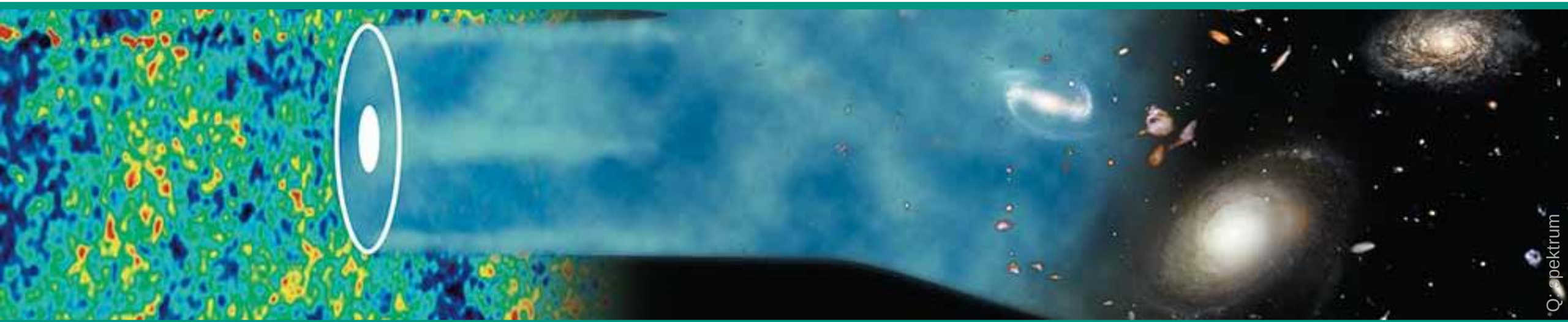


Introduction to Cosmology

Winter term 23/24 Lecture 14

Feb. 6, 2024



Recap of Lecture 13

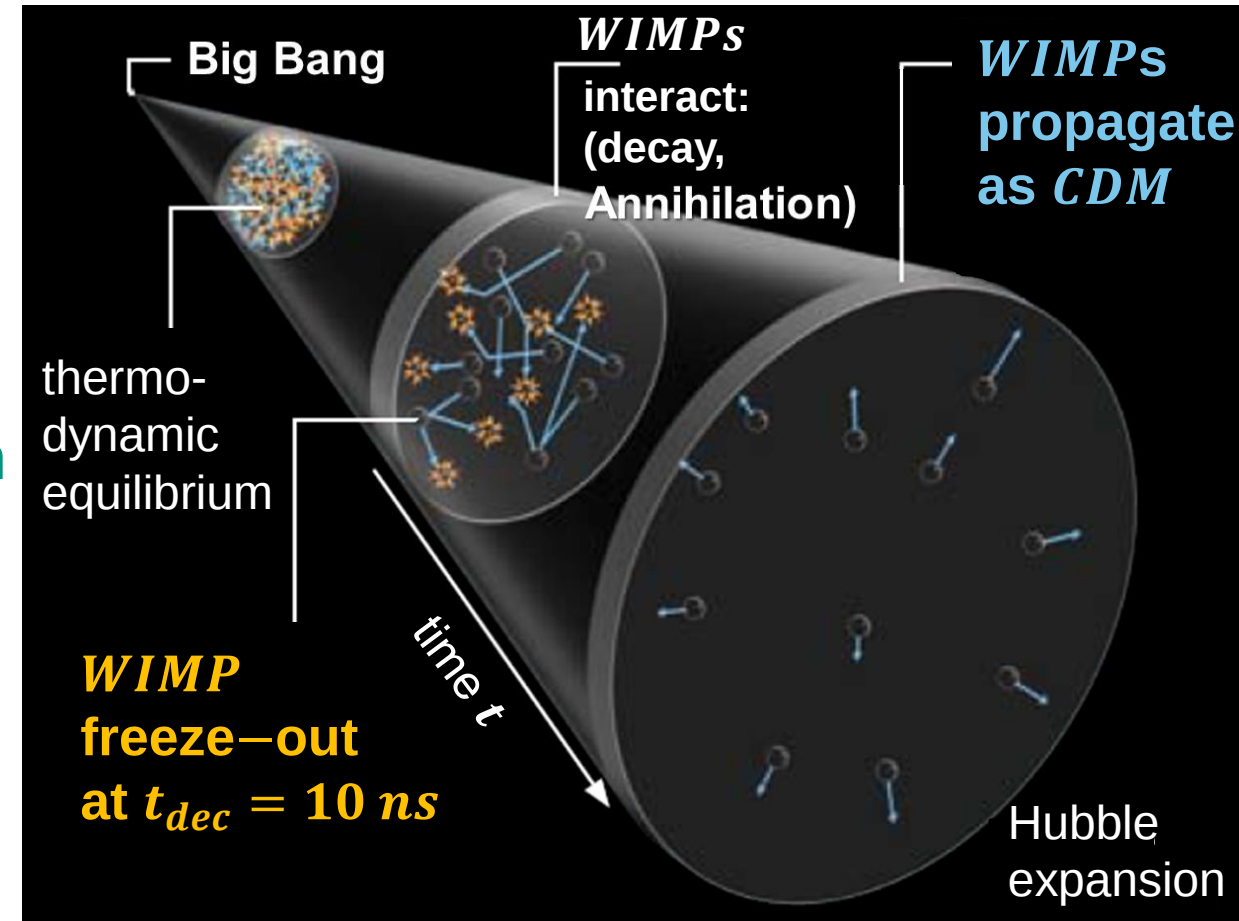
- The observed ‘clumpiness’ of the universe: **matter power spectrum $P(k)$**
 - analysis of **density contrast** – as function of **distance r** : **galaxy correlation function $\xi(r)$** $\Leftrightarrow$ as function of **wave number k** : **matter power spectrum $P(k)$**
 - **DM** – modes of specific wave number k evolve independently: important is **time of first causal contact** (in the radiation– / matter– dominated universe)
 - **small DM** – mode (**large k**): growth is delayed in radiation era – **$P(k) \sim k^{-3}$**
 - **large DM** – mode (**small k**): growth is not delayed in matter era – **$P(k) \sim k$**
 - matter power spectrum **$P(k)$** : allows to discriminate pure **HDM** vs. pure **CDM**

CDM: *WIMPs* as generic thermal relics



■ Thermal production of CDM: *WIMPs**

- *WIMPs*: massive, non–baryonic **thermal relics** left over from the Big Bang, which interact via **weak interaction** only (+ **gravity**)
- only **pair production** / **pair annihilation** of *WIMPs* ($\equiv$ *Majorana* particles with conserved *SUSY* quantum number R_P)
- very **large mass** (***TeV*** – scale):
 $\Rightarrow$ huge phase space in case of decay



CDM: *WIMPs* as generic thermal relics



■ Annihilation rate of *WIMPs*

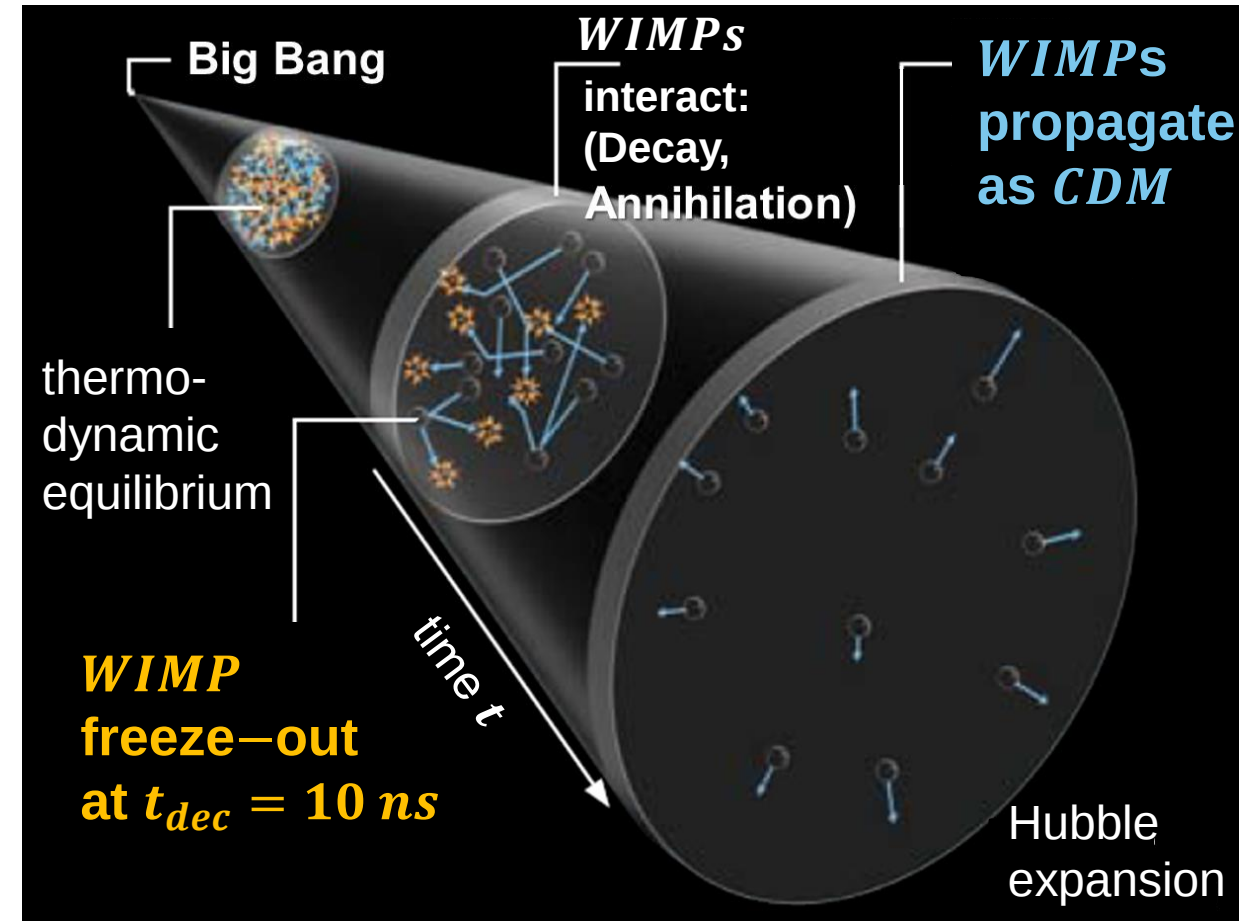
- *WIMPs*: annihilation rate given by typical **weak interaction strength**

$$\sigma_{ann} = \sigma_{weak}$$

- decoupling time t_{dec} from radiation bath is $t_{dec} \approx 10 \text{ ns}$

$$\sigma_{ann}(t_{dec}) = H(t_{dec})$$

- this will result in the so-called ***WIMP* miracle** (to be discussed later)



CDM: *WIMPs* as generic thermal relics

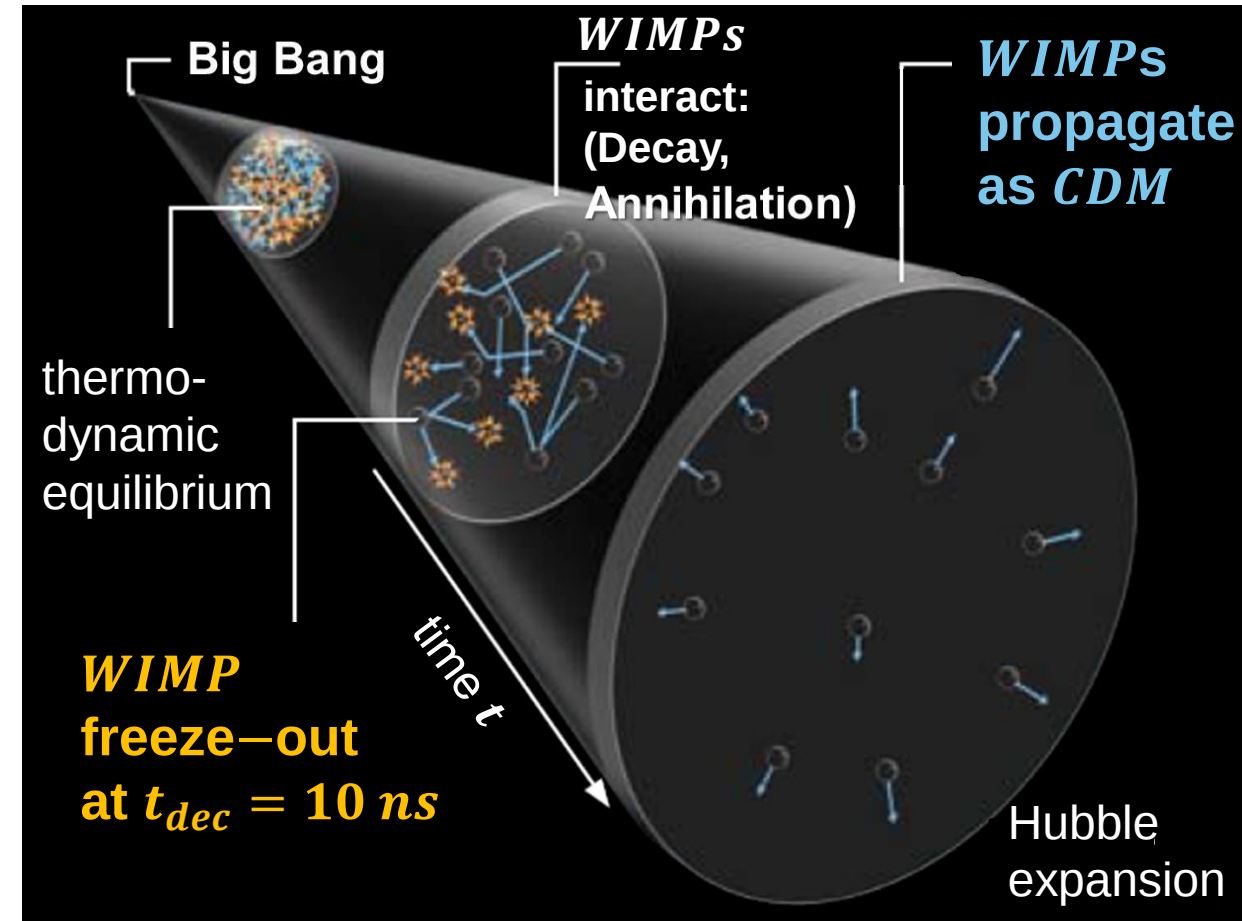


■ Long-term ‘slow’ propagation of *WIMPs*

- *WIMPs*: *non-relativistic* propagation with very **limited free-streaming range** due to kinematic relation:

$$E_{kin}(WIMP) \approx 0.05 \cdot m(WIMP)$$

- after decoupling & during propagation: χ^0 from *SUSY* are stable over long, cosmological time scales due to intrinsic symmetry: ***R* – parity R_P**



HDM: ν 's as generic light thermal relics

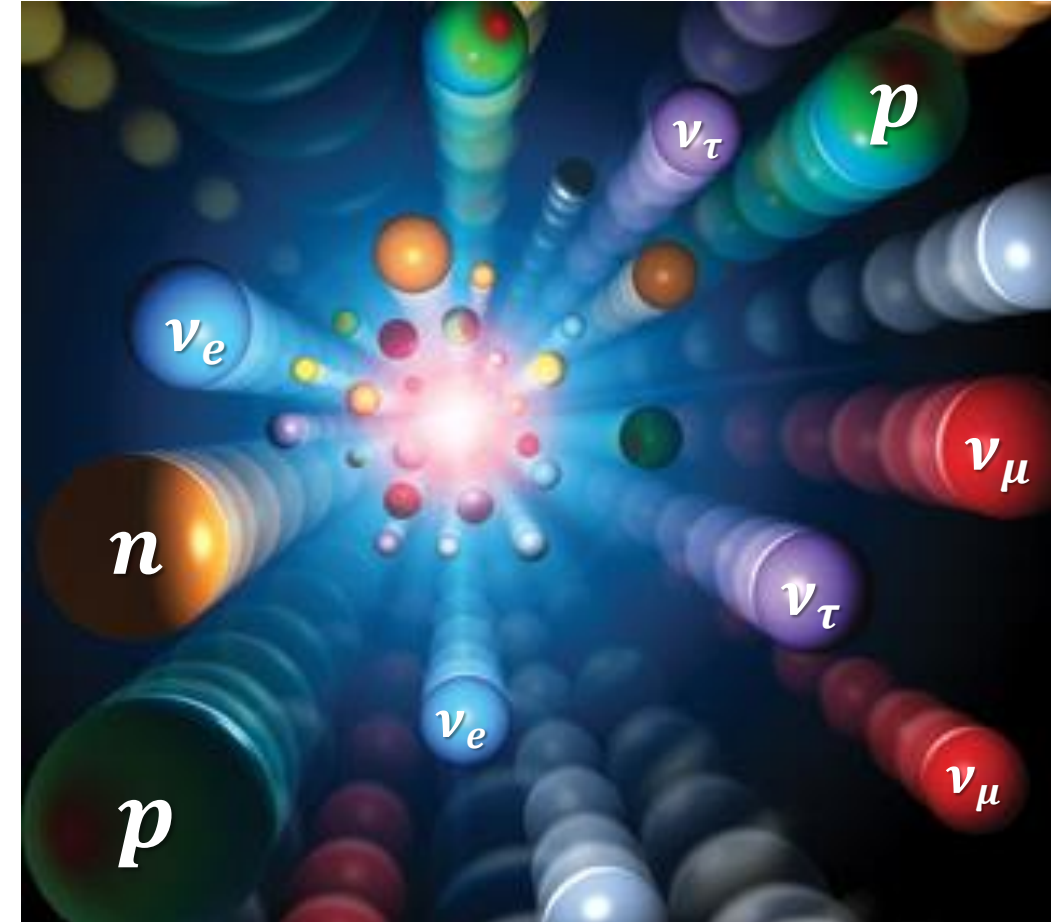


■ Hot dark matter: relativistic free–streaming

- ν 's: *ultra–relativistic* propagation over an exceedingly **large free–streaming range of $> Gpc$** due to kinematic relation

$$E_{kin}(\nu) > 10^6 \cdot m(\nu)$$

- after decoupling: stable over long, cosmological time scales due to conserved **lepton number L**

