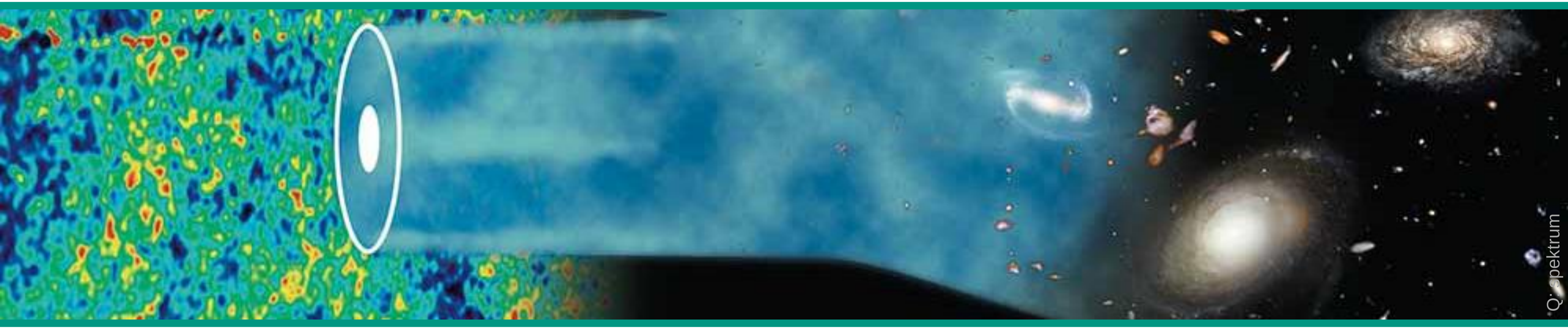
Karlsruhe Institute of Technology

Introduction to Cosmology

Winter term 23/24 Lecture 13 Jan. 30, 2024



Recap of Lecture 12

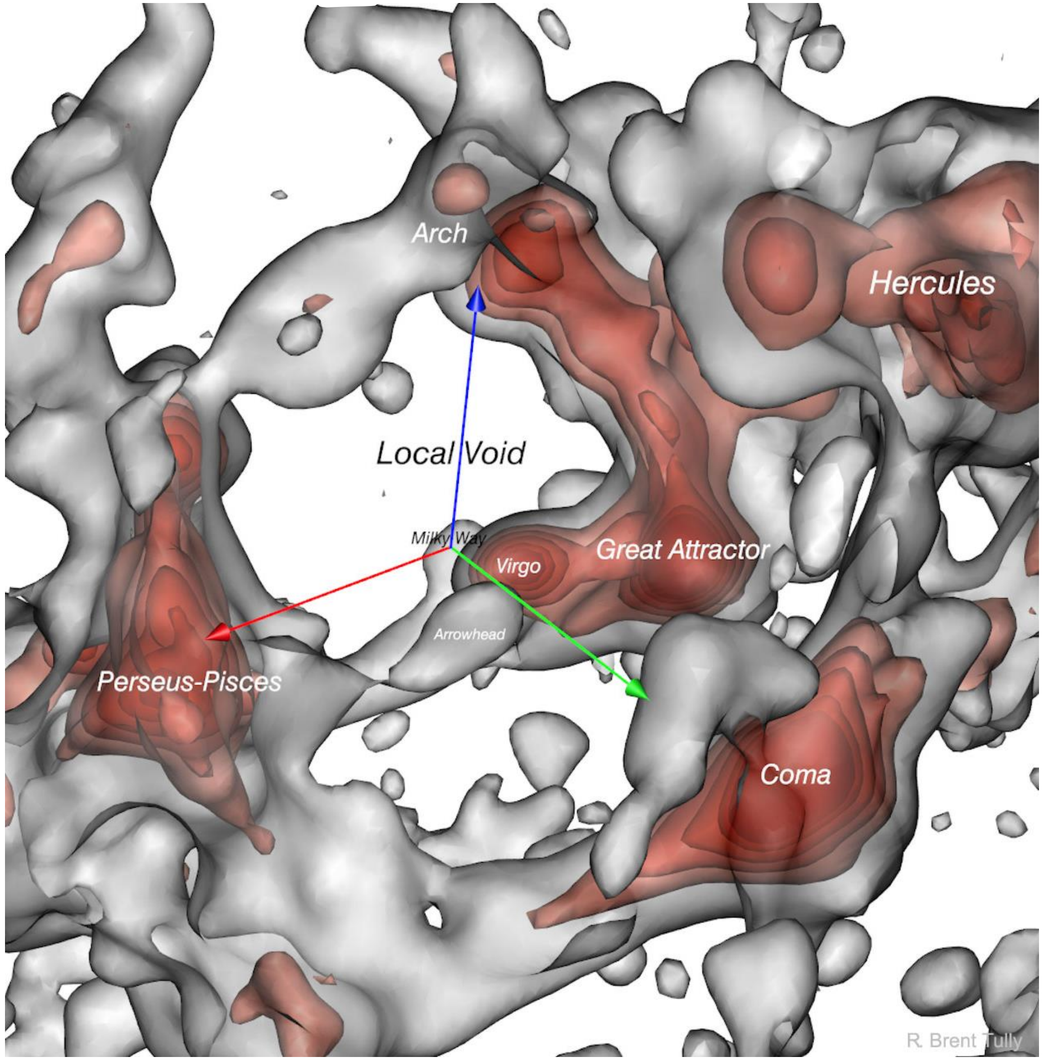
- **Evolution of $\mathcal{L}$arge $\mathcal{S}$cale $\mathcal{S}$tructures (LSS)**

- **galaxy redshift surveys** ($2dF$, $SDSS$,…):
  $3D$ – distribution of galaxies reveals
  BAO – correlation (rings) at $r \sim 140\ Mpc$

- **linear** increase of **density contrast δ**
  of matter: super–clusters ($\delta = 2 \ldots 3$) &
  super–voids ($\delta = 0.2 \ldots 0.3$)

- Hot Dark Matter (HDM): rather massive
  (few eV) neutrinos ⇨ **top–down** scenario

- Cold Dark Matter (CDM): very heavy
  (few TeV) neutralinos ⇨ **bottom–up** scenario



Cosmology: a true CMB pioneer has left us

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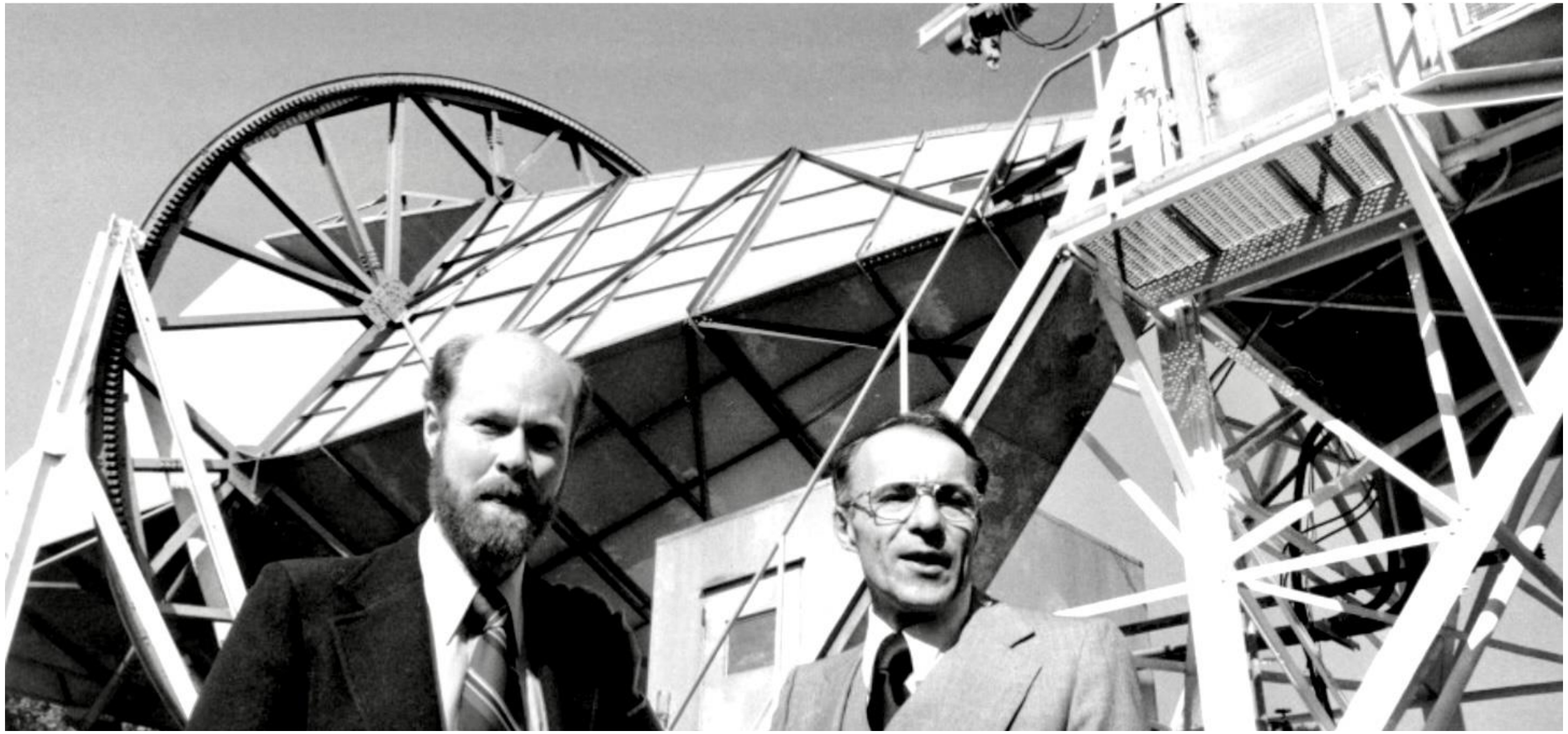
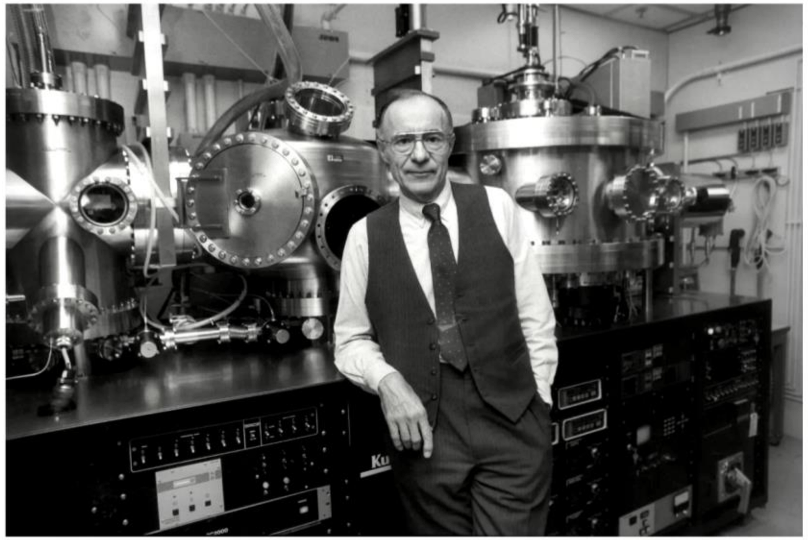
- **Penzias & Wilson: first experimental detection of the CMB radiation at Holmdel (1964)**



Arno A. Penzias, 90, Dies; Nobel Physicist Confirmed Big Bang Theory

His 1964 discovery with Robert W. Wilson settled a debate over the origin and evolution of the universe.



Dr. Arno A. Penzias in a 1991 photo at Bell Laboratories in New Jersey. He and Dr. Robert W. Wilson were researchers there in 1964 when they discovered cosmic microwave background radiation, remnants of the Big Bang. Frank C. Dougherty

By Katie Hafner

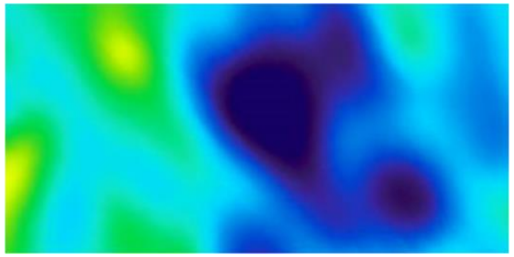
Jan. 22, 2024

Arno A. Penzias, whose astronomical probes yielded incontrovertible evidence of a dynamic, evolving universe with a clear point of origin, confirming what became known as the Big

The New York Times

Structucture formation in an expanding universe

- Increase of scale factor $a(t)$ of the universe over the Hubble time t_0



$a(CMB) \sim 10^{-3}$

$z(CMB) = 1100$

increase of **scale−factor** by 10^3

$a(t_0) = 1$

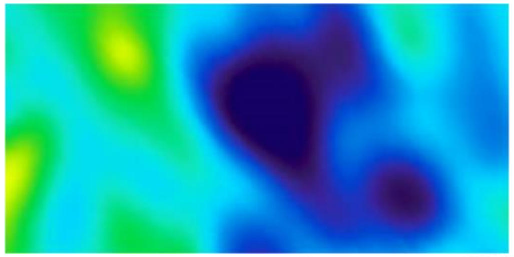
$z(t_0) = 0$

early universe
at $t = 378.000\ yr$
CMB decoupling

today´s universe
at $t = 13.8 \cdot 10^9\ yr$
present LSS

Structure formation in an expanding universe

- Linear growth of the density contrast $\delta(t)$ of LSS over the Hubble time t_0



$a(CMB) \sim 10^{-3}$

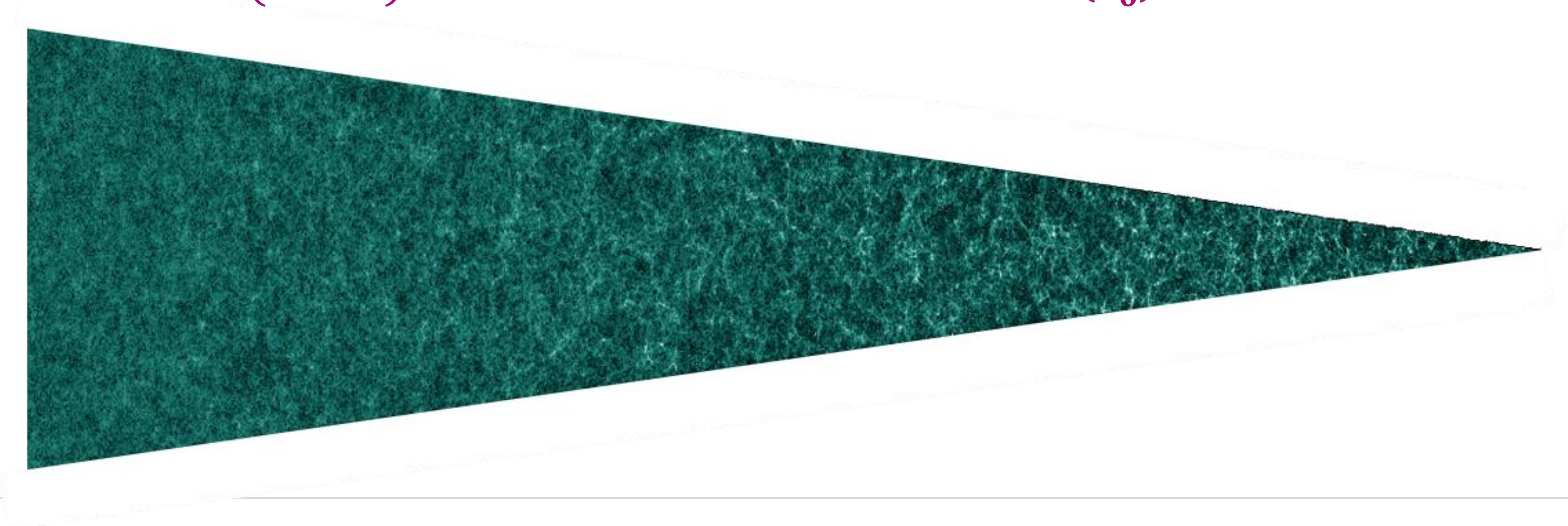
$\delta(CMB) = 10^{-5}$

increase of **density contrast**

by 10^5

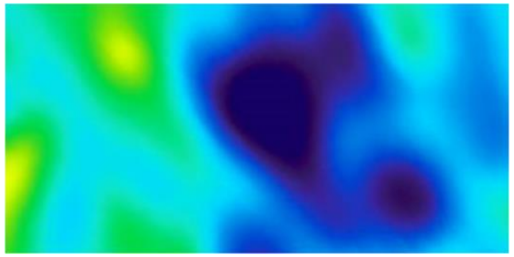
$a(t_0) = 1$

$\delta(t_0) = 2 - 3$



Structure formation in an expanding universe

- **Linear growth of the density contrast $\delta(t)$ of LSS over the Hubble time t_0**



$a(CMB) \sim 10^{-3}$

$\delta(CMB) = 10^{-5}$

increase of **density contrast**

by 10^5

$a(t_0) = 1$

$\delta(t_0) = 2 - 3$

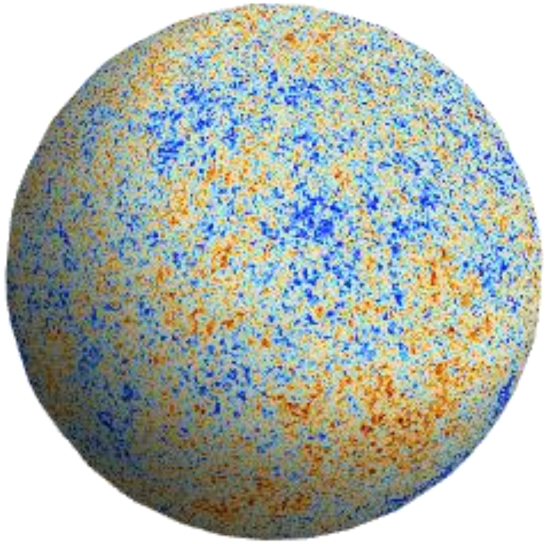
Why
- are these numbers not adding up?
- has the density contrast increased more?
- is our universe so clumpy today?

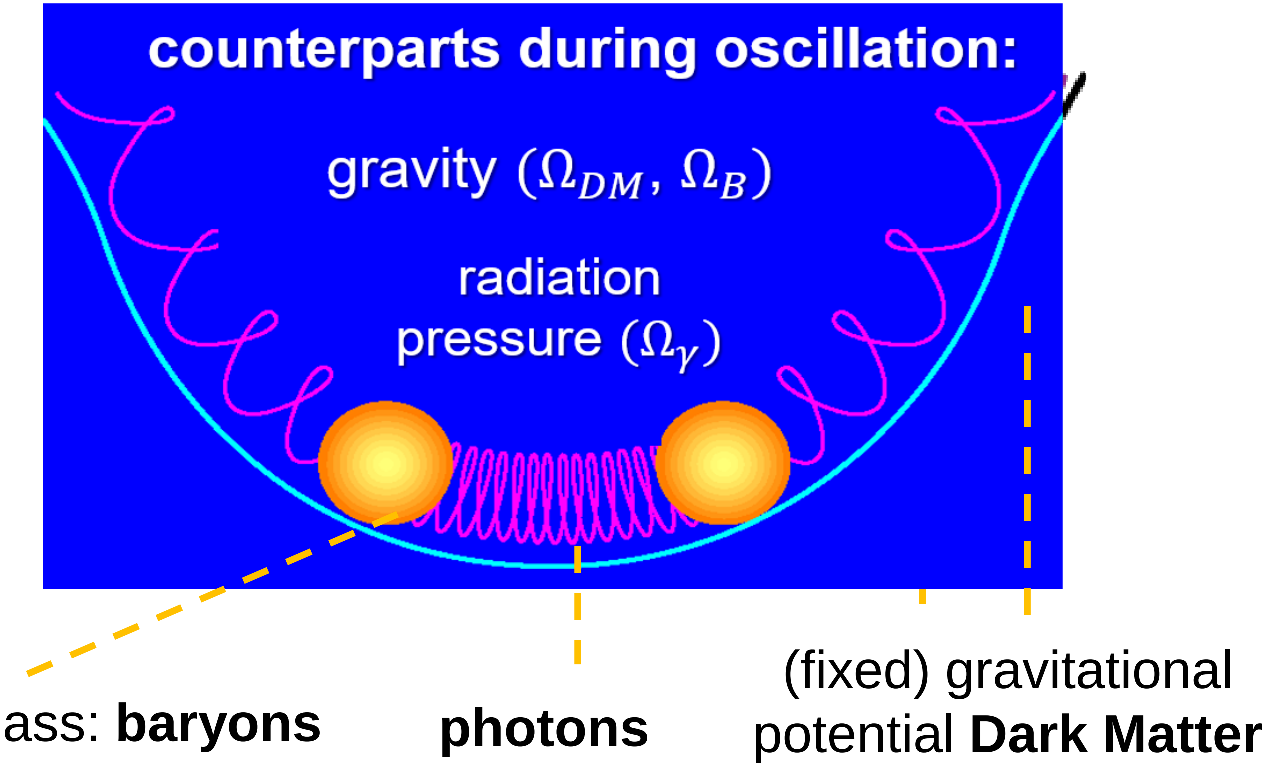
Structure formation: BAO & DM – seeds for LSS

- Baryon Acoustic Oscillations: an incontrovertible evidence for Dark Matter

  - the key importance of **Dark Matter** in structure formation: **linear increase of the density contrast** ***despite*** the strong coupling of baryons to photons

