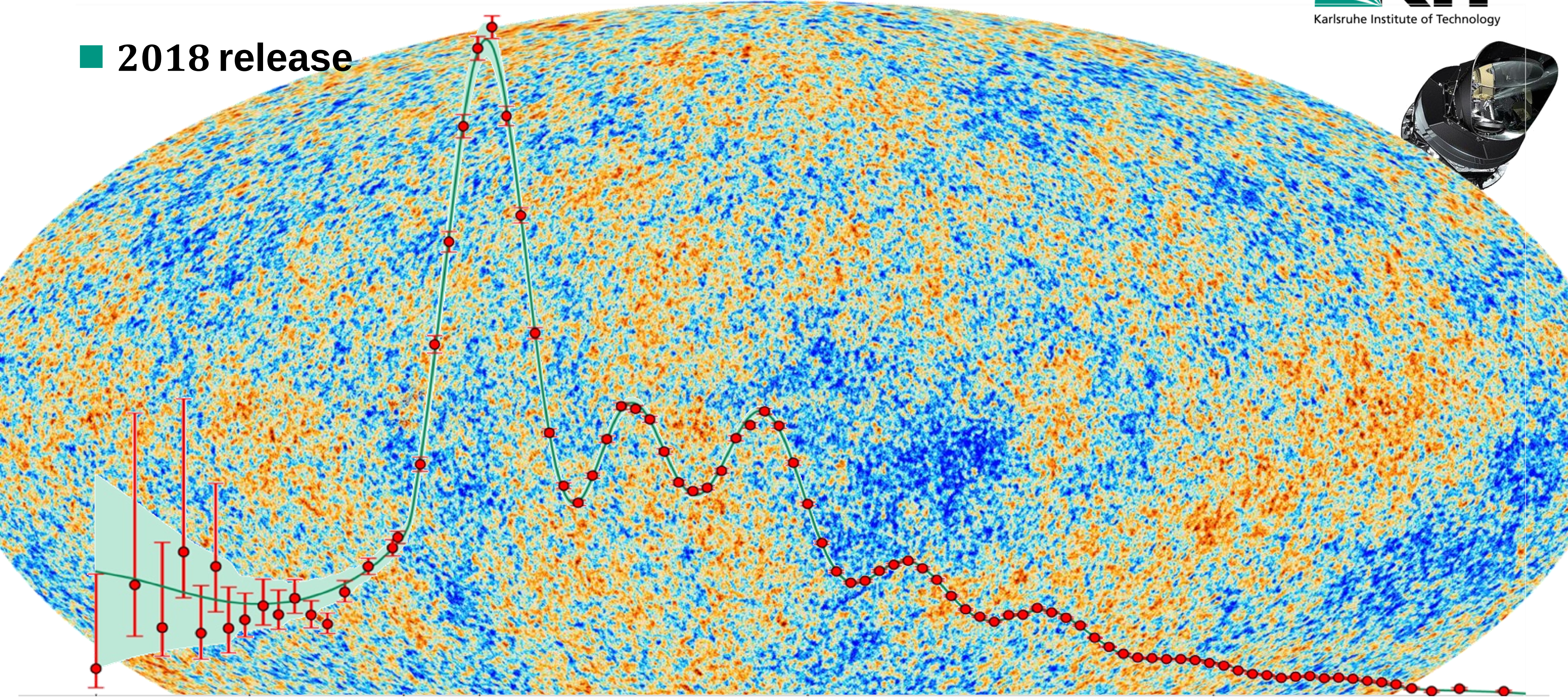


thermistor:
change of temperature T ⇨
change δR of resistivity R



$Planck$ – the ´definitive´ CMB temperature map

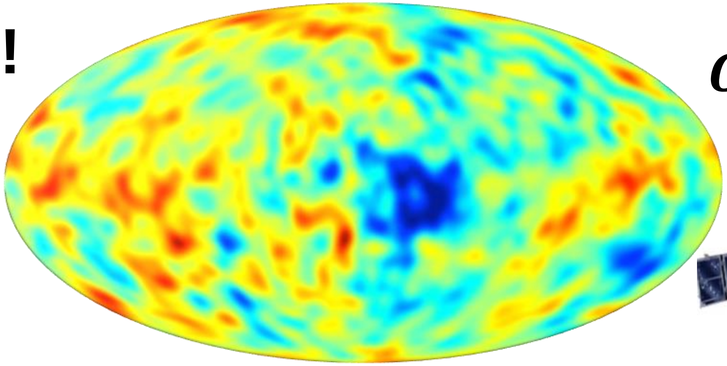
- **2018 release**



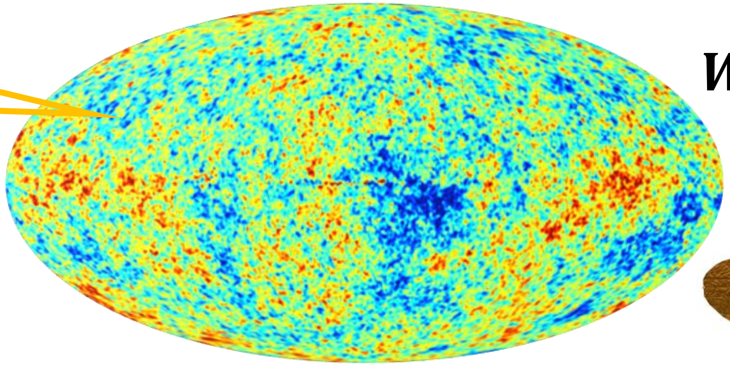
Comparing $COBE$ … $WMAP$ … $Planck$

- **Better resolution, please!**

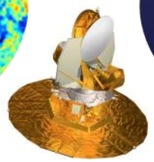
  - experimental progress
    from **1989** … **2013**