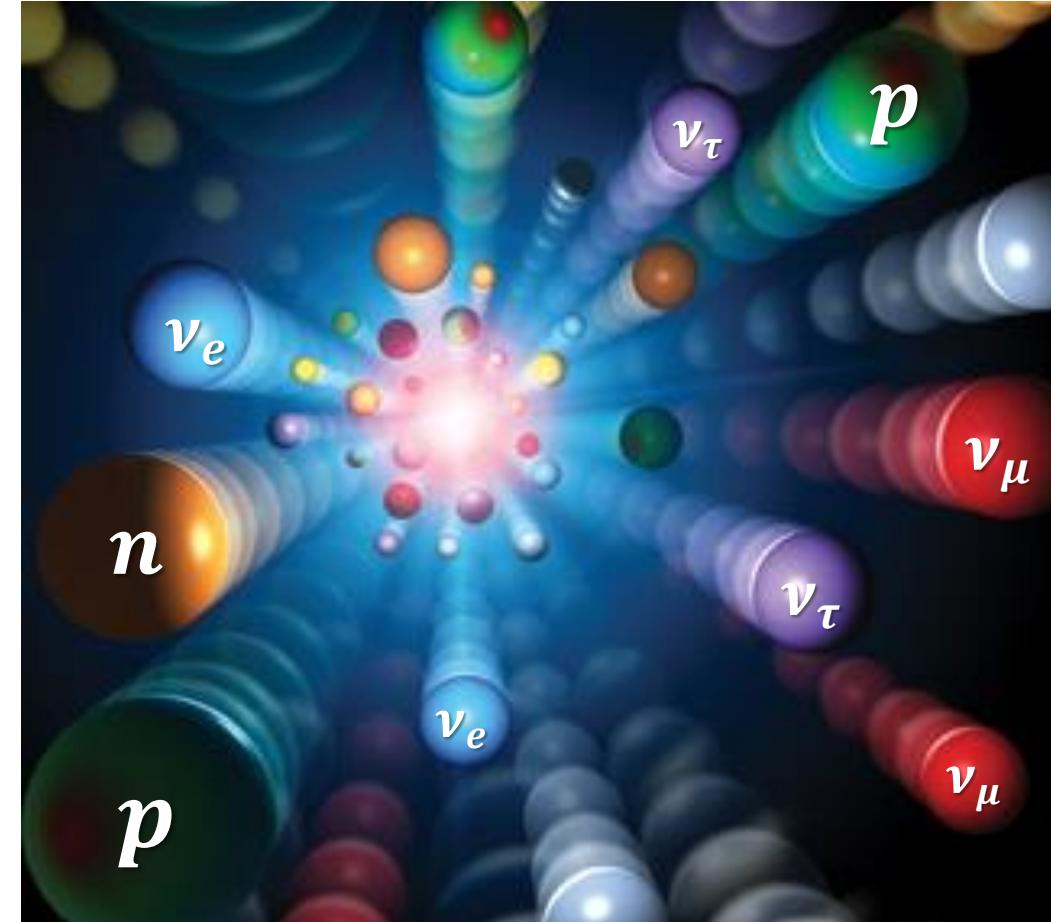


HDM: ν 's as generic light thermal relics



■ Hot dark matter: relativistic free–streaming

- ν 's: ***ultra–relativistic*** propagation is strongly suppressing annihilation in the early universe (also: there exist no lighter particles carrying lepton numbers in the *SM*)
⇒ weak interaction only (with baryonic matter)
- decoupling at $t \cong 1\text{ s}$ and $T \cong 1\text{ MeV}$
(see also: *chap. 3.1* on Big Bang Nucleosynthesis, lecture #5)



Dark Matter: hot, warm or cold (*HDM/WDM/CDM*)

■ Generic particle models for cosmological *DM* – density $\rho_{DM} \cong 1 \text{ GeV}/m^3$

Hot Dark Matter

particle candidate:

active neutrinos $\nu_{e,\mu,\tau}$
 $m \sim 0.05 \dots 0.8 \text{ eV}$

number density:

$N(\text{active}): 339/cm^3$

decoupling:

$T_{fr} = 2 - 3 \text{ MeV}$

$T_{fr}/m \sim 10^6 \dots 10^7$

impact on *LSS*:

wash-out of structure
on scales $\lambda \leq 1 \text{ Gpc}$

Warm Dark Matter

particle candidate:

sterile neutrinos ν_s
 $m \sim 1 \dots 20 \text{ keV}$

number density:

$N(\text{sterile}): < 1/cm^3$

decoupling:

no thermal process,
but via ν – oscillations

impact on *LSS*:

wash-out of structure
on scale $\lambda < 0.1 \text{ Mpc}$

Cold Dark Matter

particle candidate:

SUSY neutralinos χ^0
 $m \sim 0.1 \dots 10 \text{ TeV}$

number density:

$N(\chi^0): < 10^{-9}/cm^3$

decoupling:

$T_{fr} = \text{GeV} \dots \text{TeV}$

$T_{fr}/m \sim 1/20$

impact on *LSS*:

wash-out of structure
on scales $\lambda < 0.1 \text{ pc}$

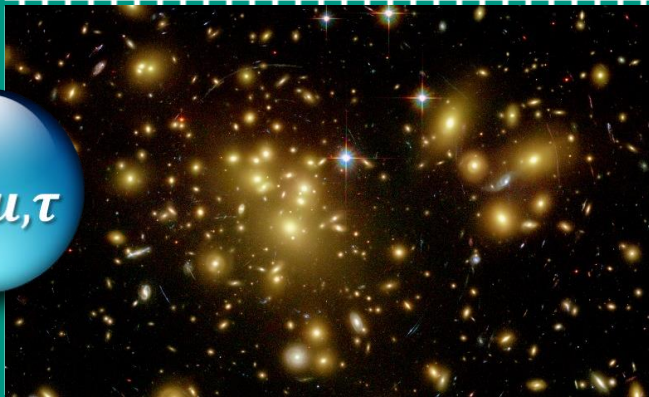
Dark Matter: hot, warm or cold (*HDM/WDM/CDM*)

- Generic particle models for cosmological *DM* – density $\rho_{DM} \cong 1 \text{ GeV}/m^3$

Hot Dark Matter

particle candidate:

active neutrinos $\nu_{e,\mu,\tau}$
 $m \sim 0.05 \dots 0.8 \text{ eV}$



impact on *LSS*:

wash-out of structure
on scales $\lambda \leq 1 \text{ Gpc}$

Warm Dark Matter

particle candidate:

sterile neutrinos ν_s
 $m \sim 1 \dots 20 \text{ keV}$



impact on *LSS*:

wash-out of structure
on scale $\lambda < 0.1 \text{ Mpc}$

Cold Dark Matter

particle candidate:

SUSY neutralinos χ^0
 $m \sim 0.1 \dots 10 \text{ TeV}$



impact on *LSS*:

wash-out of structure
on scales $\lambda < 0.1 \text{ pc}$



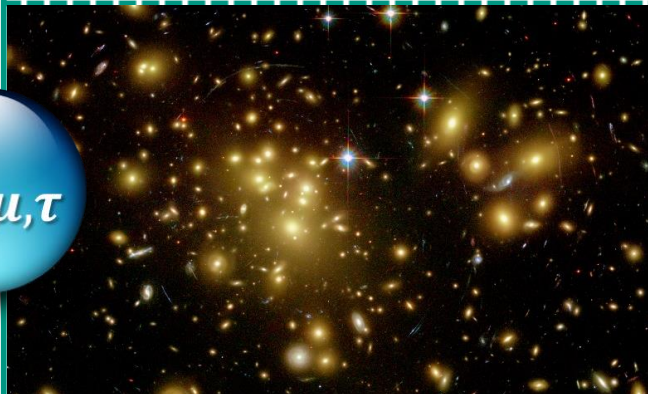
Dark Matter: hot, warm or cold (*HDM/WDM/CDM*)

■ Generic particle models: the important Lee–Weinberg curve

Hot Dark Matter

particle candidate:

active neutrinos $\nu_{e,\mu,\tau}$
 $m \sim 0.05 \dots 0.8 \text{ eV}$



impact on *LSS*:

wash-out of structure
on scales $\lambda \leq 1 \text{ Gpc}$

Lee-Weinberg curve



이휘소



Weinberg

for thermal production &
subsequent reduction due to
annihilation processes: only
two narrow mass regions
to explain $\Omega_{DM} \sim 0.25$

Cold Dark Matter

particle candidate:

SUSY neutralinos χ^0
 $m \sim 0.1 \dots 10 \text{ TeV}$



impact on *LSS*:

wash-out of structure
on scales $\lambda < 0.1 \text{ pc}$



Lee–Weinberg curve: only *HDM* or *CDM*

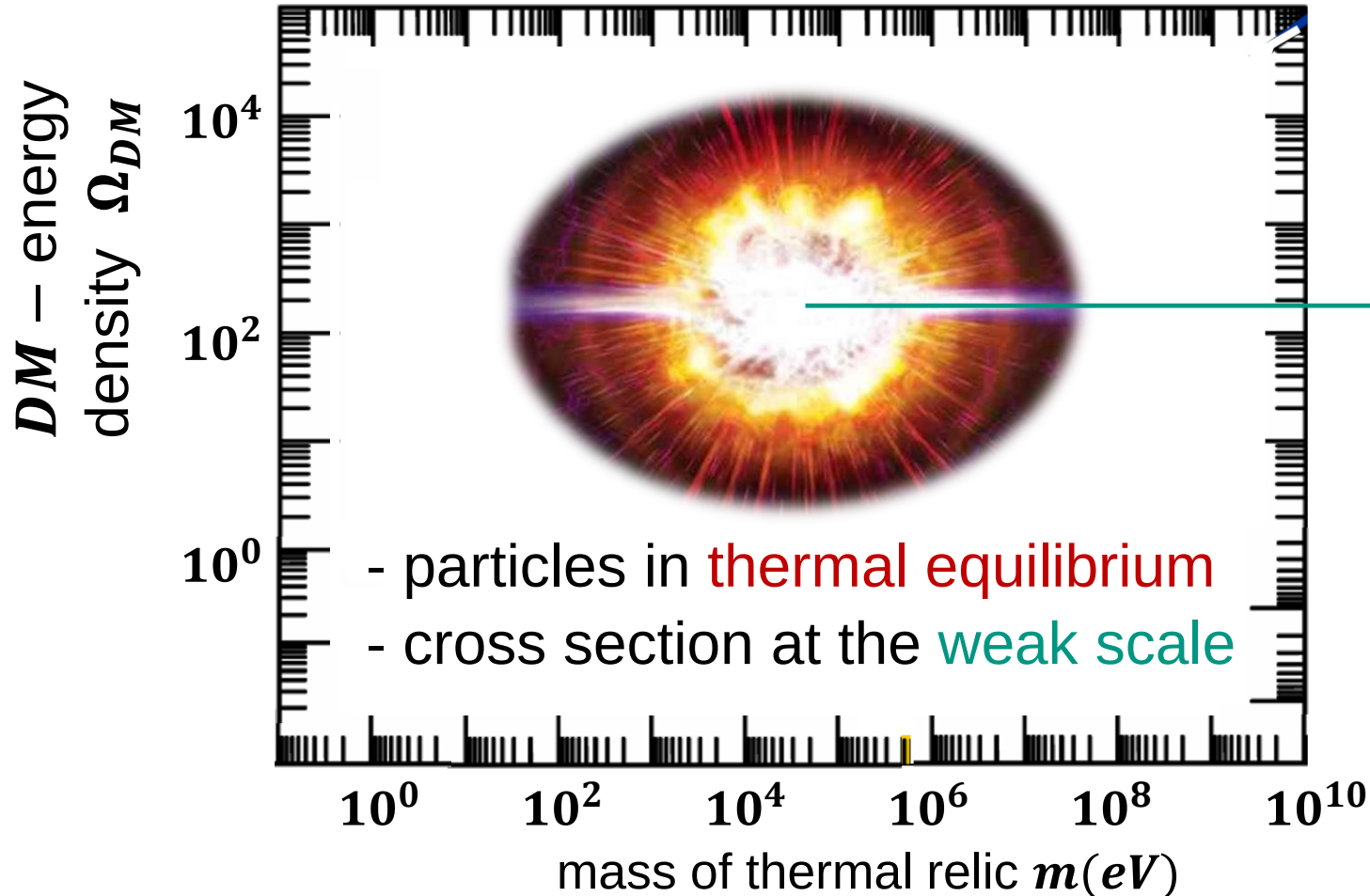
- Thermal production of *DM*: what **particle masses** can be produced?



이휘소

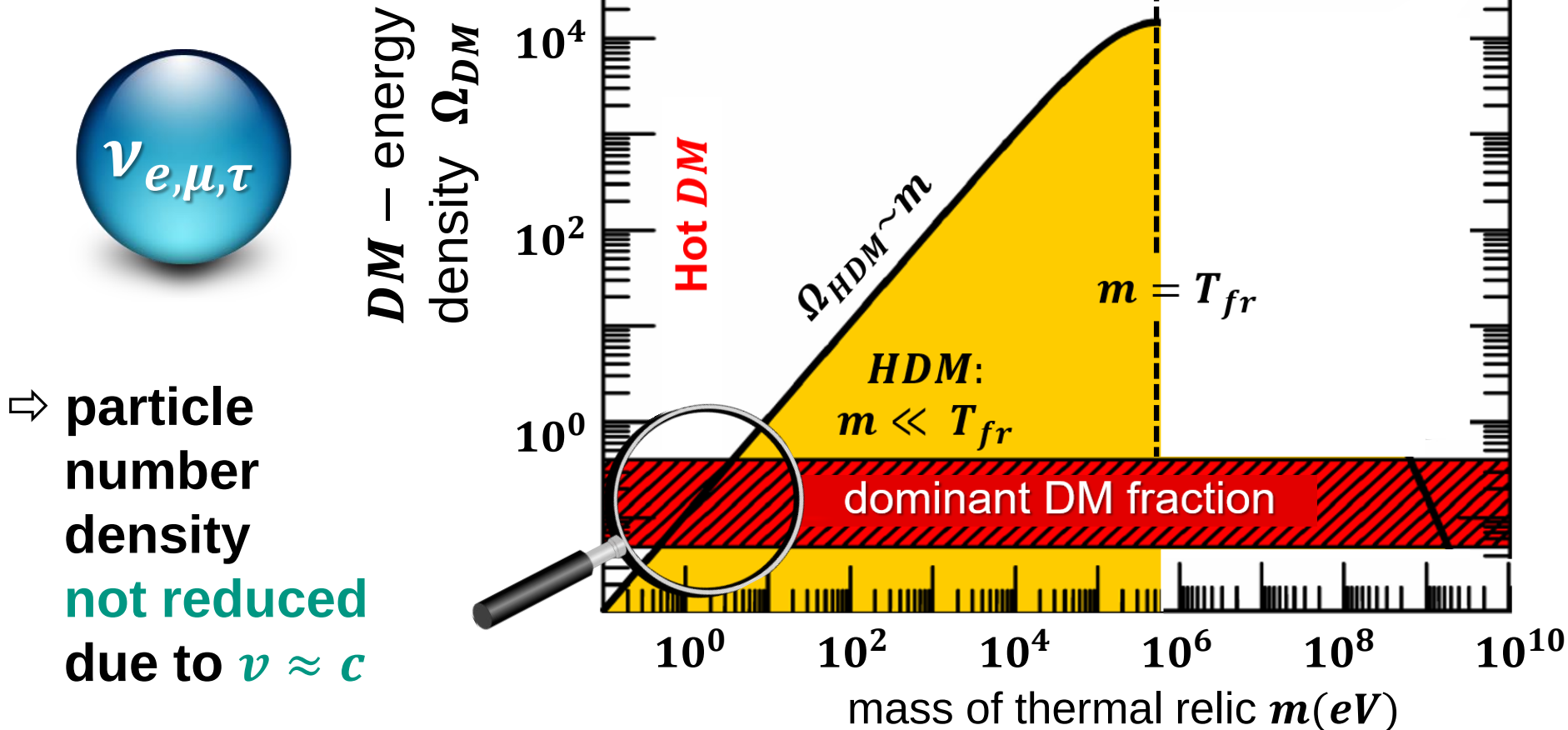


Weinberg



Lee–Weinberg curve: only *HDM* or *CDM*

- Thermal production of *DM*: **relativistic case** of neutrinos – **no annihilation**



$$\Omega_{HDM} \sim m$$

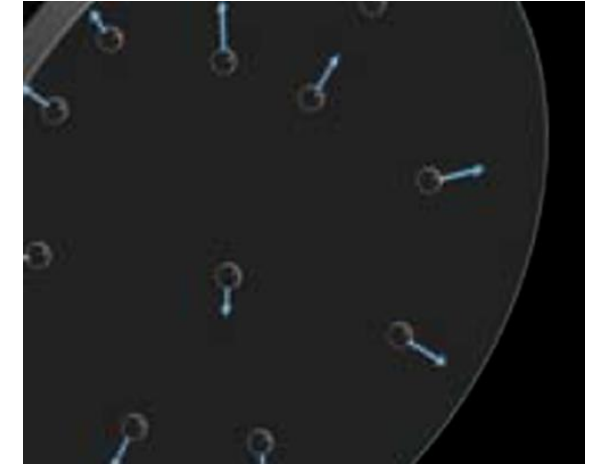
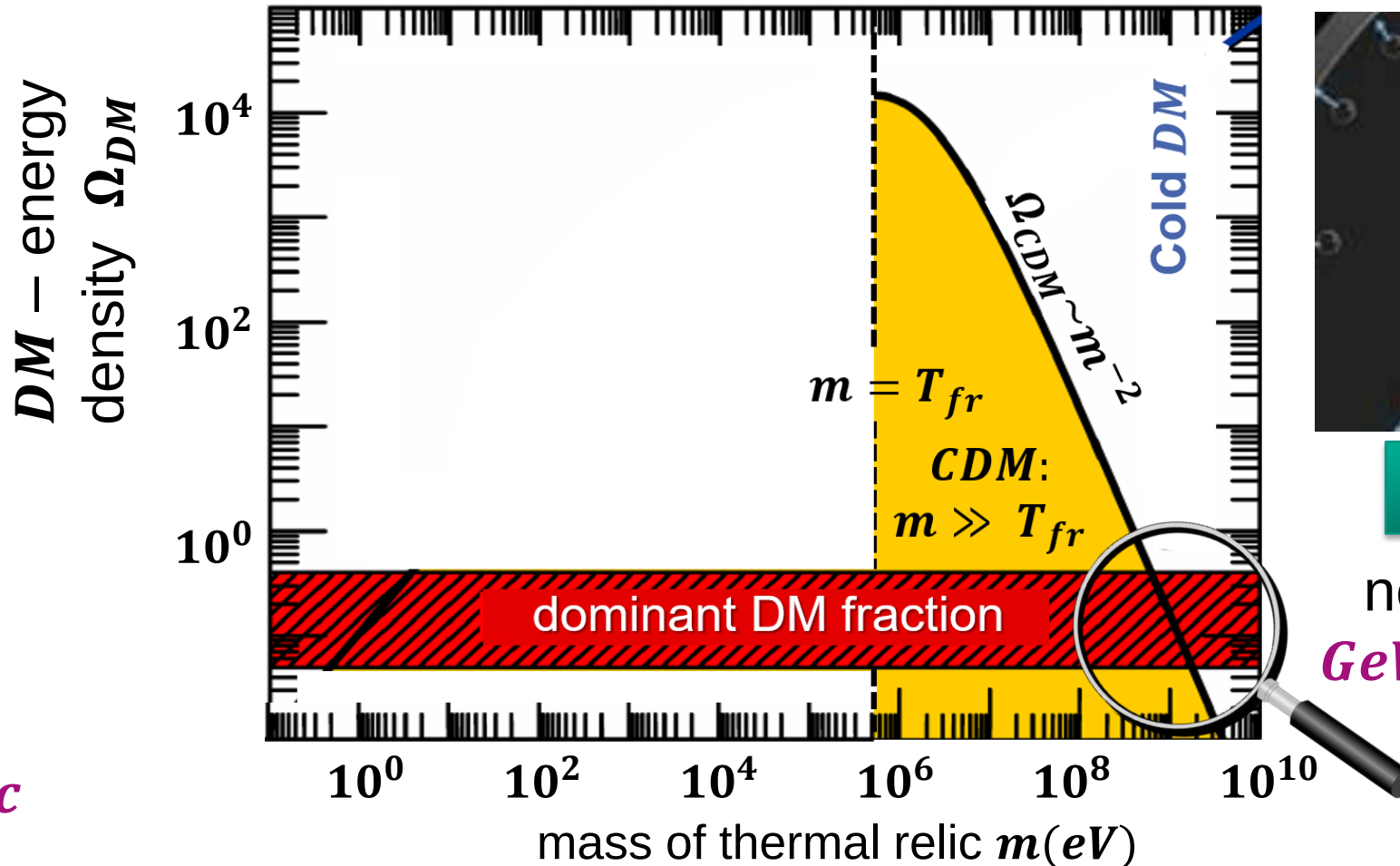
neutrinos with **sub – eV** masses