$\delta(CMB) = 10^{-5}$



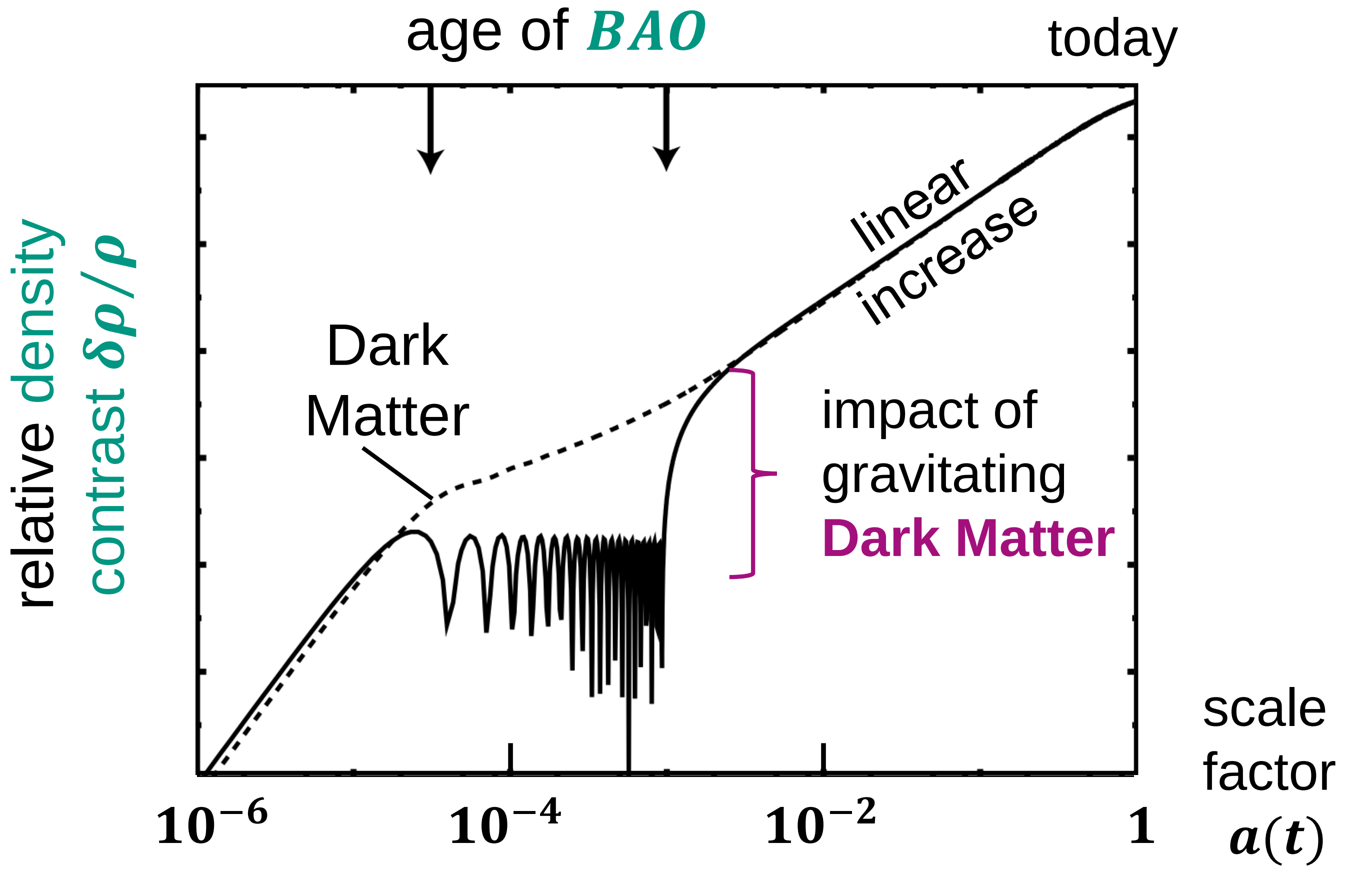


Structure formation: BAO & the DM – seed for LSS

- **During BAO phase: DM density contrast grows while baryons are stalled**

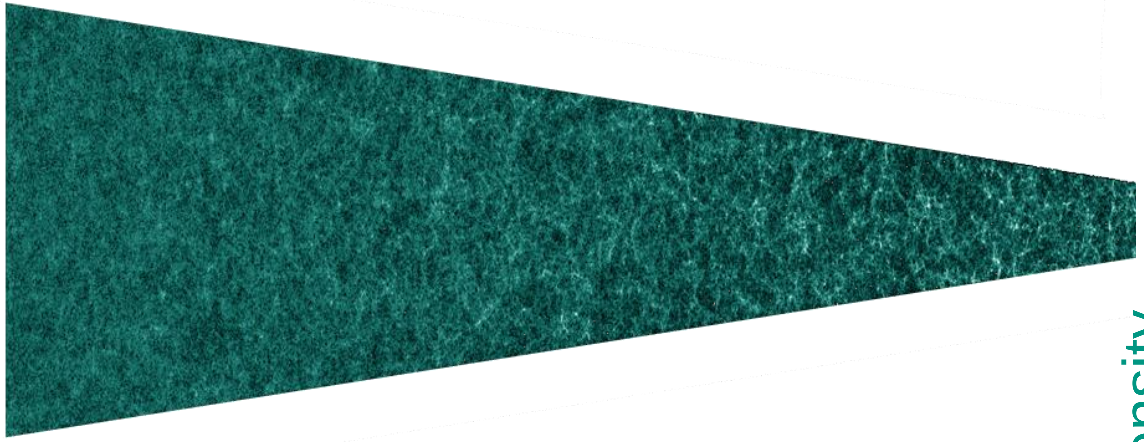
- in the primordial plasma, the **density contrast of baryons** cannot further grow and is **stalled** due to BAO

- DM, however, is unaffected by the period of BAO & its density contrast **continues** to grow

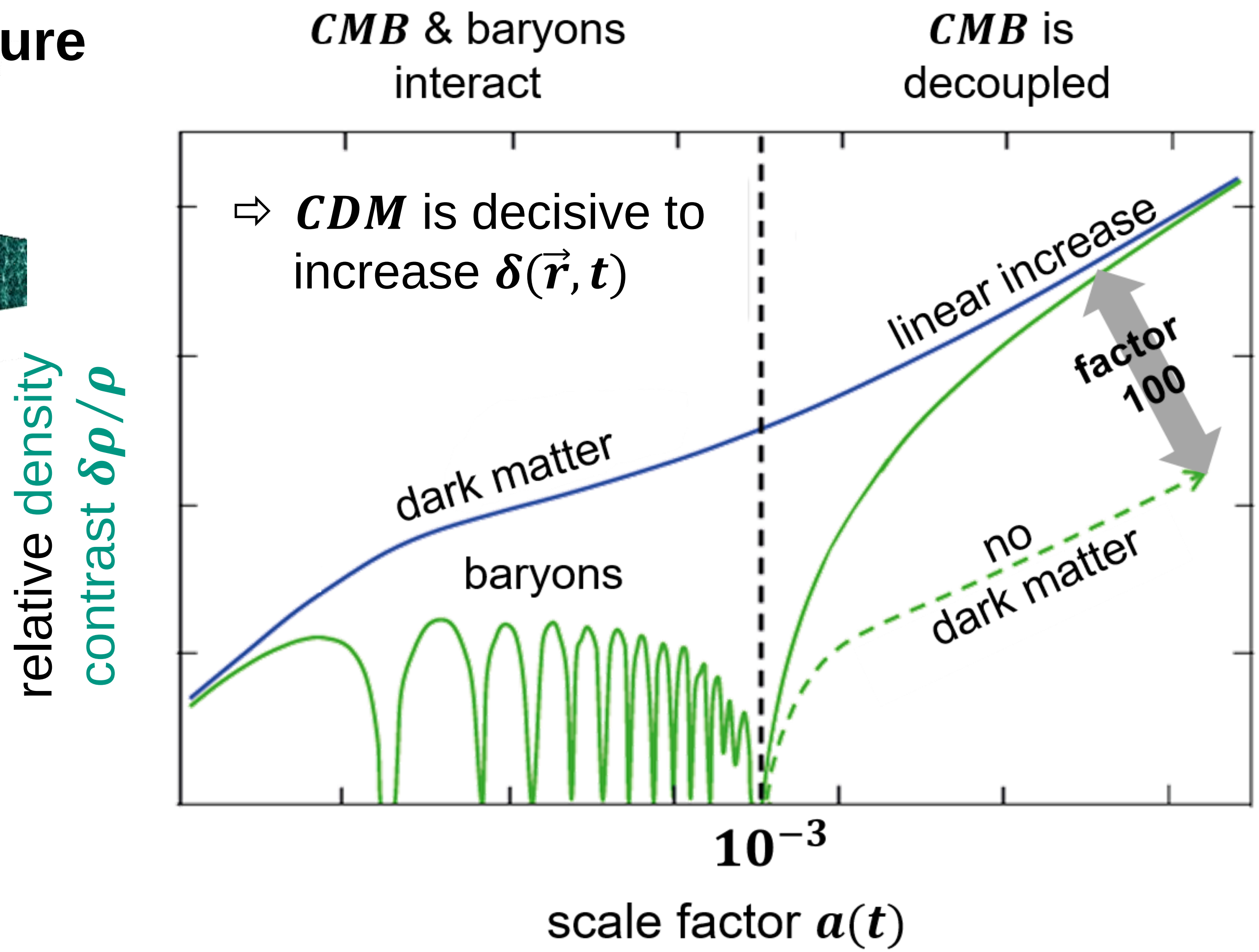


Structure formation: BAO & the DM – seed for LSS

- DM clumps as seeds for structure

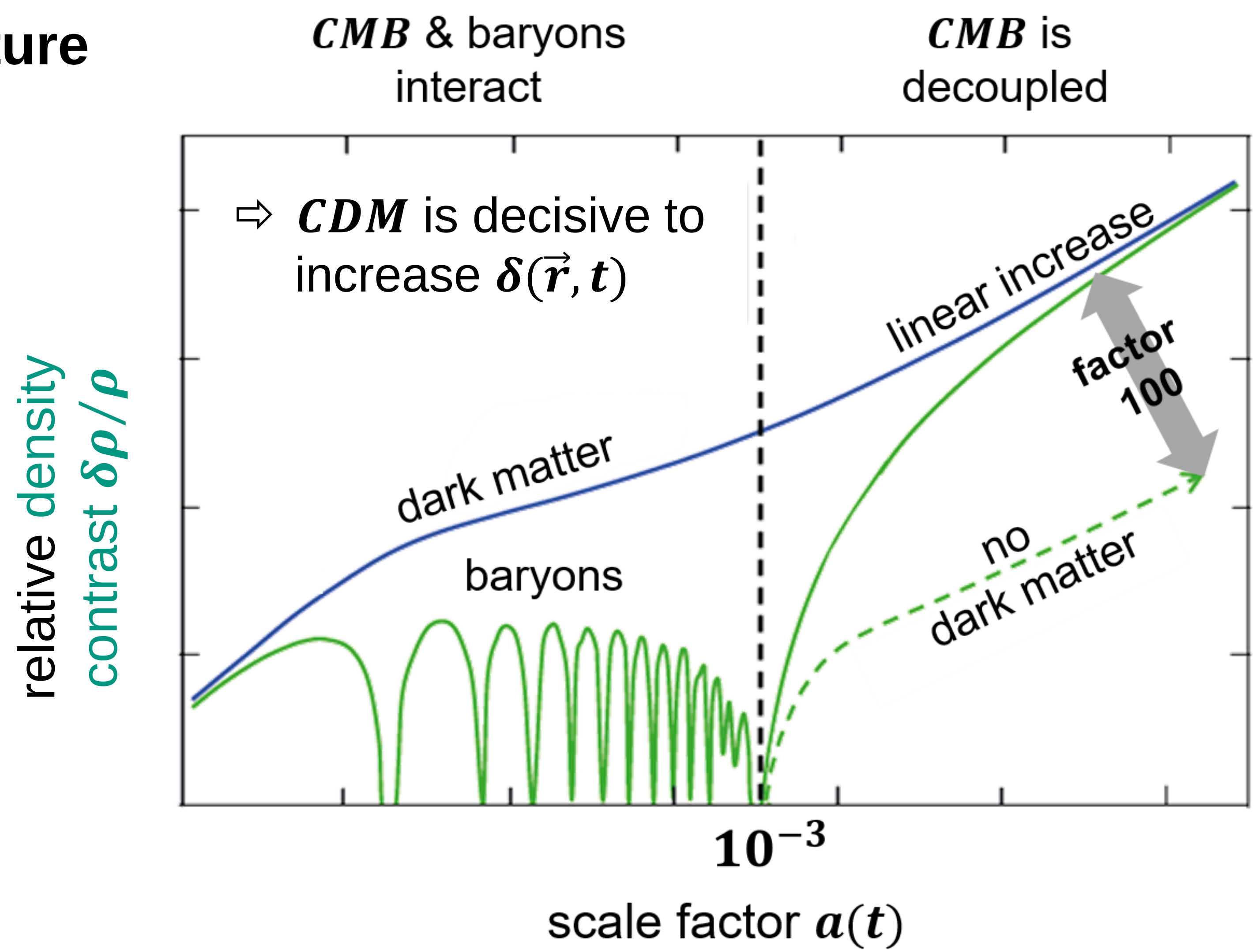


- DM, however, is unaffected by the period of BAO & its density contrast continues to grow
- no DM during BAO: the universe would be **factor 100** smoother (much less structure: no galaxies)



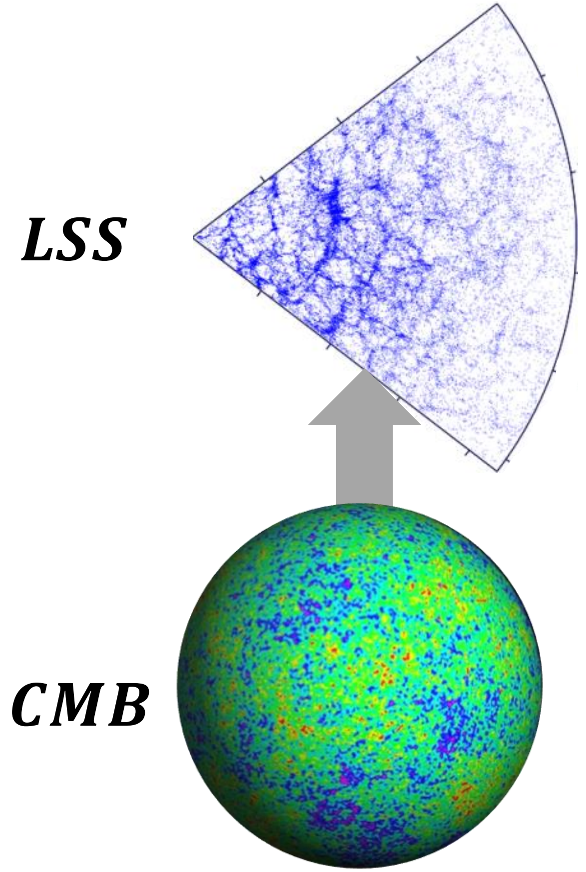
Structure formation: BAO & the DM – seed for LSS

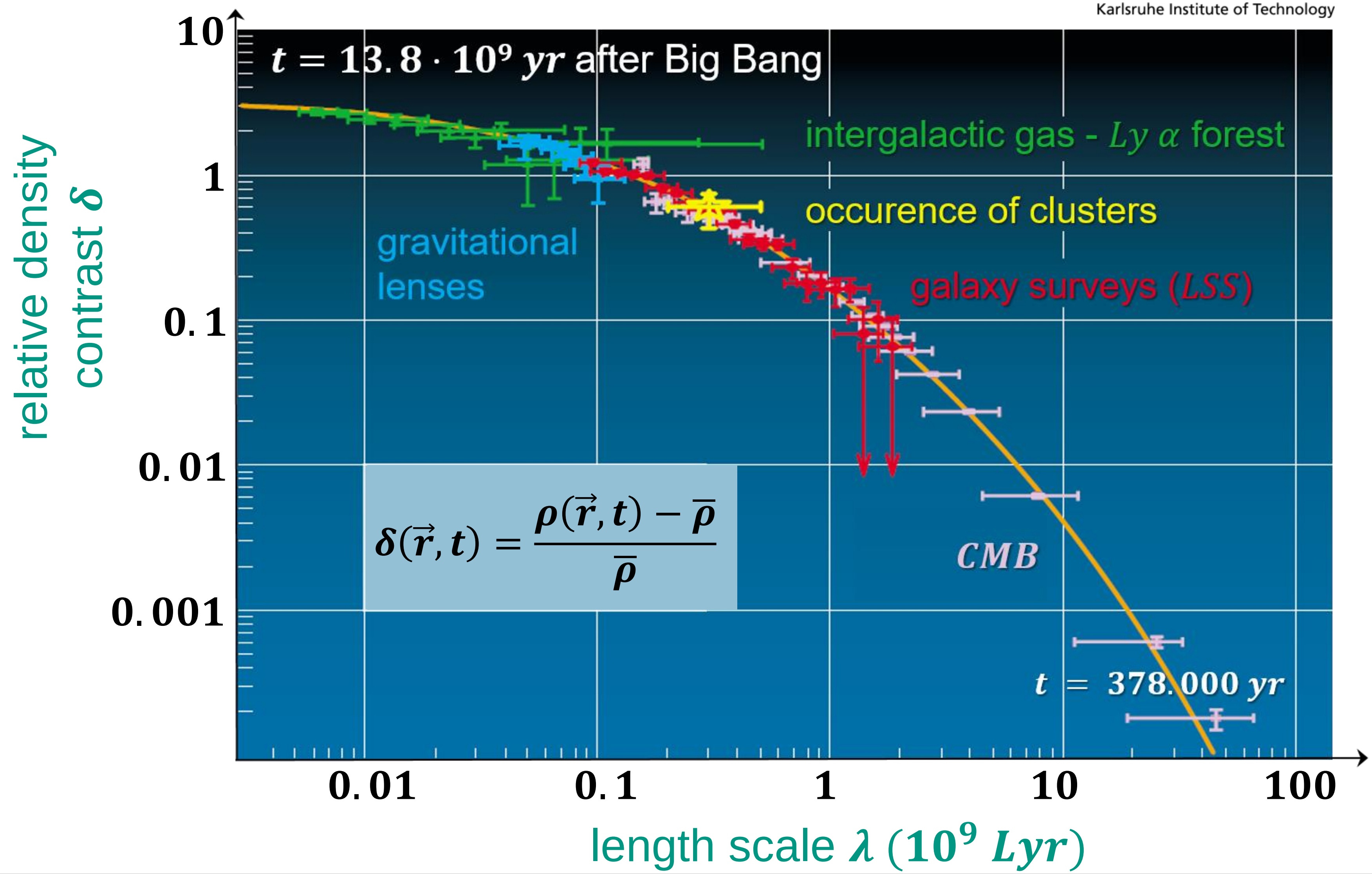
- DM clumps as seeds for structure



Observational evidence for structure growth

- Measuring the increase of $\delta(t)$





How is the density contrast δ being measured?

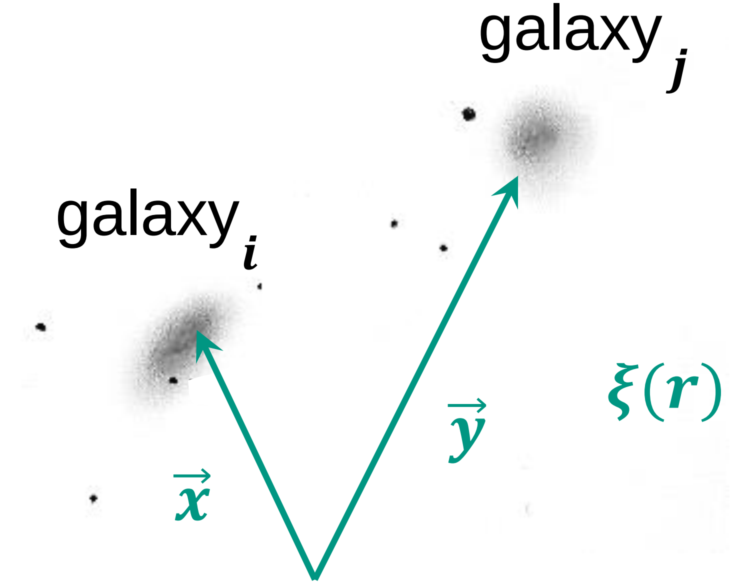
- *James Peebles*: measure the distances $\vec{r}$ of galaxy pairs in space

  - galaxies are **not** distributed randomly in space:
  use the **galaxy (auto–)** correlation function $\xi(r)$
  (as function of their relative distance $\vec{r}$) to study the
  evolution of large–scale structures

$$\delta(\vec{r}) = \frac{\rho(\vec{r}) - \langle\rho\rangle}{\langle\rho\rangle}$$

James Peebles
Nobel Prize
2019

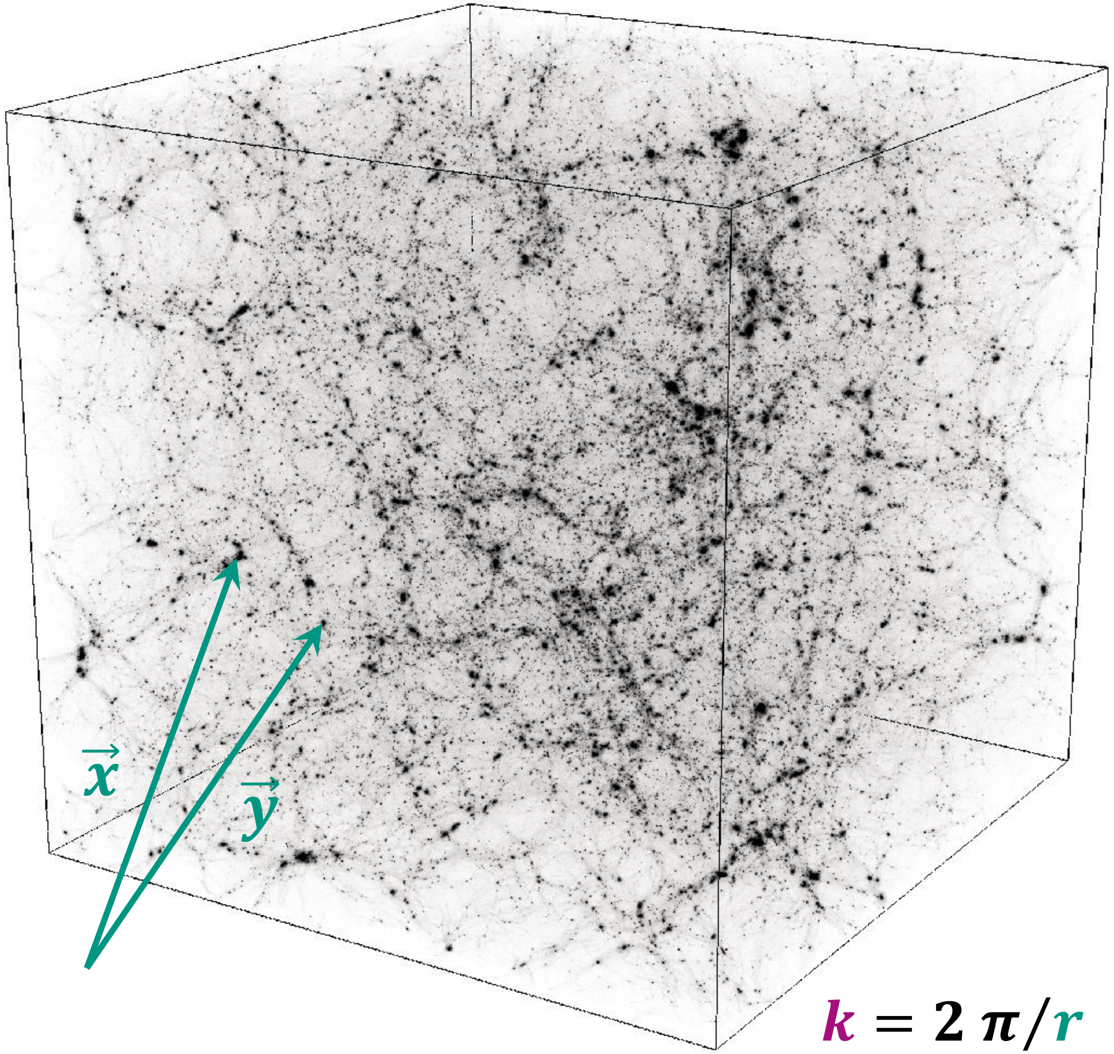
take an ´average´ galaxy i:
determine the probability of finding the next one j at a distance $\vec{r} = |\vec{x} - \vec{y}|$