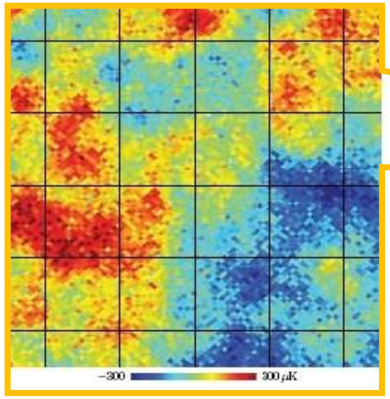


$COBE$

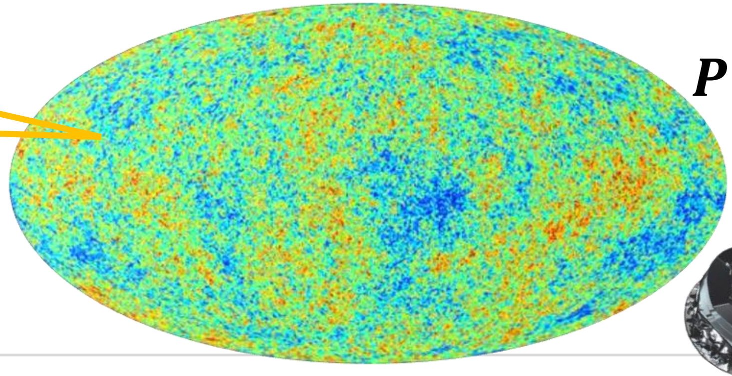


$WMAP$

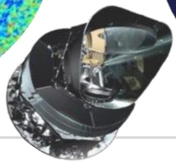


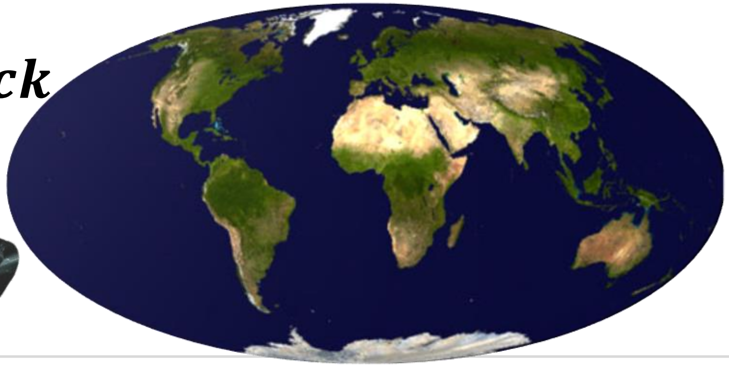


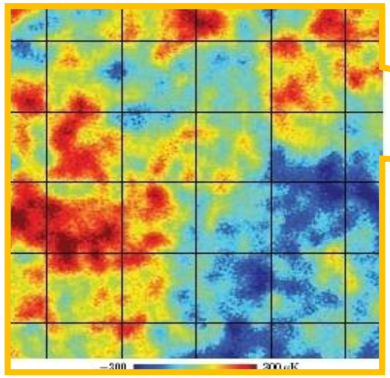
$WMAP$
9 years



$Planck$



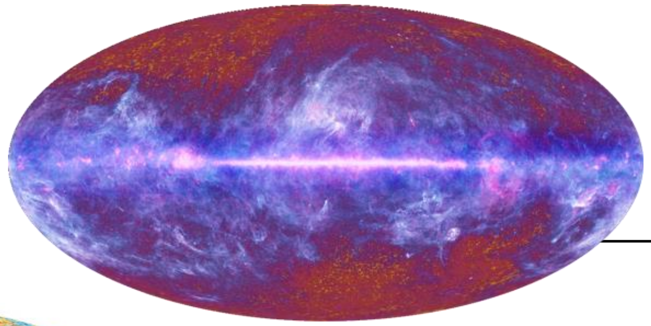


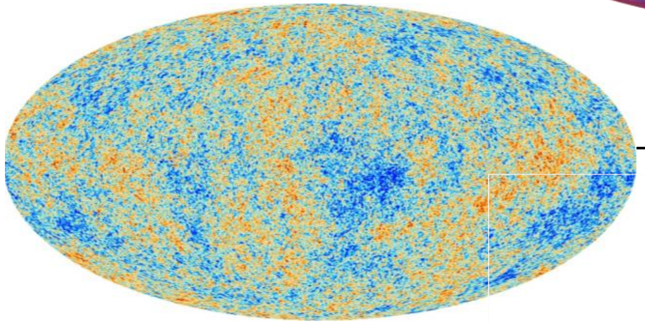


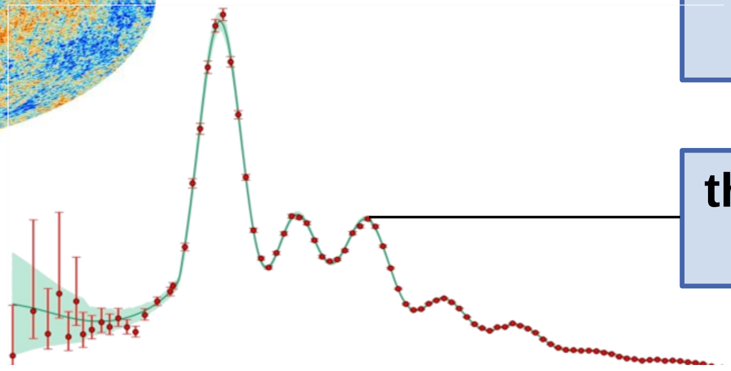
$Planck$
1 year

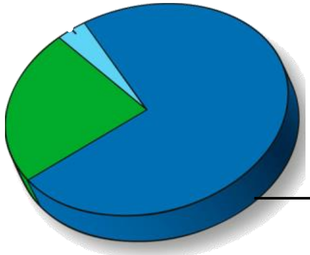
Planck – data analysis

- **Chain of analysis is complex…**









measurements: $\mathbf{72}$ detectors, $\mathbf{15000}$ data sets / $\boldsymbol{s}$ total: $\mathbf{6 \times 10^{11}}$ data sets

frequency cards: $\mathbf{9}$ channels total: $\mathbf{3 \times 10^{7}}$ pixels

experimental sensitivity systematic effects

foreground maps $\mathbf{500}$ parameters

catalog of point sources $> \mathbf{10^{4}}$ individual objects

$\boldsymbol{CMB}$ **temperature maps** $\mathbf{5 \times 10^{6}}$ pixels

$\boldsymbol{CMB}$ **polarization maps** $\mathbf{10^{7}}$ pixels

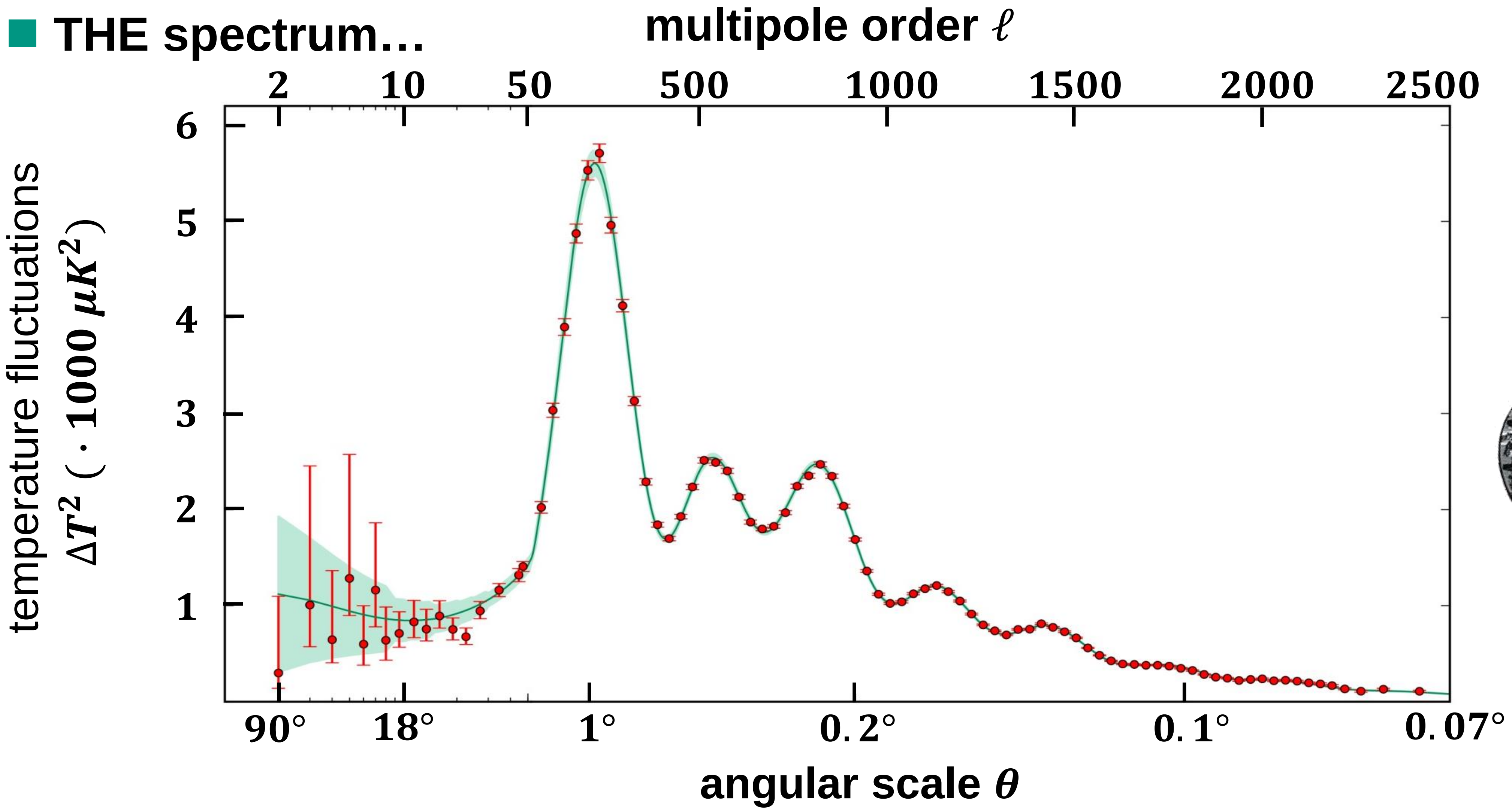
thermal power spectrum $\sim \mathbf{2000}$ numbers

power spectrum polarization $\sim \mathbf{1000}$ numbers

cosmological parameters **12 numbers**

$Planck$ – finally: THE multipole spectrum for fits

- THE spectrum…



Planck – cosmological parameters

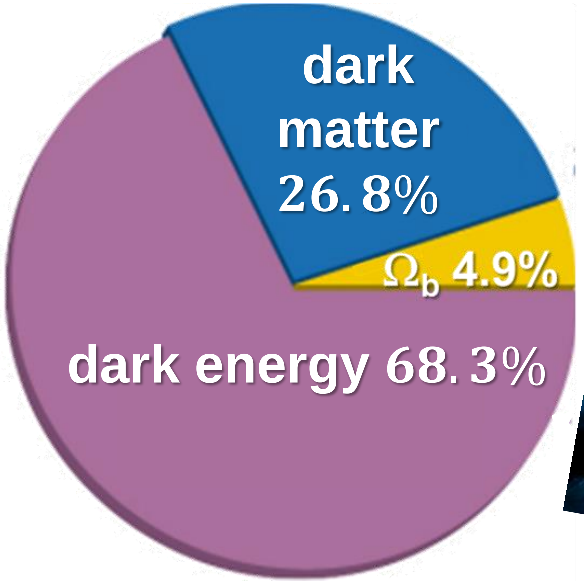
- $\boldsymbol{CMB}$ as **precision tool of cosmology**

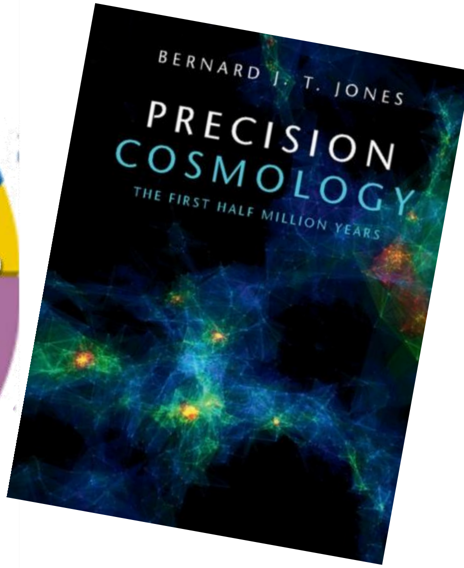
  ⇨ $\boldsymbol{CBM}$ data are described within the $\boldsymbol{\Lambda CDM}$ **– concordance model**

  ⇨ cosmological parameters based on $\boldsymbol{CMB}$ are derived from $\mathbf{2018}$ $\boldsymbol{Planck}$ data

  - $\boldsymbol{Planck}$ data are fitted together with **other data sets** (galaxy surveys,…)

parameter	best fit value
age of universe t_0	$(13.80 \pm 0.04) \cdot 10^9\ yr$
Hubble constant* H_0	$(67.8 \pm 0.9)\ km\ s^{-1}\ Mpc^{-1}$
Baryon fraction $\Omega_B\ h^2$	$0.02226\ \pm 0.000230$
Dark Matter fraction $\Omega_{DM}\ h^2$	$0.1186\ \pm 0.0020$
Dark Energy Ω_Λ	$0.685\ \pm 0.017$
time of decoupling t_{rec}	$(377730 \pm 3200)\ yr$
red–shift of decoupling z_{rec}	$1090.9\ \pm 0.7$