Lee–Weinberg curve: only *HDM* or *CDM*

- Thermal production of *DM*: **non–relativistic case** of neutralinos – **annihilation**



⇒ particle
number
density
reduced
due to $v \ll c$



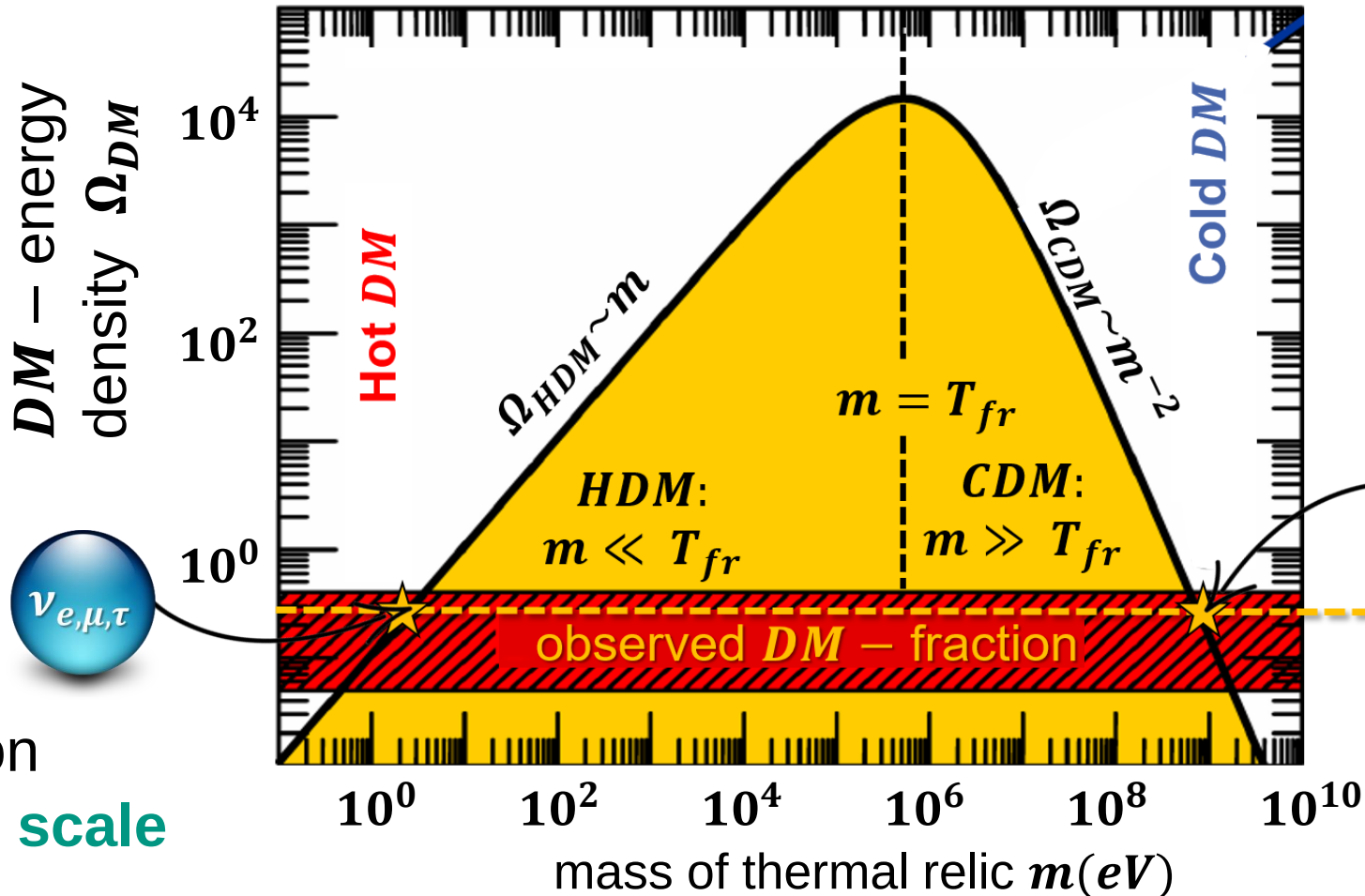
$$\Omega_{CDM} \sim m^{-2}$$

neutralinos with
GeV ... TeV masses

Lee–Weinberg curve: only *HDM* or *CDM*

- Thermal production of *DM*: only two rather narrow mass ranges – *eV* or *TeV*

relativistic
thermal relics
without
annihilation:



non–relativistic
thermal relics
with intense
annihilation:



$$\Omega_{DM} = 0.27$$

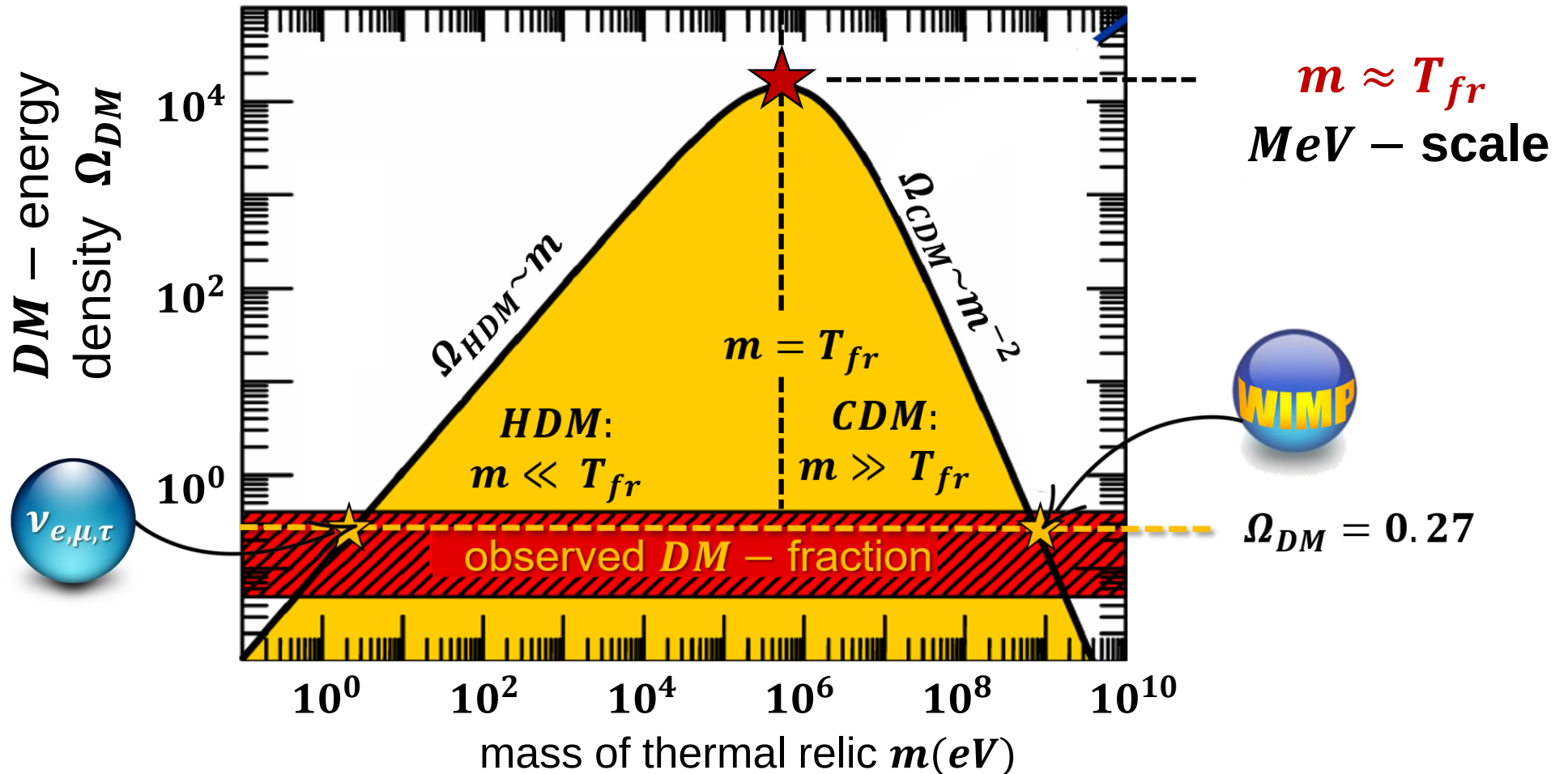
neutralinos on
TeV – mass scale

Lee–Weinberg curve: freeze–out at $T \sim 1 \text{ MeV}$

■ Thermal production of MeV – scale particles is maximum as $t_{fr} = 1 \text{ s}$

- general condition for thermal freeze–out

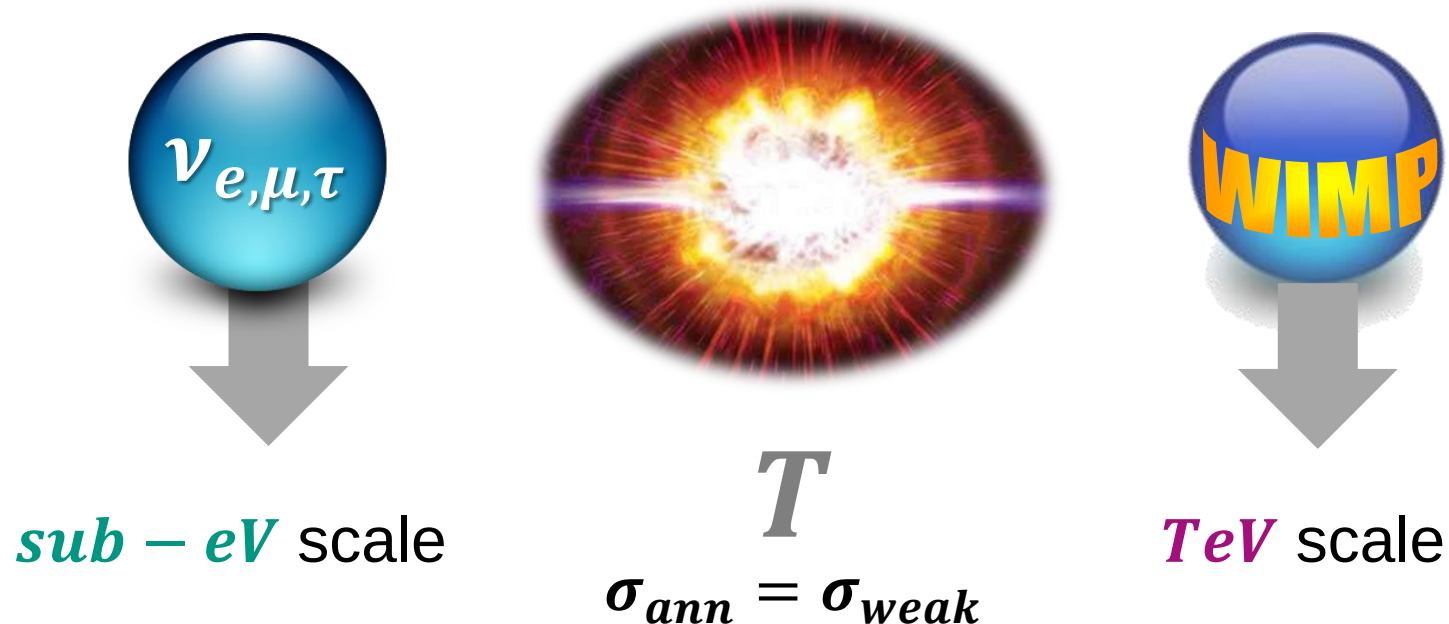
$$H(t_{fr}) = \Gamma(t_{fr})$$



Lee–Weinberg curve: only *HDM* or *CDM*

■ Thermal production of *DM*: only two rather narrow mass ranges – *eV* or *TeV*

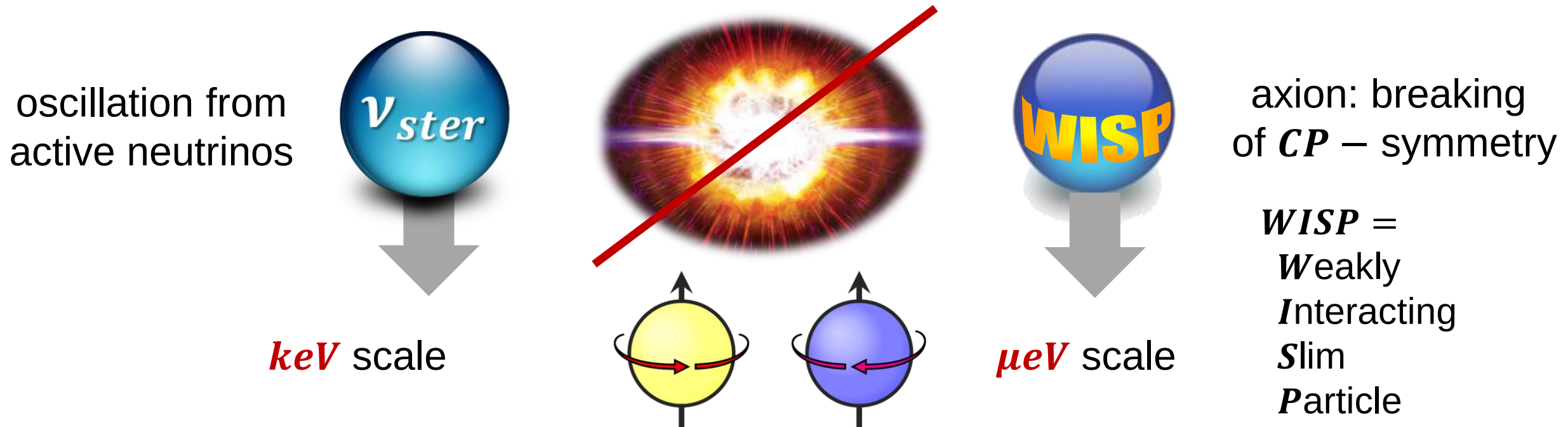
- in order to avoid overclosure of the universe due to thermally produced *DM* ($\Omega_{DM} \gg 1$) particles in **mass scale *keV ... MeV*** or $\ll eV$ are **excluded**
⇒ assumption: **weak interaction processes & subsequent freeze–out**



Lee–Weinberg curve: only *HDM* or *CDM*

■ Non–thermal production of *DM* for mass ranges: $m \sim \text{keV} \dots \text{MeV}$

- we need to to avoid overclosure of the universe in case of thermal production:
DM ($\Omega_{DM} \gg 1$) particles in **mass scale $\text{keV} \dots \text{MeV}$** (or $\text{neV} \dots \mu\text{eV}^*$) are excluded!
 $\Rightarrow$ only **non–thermal processes**: ν – oscillations, or symmetry breaking



Thermal relics: production & annihilation

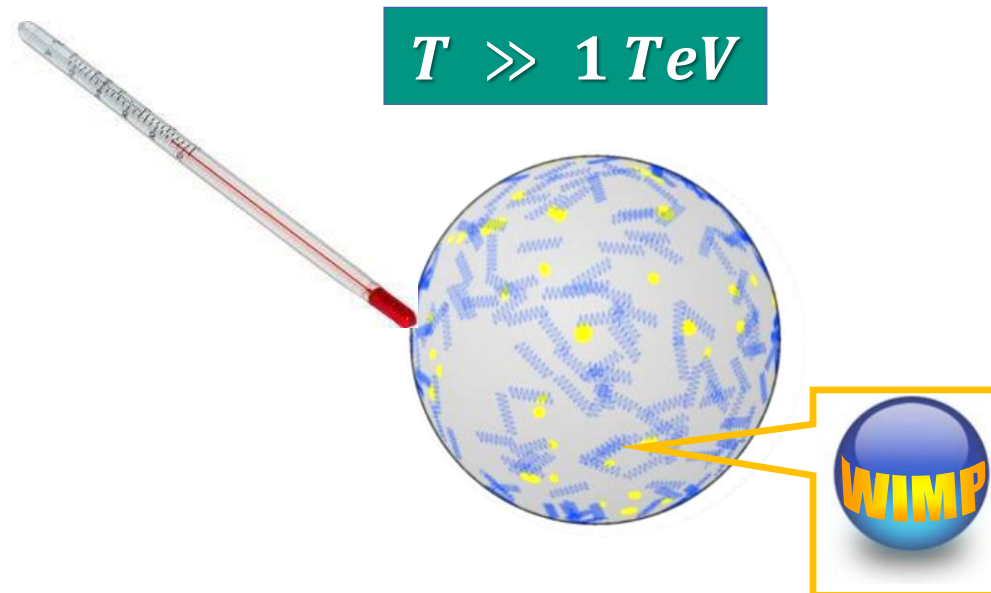
■ The *WIMP* miracle of *DM*: phase 1 – thermodynamical equilibrium

- at $T \gg TeV$: due to their weak interaction, *WIMPs* are in thermodynamical equilibrium

thermodynamical
equilibrium



$t < 1 \text{ ns}$



we consider **co-moving number densities** $n_\chi(t)$ where an increase of the scale factor $a(t)$ does not need to be accounted for

Thermal relics: production & annihilation

■ The *WIMP* miracle of *DM*: **phase 1** – thermodynamical equilibrium

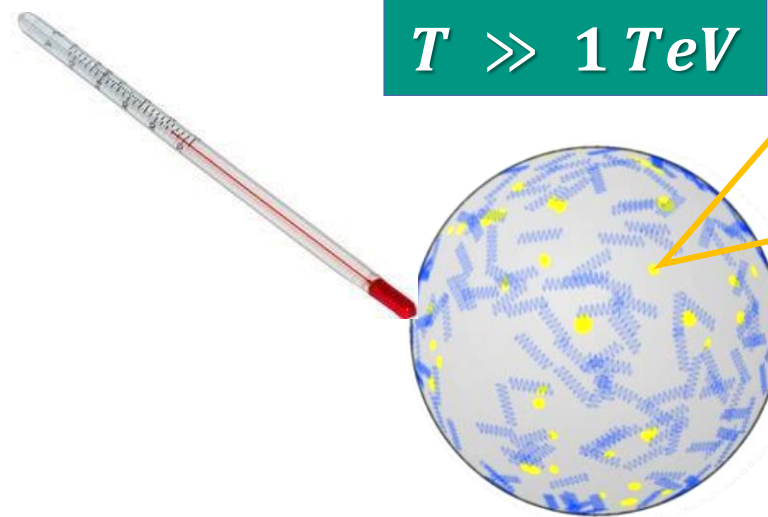
- at $T \gg TeV$: due to their weak interaction, *WIMPs* are in **thermodynamical equilibrium**

rate (*WIMP* – pair production) $\equiv$ rate (*WIMP* – pair annihilation)

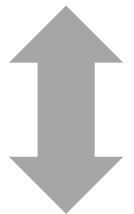
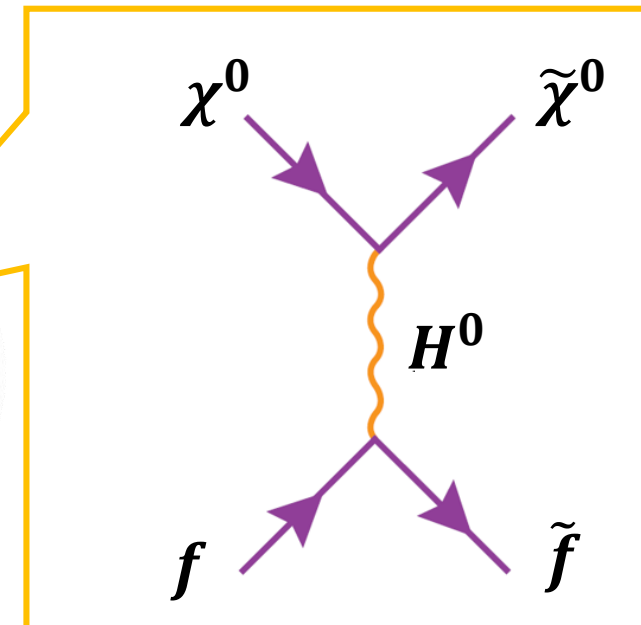
thermodynamical
equilibrium