Galaxy auto–correlation function $\xi(r)$

- **Making use of galaxy redshift surveys**
  - $3D$ correlation of galaxies only depends on their **relative distance**
    $$r = |\vec{r}| = |\vec{x} - \vec{y}|$$
  - universe is isotropic & homogenous
  - there is no ´average´ galaxy
  - large statistical ensemble yields a simple **power–law** distribution $\sim r^{-1.8}$

$$\xi(r) \sim \left(\frac{r}{5\,h^{-1}\,Mpc}\right)^{-1.8}$$



$k = 2\,\pi/r$

RECAP: the ´revealing power´ of Fourier transform

- **Example in domains of time t and frequency ω**

- $\mathcal{F}$: Fourier transform integral – definition

$$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t) \cdot e^{-i \cdot 2\pi \cdot \omega \cdot t} \cdot dt$$

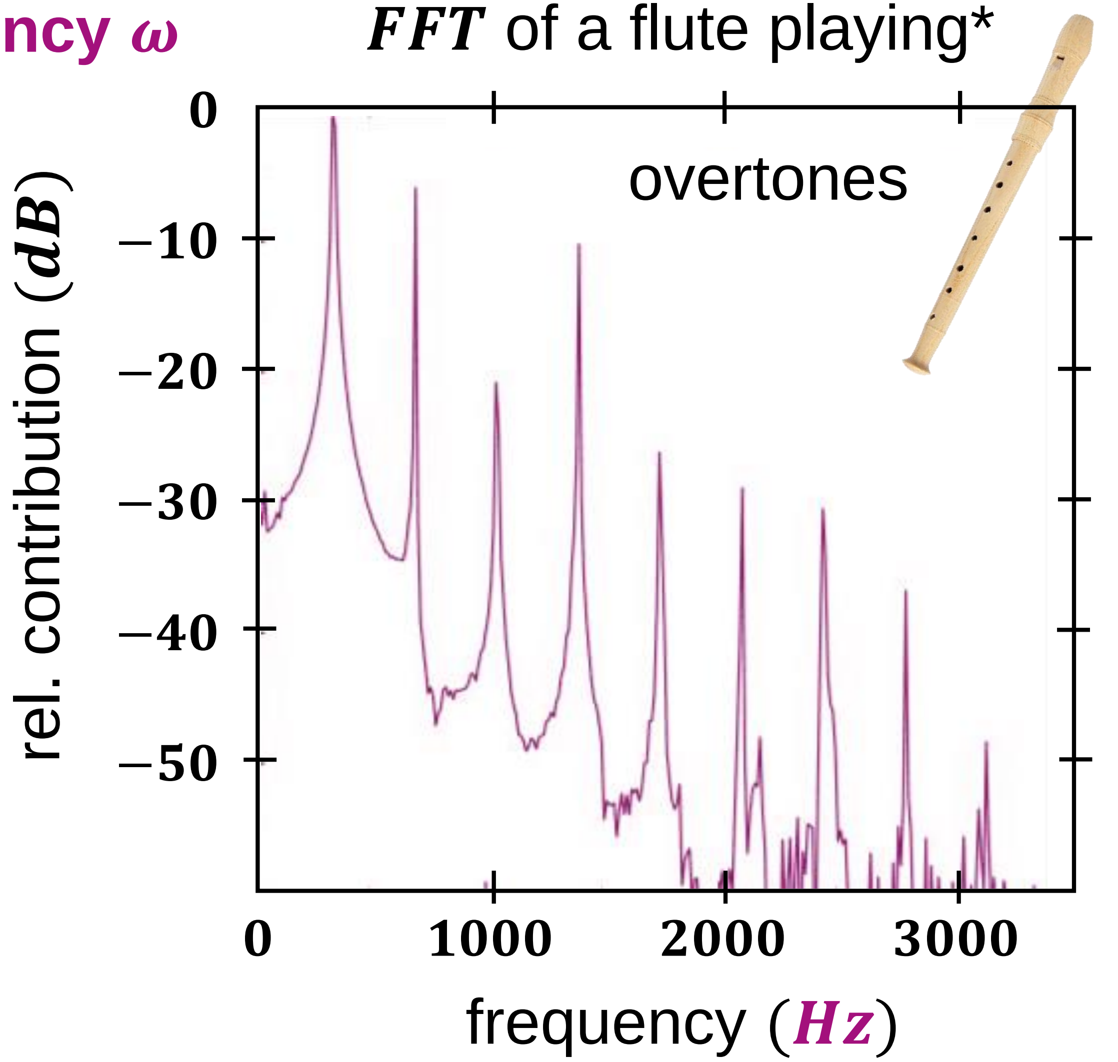
frequency → $Hz = s^{-1}$

time → s

$$\hat{f}(\omega) \overset{\mathcal{F}}{\Leftrightarrow} f(t)$$

frequency domain | time domain

FFT of a flute playing*



* see again lect. 10, p. 16 – 21

density contrast $\delta(\vec{k})$ in wave number k – space

- $\delta(\vec{k})$ **corresponds to the Fourier transform of** $\delta(\vec{r})$

  - as we now are working with wave numbers $k = |\vec{k}|$

$$\delta(\vec{r}) = \frac{\rho(\vec{r}) - \langle\rho\rangle}{\langle\rho\rangle}$$

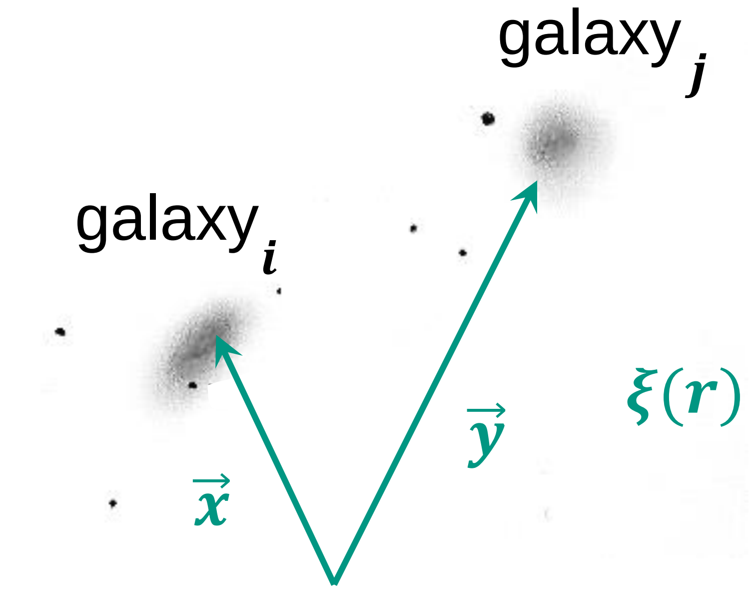
$$\delta(\vec{k}) = \sum \delta(\vec{r}) \cdot e^{-i\cdot\vec{k}\cdot\vec{r}}$$

wave number — position

  - this allows to derive the **density contrast**

$$\langle|\delta(\vec{k})|^2\rangle$$

from the **galaxy correlation function** $\xi(r)$



Galaxy correlation & matter power spectrum

- **Relation between correlation function $\xi(r)$ and power spectrum $P(k)$**

  - a key parameter for cosmology is the **matter power spectrum $P(k)$,** which is a measure of the **density contrast of matter** as function of a **wave number k**

$$\xi(r) = \frac{1}{2\pi^2} \cdot \int k^2 \cdot P(k) \cdot \frac{sin(k \cdot r)}{k \cdot r} \cdot dk$$

galaxy correlation — **matter power spectrum**

$$P(k) \sim \left\langle |\delta(\vec{k})|^2 \right\rangle$$

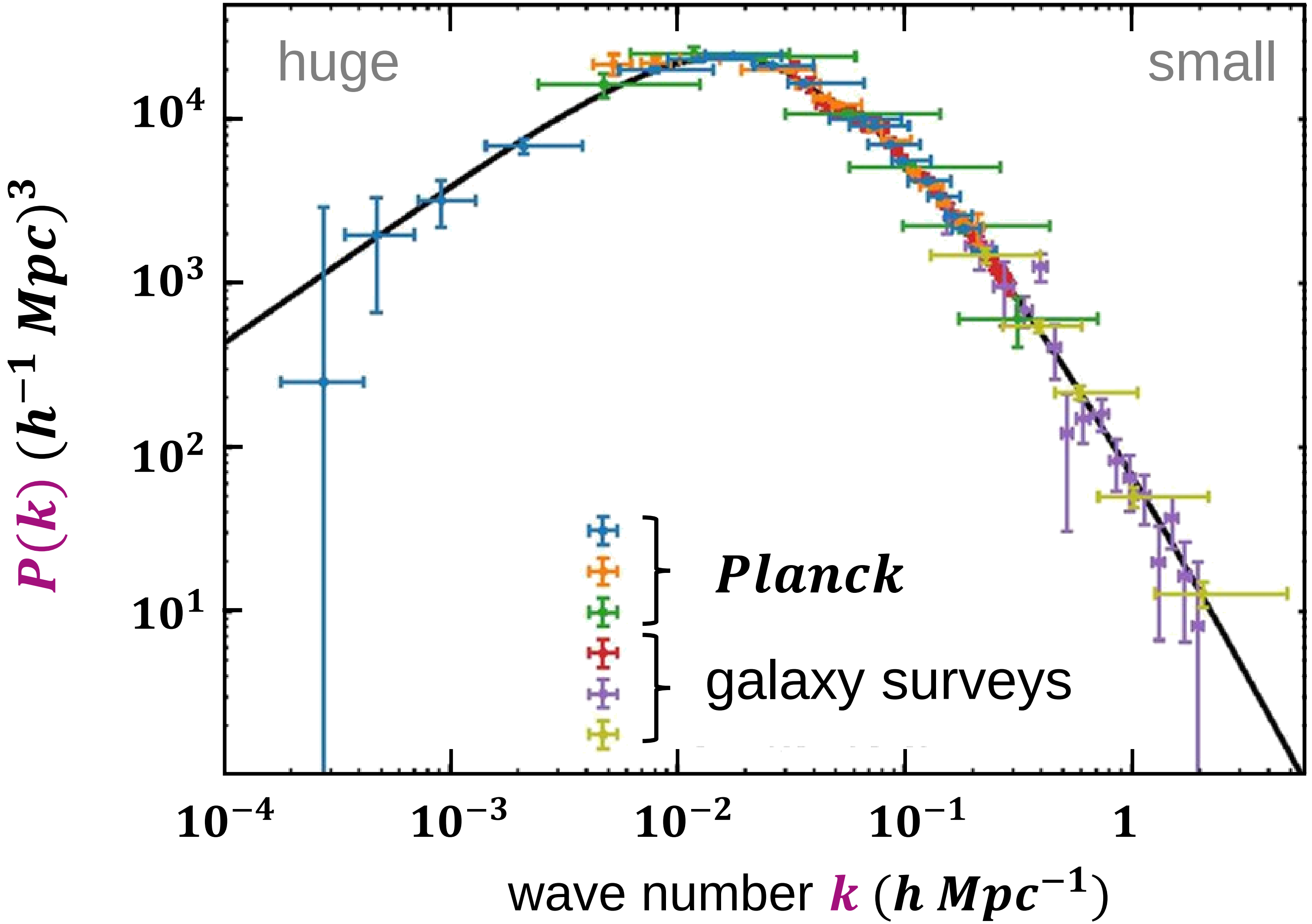
Observed matter power spectrum $P(k)$

- **Compilation of data from both the early universe (*CMB*) & the present universe (*LSS*)**

- it covers a factor ~ $\mathbf{10^6}$ in **length scales**

small k: huge length scale r

large k: small length scale r

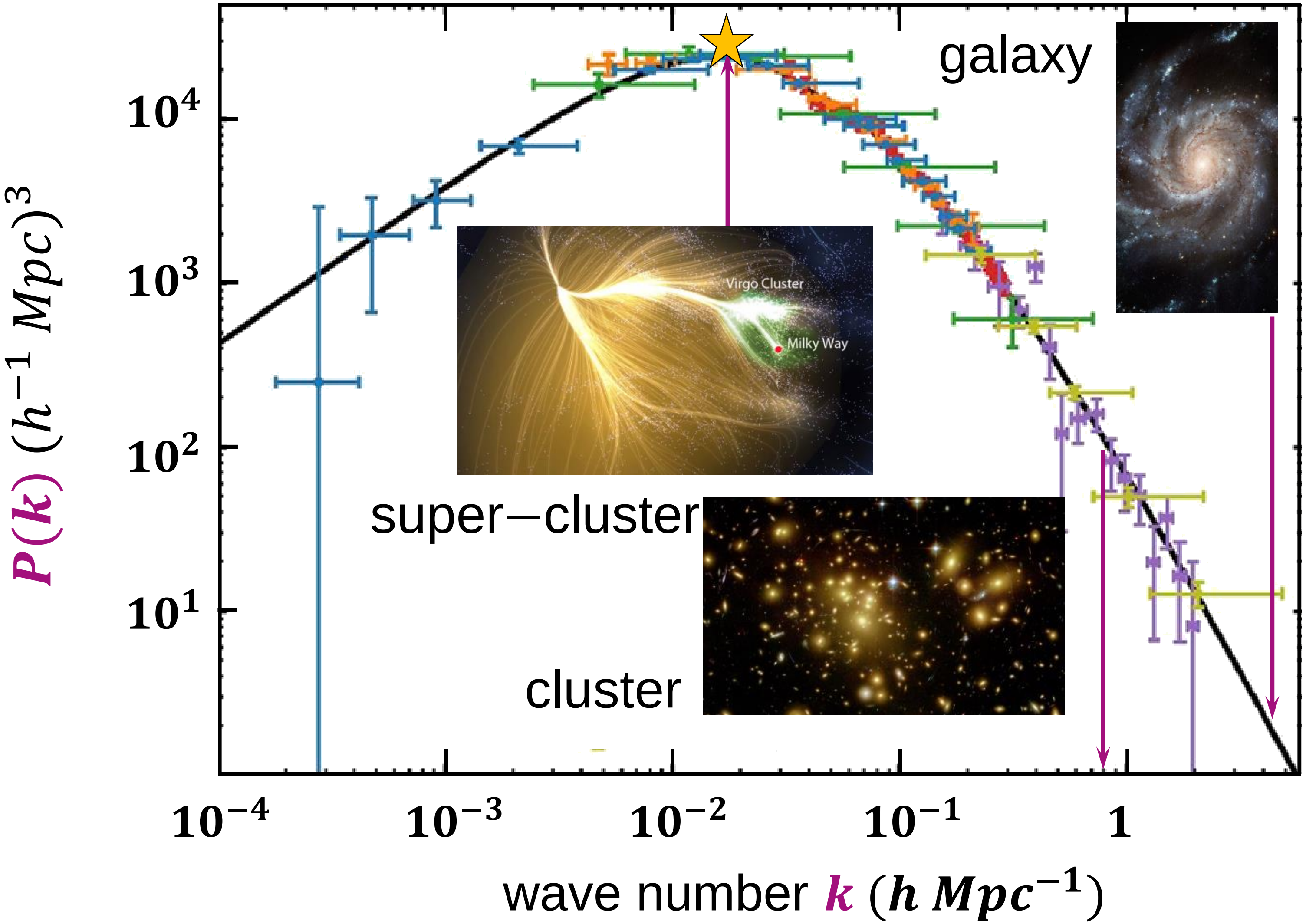


Observed matter power spectrum $P(k)$

- **Compilation of data from both the early universe (CMB) & the present universe (LSS)**

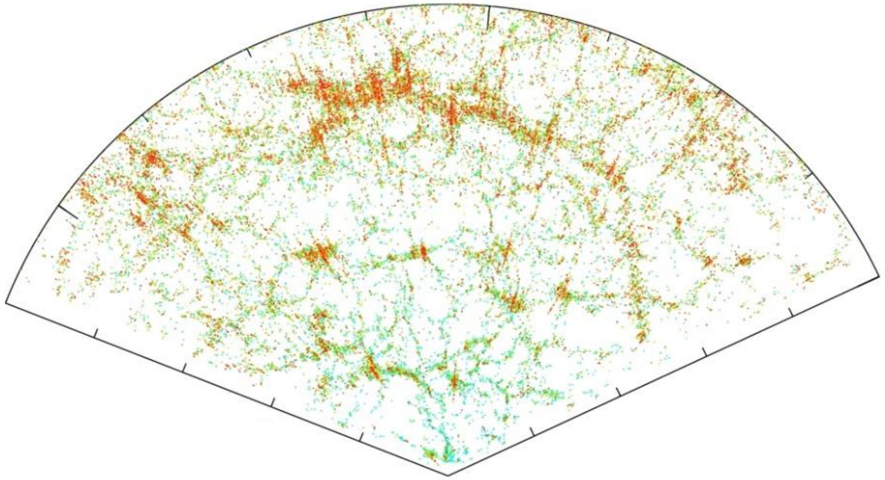
- super–clusters:
  why do they appear at the maximum of $P(k)$?