*see Hubble tension, lect. #2

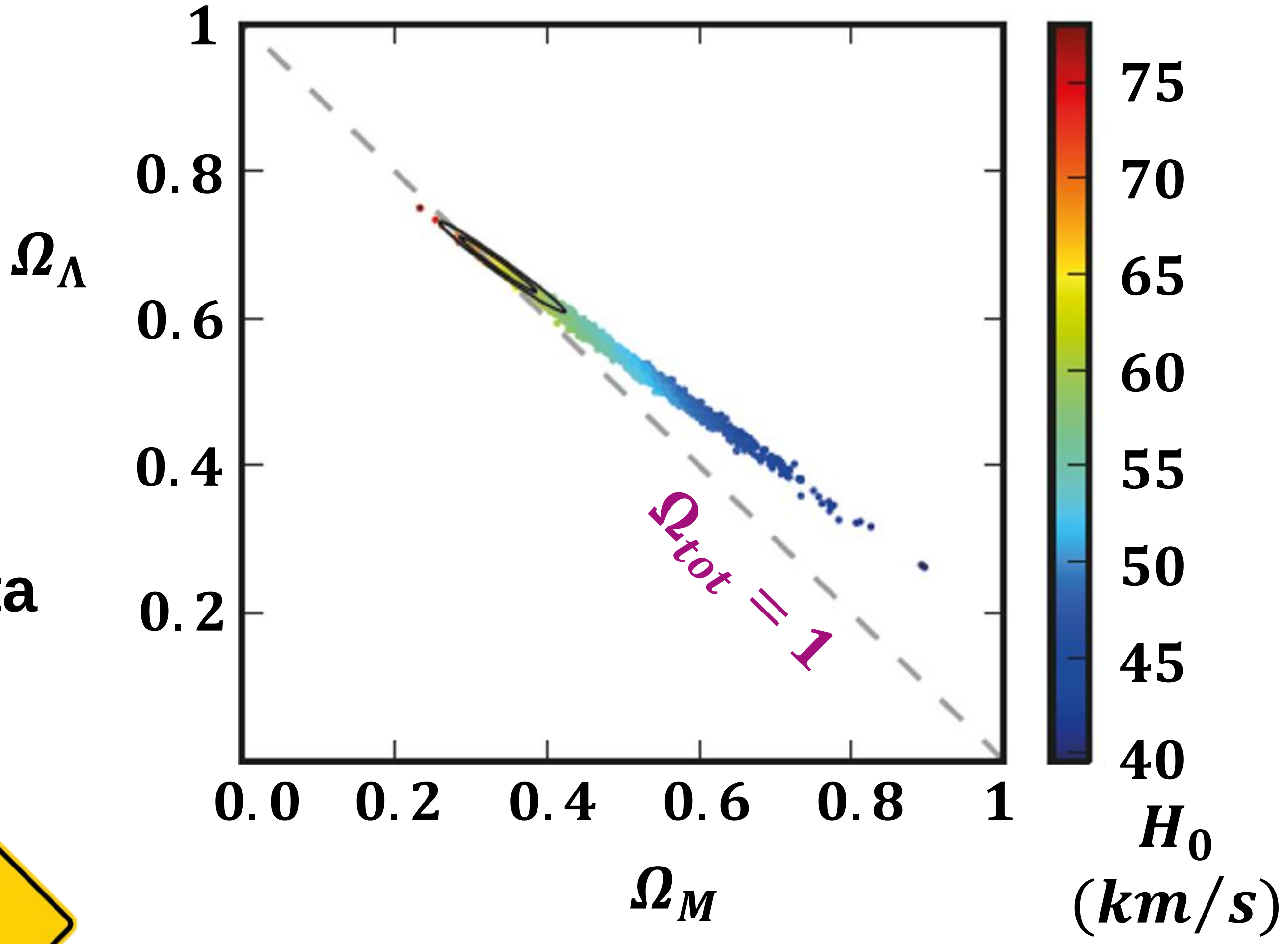
CMB parameter–fitting can be challenging

- **Model fits to CMB & other data: ⇨ sets of degenerate parameters**

  - example:
    degeneracy of the three
    key cosmological parameters
    Ω_Λ Ω_M & H_0

  - **degeneracy of parameters**:
    different combinations of
    parameters yield **identical fits to data**

    ⇨ needs to be broken by additional
    information (**orthogonal data sets**,...)



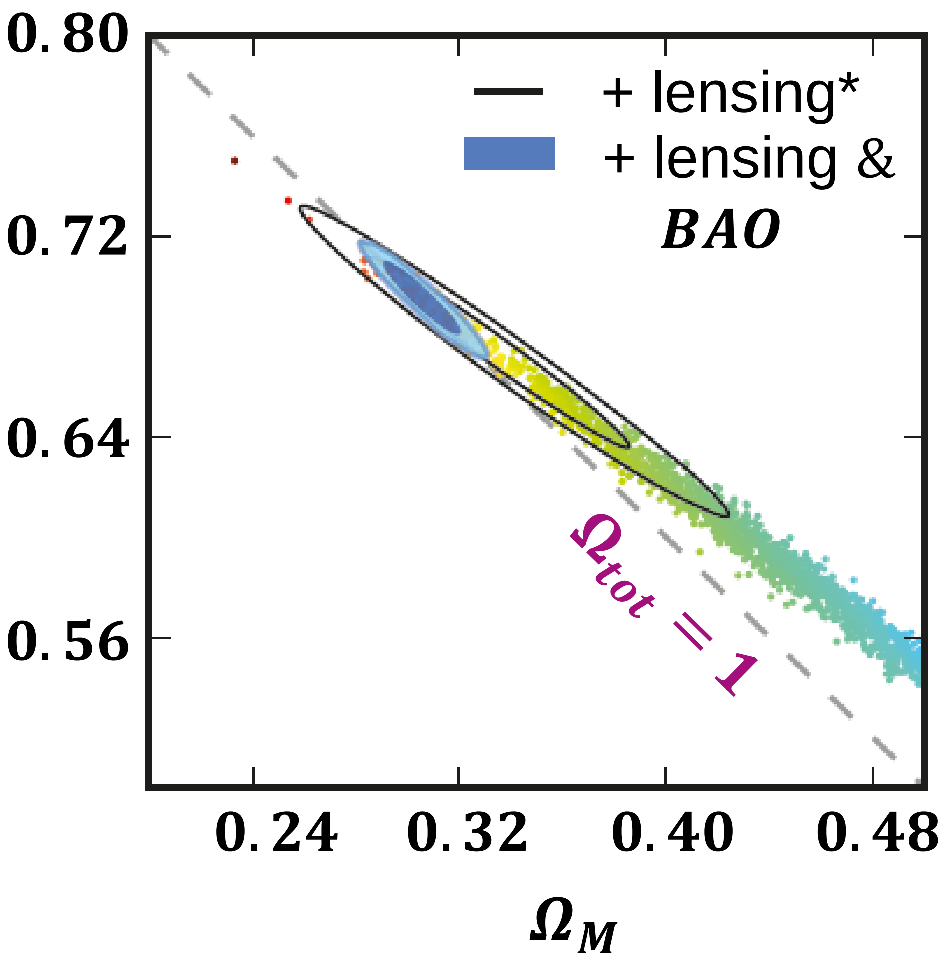
Cosmological parameters: degeneracy broken

- **Model fits to CMB & other data: ⇨ sets of degenerated parameters**

- example with additional data: **degeneracy** of the **two** key cosmological parameters Ω_Λ Ω_M

- **degeneracy of parameters reduced:** different combinations of parameters still good fits, but smaller region

⇨ additional information (orthogonal data sets,…): breaks/reduces degeneracy here – **weak lensing effect***



*see chapter 5.2 (lecture #14)

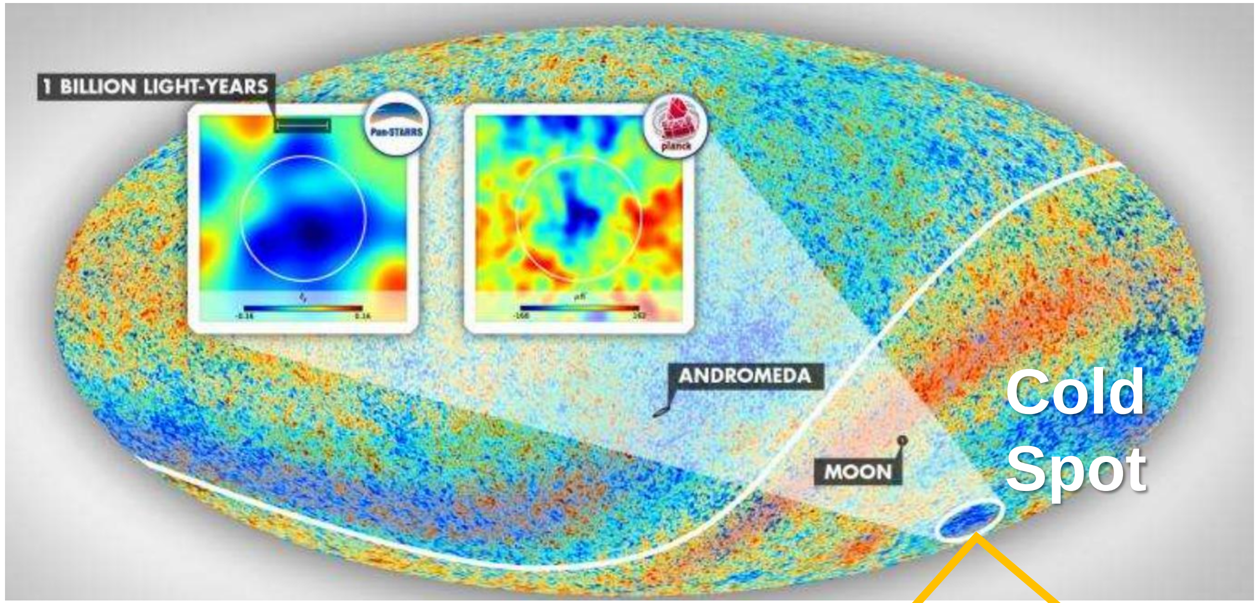
Anomalies in the CMB: $WMAP$ ´Cold Spot´