$t < 1 \text{ ns}$



$T \gg 1 \text{ TeV}$



Thermal relics: production & annihilation

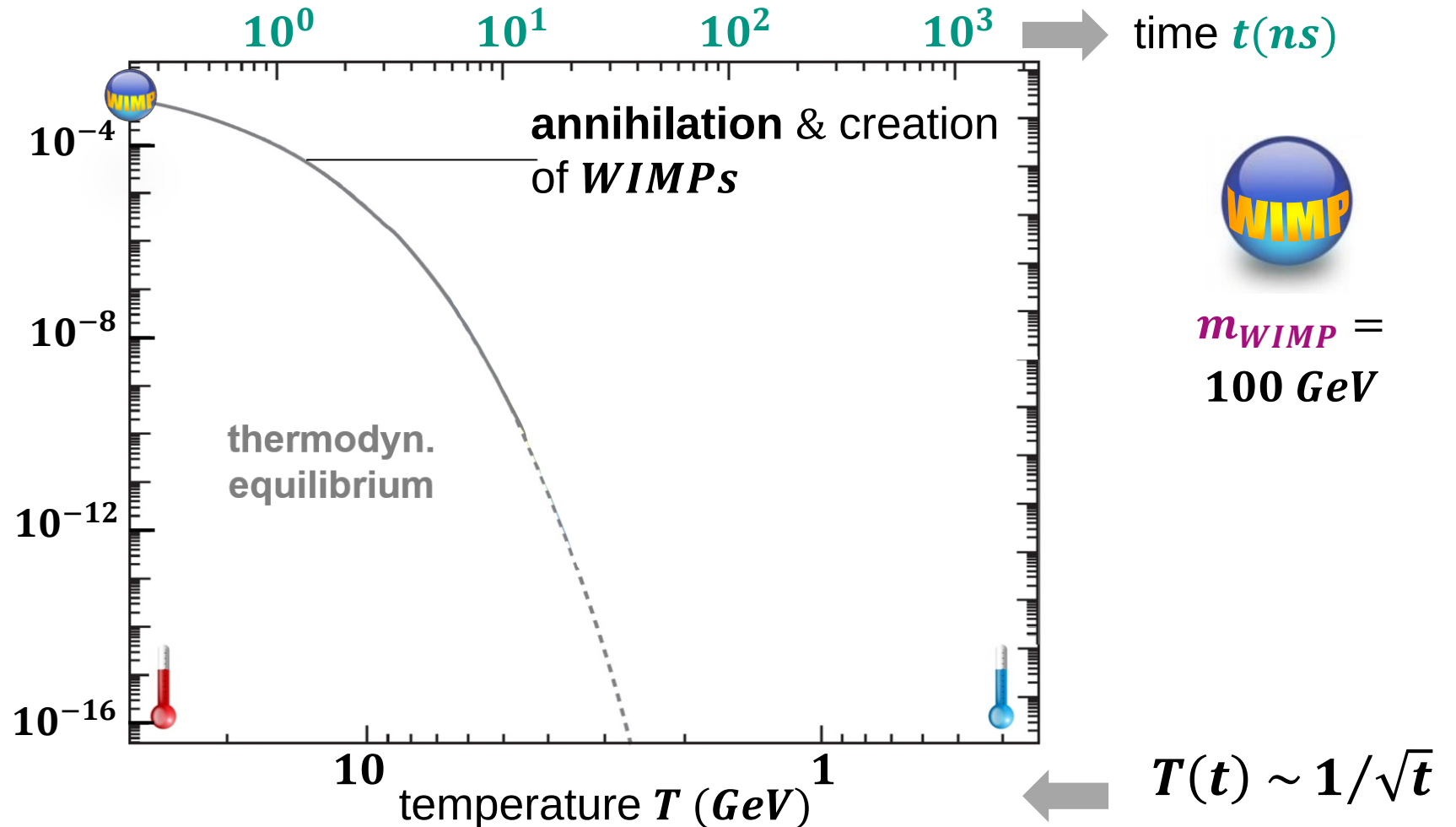
■ The *WIMP* miracle of *DM*: phase 2 – annihilations reduce number density

$$n_{\chi}(t) \sim e^{-m_{WIMP}/k_B T}$$

exponential
decrease of $n_{\chi}(t)$



$$t < 30 \text{ ns}$$



Thermal relics: freeze-out & final number density

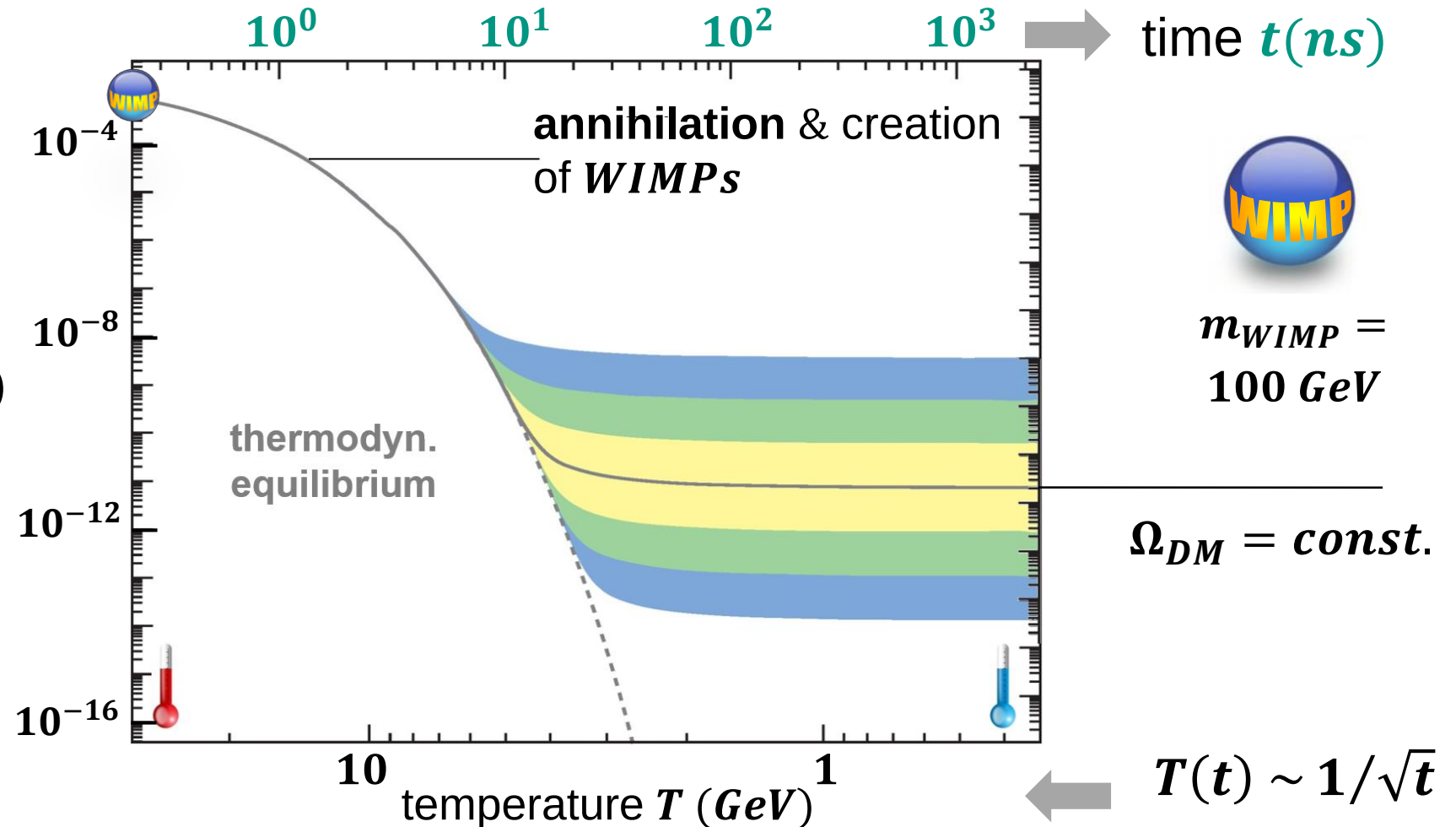
■ The *WIMP* miracle of *DM*: phase 3 – freeze out & decoupling of *WIMPs*

$$\Gamma(t) = H(t)$$

after **decoupling** of
WIMPs: constant $n_\chi(t)$



$$t > 30 \text{ ns}$$



Thermal relics: freeze-out & final number density

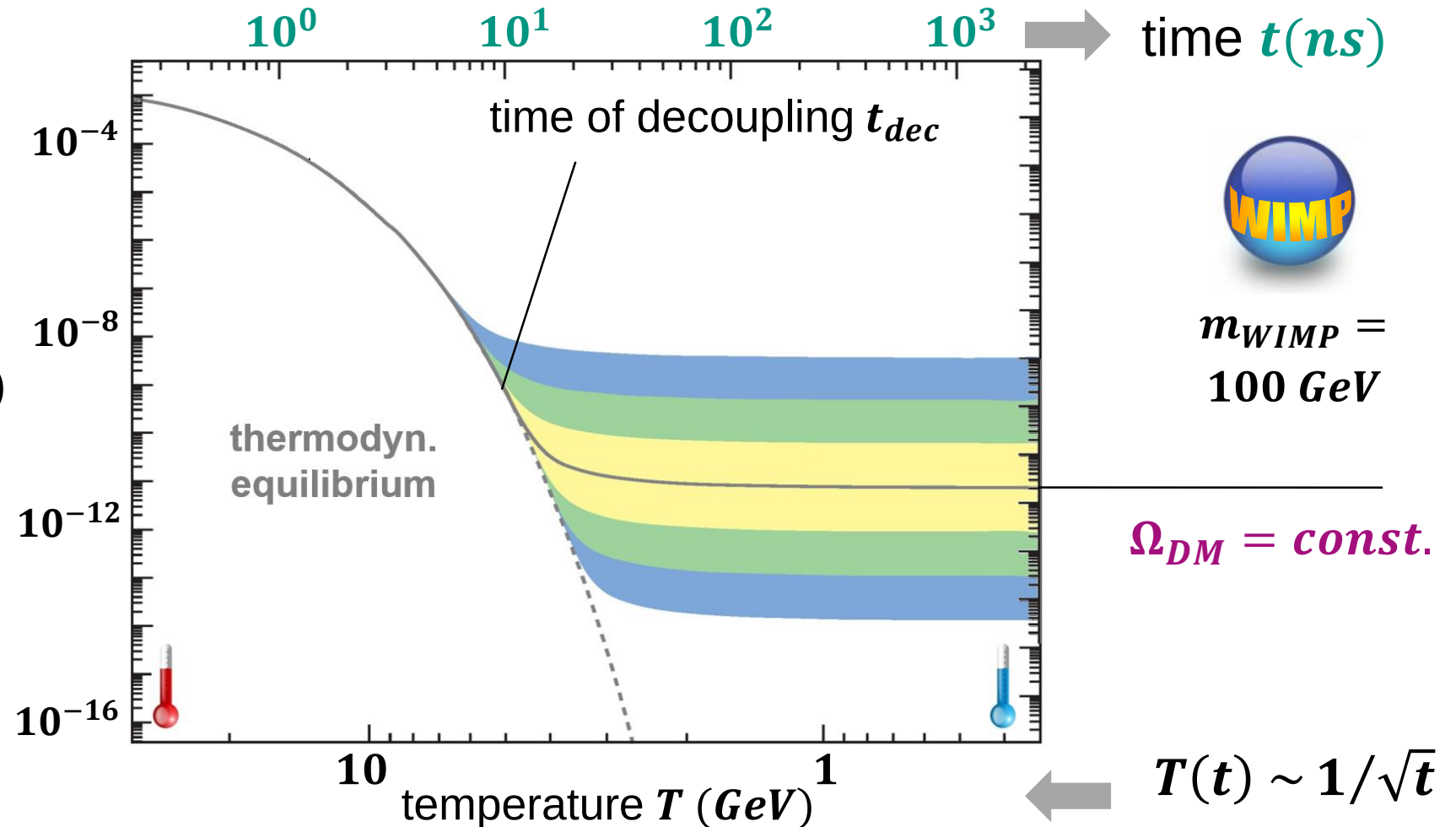
■ The *WIMP* miracle of *DM*: phase 3 – freeze out & decoupling of *WIMPs*

$$\Gamma(t_{dec}) = H(t_{dec})$$

after **decoupling** of
WIMPs: constant $n_\chi(t)$



distance too large
for annihilations

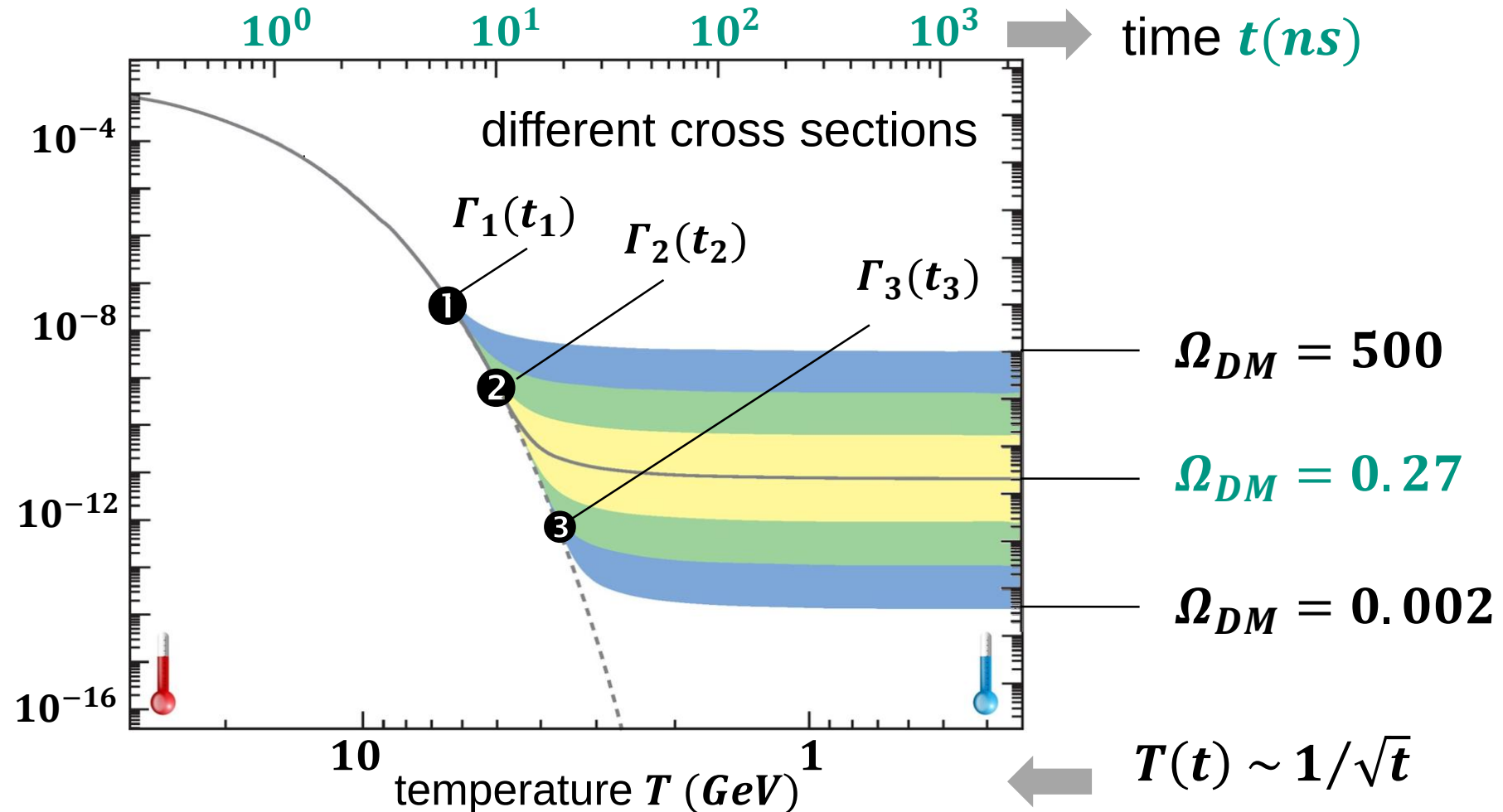
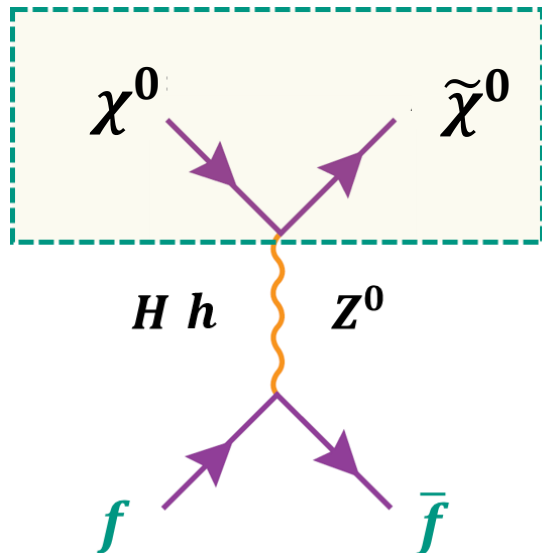