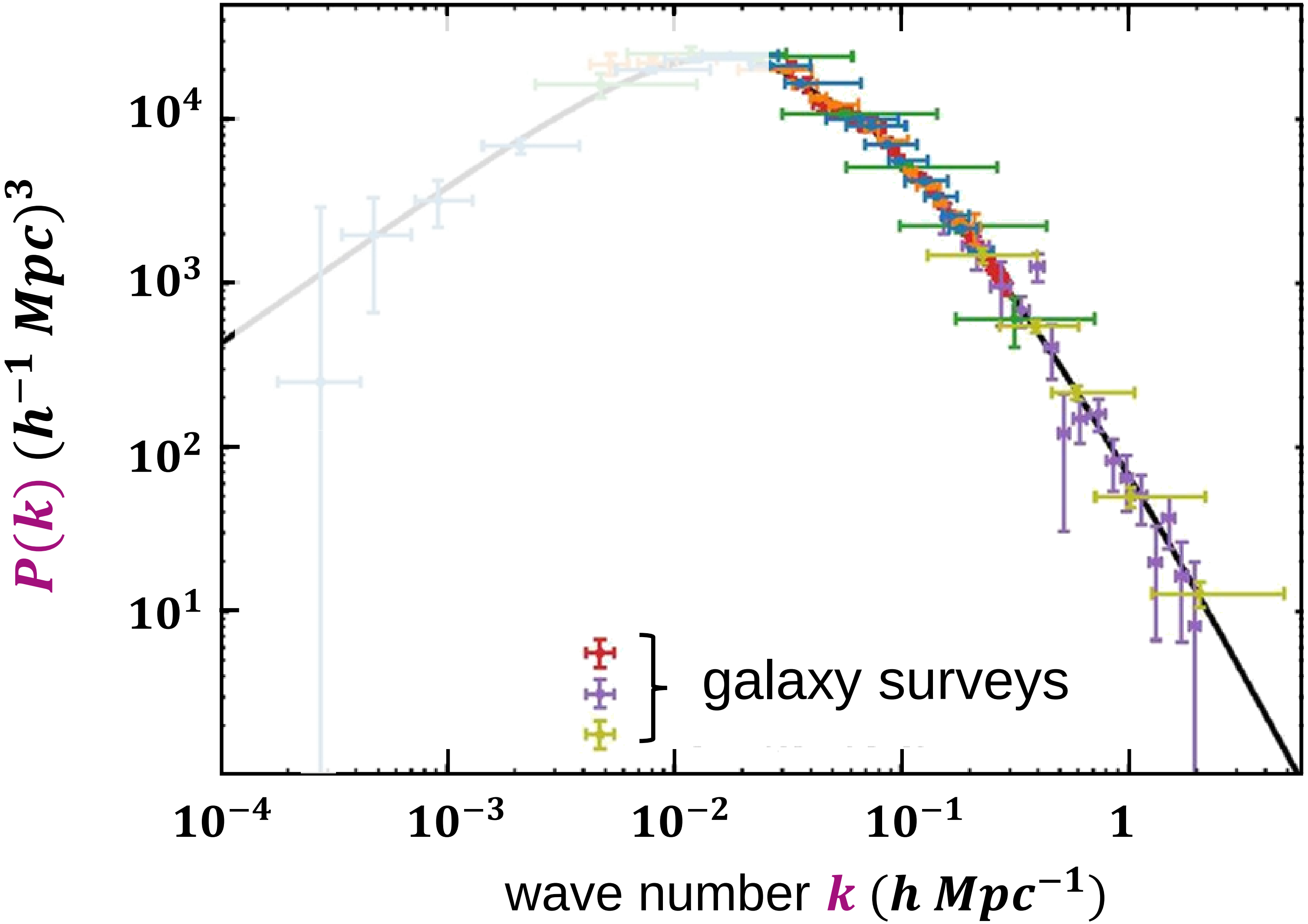
To Be Discussed Later



Observed matter power spectrum $P(k)$

- **Compilation of data from the present universe (LSS)**

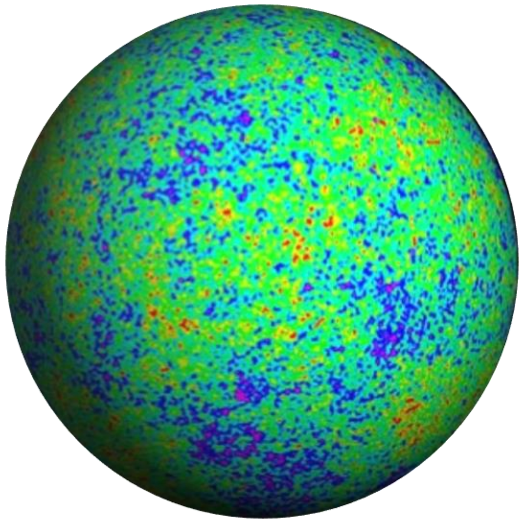




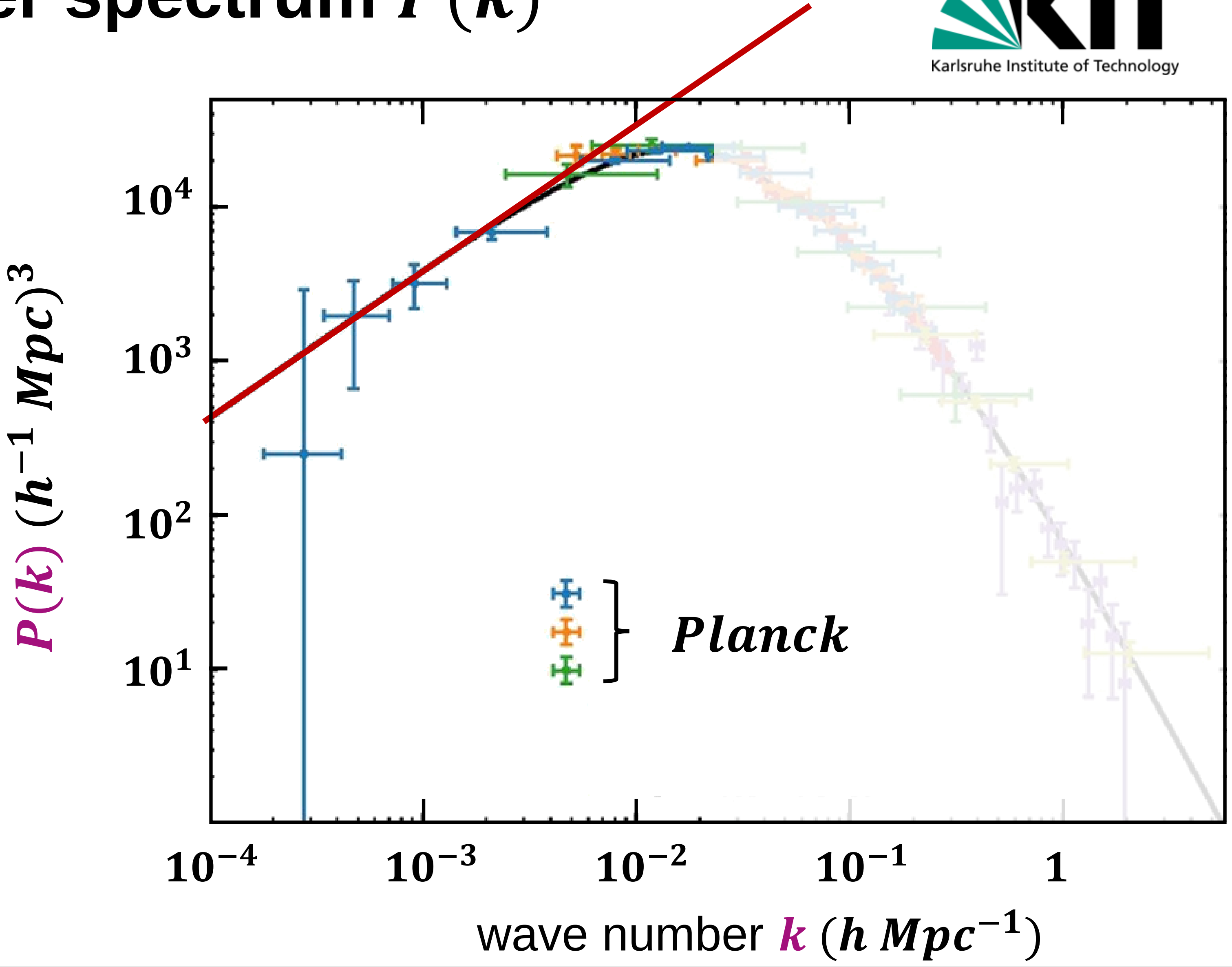
large k: small length scale r

Observed matter power spectrum $P(k)$

- Compilation of data from early universe (CMB)



small k: huge length scale r

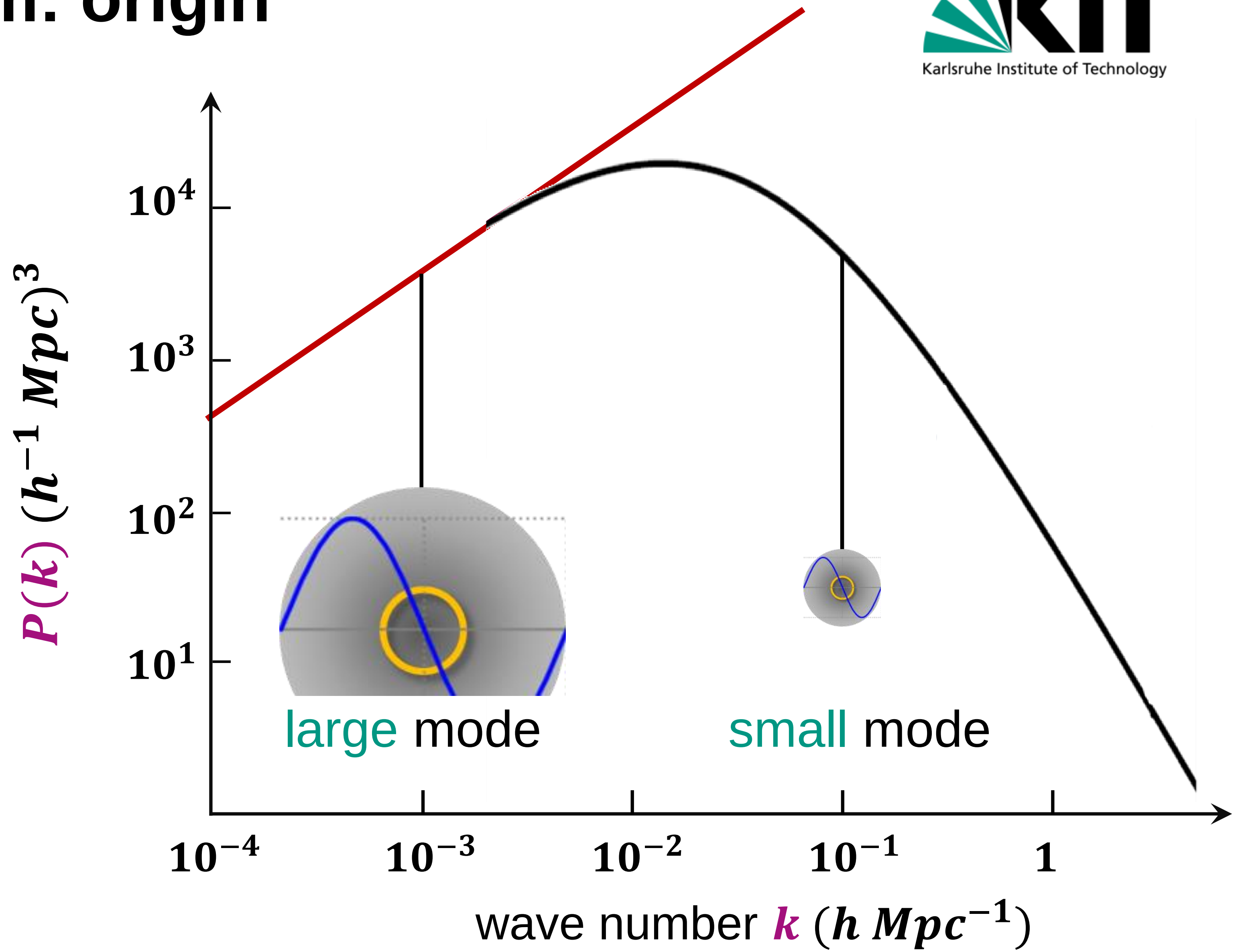


Matter power spectrum: origin

- **Density contrast** & ***D*ark *M*atter (*DM*) modes**

- $P(k)$ arises from the superposition of a large number of DM – **modes**, which grow **independently** from each other

- **single DM – mode**:
  wave number k
  length scale r



Matter power spectrum: discussion

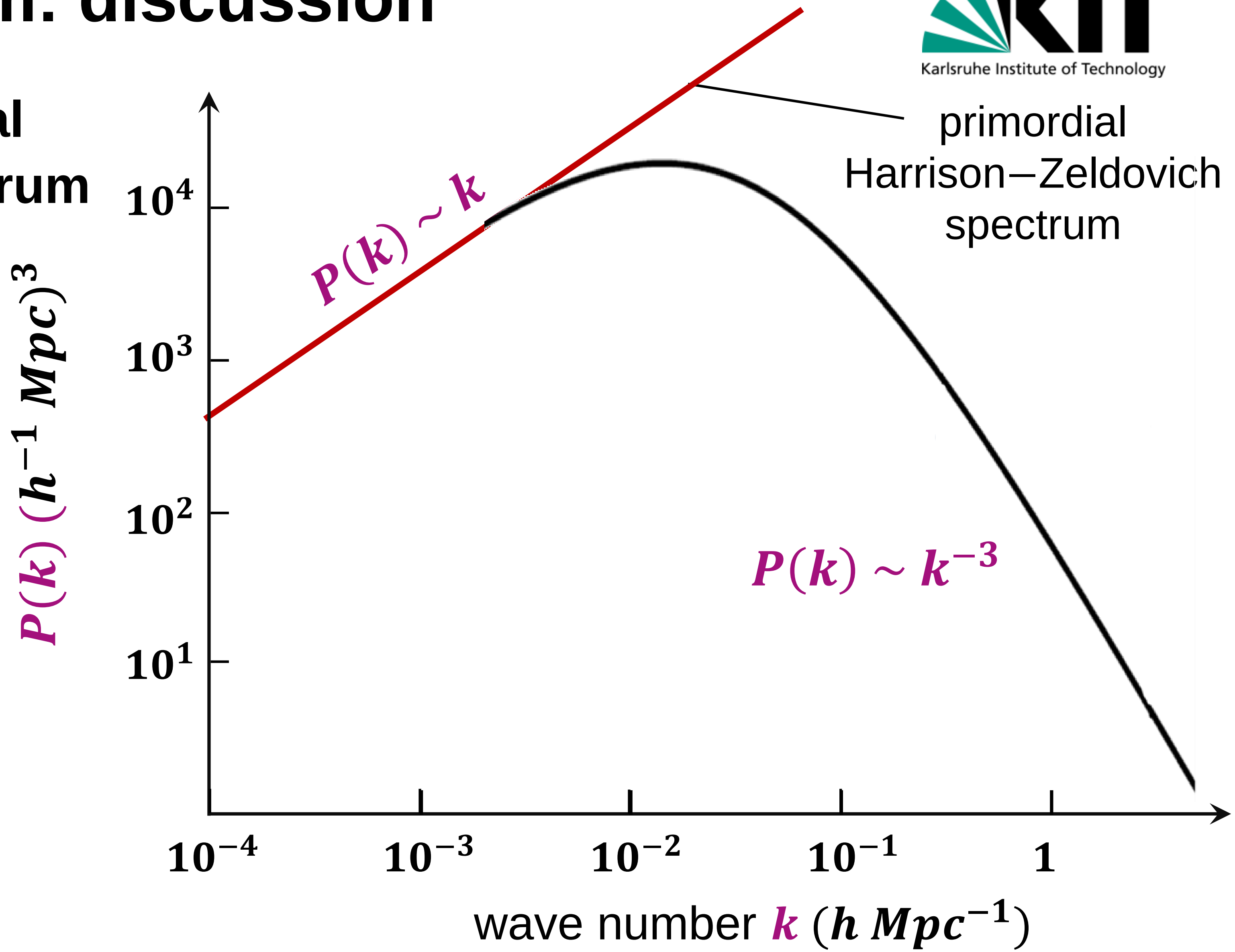
- $\boldsymbol{CMB}$: **we see the primordial Harrison–Zeldovich spectrum**

largest scales:
´scale–invariant´ spectrum with **linear growth** of density contrast

$$P(k) \sim k$$

smaller scales:
strong reduction in power with

$$P(k) \sim k^{-3}$$



Matter power spectrum: discussion

- $\boldsymbol{CMB}$: **we see the primordial Harrison–Zeldovich spectrum**

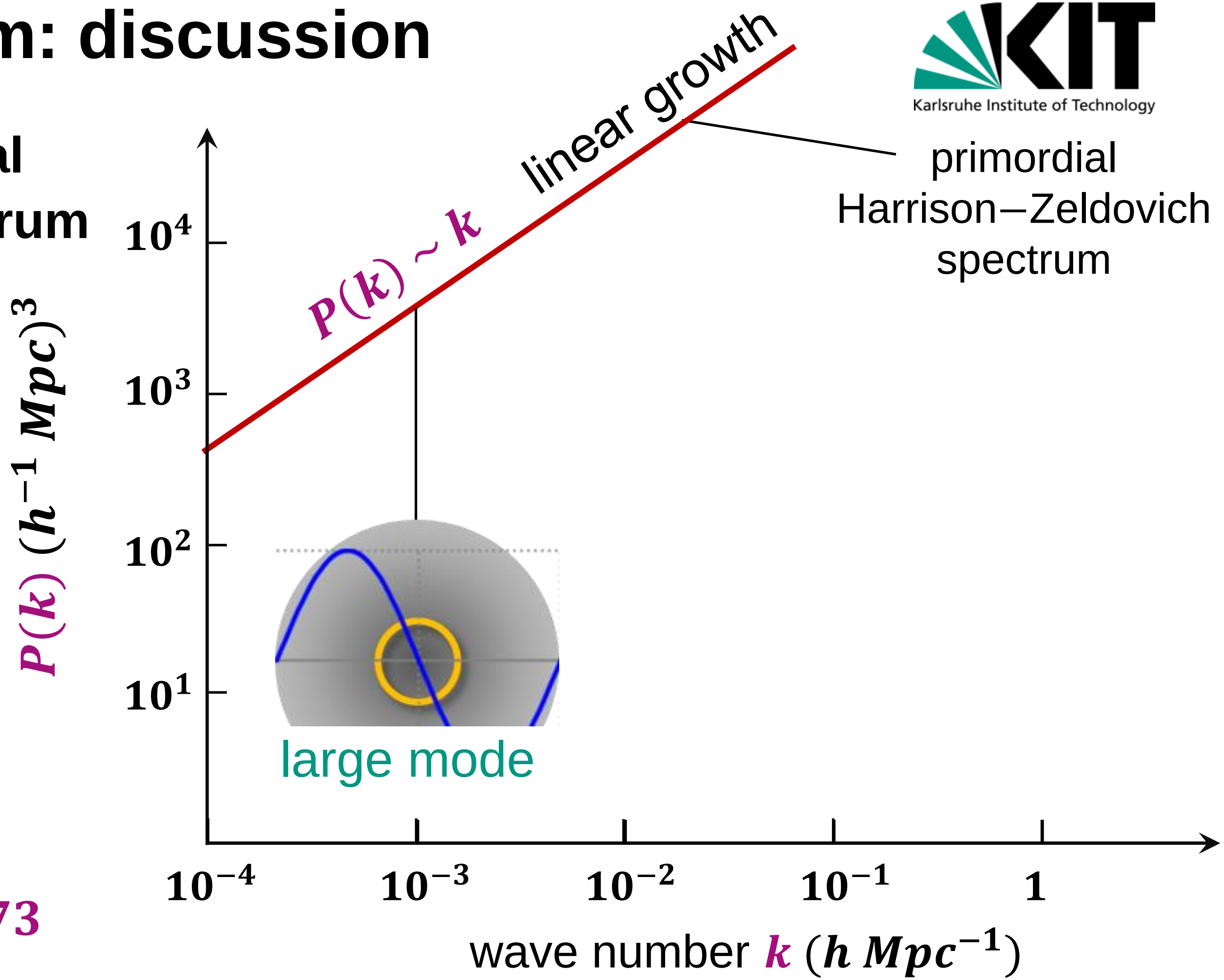
\- expectation from **inflation**:

$\boldsymbol{DM}$ – **modes** increased to a scale–invariant Harrison–Zeldovich power spectrum $\boldsymbol{k^n}$

$\boldsymbol{n}$ = (scalar) spectral index

theory: $\boldsymbol{n = 0.92 \ldots 0.98}$

$\boldsymbol{Planck}$: $\boldsymbol{n = 0.9603 \pm 0.0073}$

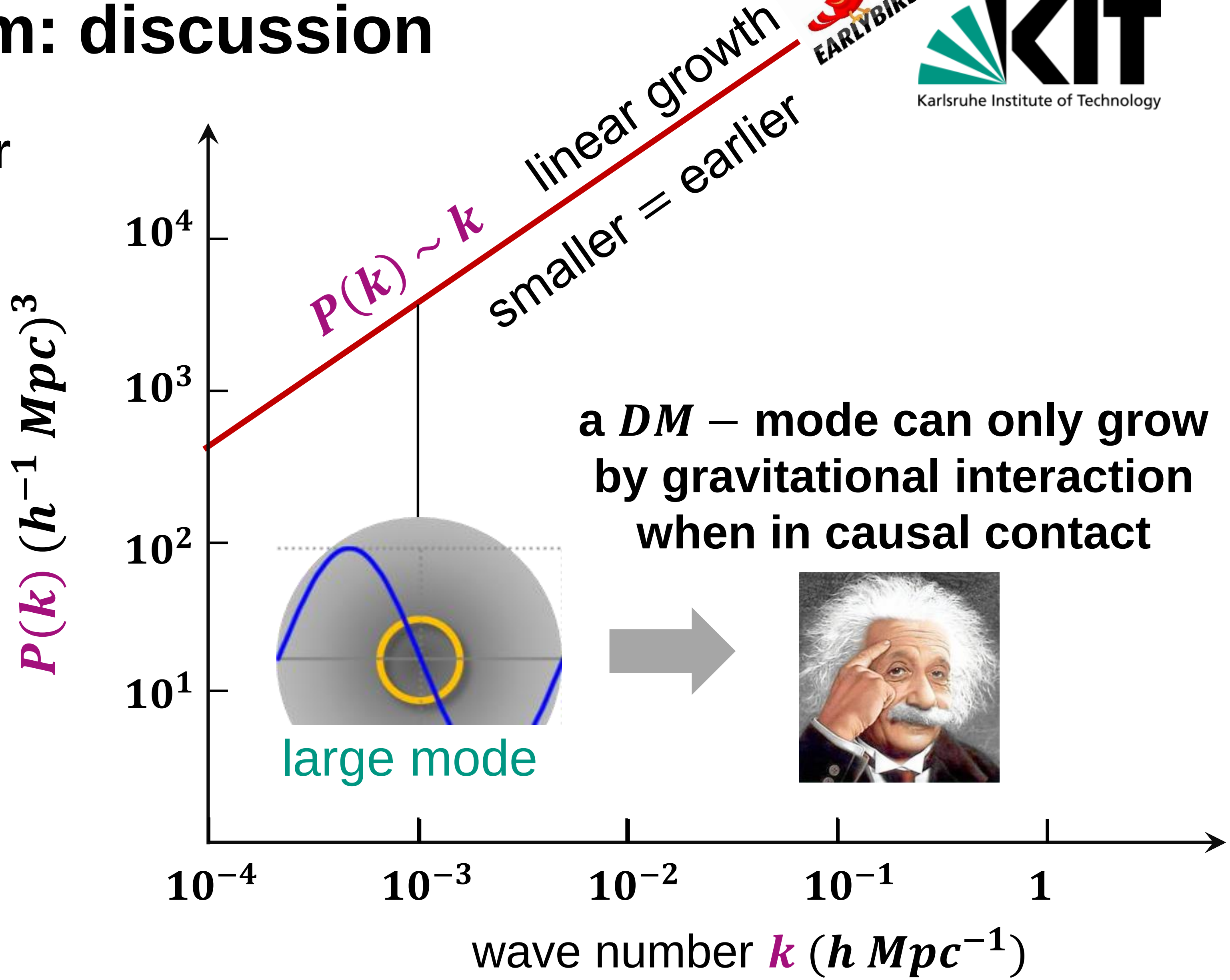


Matter power spectrum: discussion

- **Density contrast of smaller modes increases**

- DM – modes need to be in causal contact to start gravitational interaction against cosmological expansion
  then: linear growth regime

smaller DM – modes:
earlier in causal contact
& longer in linear growth



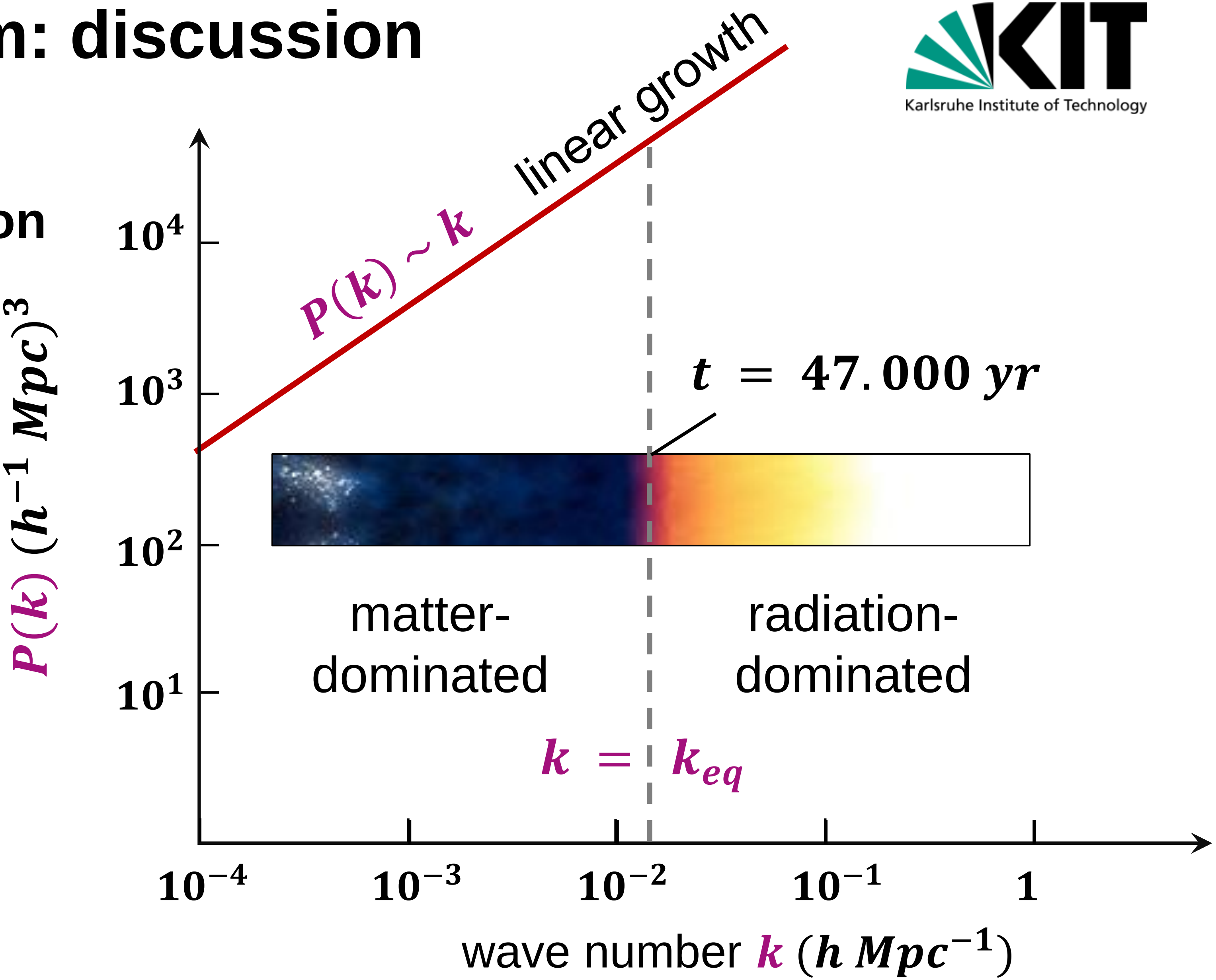
a DM – mode can only grow by gravitational interaction when in causal contact