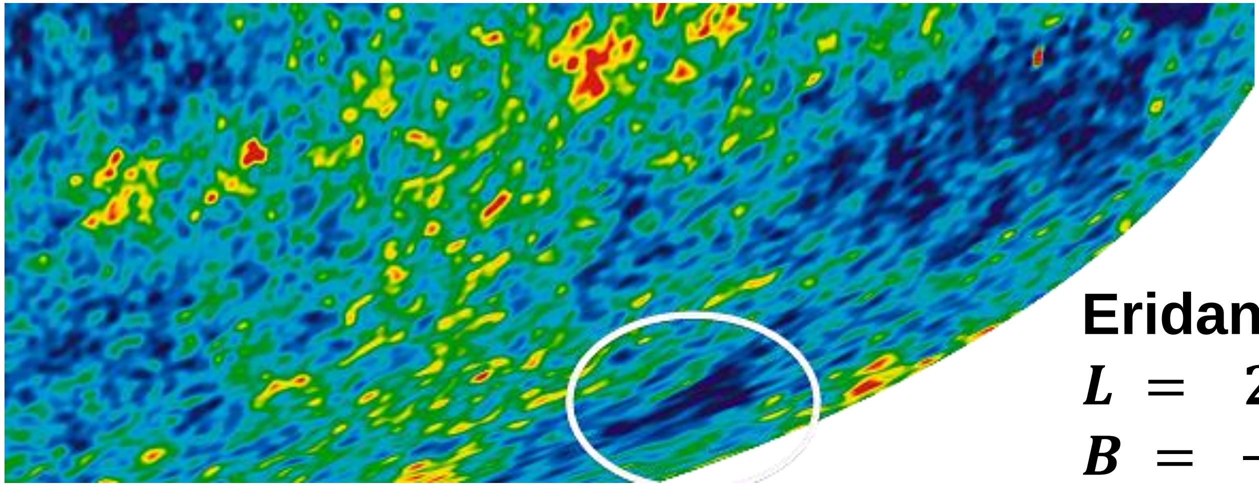
Karlsruhe Institute of Technology

■ **A large cold region revealed by $WMAP$: is it a cosmic ´super–void´ ?**

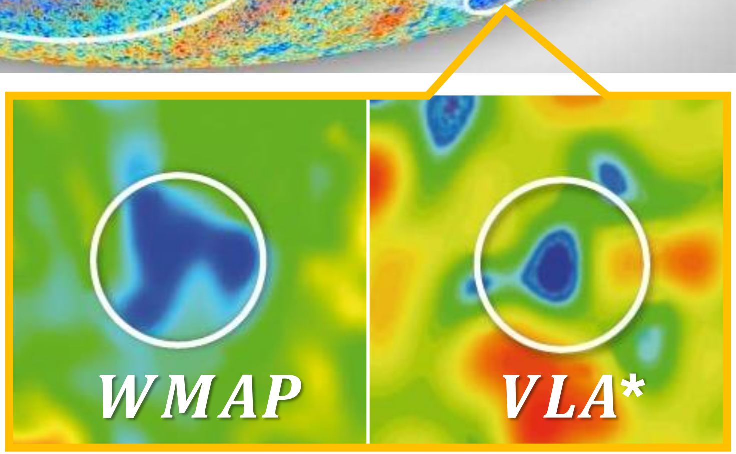
- $\boldsymbol{WMAP}$ data have yielded a surprise:
  a **5° large cold $\boldsymbol{CMB}$ region**, where
  $\boldsymbol{\Delta T \sim -70\ \mu K}$ (below $\boldsymbol{CMB}$ – average)

- also seen in $\boldsymbol{VLA}$* data & others





Eridanus:
$$\boldsymbol{L = 207.8°}$$
$$\boldsymbol{B = -56.3°}$$



*$\boldsymbol{V}$ery $\boldsymbol{L}$arge $\boldsymbol{A}$rray (radio dishes)

Anomalies in the CMB: $WMAP$ ´Cold Spot´

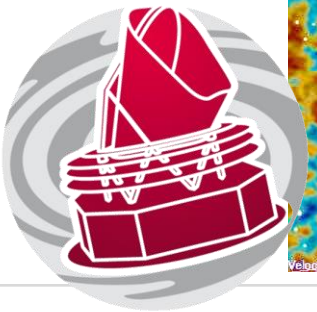
Karlsruhe Institute of Technology

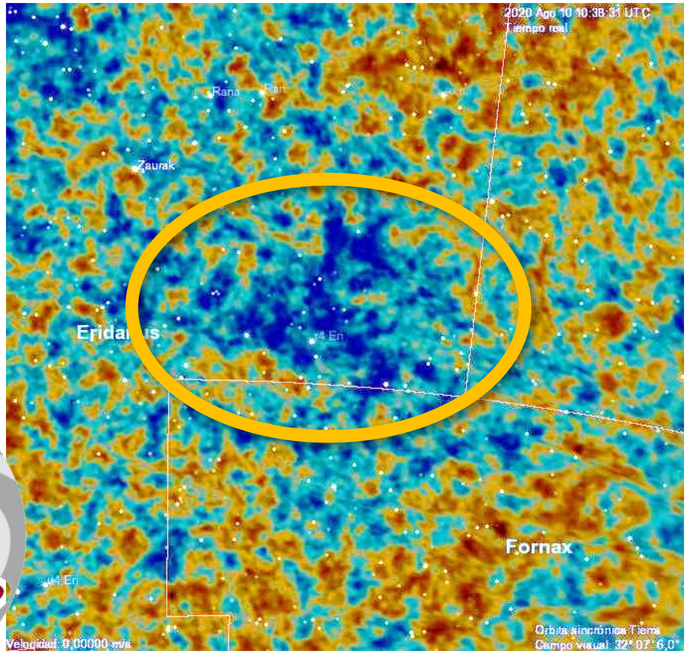
- **Does the CMB indicate a super–void ?**

- $WMAP$ data have yielded a surprise:
  a **5° large cold CMB region**, where
  $\Delta T \sim -70\ \mu K$ (below CMB – average)

- also seen in $Planck$ data

- probability in case of
  Gaussian fluctuations
  $p = 1.85\ \%$





1/2024

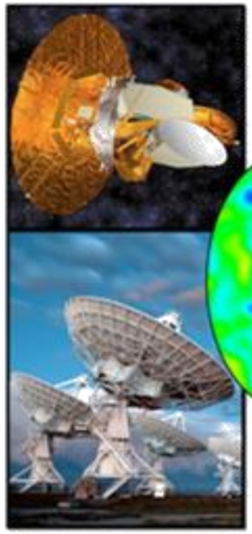
Anomalies in the CMB: $WMAP$ ´Cold Spot´

Karlsruhe Institute of Technology

- **Does the CMB indicate a super–void with $d = 300\ Mpc$ at $z = 1$?**

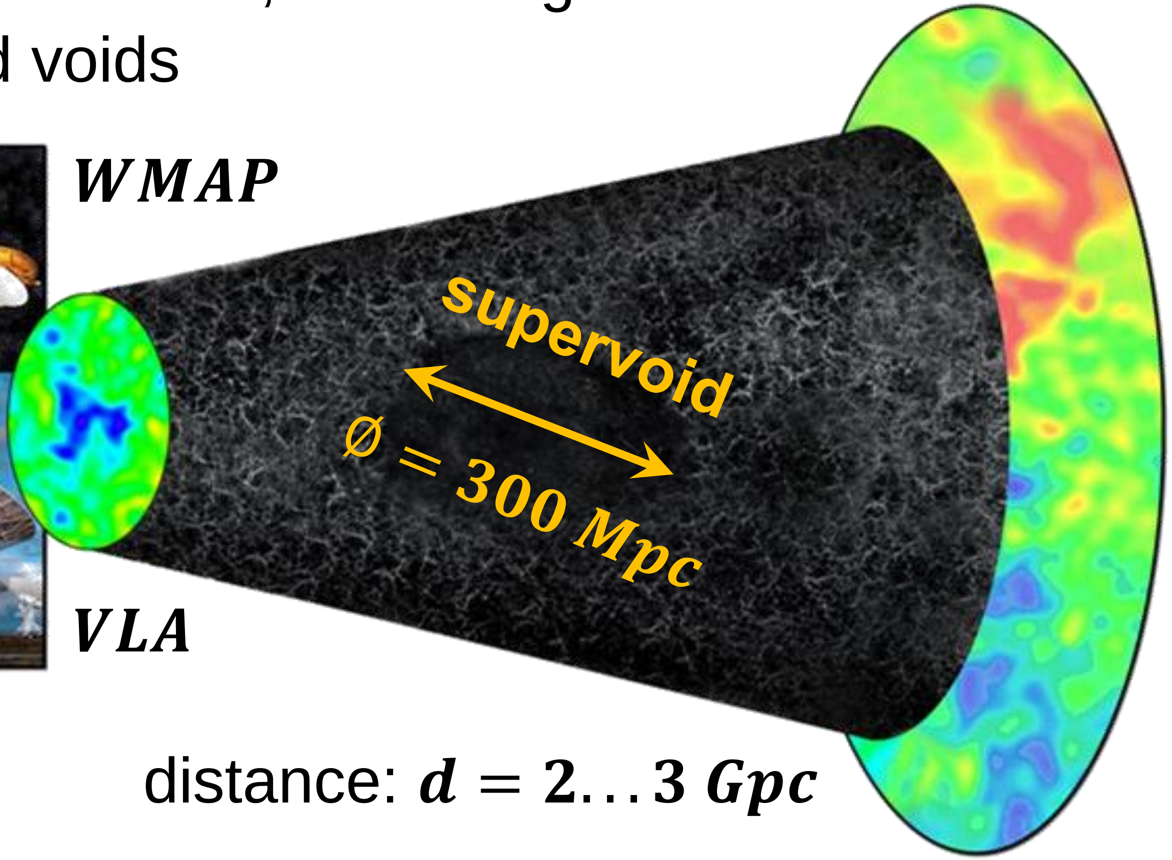
- this **super–void**, if it is really existing, would be much, much larger ($V \approx 30 \ldots 100$) than all other, average–sized voids

- (**highly speculative**) interpretation of data: we are witnessing the ongoing collision with another universe ´next door´