Thermal relics: freeze-out & final number density

■ The *WIMP* miracle of *DM*: phase 3 – freeze out & decoupling of *WIMPs*

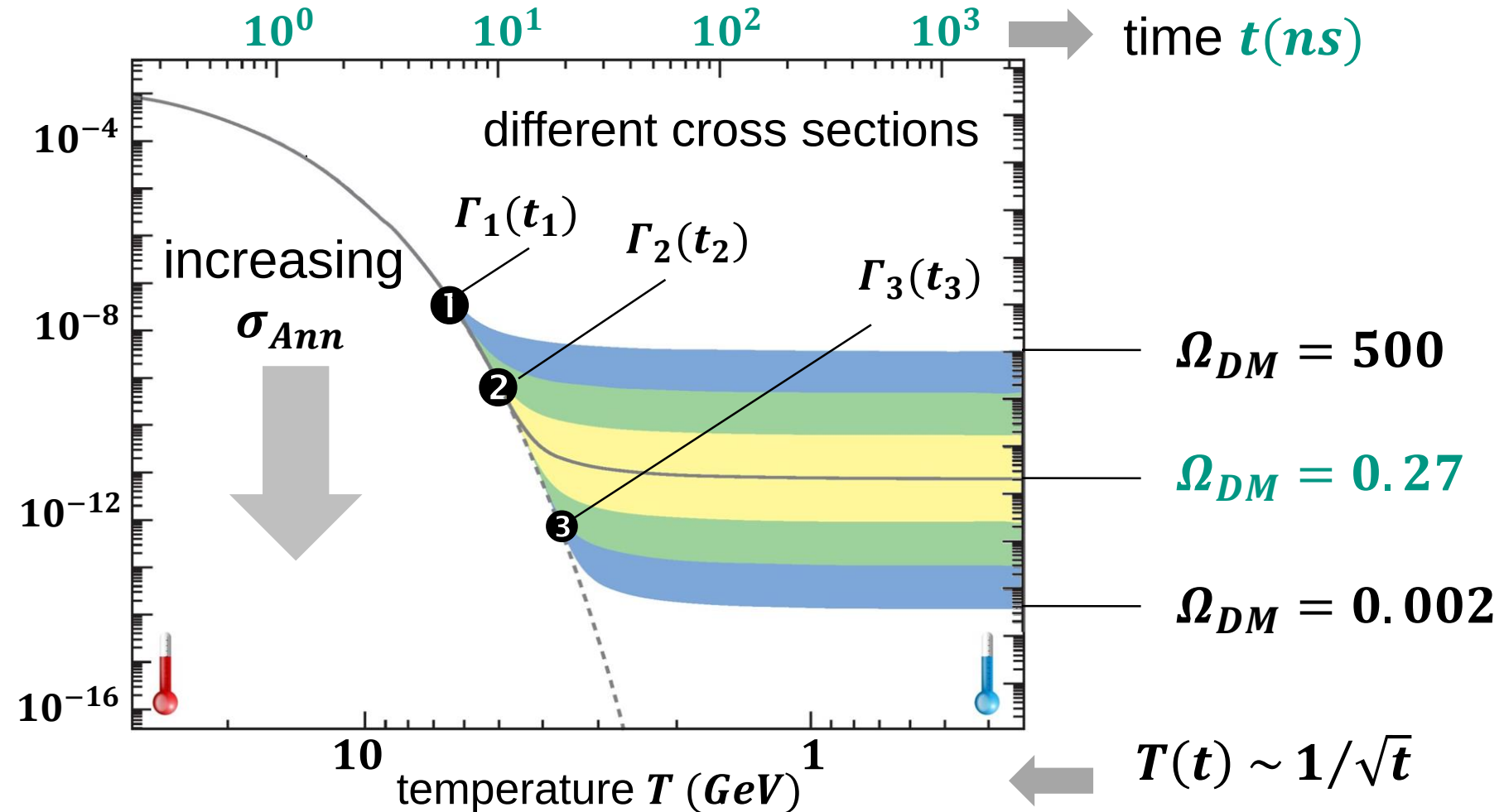
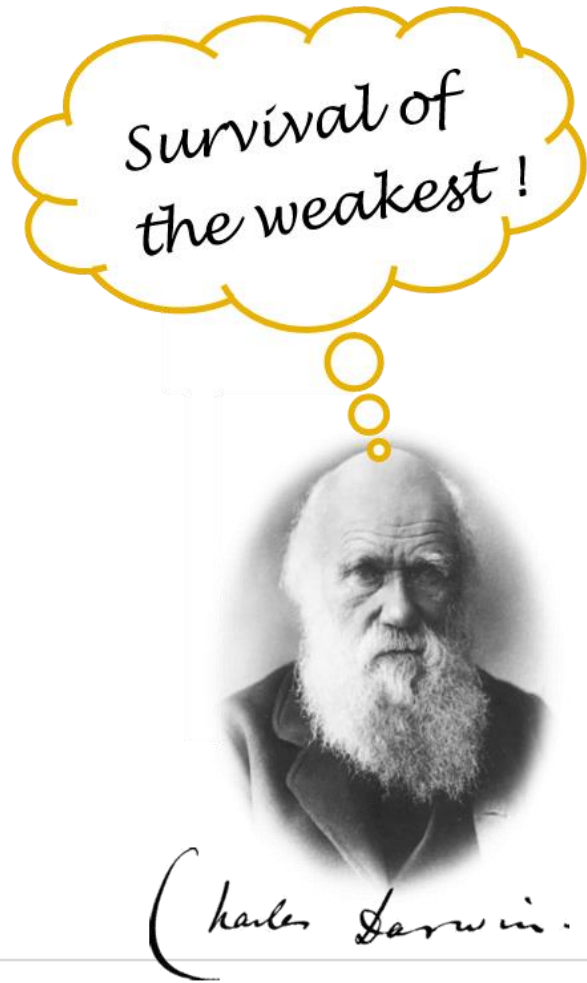
$$\Gamma_i(t_{dec,i}) = H(t_{dec,i})$$

Γ_i = annihilation *xsecs*
of *SUSY* – *WIMPs*



Thermal relics: freeze-out & final number density

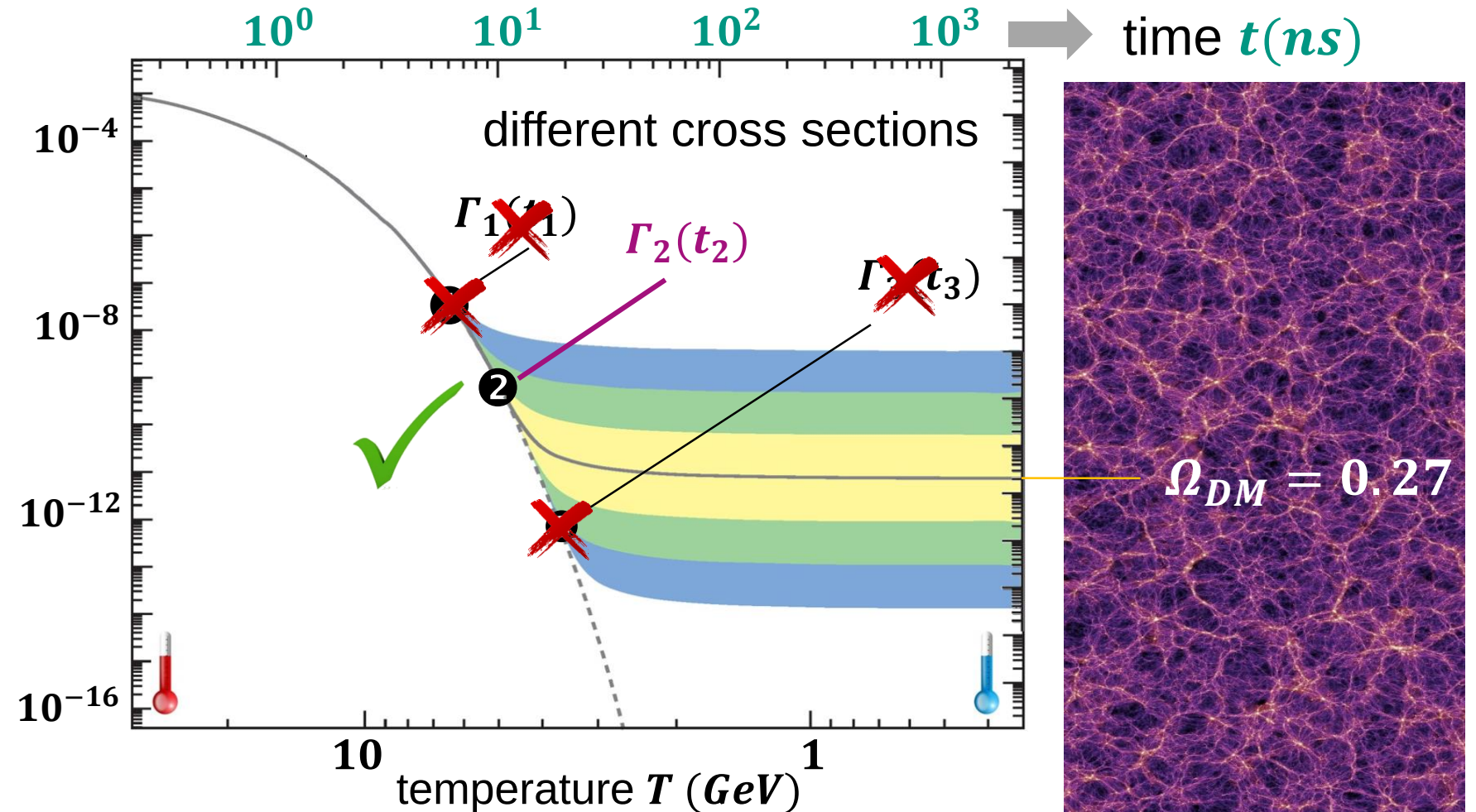
■ The *WIMP* miracle of *DM*: phase 3 – freeze out & decoupling of *WIMPs*



Thermal relics: freeze-out & final number density

■ The *WIMP* miracle of *DM*: phase 3 – freeze out & decoupling of *WIMPs*

- is the observed *DM* – content of the universe of $\Omega_{DM} = 0.27$ just a chance value?

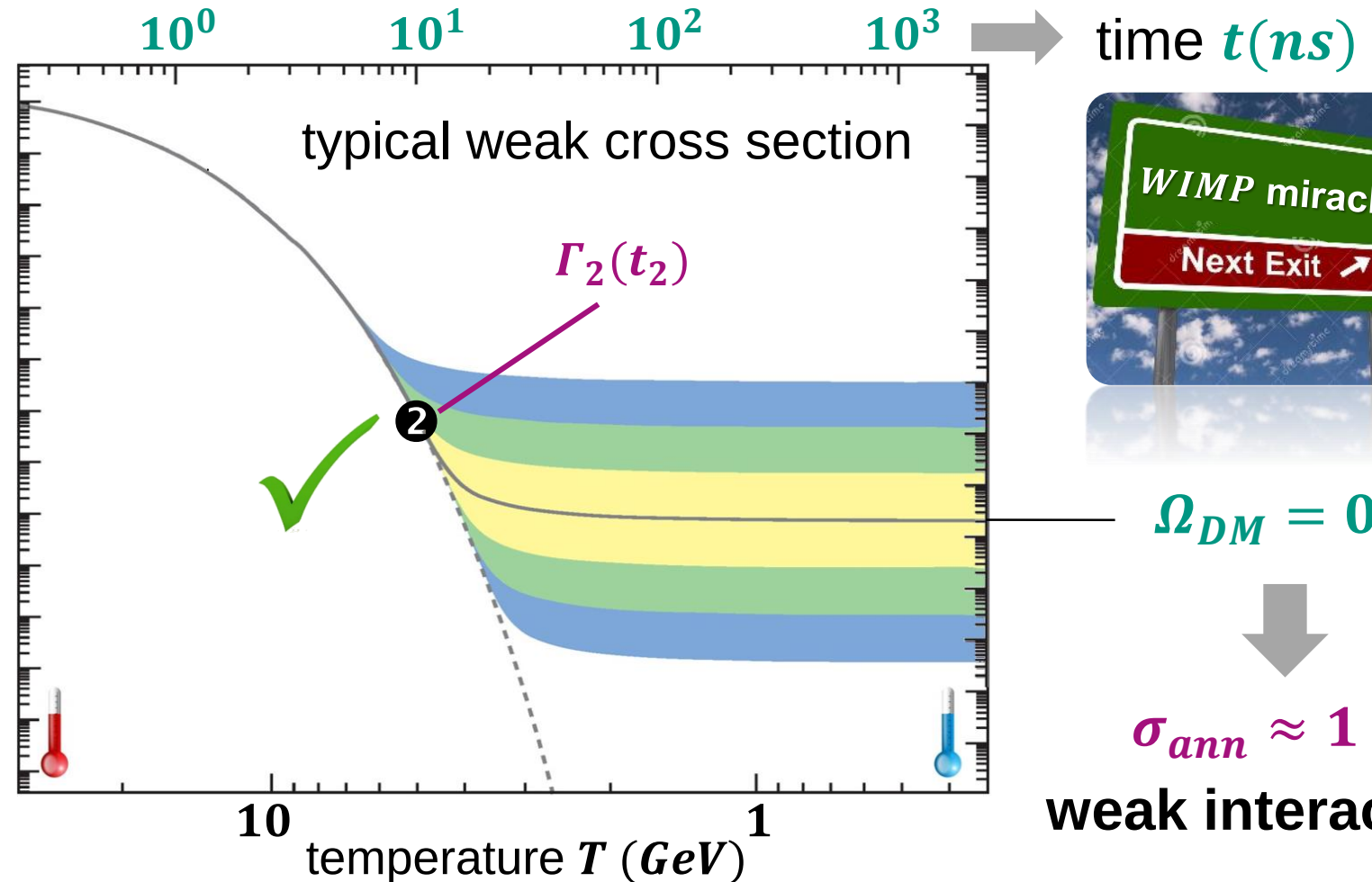
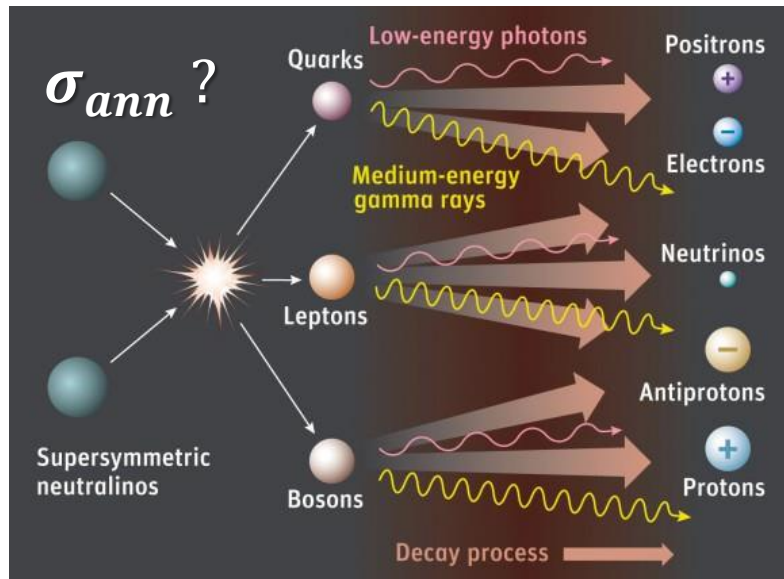


Thermal relics: freeze-out & annihilation $xsec$

■ The *WIMP* miracle of *DM*: we 'automatically' obtain Ω_{DM} if $\sigma_{ann} \approx 1 pb$

- *DM* – fraction today:

$$\Omega_{DM}(t = t_0) \sim \frac{1}{\langle \sigma_{ann} \cdot v \rangle}$$



v : *WIMP* velocity at decoupling

Thermal relics: freeze-out as *CDM*

■ The *WIMP* miracle of *DM*: we ‘automatically’ obtain **Cold Dark Matter**

- *WIMP* – velocity

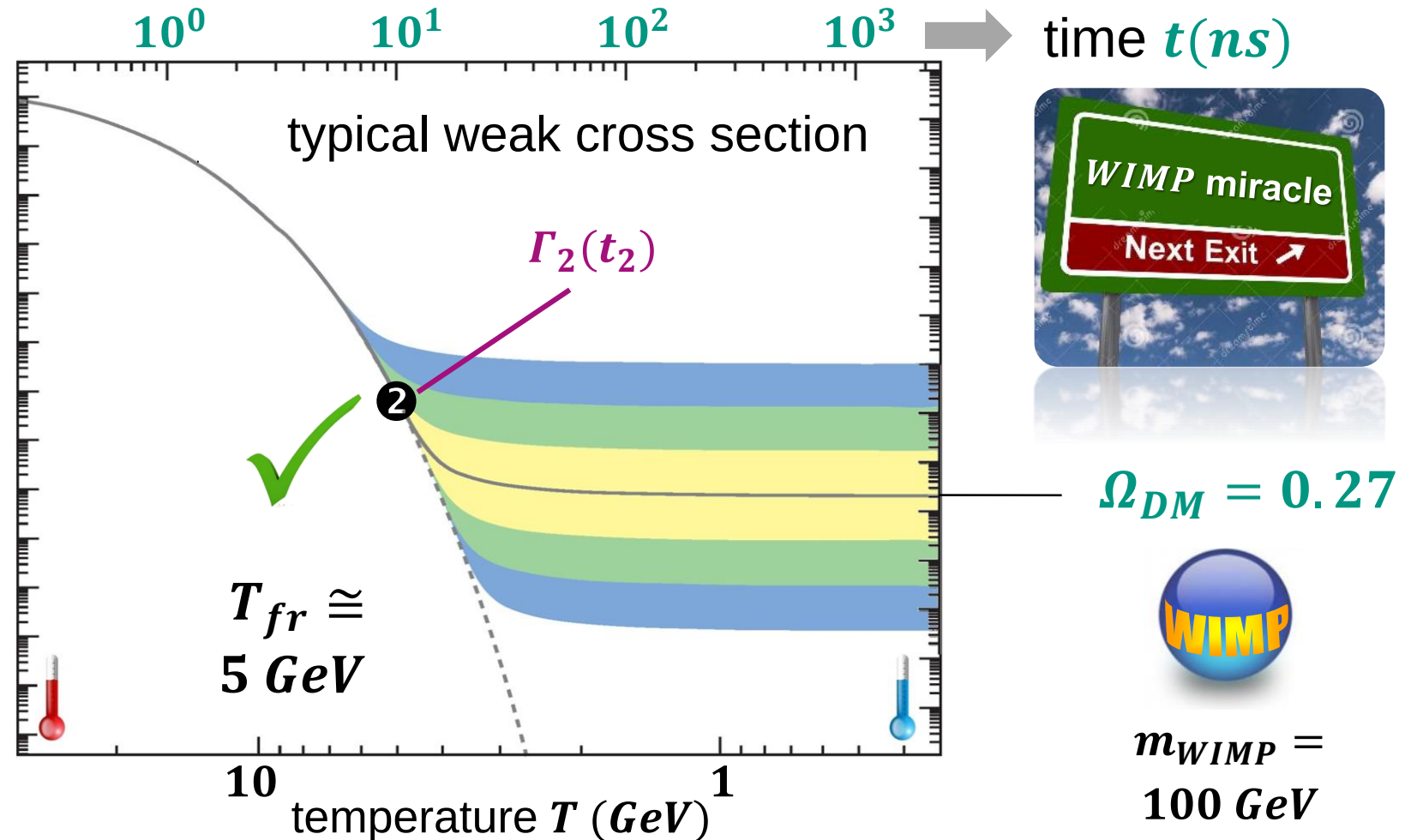
$$x_{fr} = \frac{T_{fr}}{M_{WIMP}} = \frac{1}{20}$$

is **non-relativistic**

due to relation:

$$T_{fr} \ll M_{WIMP}$$

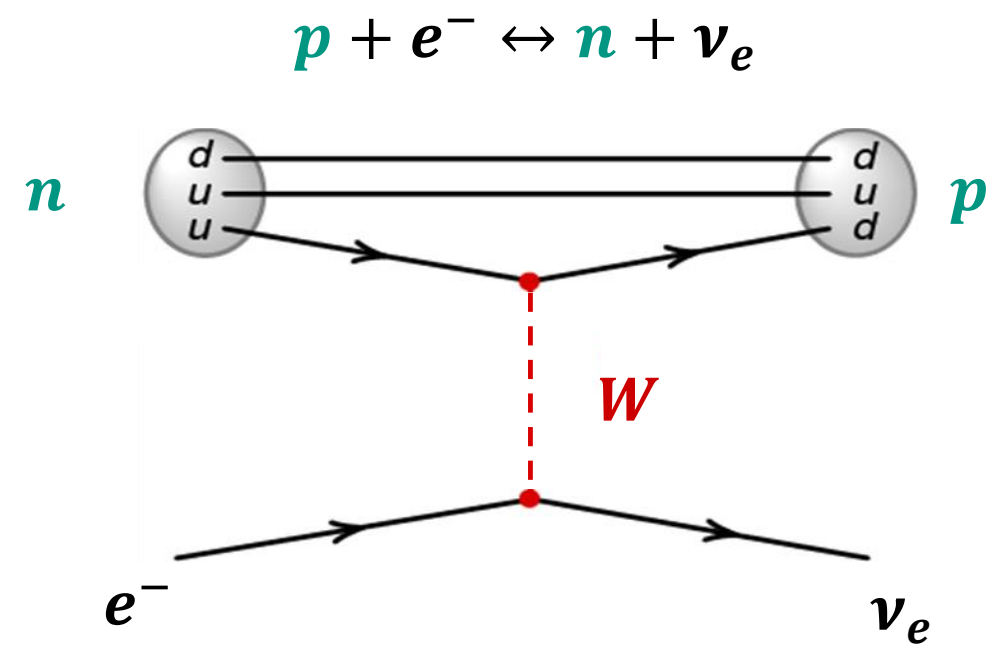
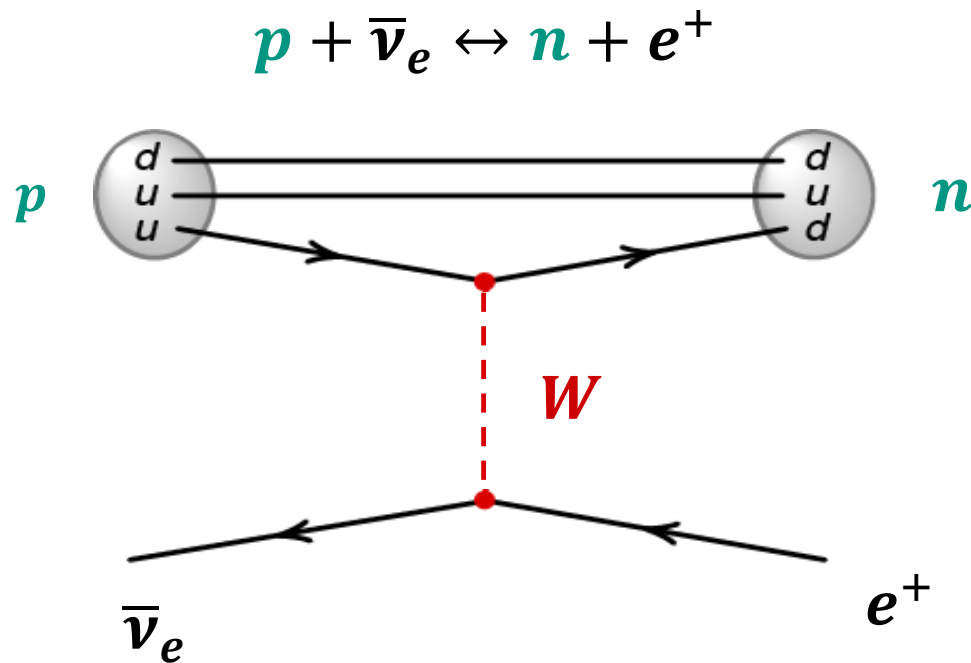
↓
CDM