Matter power spectrum: discussion

- **Very small modes: causal contact in the early radiation dominated epoch**

- now we investigate what happens to very small DM – modes that came into causal contact at $t < 47.000\ yr$ in radiation dominated epoch

very small DM – modes:
during radiation dominated epoch: no linear growth (**stopped**)

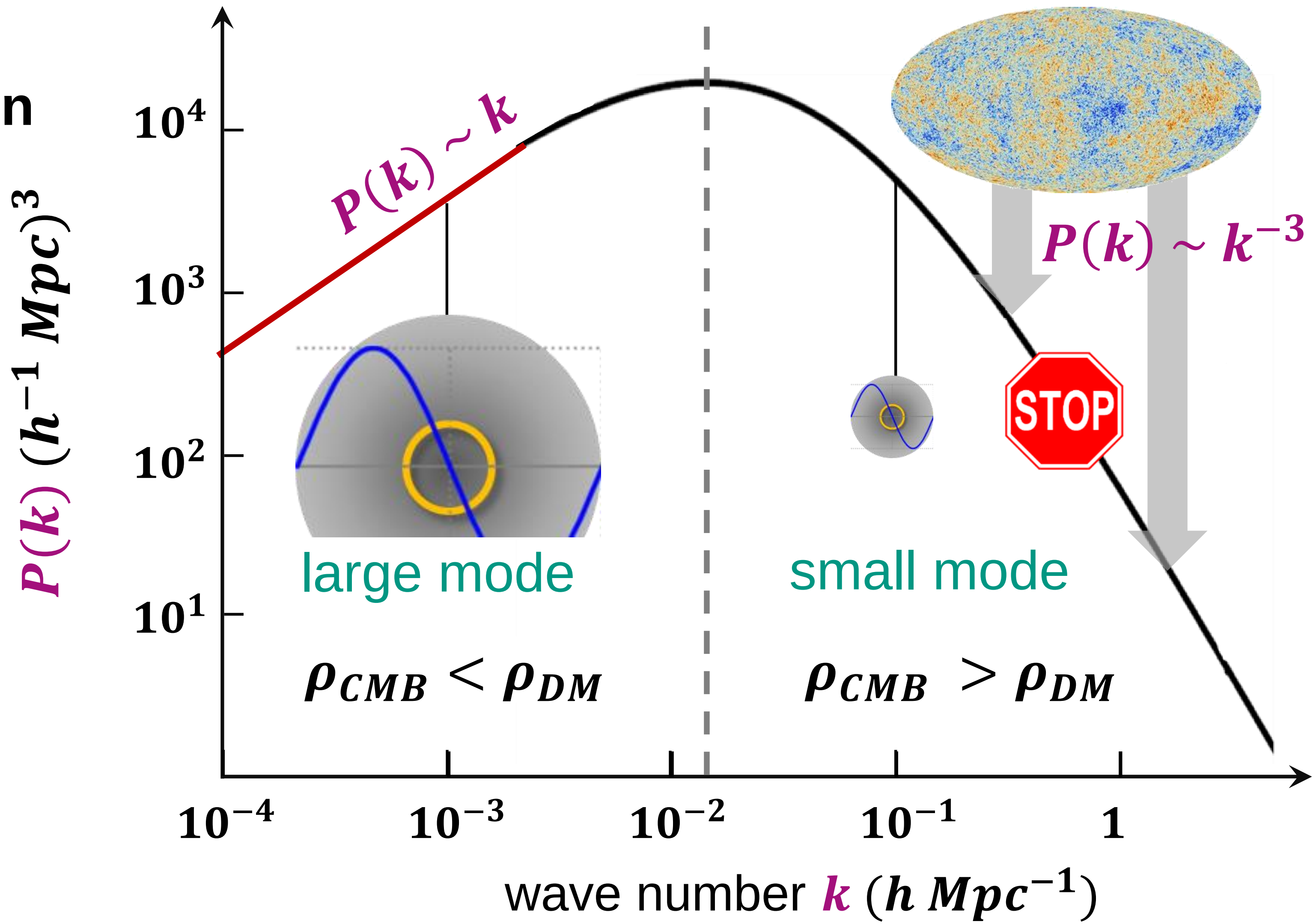


Matter power spectrum: discussion

- **The size of a DM – mode determes its causal horizon & density contrast**

- relativistic photons inhibit the growth of the density contrast of small DM – modes during $t < 47.000\ yr$

very small DM – modes:
during radiation dominated epoch: **photons are diffusing!**

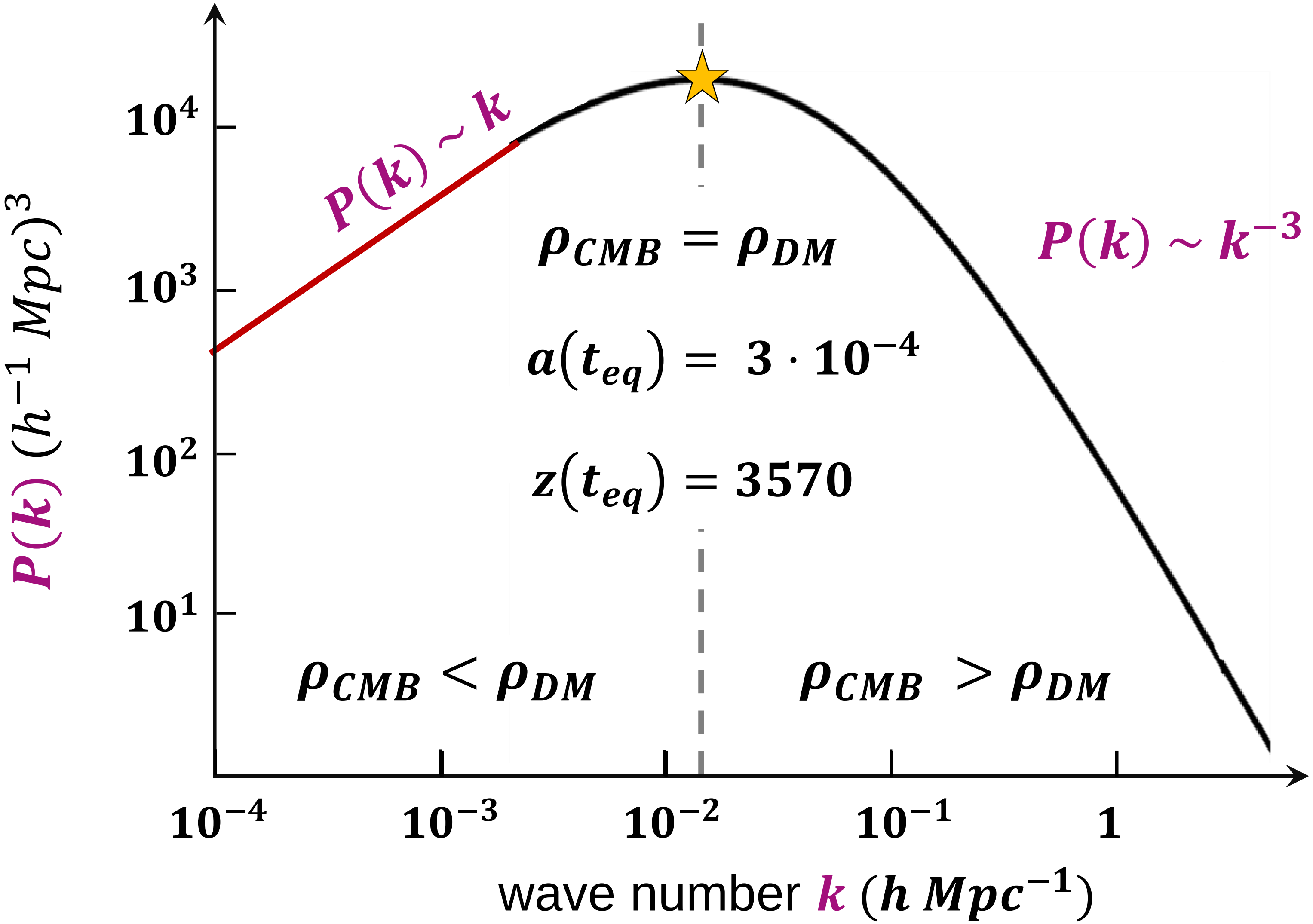


Matter power spectrum: discussion

- **The power spectrum $P(k)$ is maximal at $k = k_{eq}$**

- ´optimum´ case for $P(k)$ is reached for all DM – modes $k = k_{eq}$ which come into **causal contact** at $t = t_{eq}$

´optimum´ DM – mode:
wavelength $\lambda_{eq} = 350\ h^{-1} Mpc$



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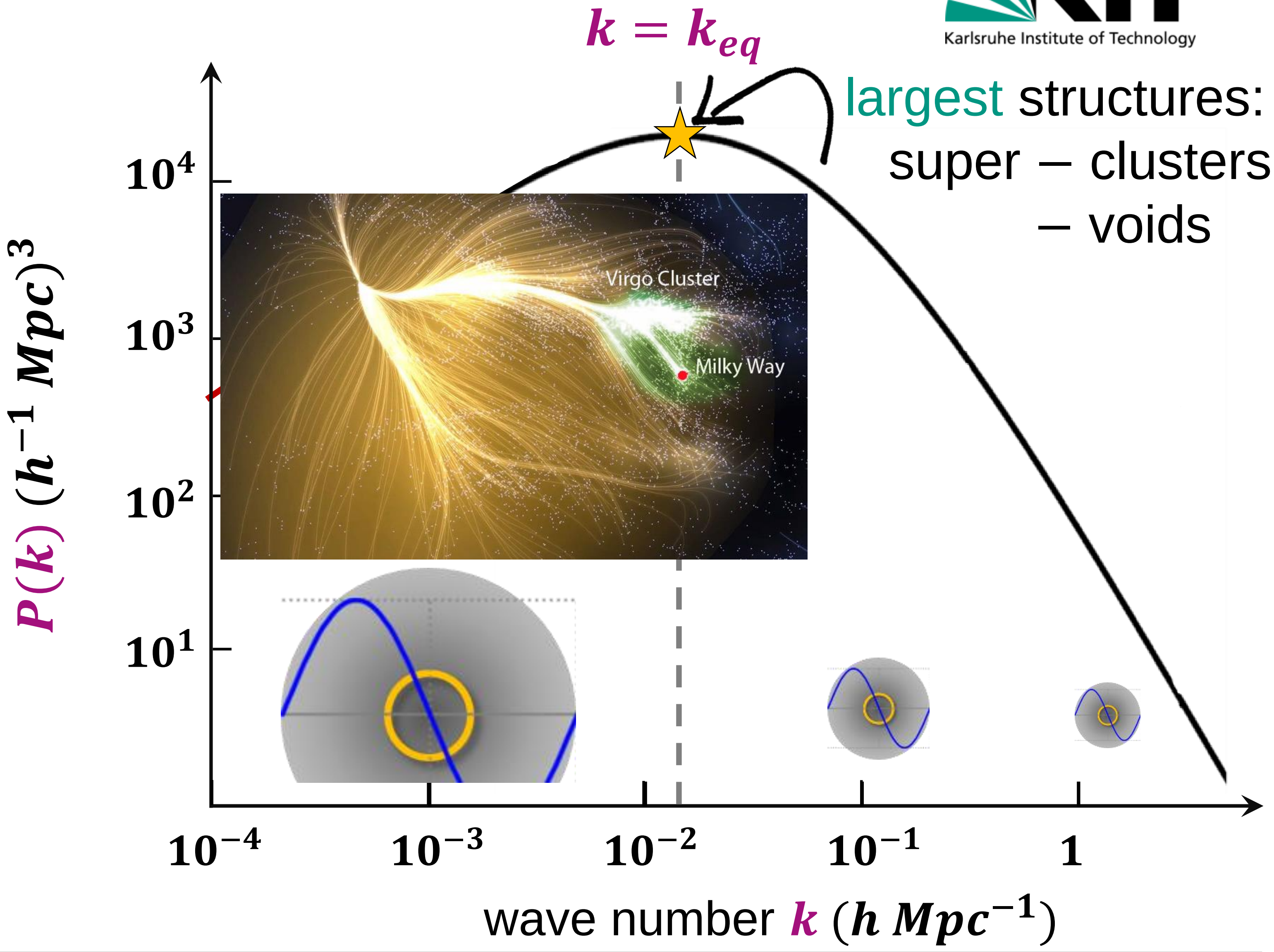

Comparing CMB & LSS analyses

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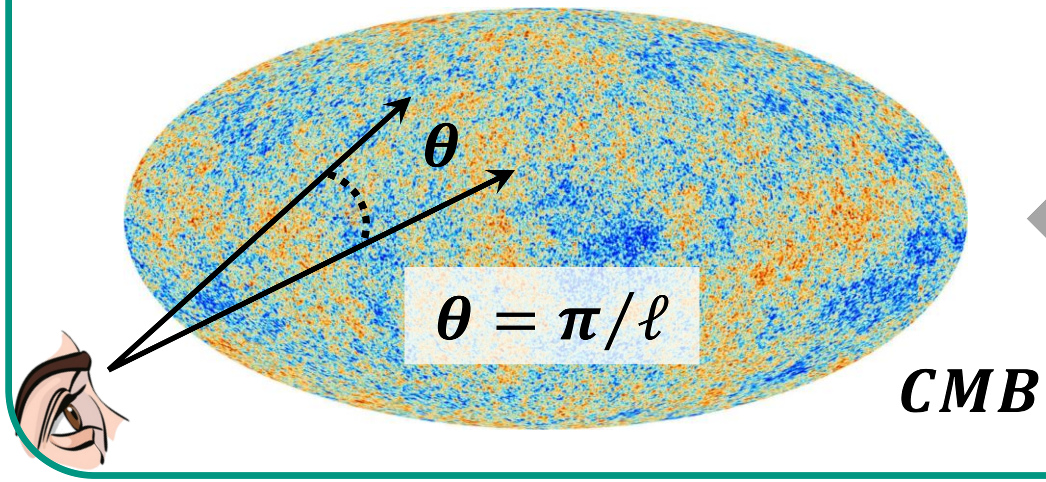
- **What can we learn from CMB multipoles and from matter power spectrum ?**

CMB – power spectrum: **coefficients C_ℓ**

$2 - dim.$: surface on a sphere

angle θ ⇔ multipole ℓ

$$C(\theta) = \frac{1}{4\pi} \sum (2\ell + 1) \cdot C_\ell \cdot P_\ell(cos\theta)$$



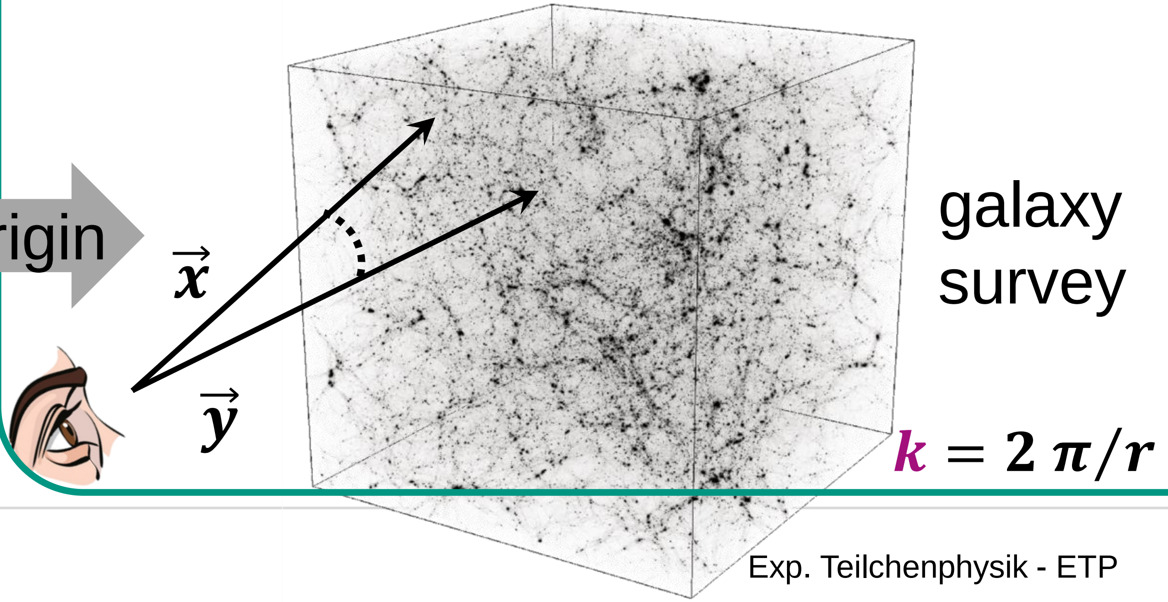
same origin

matter density contrast: **power P_k**

$3 - dim.$: volume within a sphere

distance r ⇔ wave number k

$$\langle \delta^2 \rangle = \frac{1}{2\pi^2} \int k^2 \cdot P(k) \cdot dk$$



Comparing CMB & LSS analyses

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angle $\theta \Leftrightarrow$ multipole ℓ

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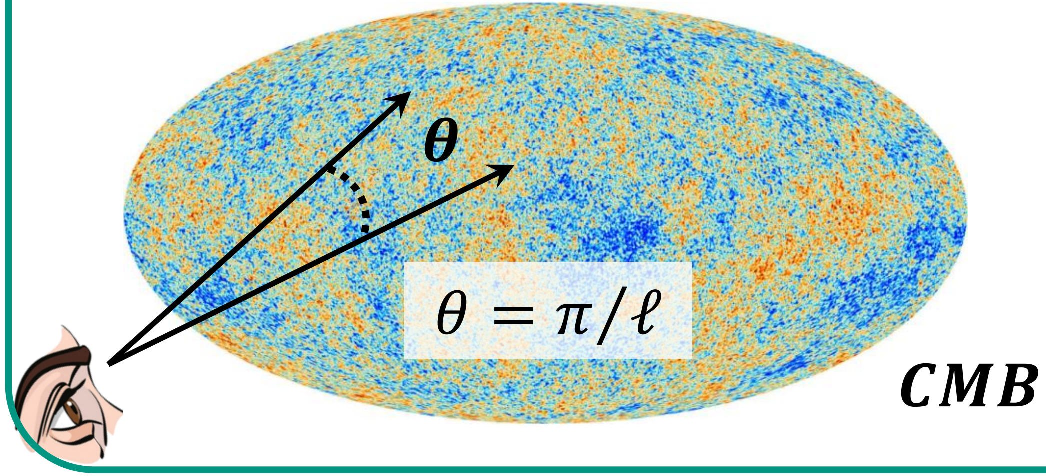
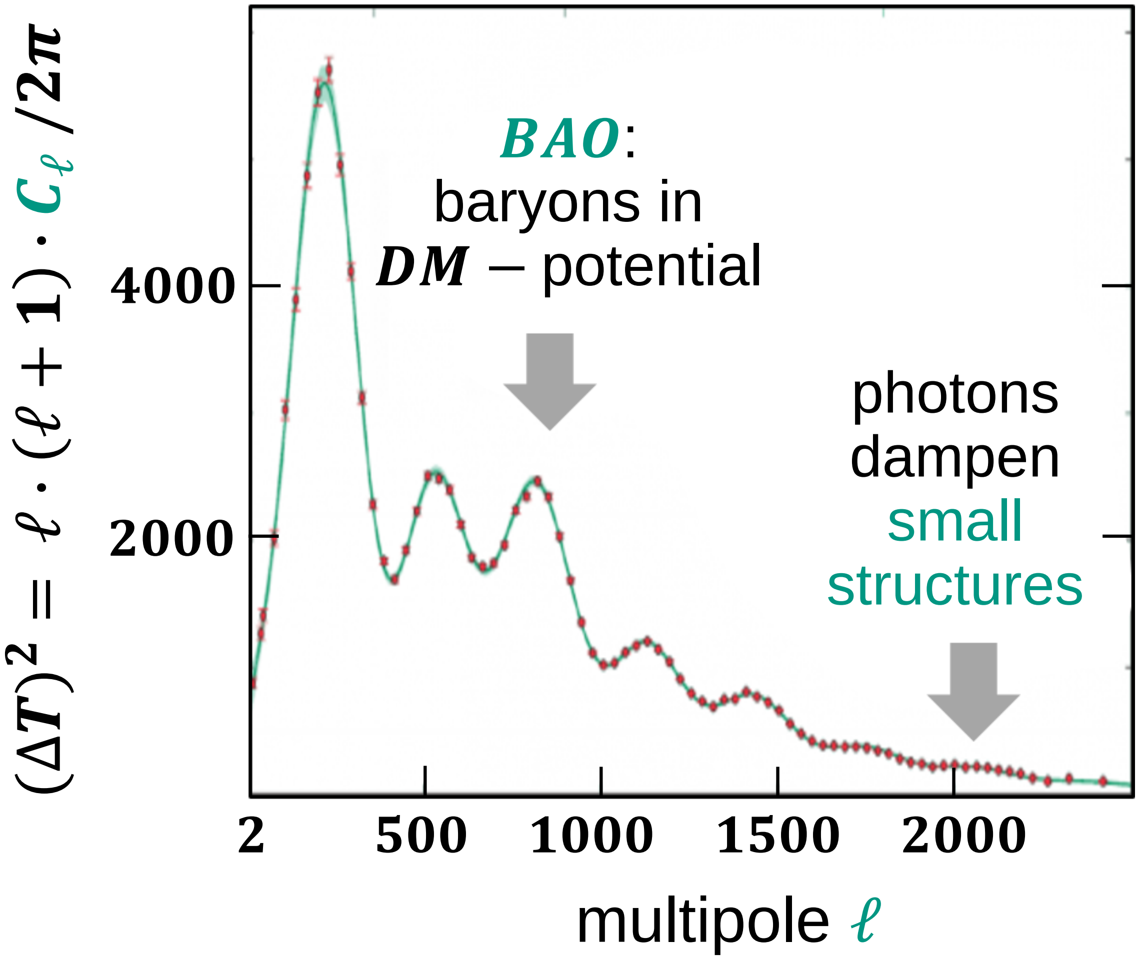