$WMAP$

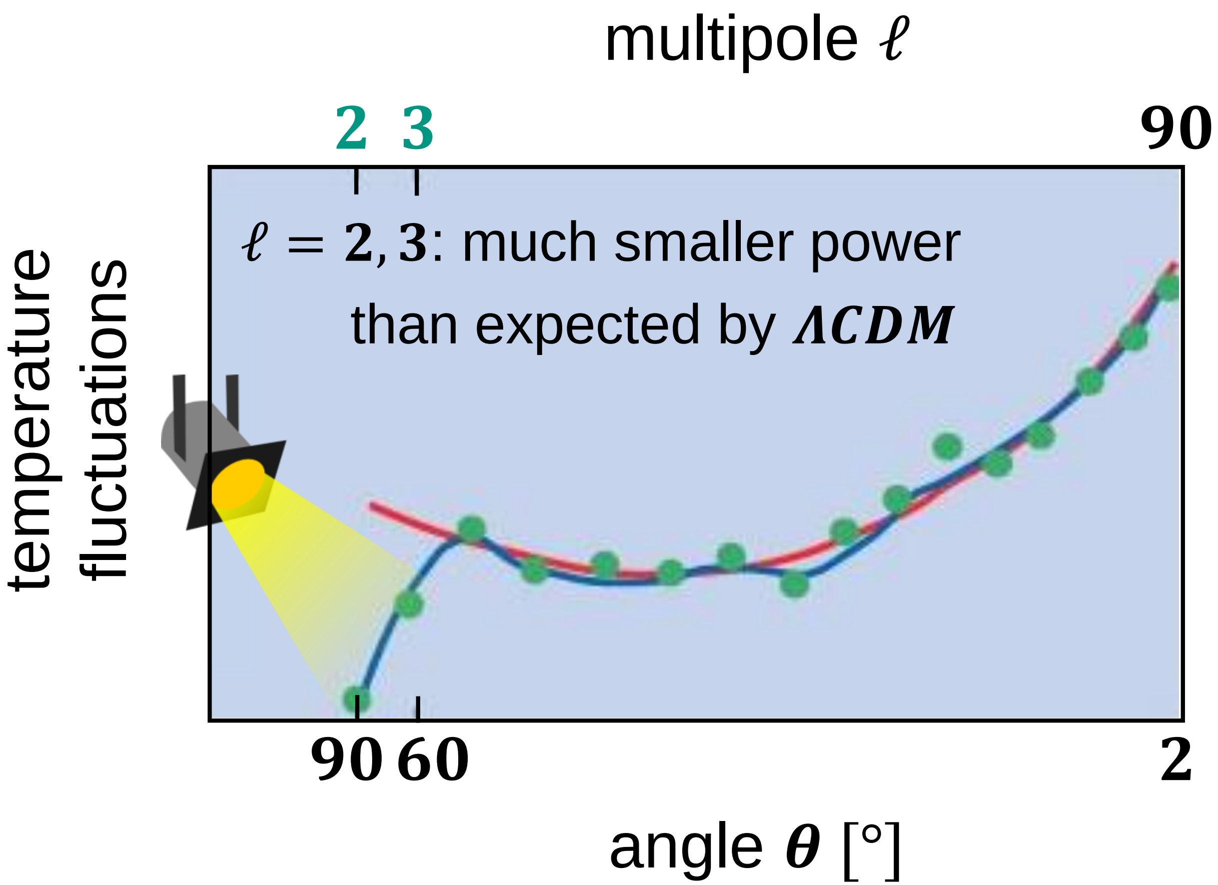
VLA



distance: $d = 2 \ldots 3\ Gpc$

Anomalies in the CMB: quadrupole & octopole

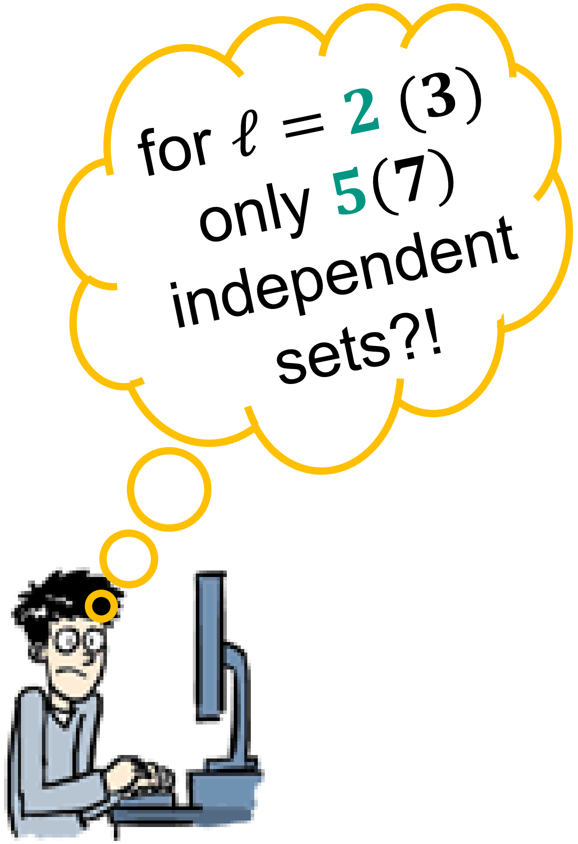
- **Why is there so little power in the two lowest−order multipoles?**



cosmic variance

$$\pm\sqrt{2/(2\ell+1)} \cdot C_\ell$$

- an intrinsic limit to cosmology with the CMB, does **NOT** depend on the resolution of my experiment!



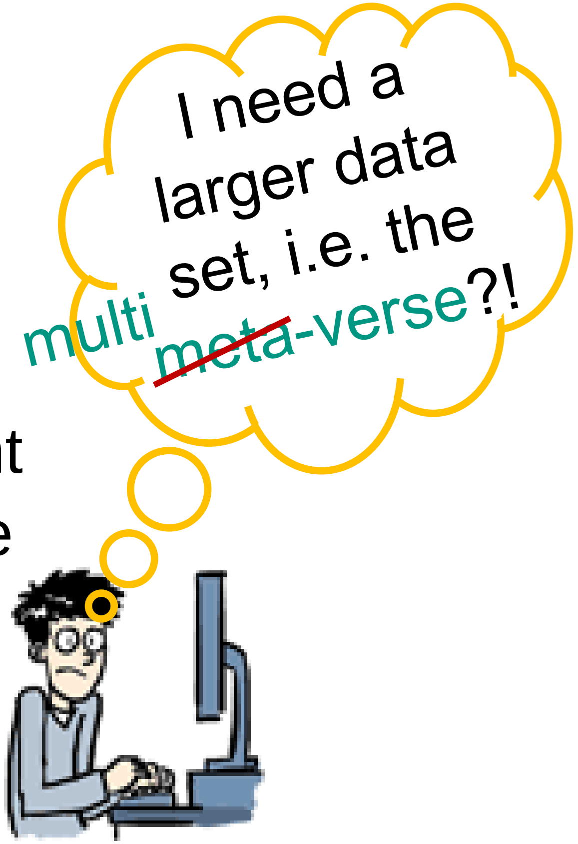
Anomalies in the CMB: quadrupole & octopole

- **Why is there so little power in the two lowest−order multipoles?**

cosmic variance

$$\pm\sqrt{2/(2\ell+1)}\cdot C_\ell$$

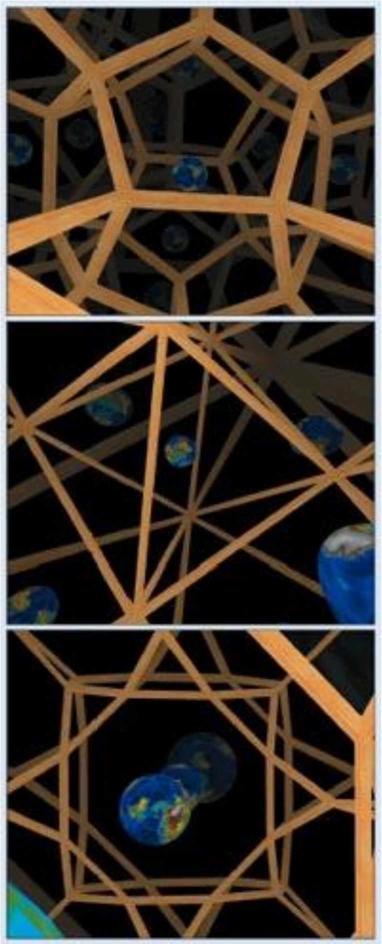
- we would need to sample* many different universes to overcome cosmic variance!