Thermal Relicts: relativistic neutrinos

■ RECAP: neutrinos remain in thermal equilibrium until $t = 1\text{ s}$

- **semi-leptonic** reactions with protons, neutrons via **CC** (**C**harged **C**urrent) and **NC** (**N**eutral **C**urrent) processes: important to fix **n/p – ratio** for **BBN**



Thermal Relicts: relativistic neutrinos as *HDM*

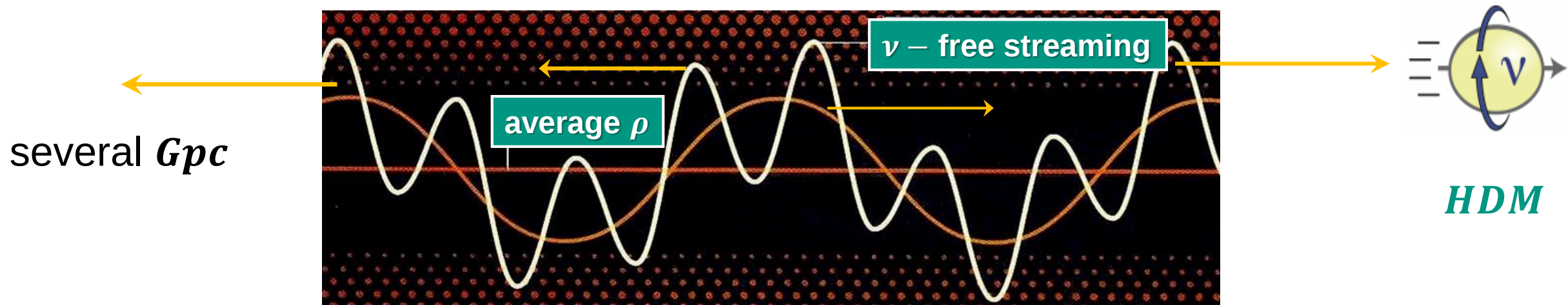
■ neutrinos free-stream in an evolving universe over distances $d \sim Gpc$

- neutrinos decouple after $t = 1\text{ s}$ at freeze-out temperature $T_{fr} \sim MeV$

sub – *eV* mass (*KATRIN* 2022: $m(\nu) < 0.8\text{ eV}$ (90% *CL*))

$\Rightarrow$ resulting Lorentz- $\gamma = 10^6 \dots 10^7$

$\Rightarrow$ free-streaming distance $d \sim Gpc$



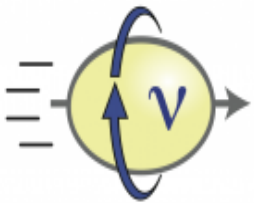
KIT
Karlsruhe Institute of Technology

- $$m(\nu) = meV$$

$$m(\nu) = \text{few } eV$$
several *Gpc*

v – free streaming

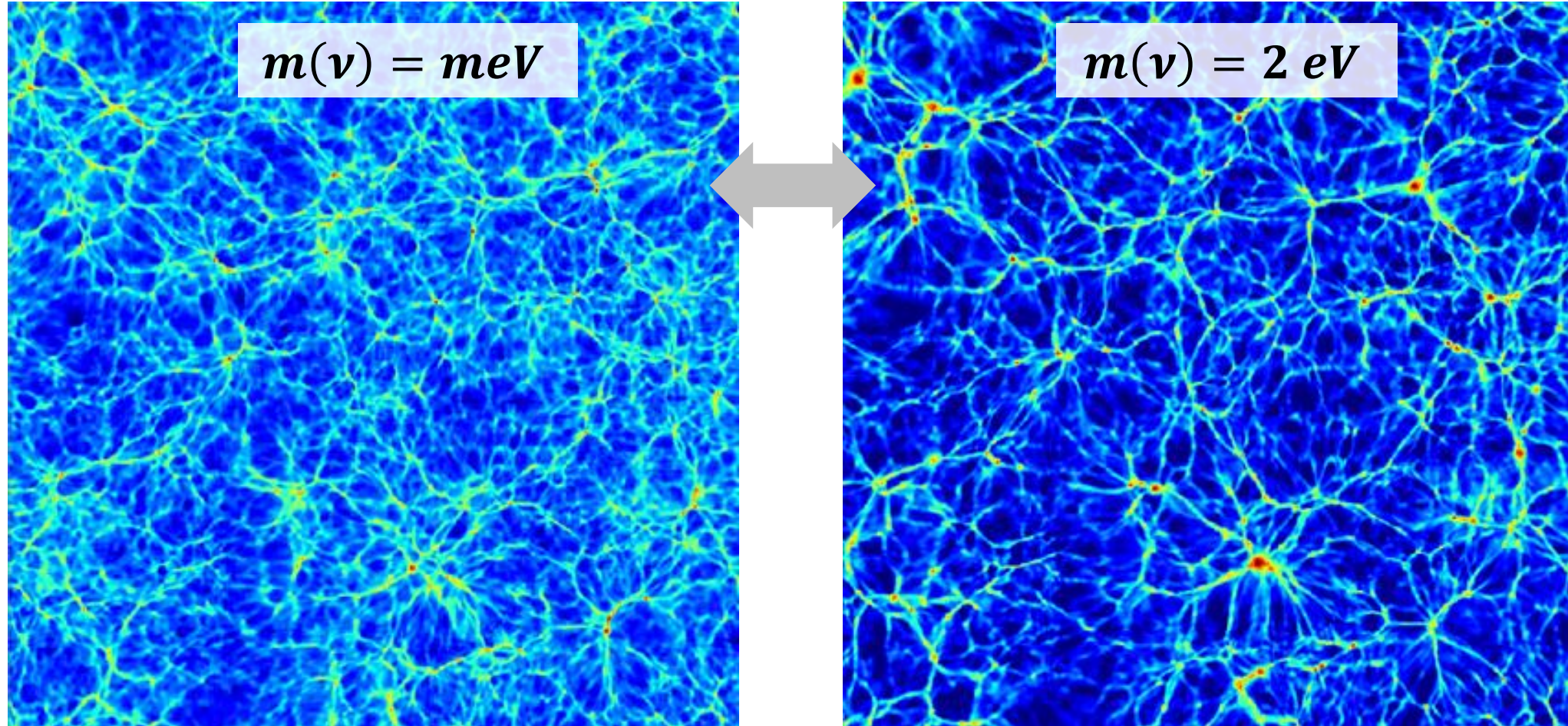
average ρ



HDM

Thermal Relicts: relativistic neutrinos as *HDM*

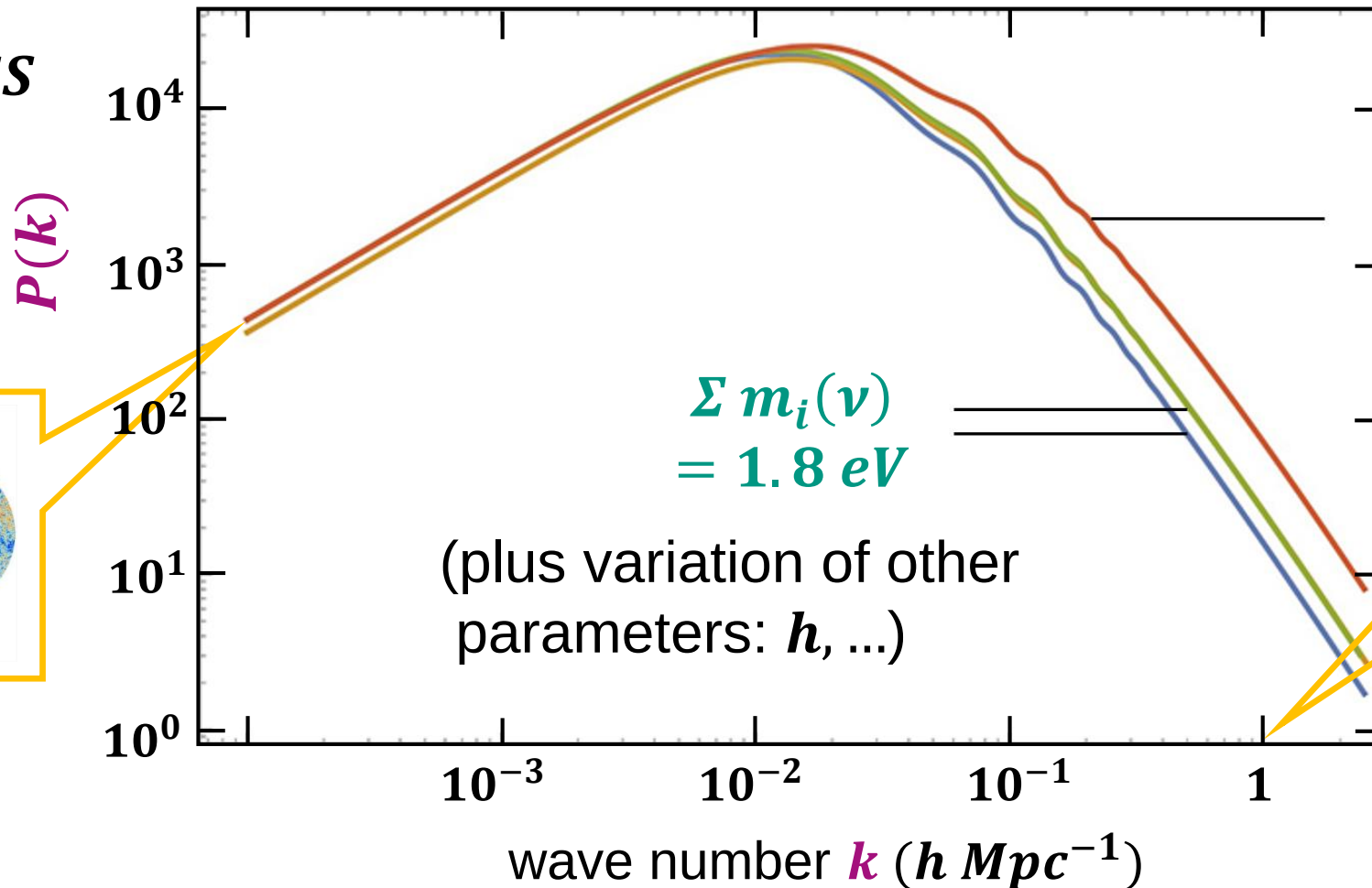
- neutrinos **free-stream**: wash-out of small-scale structures in the universe



Massive ν 's (*HDM*) & matter power spectrum $P(k)$

- Imprint of massive neutrinos on large wave numbers k of spectrum $P(k)$

- *CMB* + *LSS*



Massive ν 's (*HDM*) & matter power spectrum $P(k)$

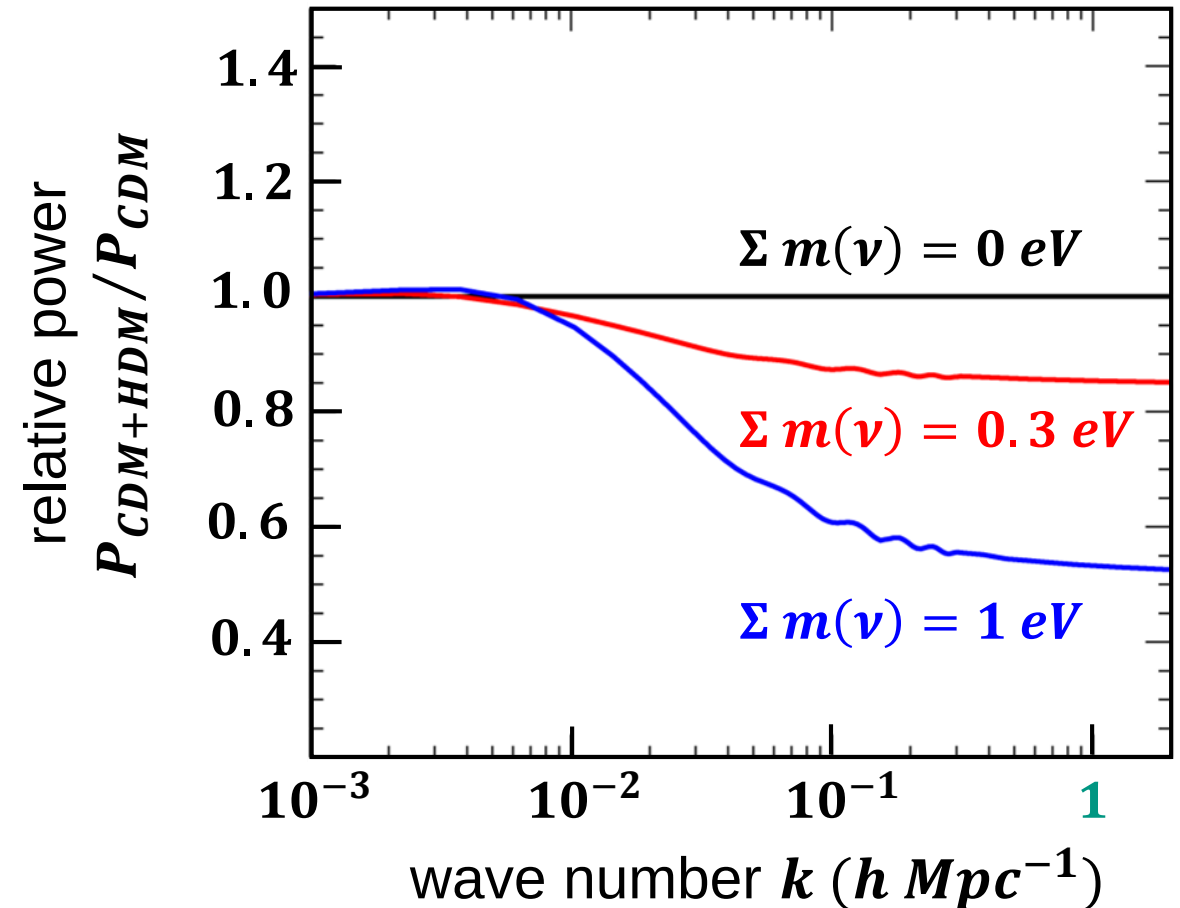
■ Imprint of massive neutrinos on large wave numbers k of spectrum $P(k)$

- adding small amounts of *HDM* to evolution of *LSS* reduces $P(k)$

$$P(k) \sim P_{CDM}(k) \cdot \left(1 - \frac{8 \cdot \Omega_\nu}{\Omega_M}\right)$$

large factor

- example: $\Sigma m(\nu) = 0.3 \text{ eV}$
~ 15 % reduction of power for 'small' structures at
 $k = 1 \text{ h Mpc}^{-1}$



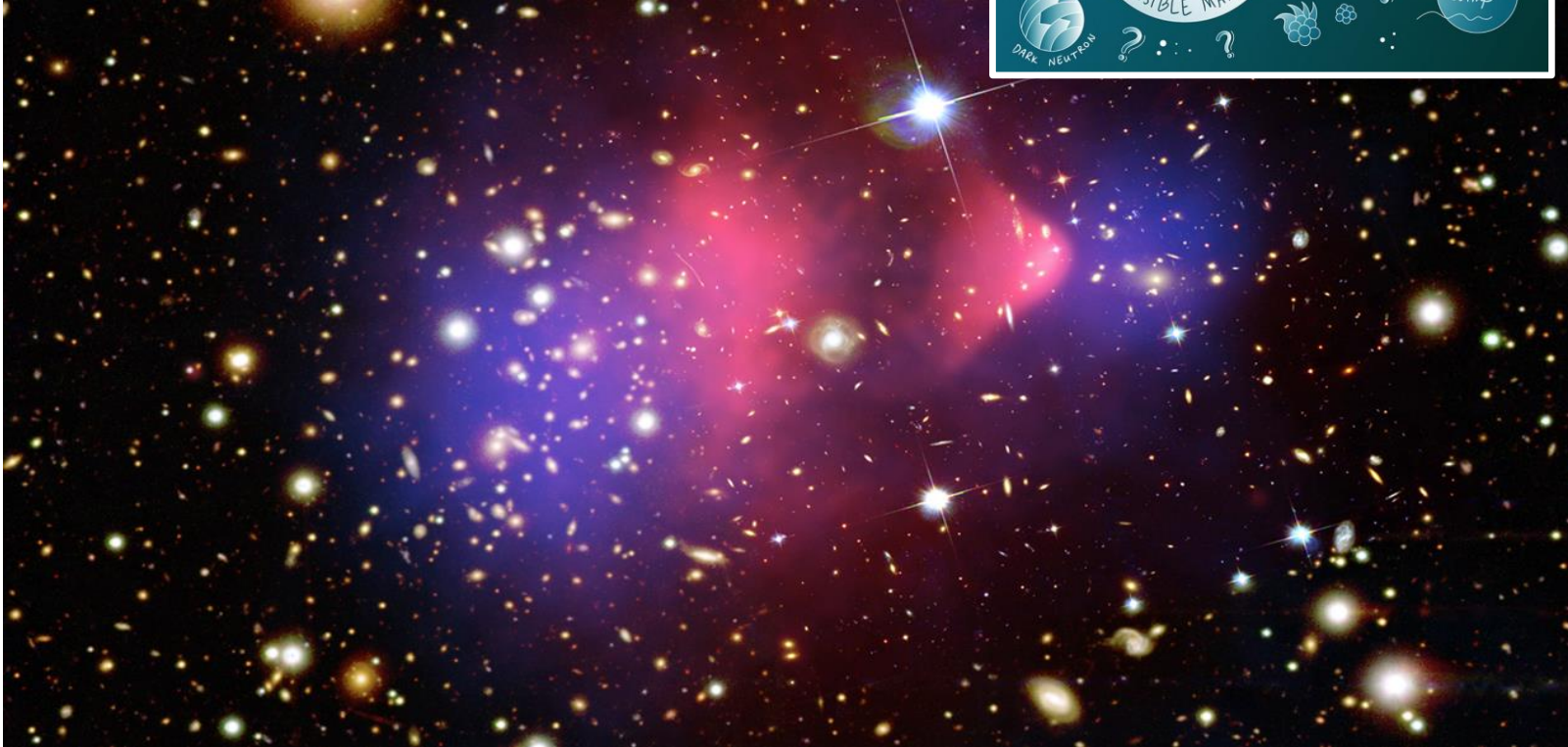
Massive ν 's (*HDM*): mass eigenstates $m_{1,2,3}$