Comparing CMB & LSS analyses

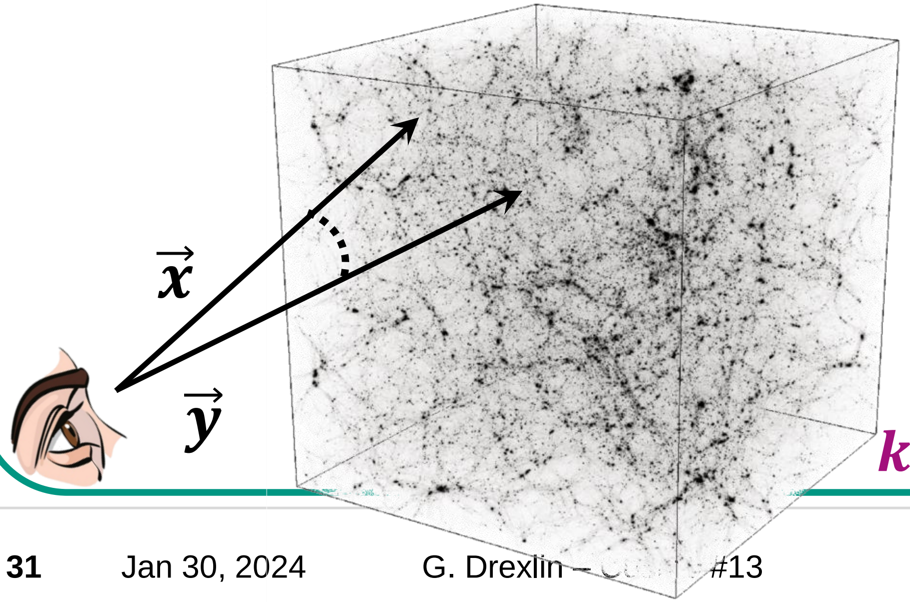
- **What can we learn from CMB multipoles and from matter power spectrum ?**

matter density contrast: **power P_k**

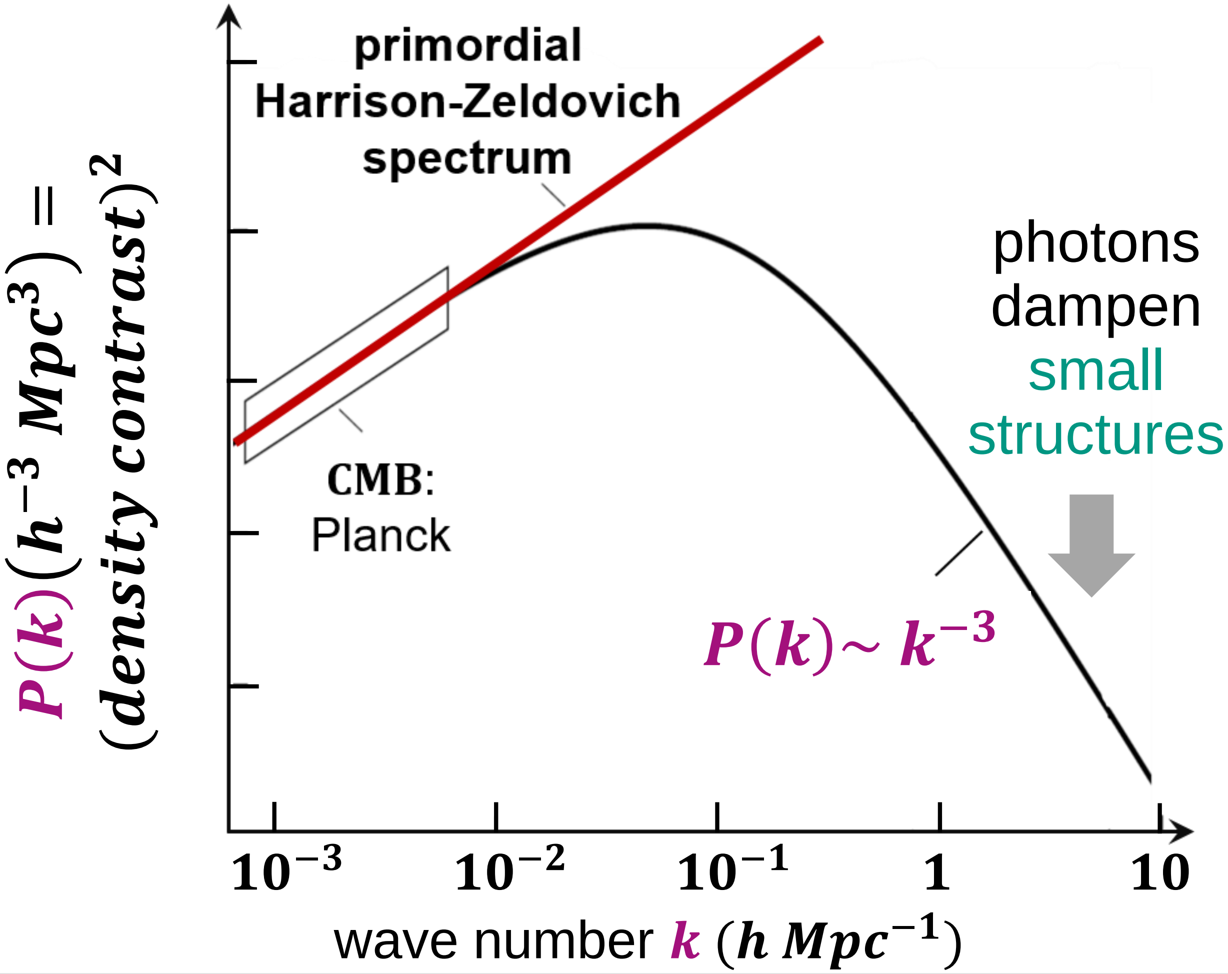
$3 - dim.$ **: volume within a sphere**

distance $r \Leftrightarrow$ wave number k

$$\langle \delta^2 \rangle = \frac{1}{2\pi^2} \int k^2 \cdot P(k) \cdot dk$$



$k = 2\,\pi / r$

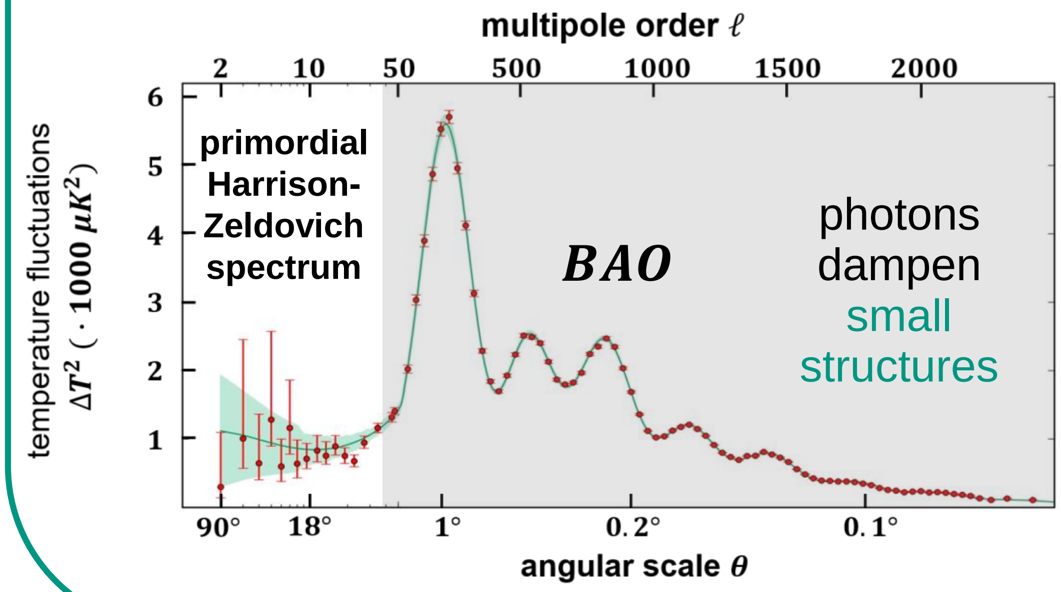


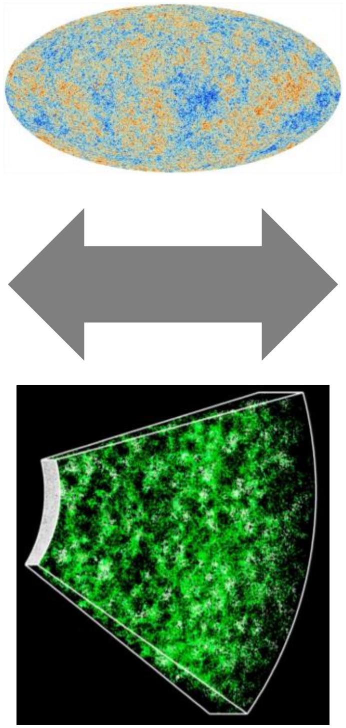
Comparing CMB & LSS analyses

- What we can learn from CMB multipoles & from matter power spectrum

CMB – power spectrum: C_ℓ

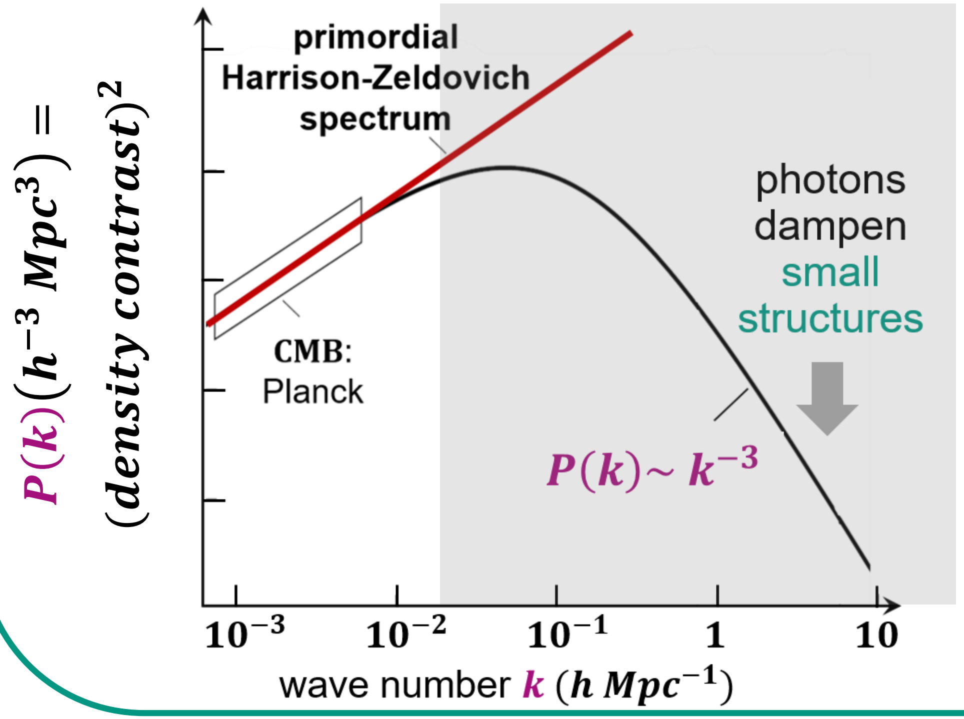
- C_ℓ: structure information on **surface of sphere** up to CMB decoupling





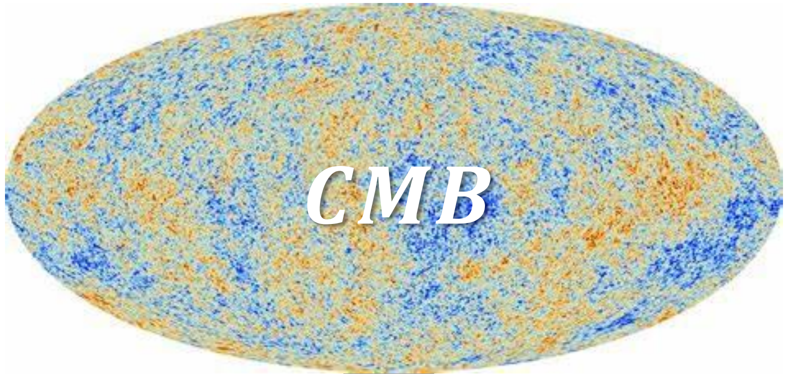
matter power spectrum: $P(k)$

- $P(k)$: structure information in **volume of sphere** up to present universe

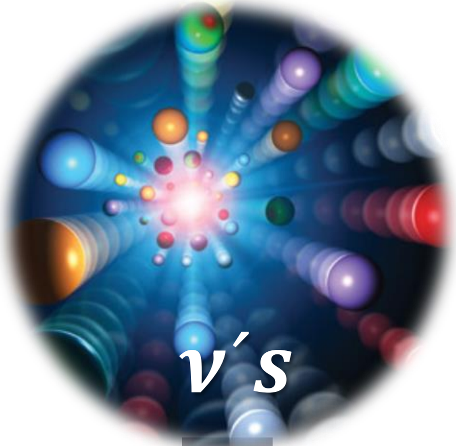


Matter power spectrum & relativistic particles

- $P(k)$ and the wash–out of structures at small scales due to photons & ν´s



(dark) matter:
$P(k)$ reduced due to gravitational interaction

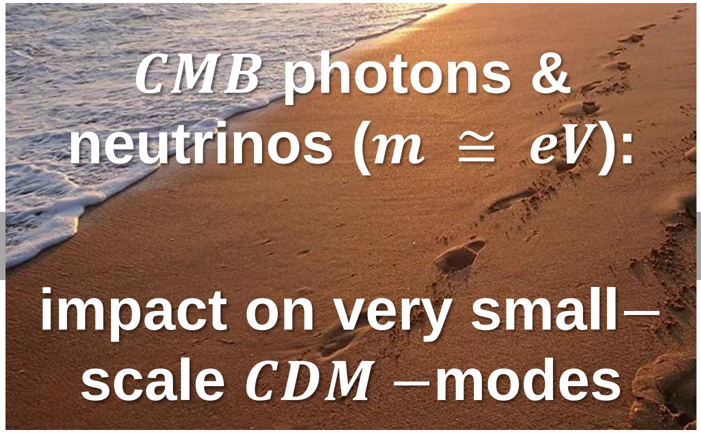


relativistic **BAO &**

CMB – photons: **Silk–damping**

neutrinos with **´freestreaming´**

eV – masses: over Gpc



wash–out of **small–scale** structures

Matter power spectrum & scales of structures

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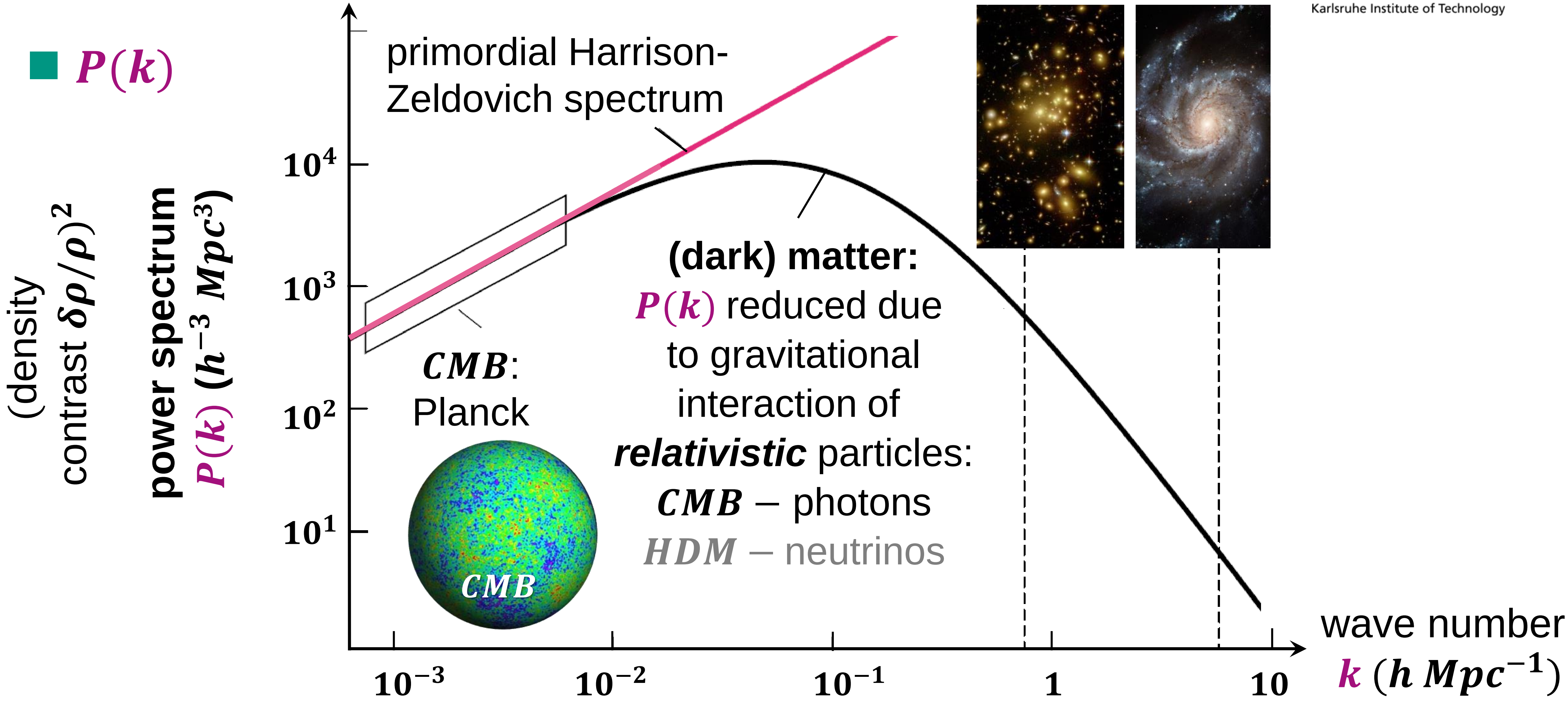
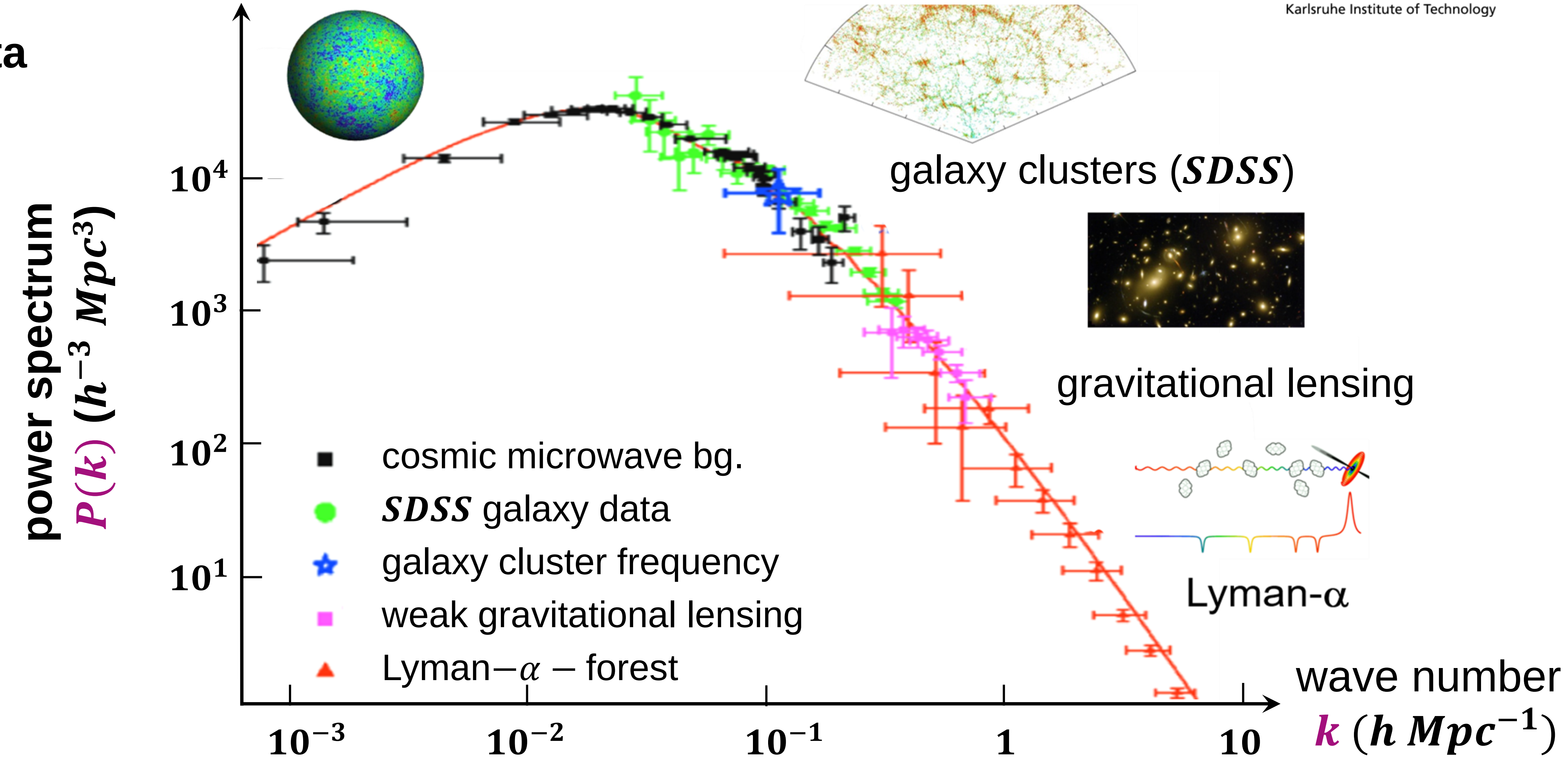
- $P(k)$