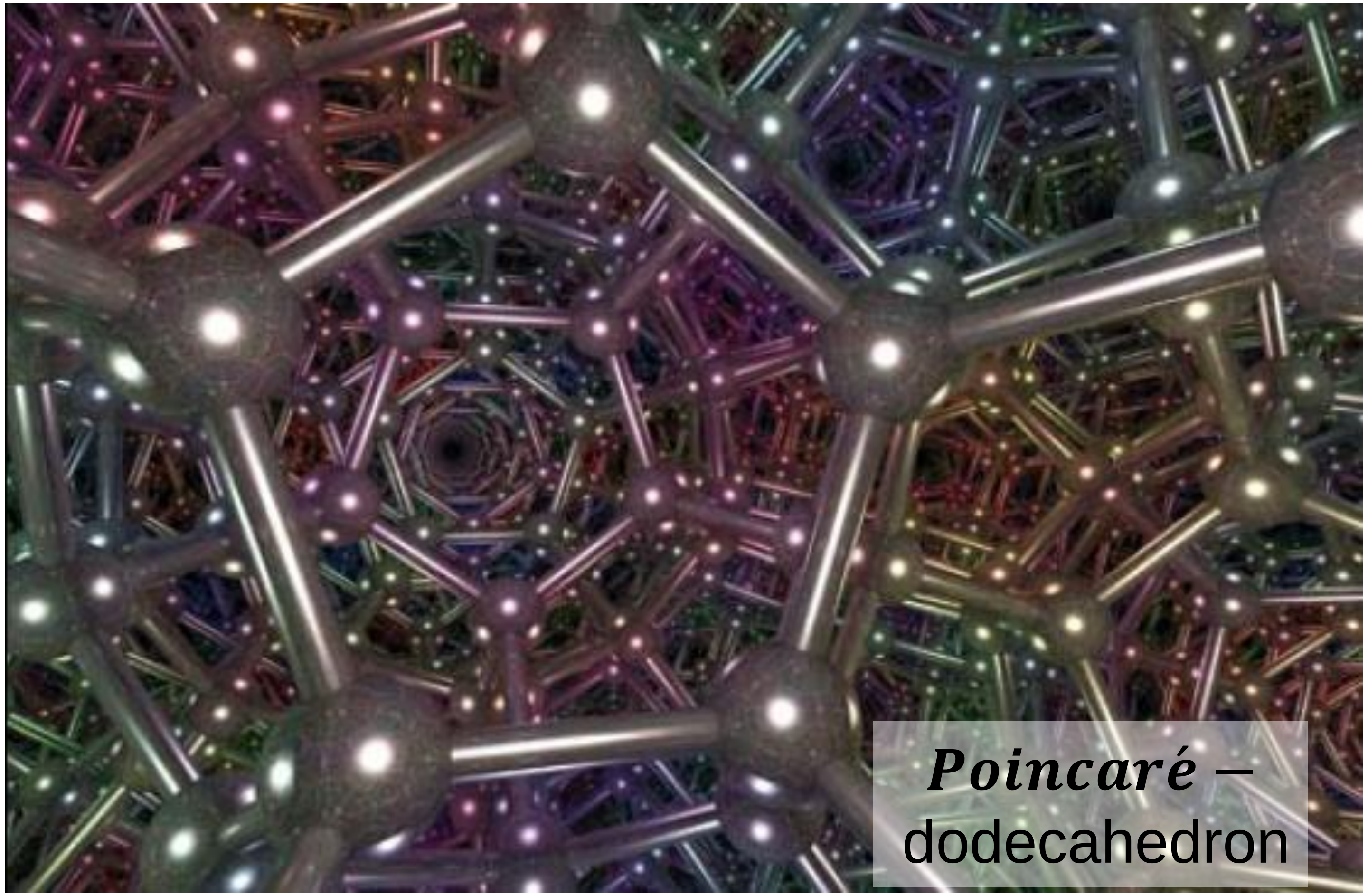


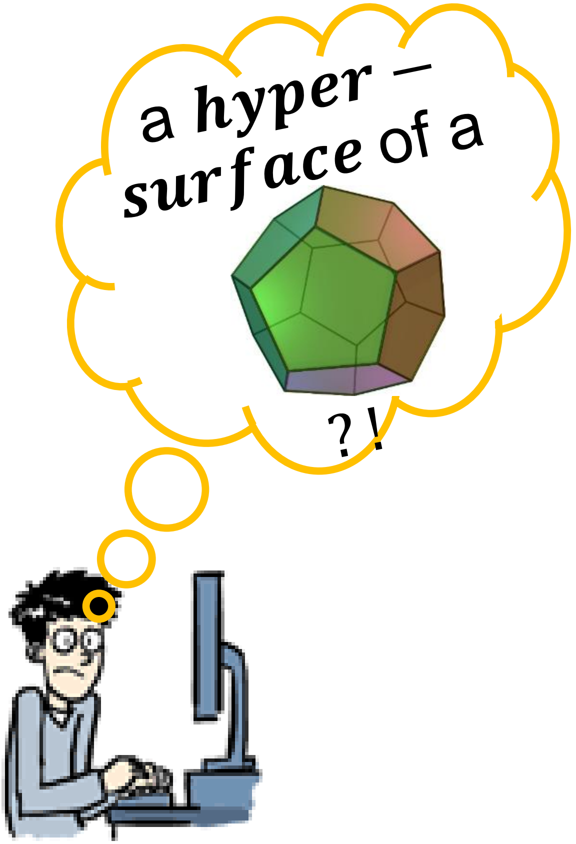
* or, in our universe, at different, far−off locations

Anomalies in the CMB: quadrupole & octopole

- Answer 1: it could point to the universe being a *Poincaré* manifold

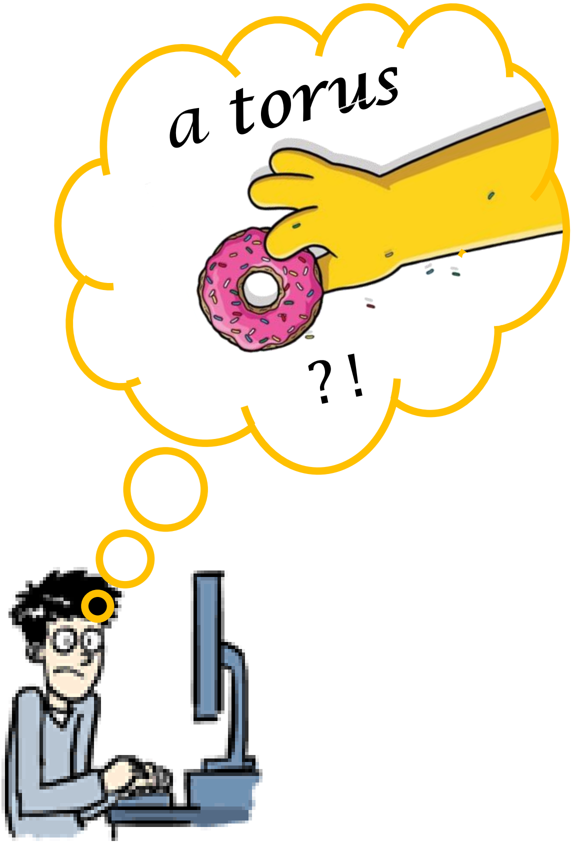






Anomalies in the CMB: quadrupole & octopole

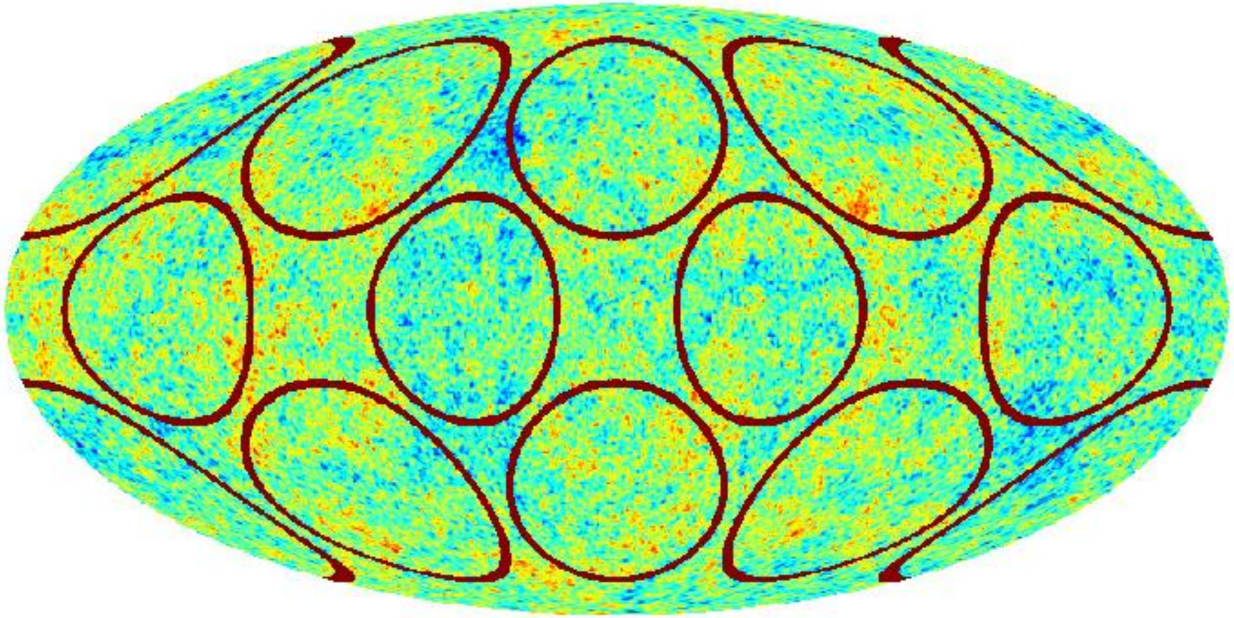
- **Answer 2: it could point to the universe being a *torus* (not all dimensions $a_i(t)$ did increase equally)**



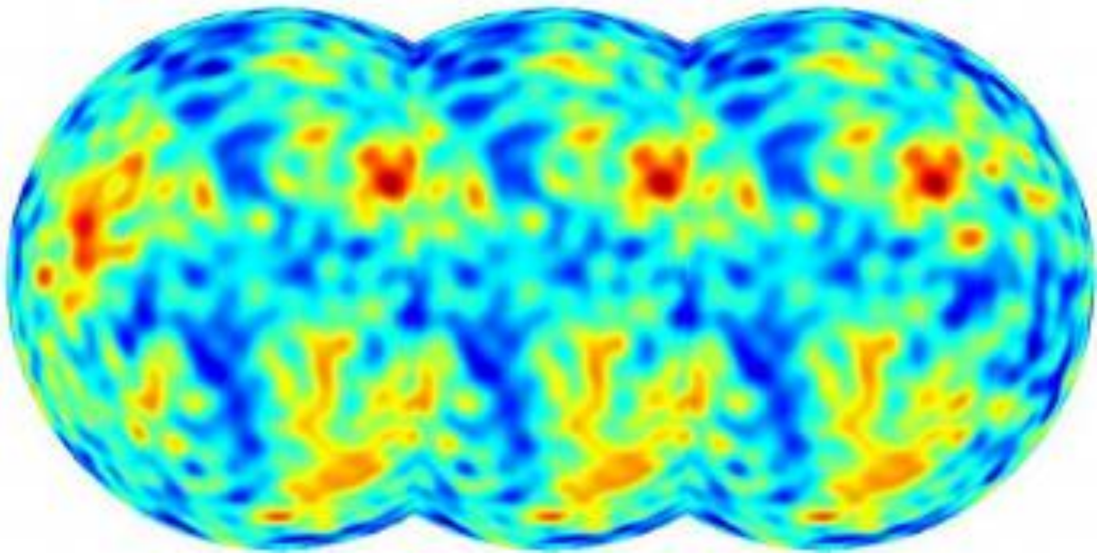
Anomalies in the CMB: circles in the sky?