- primordial ν 's have cooled down to $T = 1.9 \text{ K}$ in today's universe
 - neutrinos from *Big Bang* with masses $m \approx 50 \text{ meV}$ today are **bound** gravitationally in galaxy clusters (i.e. on scales $d \approx 50 \text{ Mpc}$)
 - **flavour states** $\nu_{e,\mu,\tau}$ produced up to $t = 1 \text{ s}$: today they have fully 'decoupled' to **mass eigenstates** $\nu_{1,2,3}$ (very long *de Broglie* wavelengths)

$\nu_e =$



incoherent
mass eigenstates
of a former ν_e



CHAPTER 5 – DARK UNIVERSE

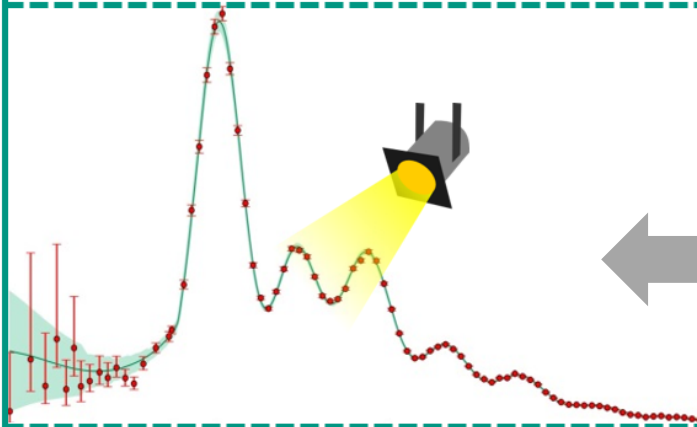


5.1 Evidences for Dark Matter

cosmological scale

method:

- 2./3. acoustic peak of **CMB** multipoles



result:

- (Cold) Dark Matter is dominant $\Omega_{DM} \sim 0.27$

galaxy clusters

method:

- virial theorem (*Zwicky*)
- hot gas, gravit. lensing



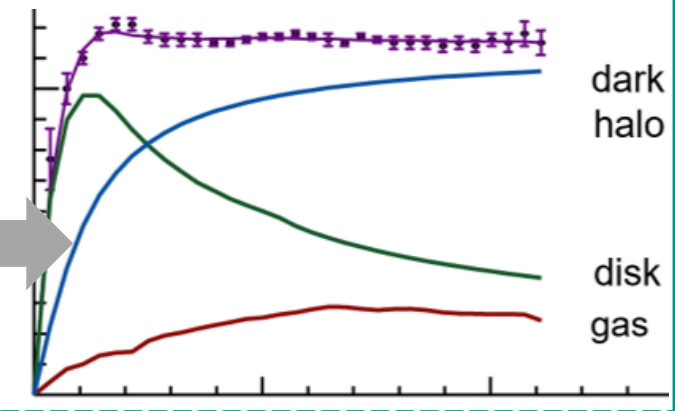
result:

- extended **DM** – halos in galaxy clusters

galaxy (local)

method:

- rotation curves of galaxies (*V. Rubin*)



result:

- dark galactic halo
- local value Ω_{DM}

Gpc

Mpc

Mpc

kpc

Dark Matter & galaxy clusters

■ **Fritz Zwicky** proposes the existence of Dark Matter (from the **Coma cluster**)

- **observation**: (too) high peculiar velocities of single galaxies in the very large **Coma cluster** of galaxies !



Virial theorem:

$$\langle E_{kin} \rangle = -\frac{1}{2} \langle U_{pot} \rangle$$

nichtleuchtende Materie
~ 90% der Masse
im Coma-Cluster...



F. Zwicky

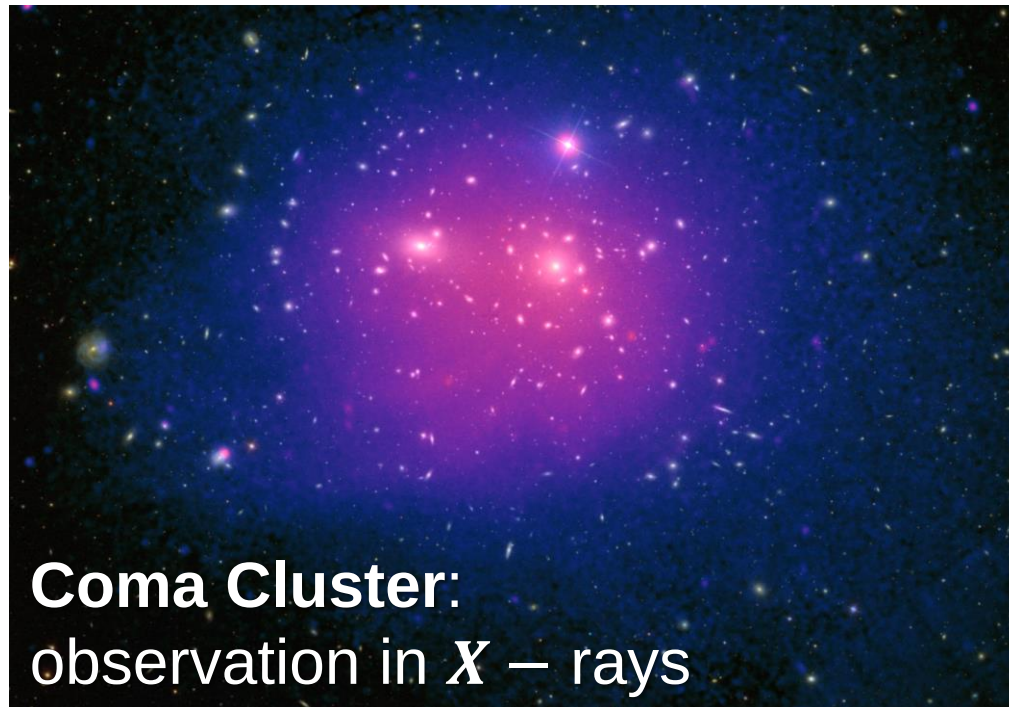
Helv. Phys. Acta 6 110 – 127 (1933)

‘Die Rotverschiebung von
extragalaktischen Nebeln’

Dark Matter & galaxy clusters

■ *Fritz Zwicky* proposes the existence of Dark Matter (from the **Coma cluster**)

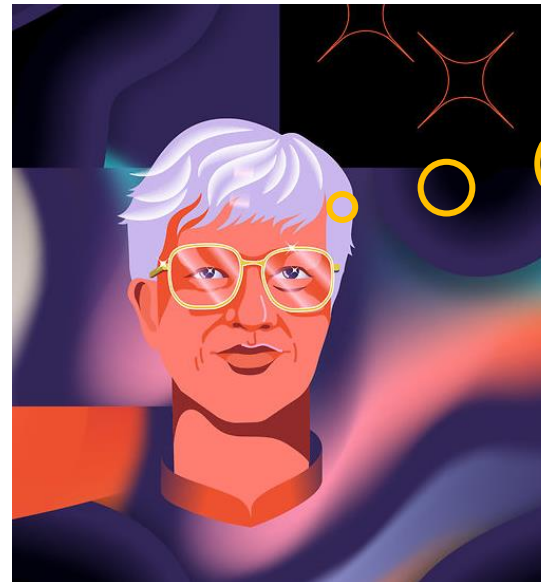
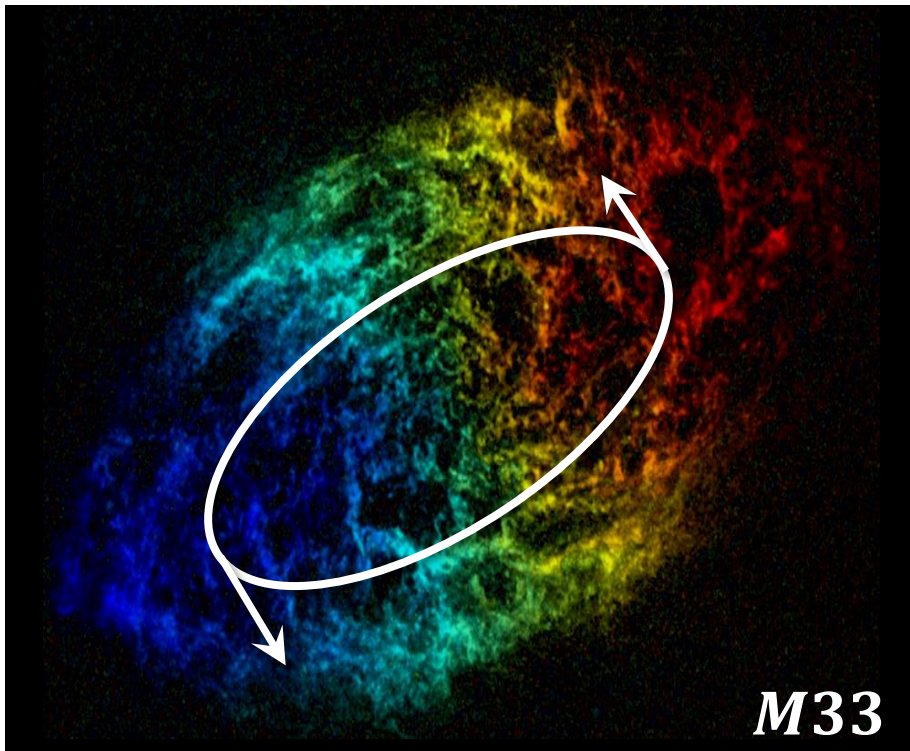
- **explanation**: non-luminous form of matter (**Dark Matter**) which interacts only via its dominant **gravitational potentials**!



Dark Matter & rotational curves of galaxies

■ Vera Rubin observes **flat rotational profiles** of galaxies

- **observation**: (too) high velocities of **single stars** & **gas clouds** in the very large **Andromeda spiral galaxy**!

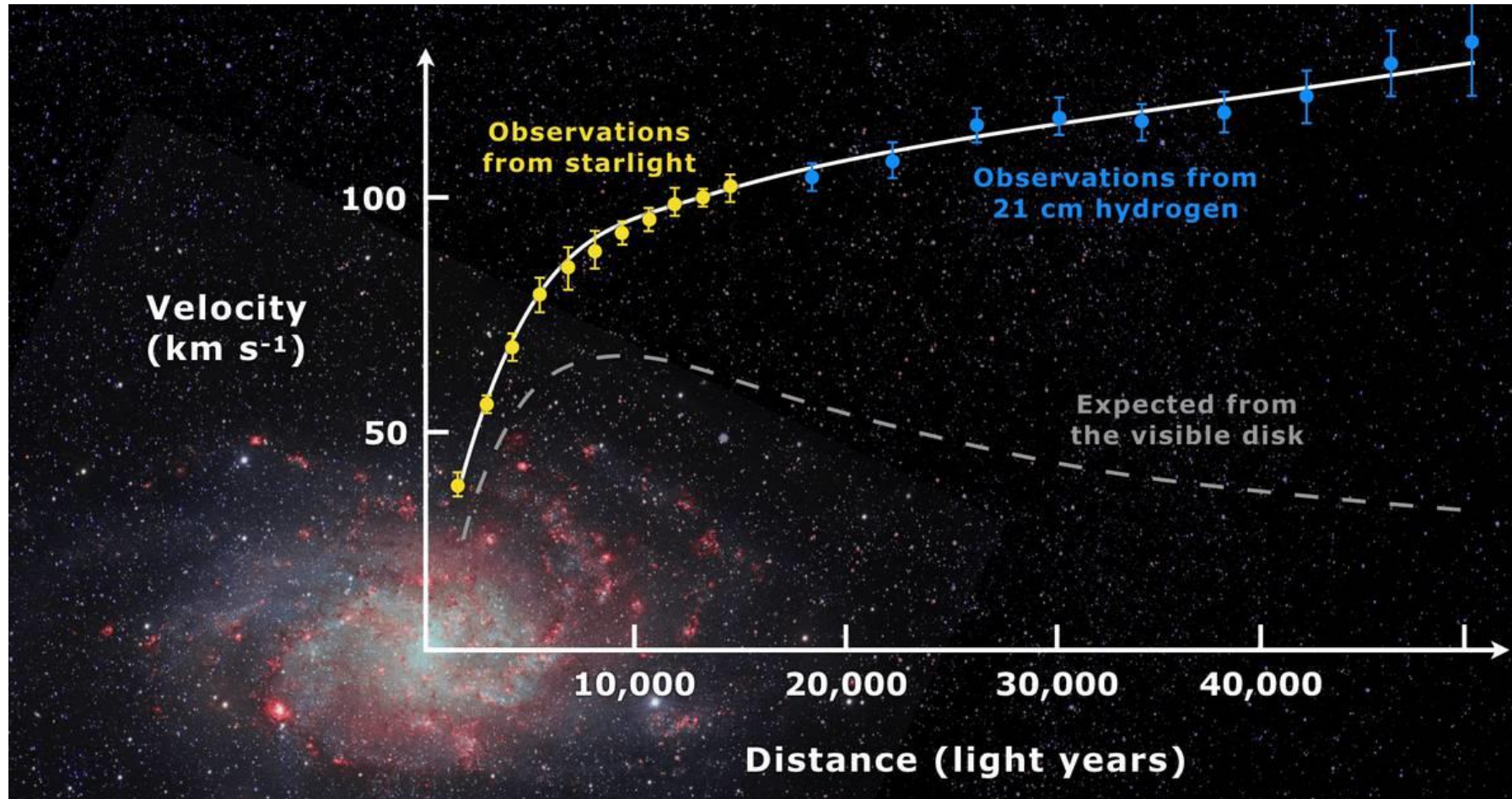


non-luminous matter
~90% of the mass
in galaxies...

[Vera Rubin, giant of astronomy | symmetry magazine](#)

Dark Matter & rotational curves of galaxies

■ *Vera Rubin* observes **flat rotational profiles** of galaxies

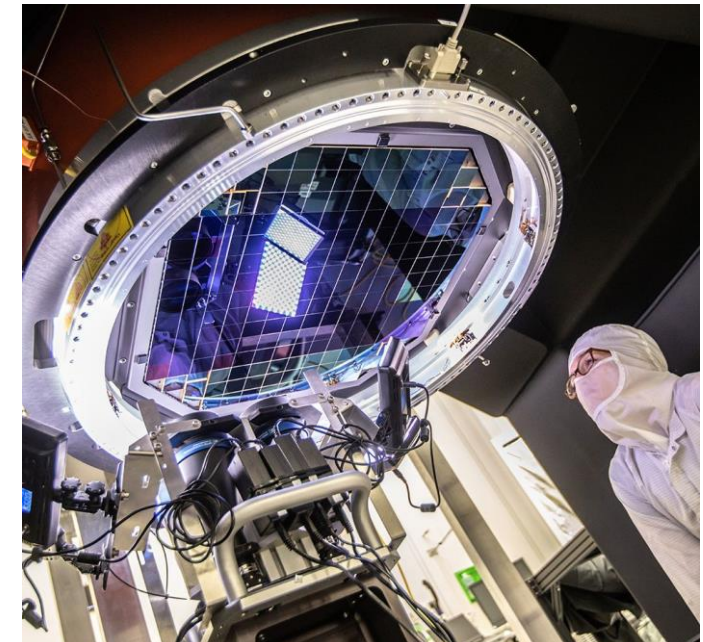


*V. Rubin et al.,
ApJ 159 379 (1970)
‘Rotation of the
Andromeda Nebula’*

Dark Matter – future surveys

■ *Vera Rubin* observatory: Legacy Survey of Space and Time

- 8.4 *m* mirror telescope with 3.2 *giga – pix CCD* (2025 *ff*)



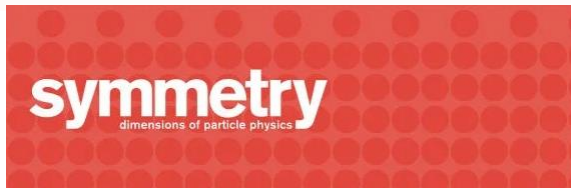
Dark Matter – future surveys

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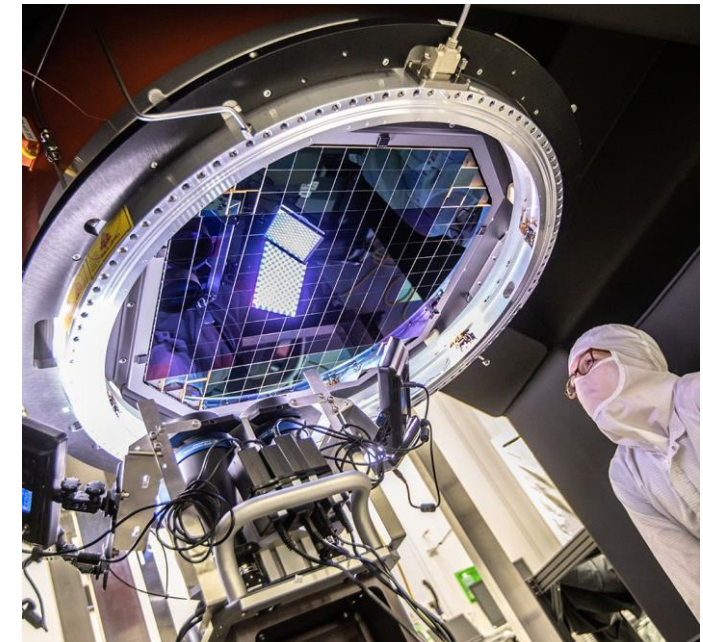
Courtesy of Margaux Lopez



A day in the life of a mountaintop telescope builder

01/18/24 | By Joe Howlett

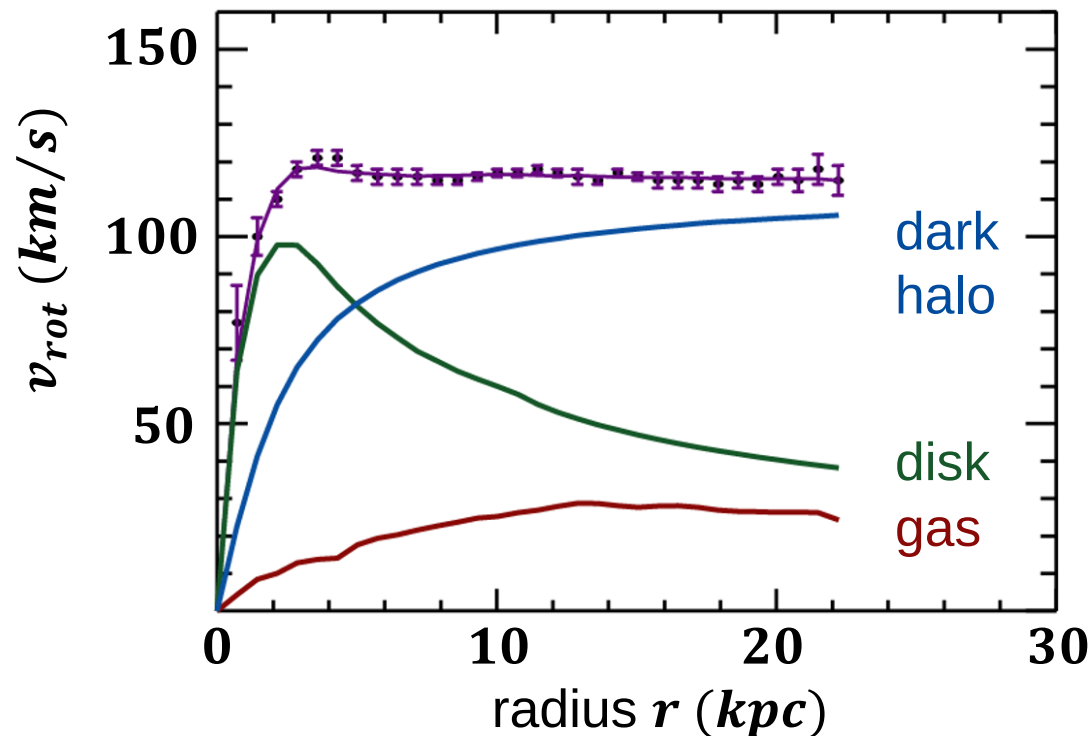
Margaux Lopez is one of a team of engineers preparing the Vera Rubin Observatory in Chile for the arrival of the largest digital camera ever built for astrophysics and cosmology.