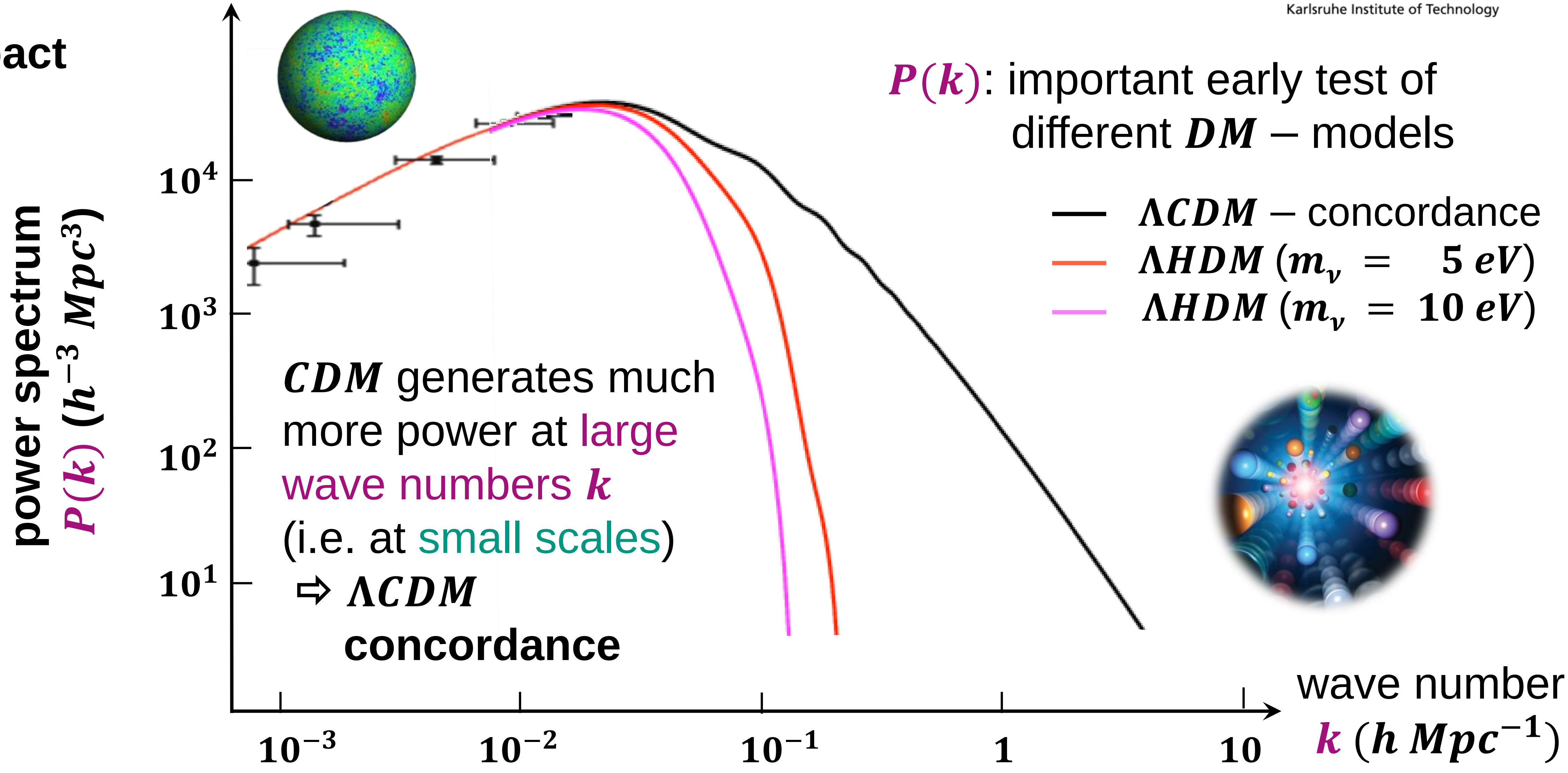
Matter power spectrum & measurements

- **data**



Matter power spectrum: CDM vs. HDM only

- impact



$P(k)$: important early test of different DM – models

- ΛCDM – concordance
- ΛHDM $(m_\nu = 5\ eV)$
- ΛHDM $(m_\nu = 10\ eV)$

Matter power spectrum: $sub - eV$ limits for m_ν

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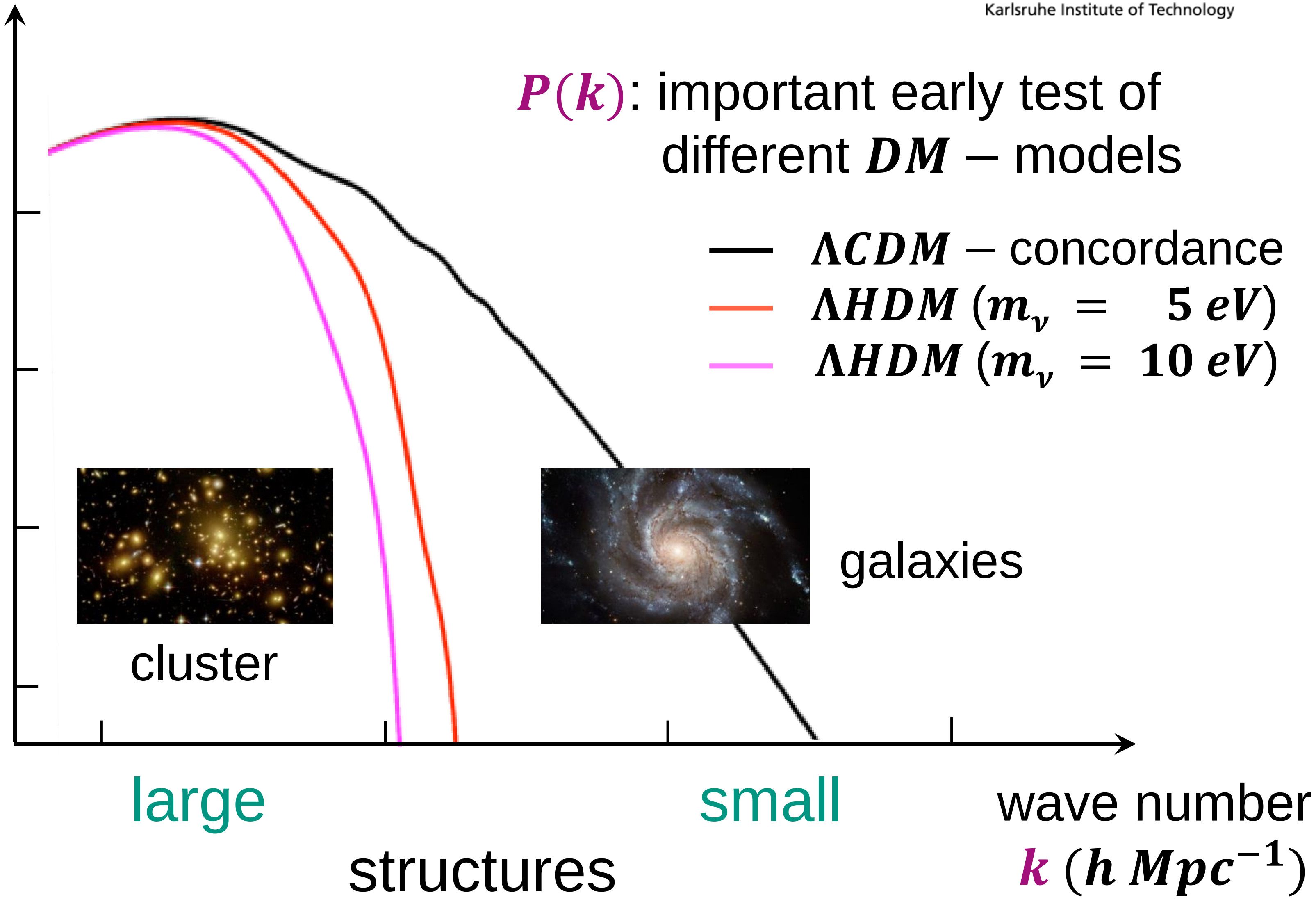
- **Cosmological limits for neutrino masses**

- interplay of LSS **data** from **cosmology** & **elementary particle physics**!

$P(k)$: important early test of different DM – models



Matter power spectrum: new conclusions ...

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- **Cosmology and the Standard Model**

- interplay of ***LSS*** **data** from **cosmology** & **elementary particle physics**!

Astronomy

‘Less clumpy’ universe may suggest existence of mysterious forces

Survey could mean there is a crucial component missing from so-called standard model of physics

[Joint analysis of Dark Energy Survey Year 3 data and CMB lensing from SPT and Planck. III. Combined cosmological constraints (aps.org)]

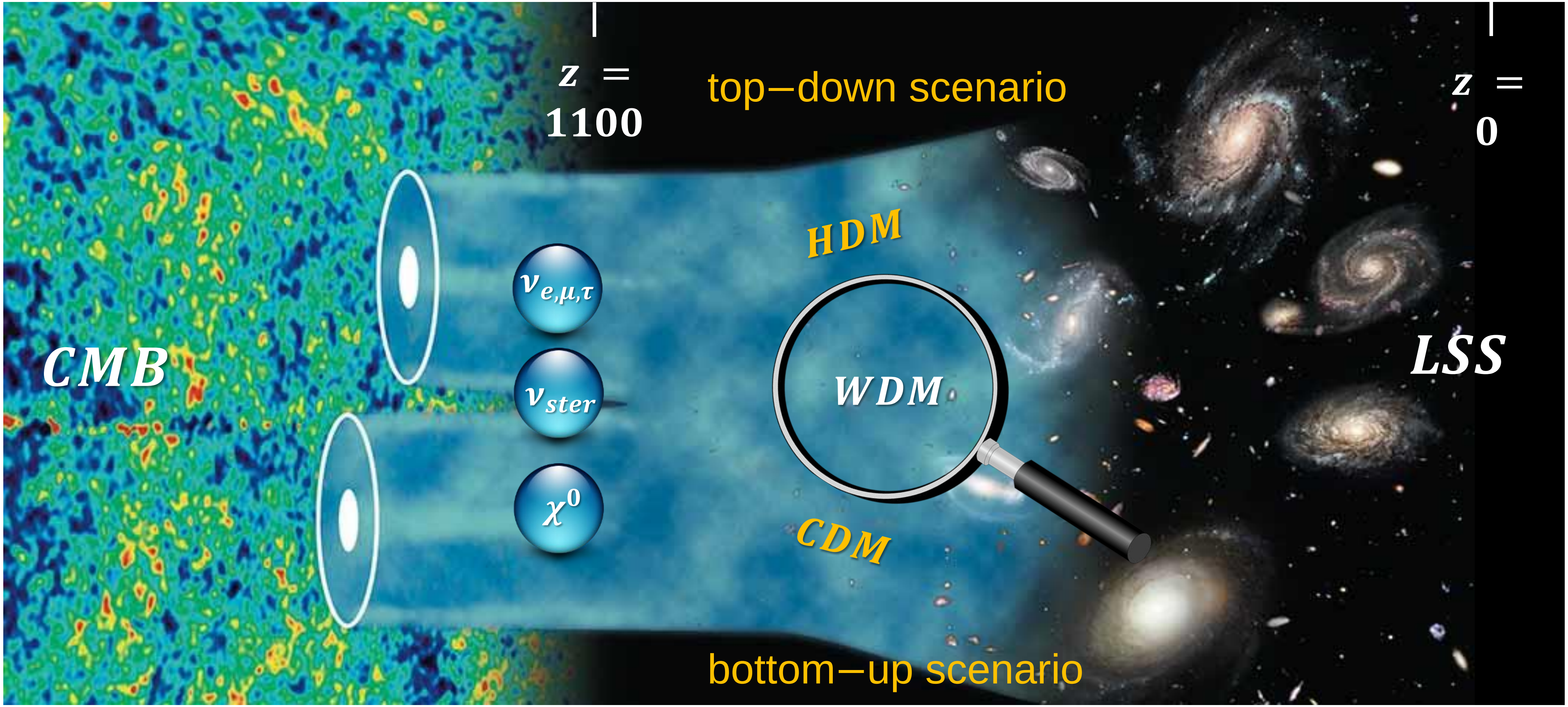
Hannah Devlin *Science correspondent*

@hannahdev

Tue 31 Jan 2023 18.22 GMT

One of the most precise surveys of the structure of the universe has suggested it is “less clumpy” than expected, in findings that could indicate the existence of mysterious forces at work.

A closer look at properties of CDM, WDM & HDM



thermal relics vs. non–thermal DM production

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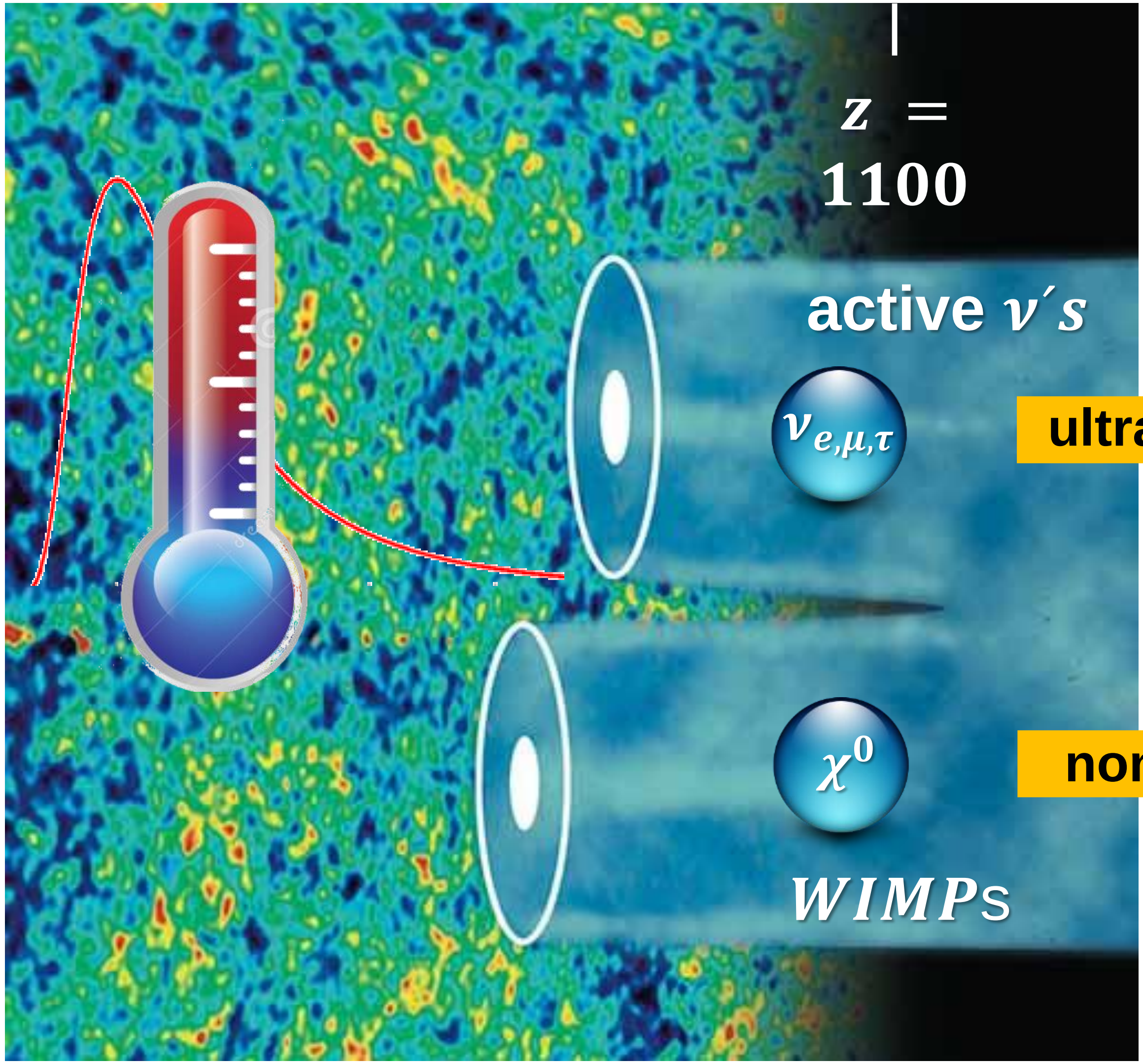
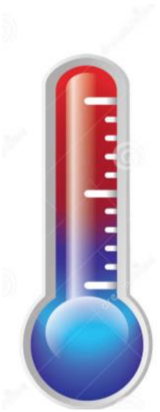


DM – particles as **thermal relics**: produced via thermal processes in **radiation bath** of early universe

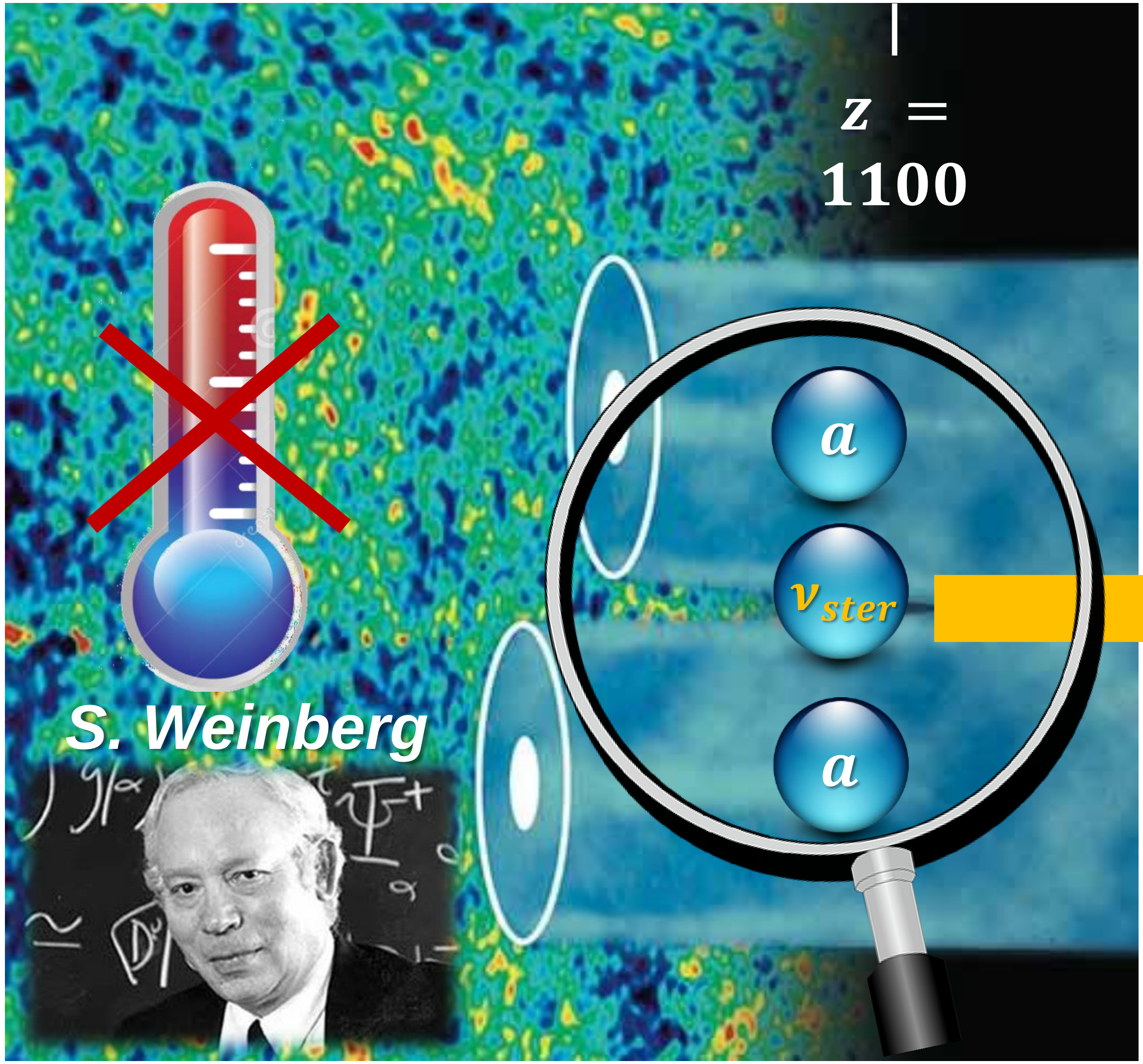
ultra–relativistic → HDM: $T_{fr} \gg m$



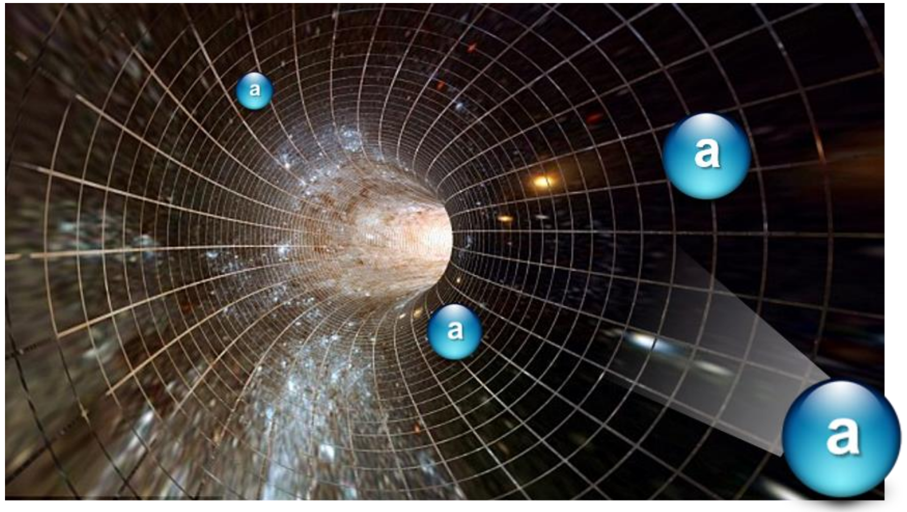
non–relativistic → CDM: $T_{fr} \ll m$

⇨ **characteristic freeze–out temperature T_{fr}**

thermal relics vs. non–thermal DM production



DM – particles as **non–thermal relics**:
produced via **symmetry breaking (*axion*)**
or due to **active–sterile ν – oscillations**