- **Answer 3: universe with non$-$trivial topology : pairs of matching circles?**

- simulated $\boldsymbol{Planck}$ map with a $[2,2,2]$ **toroidal symmetry**



- **none** found so far …



Anomalies in the CMB: quadrupole & octopole

- The ´axis of evil´: pure coincidence or systematic effect in the analysis?

The universe lines up along the 'axis of evil'. Coincidence?

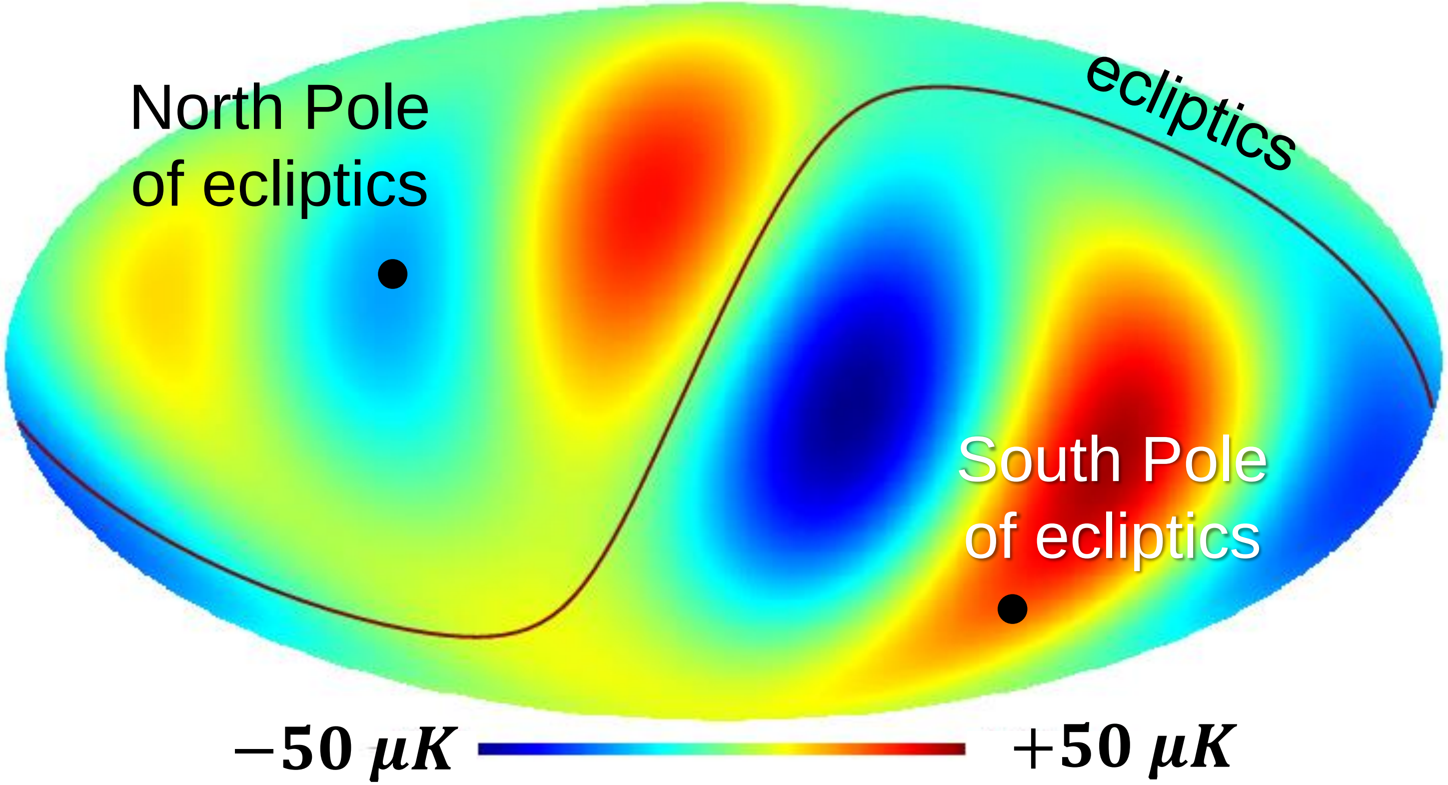
From the rotation of galaxies to cosmic expansion everything points in one direction. If only we knew why

SPACE 26 October 2016

By Stuart Clark

ESO/B. Tafreshi (twanight.org)

COSMOLOGISTS called it the axis of evil. Spotted in 2005 in the cosmic microwave background, the all-pervading afterglow of the big bang, the axis was a peculiar alignment of features where we would have



quadrupole & octopole alignment

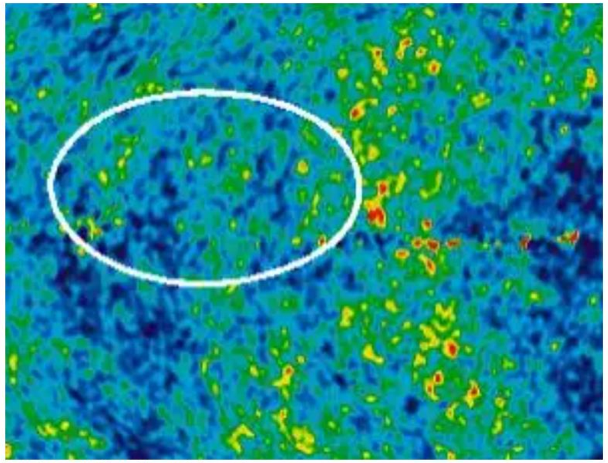
Anomalies in the CMB

- **Cosmic variance** & **the element of coincidence* in large data sets**

Found: Hawking's initials written into the universe

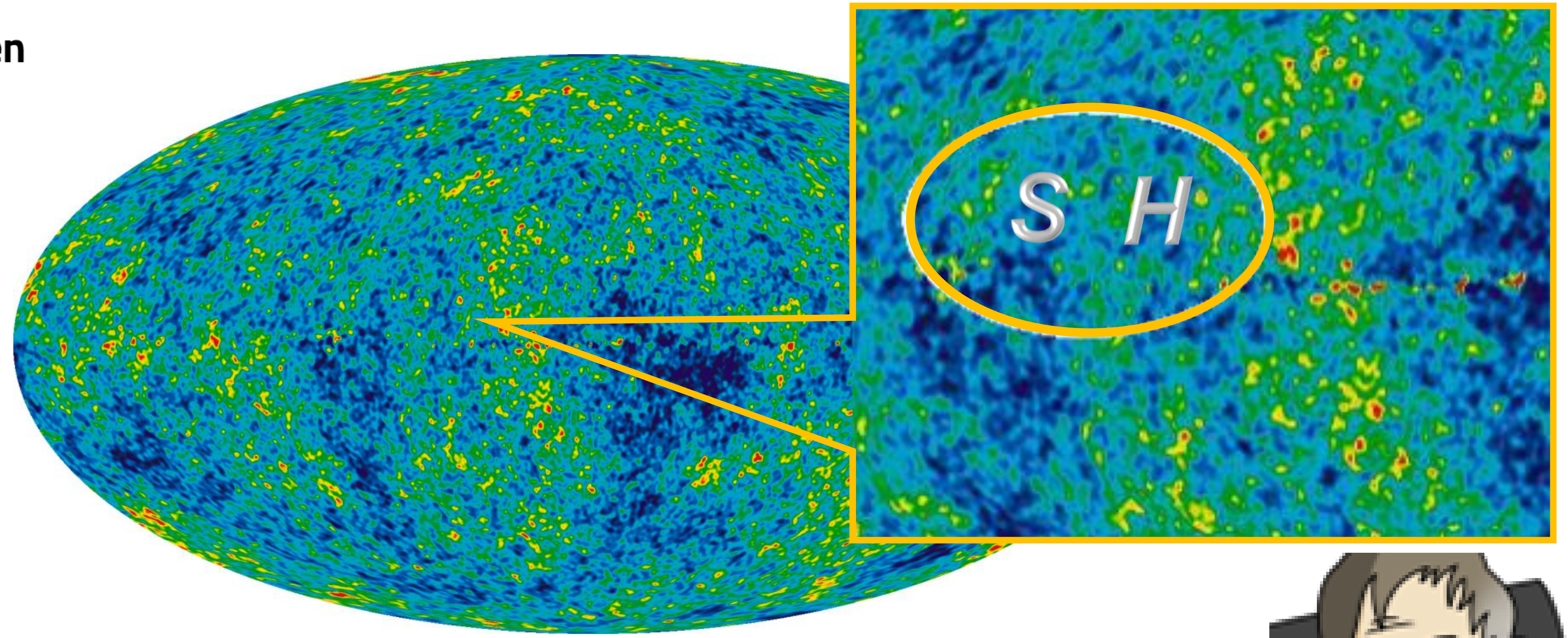
SPACE 7 February 2010

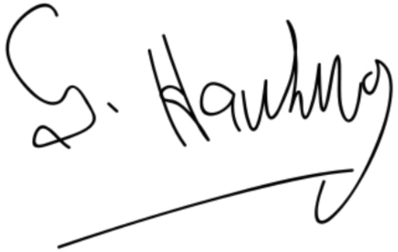
By Richard Fisher and Rachel Courtland



Stephen Hawkings leaves his mark
(Image: NASA/WMAP Science Team)

Is Stephen Hawking a galactic graffiti artist? Hidden away in the cosmic microwave background, the afterglow of the big bang, the initials "SH" are clear to view (see picture, right). We took a closer look and spotted