***WDM*:**
sterile ν

***´HDM, CDM´*:**
axion a

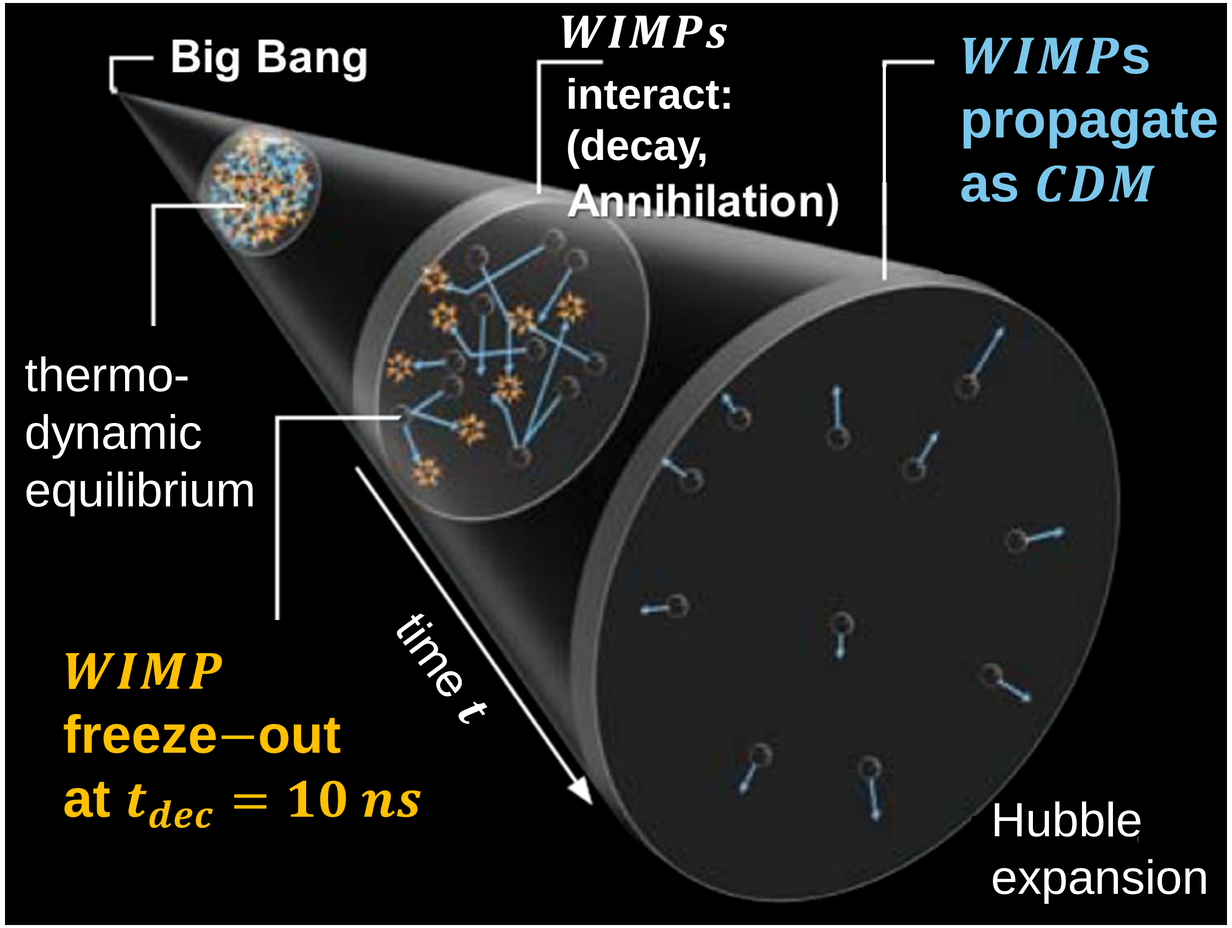
CDM: $WIMPs$ as generic thermal relics

■ Thermal production of CDM: $WIMPs$*

- $WIMPs$: massive, **non−baryonic** **thermal relics** left over from the Big Bang, which interact via **weak interaction** only (+ **gravity)**

- only **pair production** / **pair annihilation** of $WIMPs$ ($\equiv$ $Majorana$ particles with conserved $SUSY$ quantum number R_P)

- very **large mass** (TeV – scale):
  ⇨ huge phase space in case of decay



*Weakly Interacting Massive Particle

CDM: $WIMPs$ as generic thermal relics

Annihilation rate of $WIMPs$

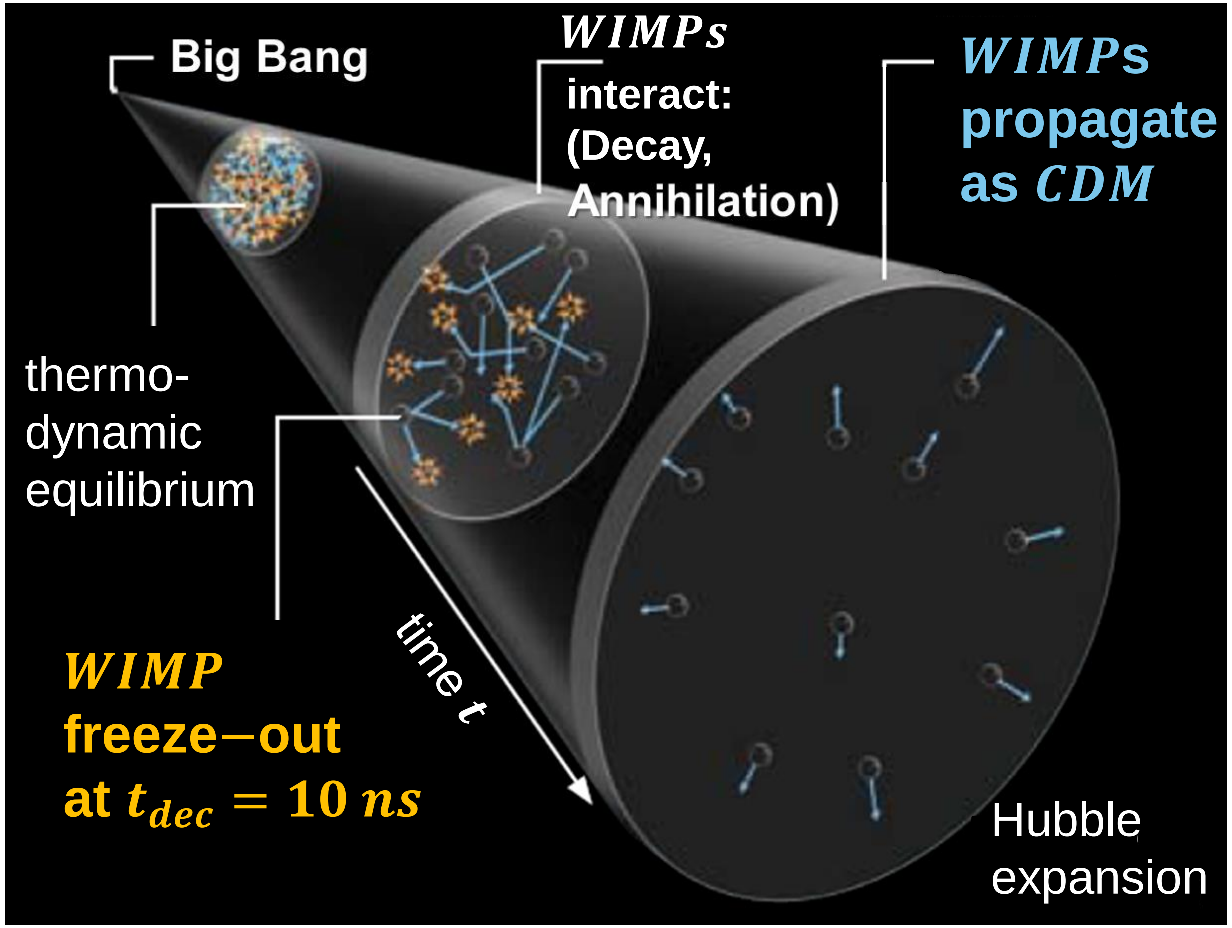
- $WIMPs$: annihilation rate given by typical **weak interaction strength**

$$\sigma_{ann} = \sigma_{weak}$$

- decoupling time t_{dec} from radiation bath is $t_{dec} \approx 10\ ns$

$$\sigma_{ann}(t_{dec}) = H(t_{dec})$$

- this will result in the so–called $WIMP$ **miracle** (to be discussed later)



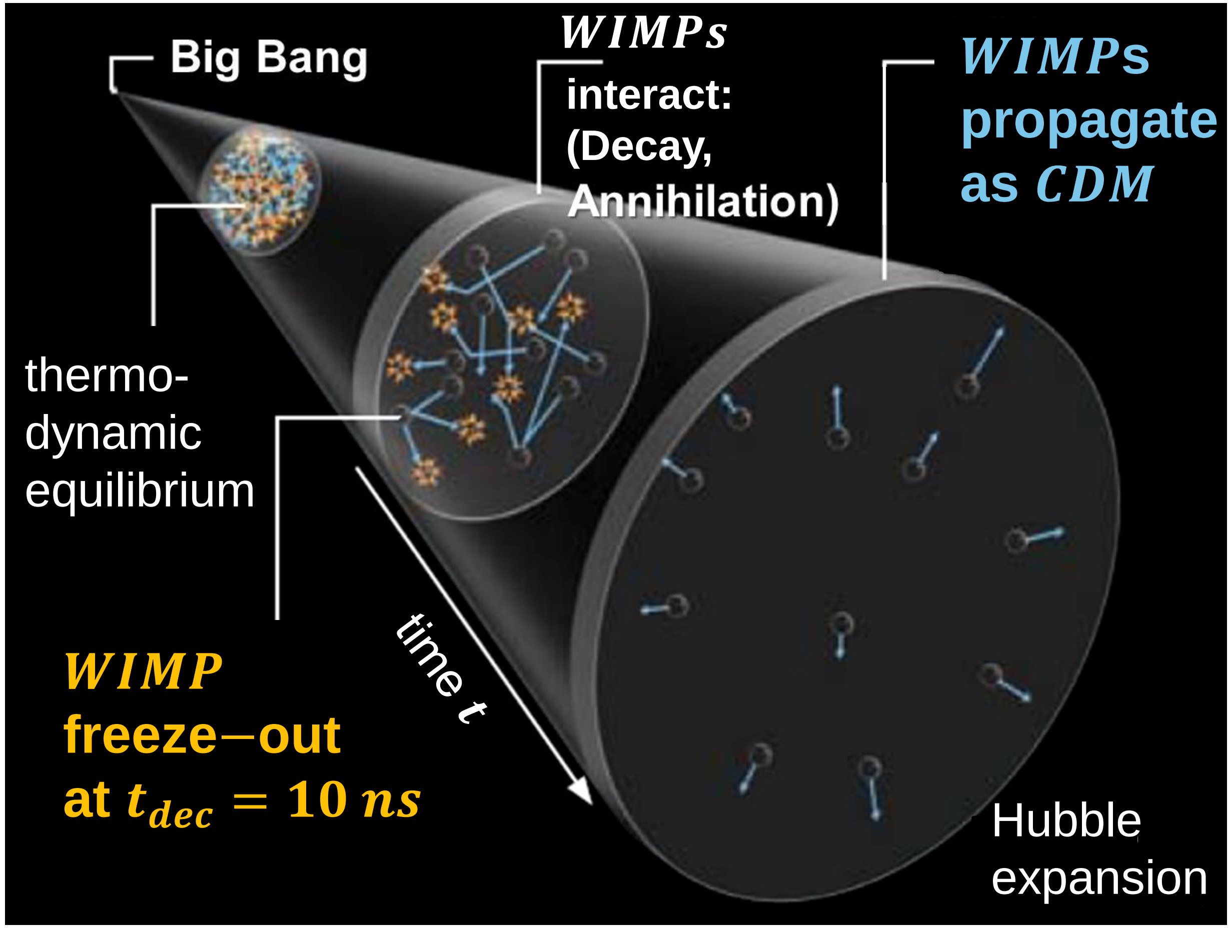
CDM: $WIMPs$ as generic thermal relics

Long–term ´slow´ propagation of $WIMPs$

- $WIMPs$: ***non–relativistic*** propagation with very **limited free–streaming range** due to kinematic relation:

$$E_{kin}(WIMP) \approx 0.05 \cdot m(WIMP)$$

- after decoupling & during propagation: χ^0 from $SUSY$ are stable over long, cosmological time scales due to intrinsic symmetry: R – parity R_P



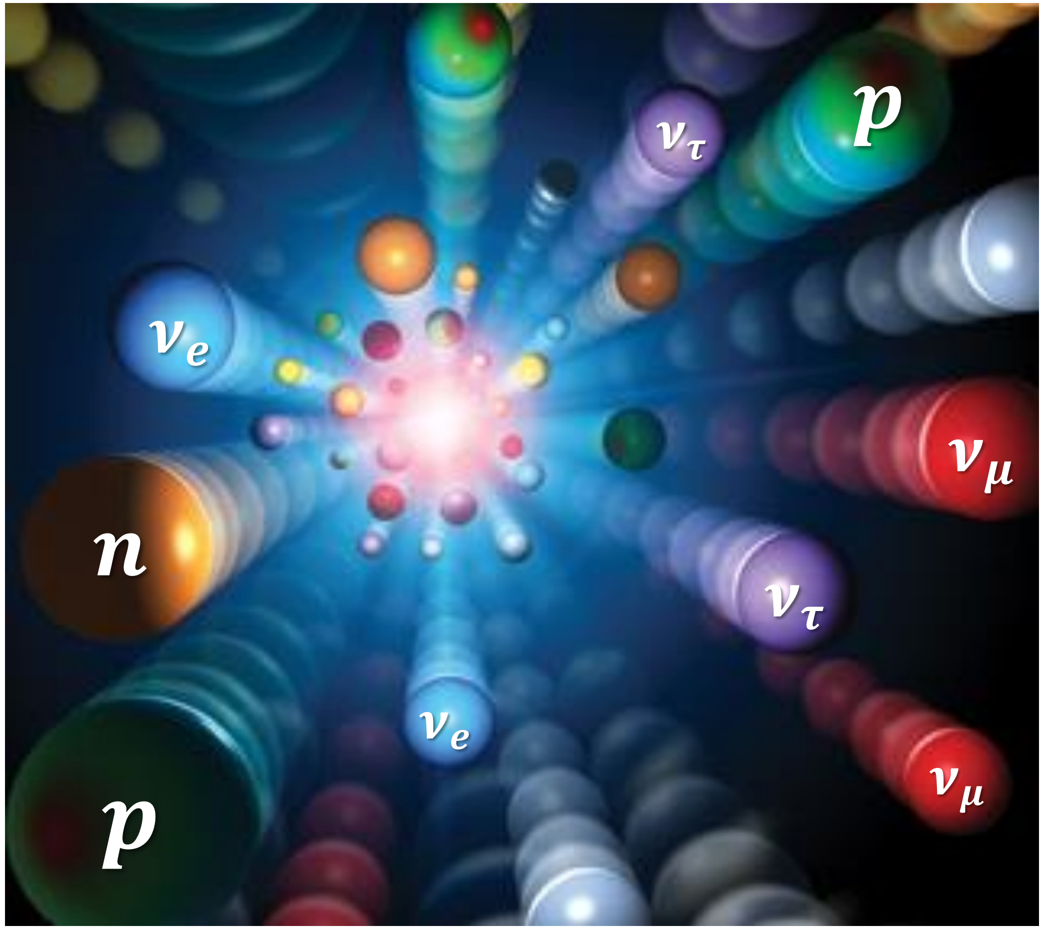
HDM: ν´s as generic light thermal relics

Hot dark matter: relativistic free−streaming

- ν´s: ***ultra−relativistic*** propagation over an exceedingly **large free−streaming range** of $> Gpc$ due to kinematic relation

$$E_{kin}(\nu) > 10^6 \cdot m(\nu)$$

- after decoupling: stable over long, cosmological time scales due to conserved **lepton number** L



HDM: ν´s as generic light thermal relics

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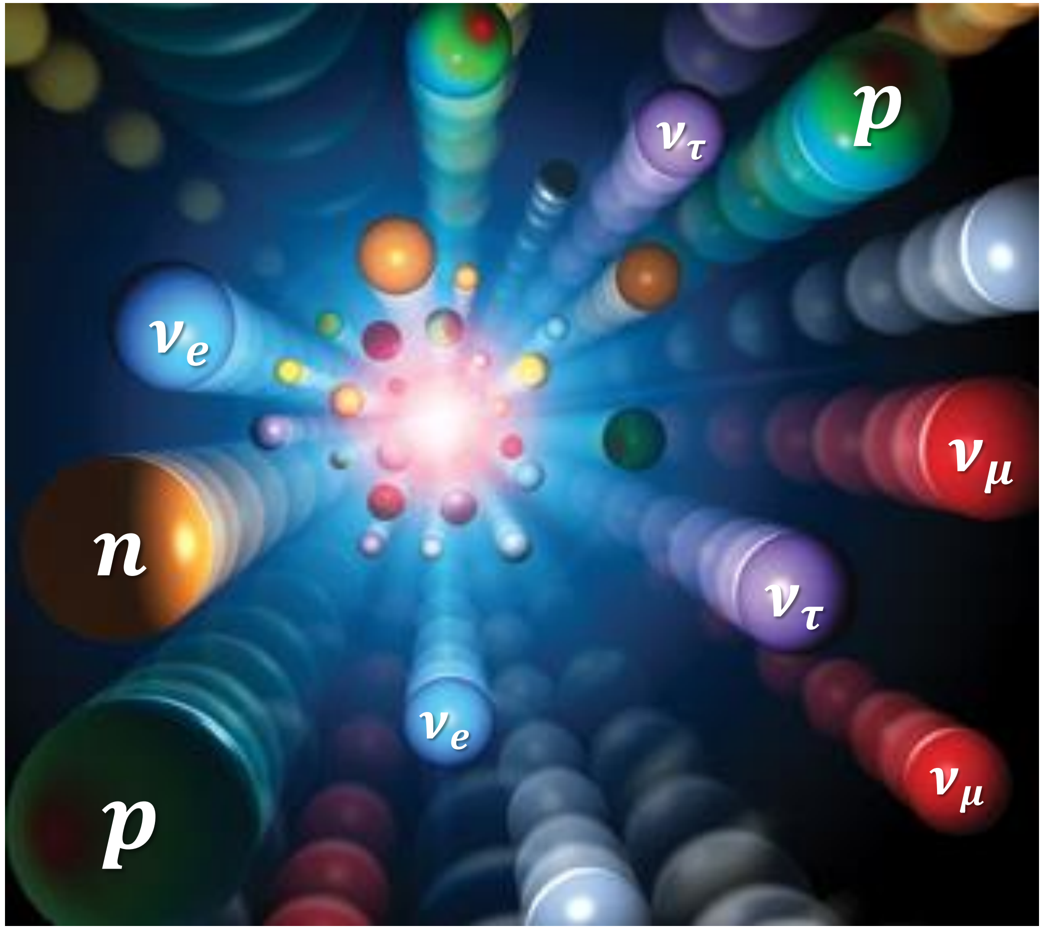
Hot dark matter: relativistic free–streaming

- ν´s: ***ultra–relativistic*** propagation is strongly suppressing annihilation in the early universe (also: there exist no lighter particles carrying lepton numbers in the SM)
  ⇨ weak interaction only (with baryonic matter)

- decoupling at $t \cong 1\ s$ and $T \cong 1\ MeV$
  (see also: $chap.\,3.1$ on Big Bang Nucleosynthesis, lecture #5)



Dark Matter: hot, warm or cold $(HDM/WDM/CDM)$

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- Generic particle models for cosmological $DM-$ density $\rho_{DM} \cong 1\,GeV/m^3$

Hot Dark Matter	Warm Dark Matter	Cold Dark Matter
particle candiate: ***active*** neutrinos $\nu_{e,\mu,\tau}$ $m \sim 0.05 \dots 0.8\ eV$	particle candiate: ***sterile*** neutrinos ν_s $m \sim 1 \dots 20\ keV$	particle candiate: ***SUSY*** neutralinos χ^0 $m \sim 0.1 \dots 10\ TeV$
number density: N(active): $339/cm^3$ decoupling: $T_{fr} = 2-3\ MeV$ $T_{fr}/m \sim 10^6 \dots 10^7$	number density: N(sterile): $< 1/cm^3$ decoupling: no thermal process, but via ν –oscillations	number density: $N(\chi^0)$: $< 10^{-9}/cm^3$ decoupling: $T_{fr} = GeV \dots TeV$ $T_{fr}/m \sim 1/20$
impact on LSS: wash-out of structure on scales $\lambda \le 1\ Gpc$	impact on LSS: wash-out of structure on scale $\lambda < 0.1\ Mpc$	impact on LSS: wash-out of structure on scales $\lambda < 0.1\ pc$

Dark Matter: hot, warm or cold $(HDM/WDM/CDM)$

- Generic particle models for cosmological $DM-$ density $\rho_{DM} \cong 1\,GeV/m^3$

Hot Dark Matter

particle candiate:

active neutrinos $\nu_{e,\mu,\tau}$

$m \sim 0.05 \ldots 0.8\ eV$

impact on LSS:

wash-out of structure

on scales $\lambda\ \leq\ 1\,Gpc$

Warm Dark Matter

particle candiate:

sterile neutrinos ν_s

$m \sim 1 \ldots 20\ keV$

impact on LSS:

wash-out of structure

on scale $\lambda <\ 0.1\,Mpc$

Cold Dark Matter

particle candiate:

SUSY neutralinos χ^0

$m \sim 0.1 \ldots 10\ TeV$

impact on LSS:

wash-out of structure

on scales $\lambda <\ 0.1\,pc$