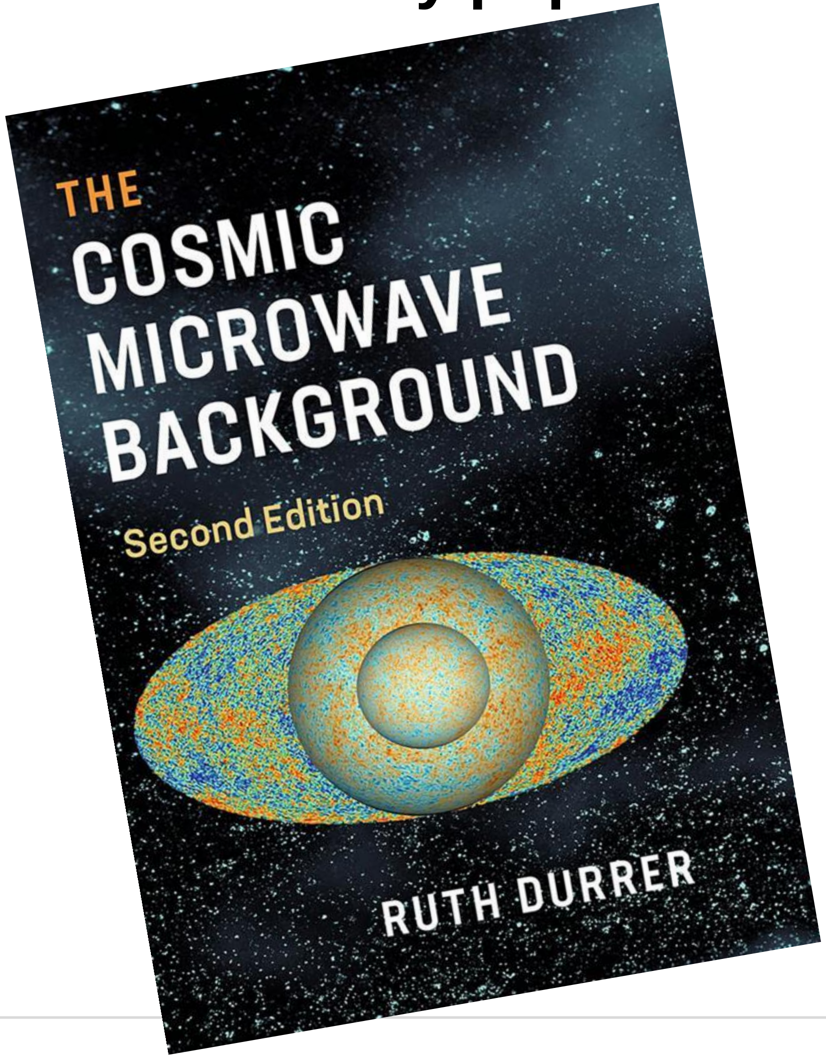


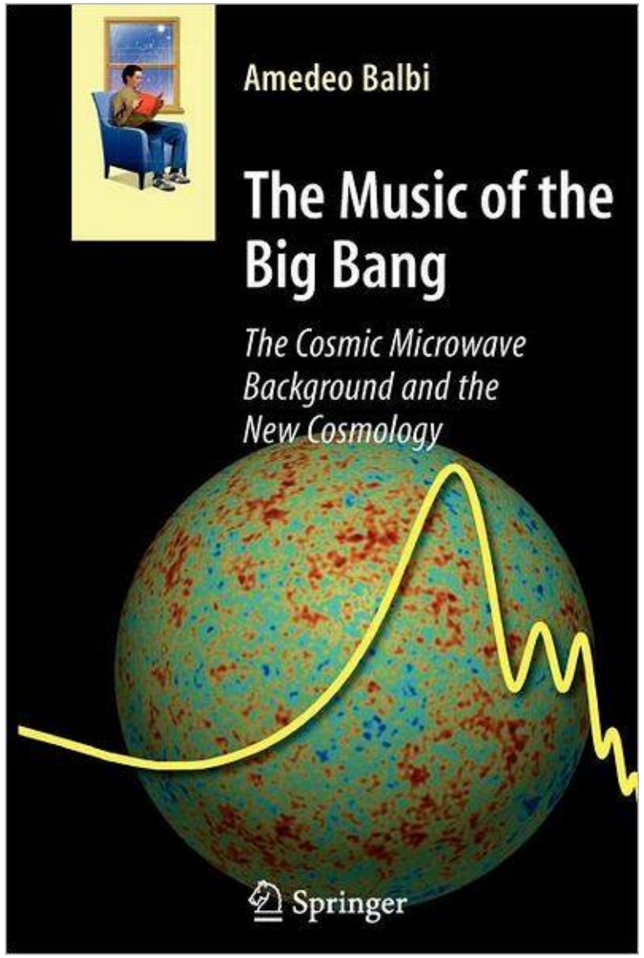


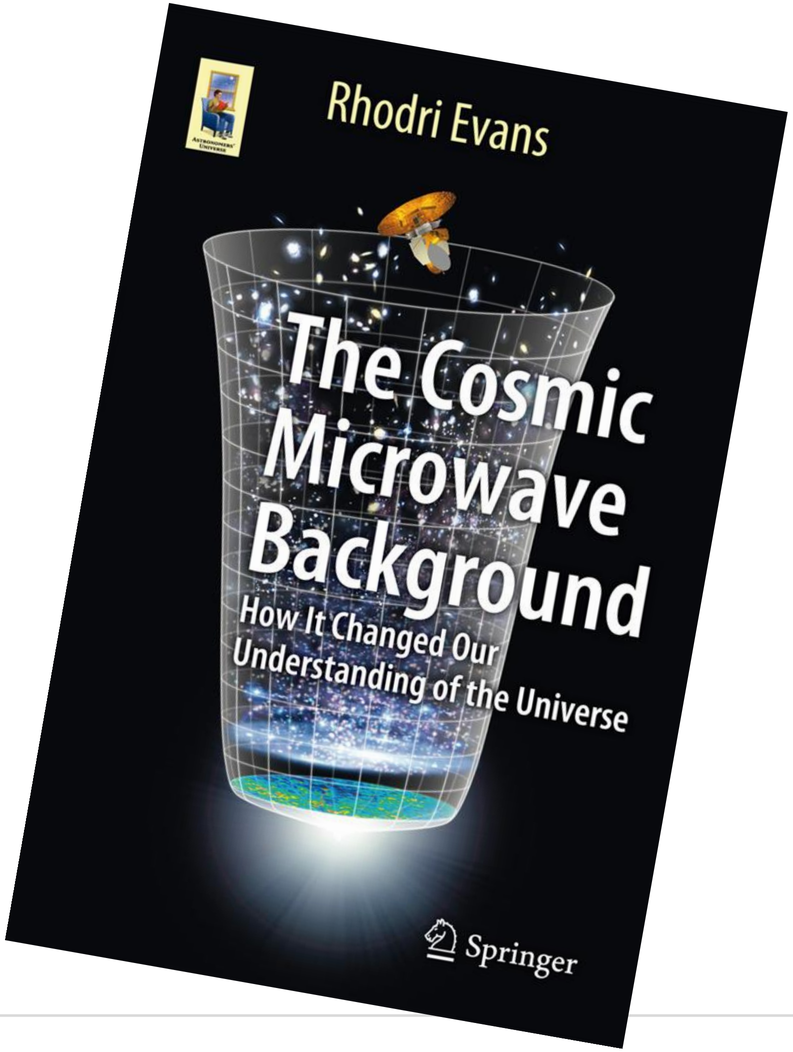
*find your own initials in the CMB

Riding early waves: interesting (new) books

- CMB is fairly popular: many books on the market

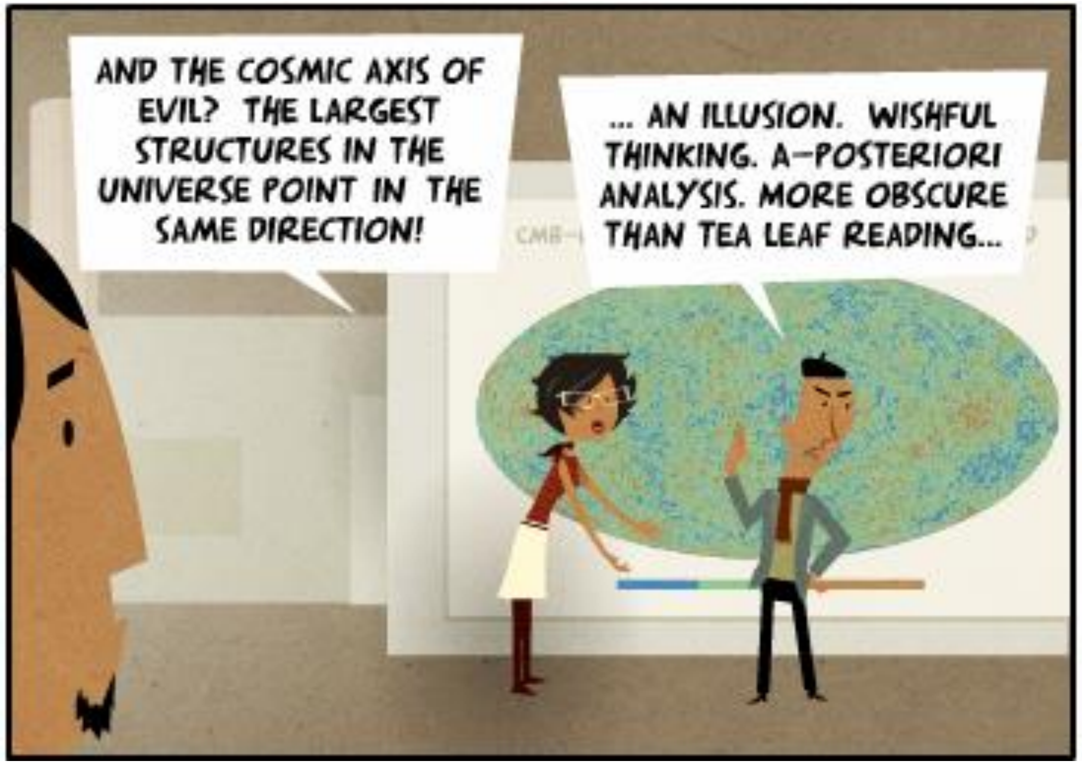






Riding early waves: two Post–docs at MPA

- Do NOT copy: surfing on acoustic (sound) waves in the Early Universe …



[Riding the Cosmic Waves – Pressepaket (mpg.de)]