Dark Matter & flat rotational curves of galaxies

■ Sir Isaac: rotational velocity profile of a galaxy should fall off

- **explanation**: non-luminous form of matter (**Dark Matter**) which interacts only via **gravitational potential**!



$$F = G \cdot \frac{M_r}{r^2} \cdot m = m \cdot a$$

galactic mass enclosed in r

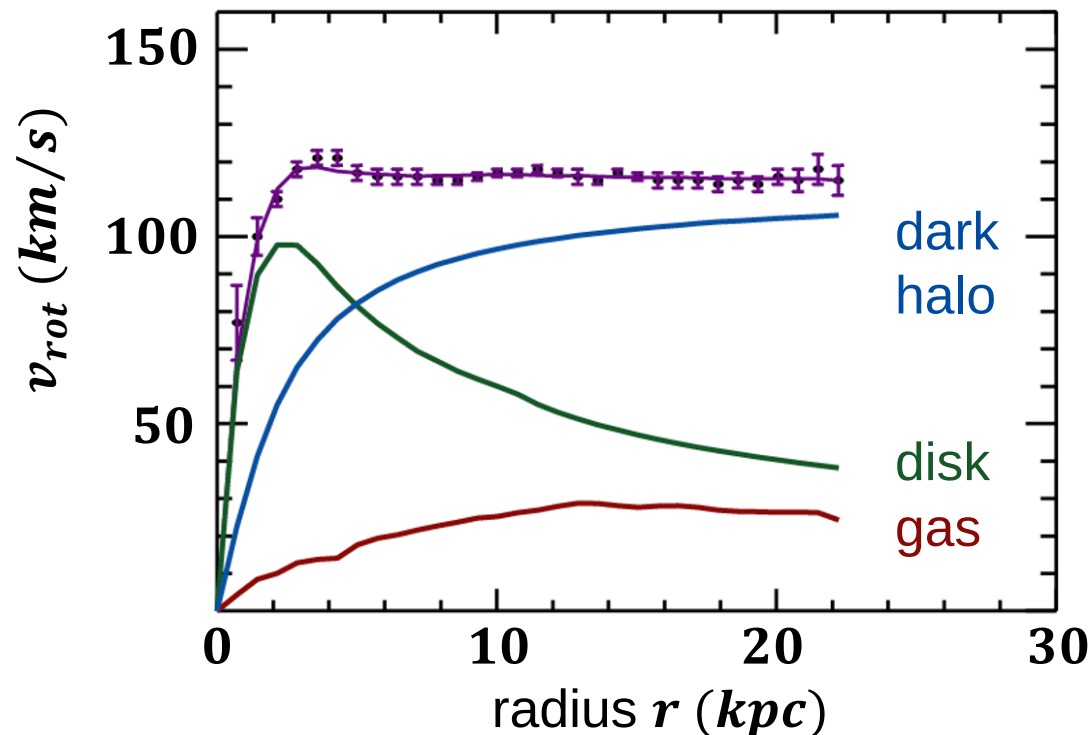
'test mass' (star, gas cloud,...)



Dark Matter & flat rotational curves of galaxies

■ Sir Isaac: rotational velocity profile of a galaxy should fall off as $v_{rot} \sim 1/\sqrt{r}$

- **explanation**: non-luminous form of matter (**Dark Matter**) which interacts only via **gravitational potential**!



- expected rotation curve for stars

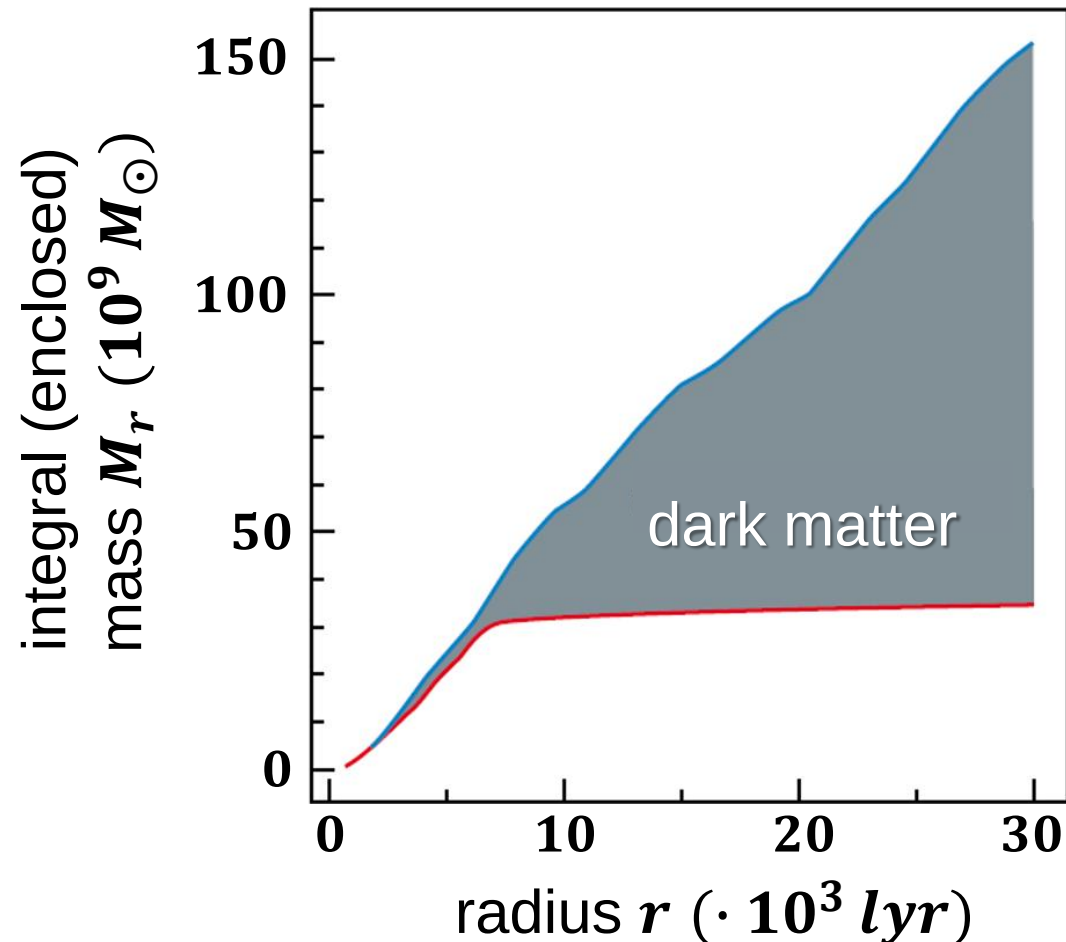
$$a = G \cdot \frac{M_r}{r^2} = \frac{v_{rot}^2}{r}$$

$$\Rightarrow v_{rot}(r) = \sqrt{\frac{G \cdot M_r}{r}}$$

- falling curve expected (if there is **no DM**)

Flat rotational curves reveal a Dark Matter halo

- Today's observations: **linear increase** of enclosed mass M_r up to $r = 50 \text{ kpc}$



$$v_{\text{rot}}(r) = \text{const.}$$

$$\Rightarrow M_r \sim r$$

$$\Rightarrow \rho(r) \sim \frac{1}{r^2}$$

linear increase of enclosed mass M_r

halo profile

- **DM – halo** with
(80 ... 90)% of entire mass



based on validity of
Newtonian Gravity

DM halo: characteristic $1/r^2$ density profile

- Today's observations: **linear increase** of enclosed mass M_r up to $r = 50 \text{ kpc}$



$$v_{\text{rot}}(r) = \text{const.}$$

$$\Rightarrow M_r \sim r$$

linear increase of
enclosed mass M_r

$$\Rightarrow \rho(r) \sim \frac{1}{r^2}$$

halo profile

- **DM – halo** with
(80 ... 90)% of entire mass

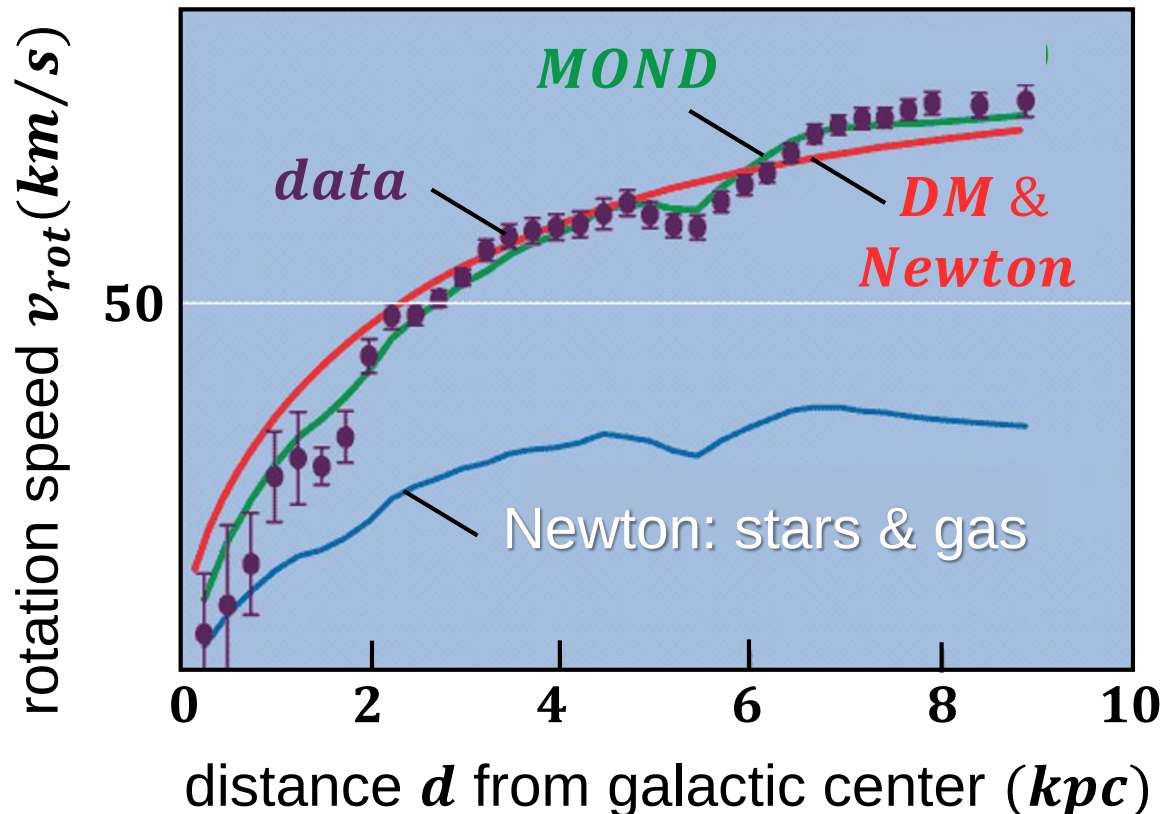


based on validity of
Newtonian Gravity

DM 'alternative': the 'ad hoc' MOND theory

■ *MOND*: *MO*dified *N*ewton *D*ynamics - a rather unlikely *DM* 'competitor'

- galactic **rotation profiles** can be 'reproduced' by **modifying Newtonian gravity**



$$F = m \cdot \mu\left(\frac{a}{a_0}\right) \cdot a$$

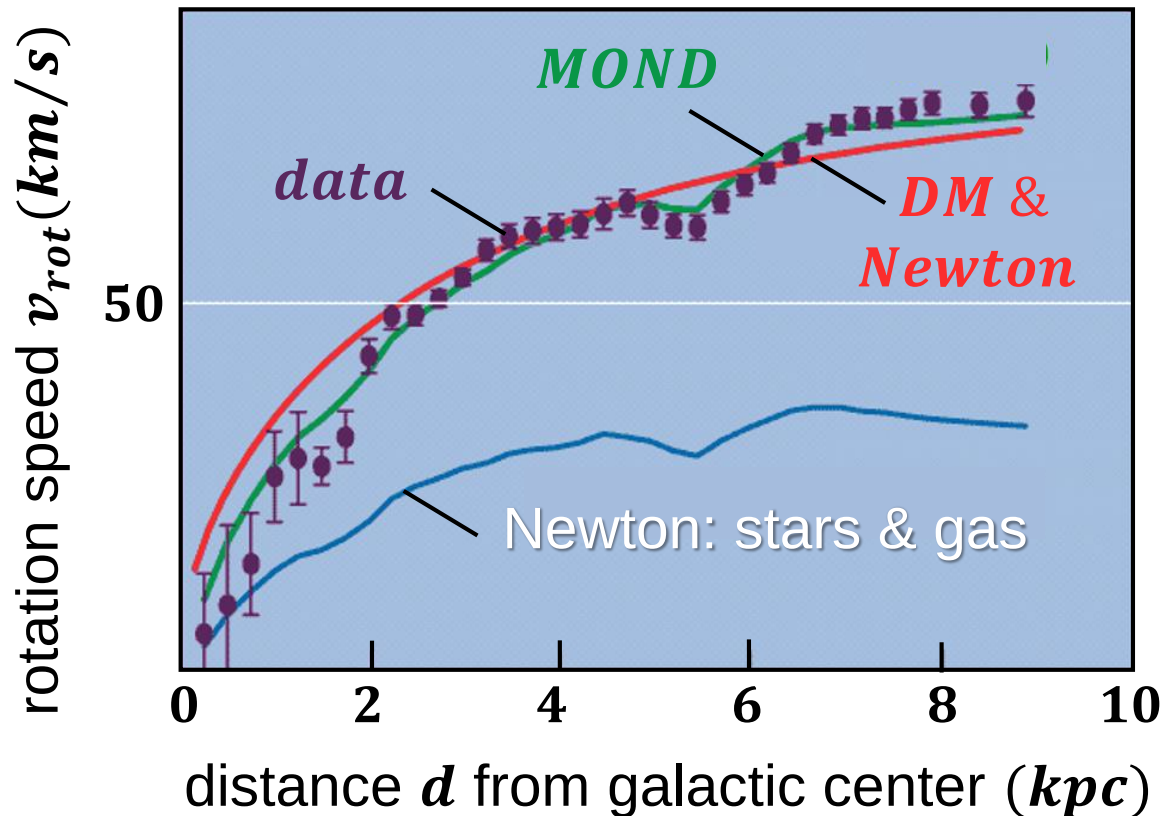
- case #1: $a/a_0 \ll 1$: $\mu = \frac{a}{a_0}$
- case #2: elsewhere: $\mu = 1$
- introduction of 'fundamental acceleration'

$$a_0 \approx 1.2 \cdot 10^{-10} \text{ m s}^{-2}$$

MOND theory: a one-trick pony ?

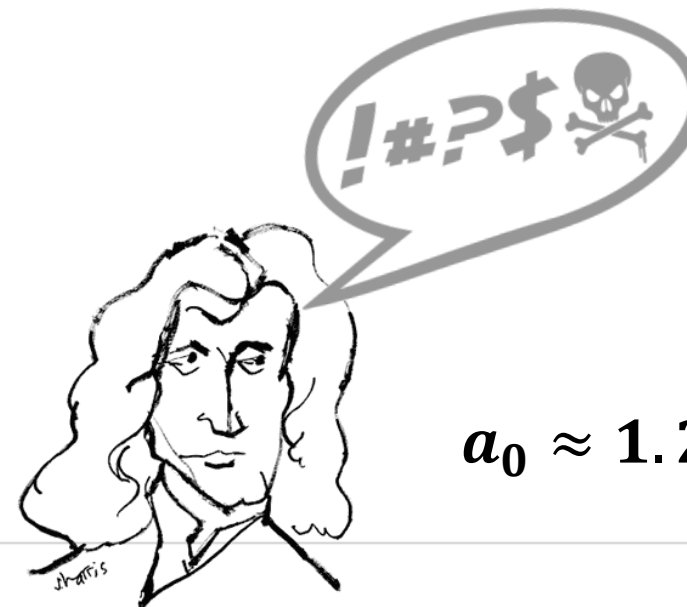
■ *MOND*: *M*odified *N*ewton *D*ynamics as an 'alternative' to Dark Matter

- galactic rotation profiles can be 'reproduced' by modifying Newtonian gravity



- an '*ad hoc*' theory only (so far)

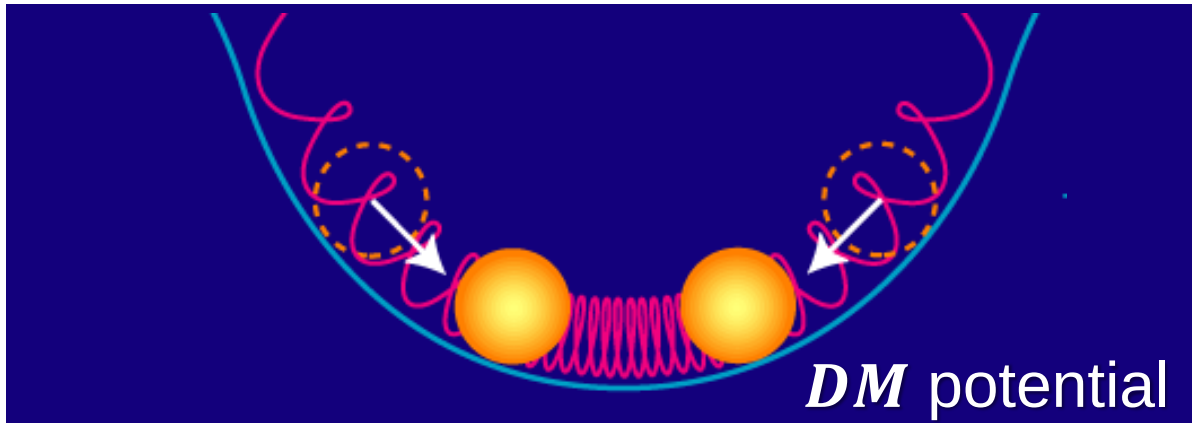
- *MOND* cannot explain galaxy clusters, or *BAO*, or...



$$a_0 \approx 1.2 \cdot 10^{-10} \text{ m s}^{-2}$$

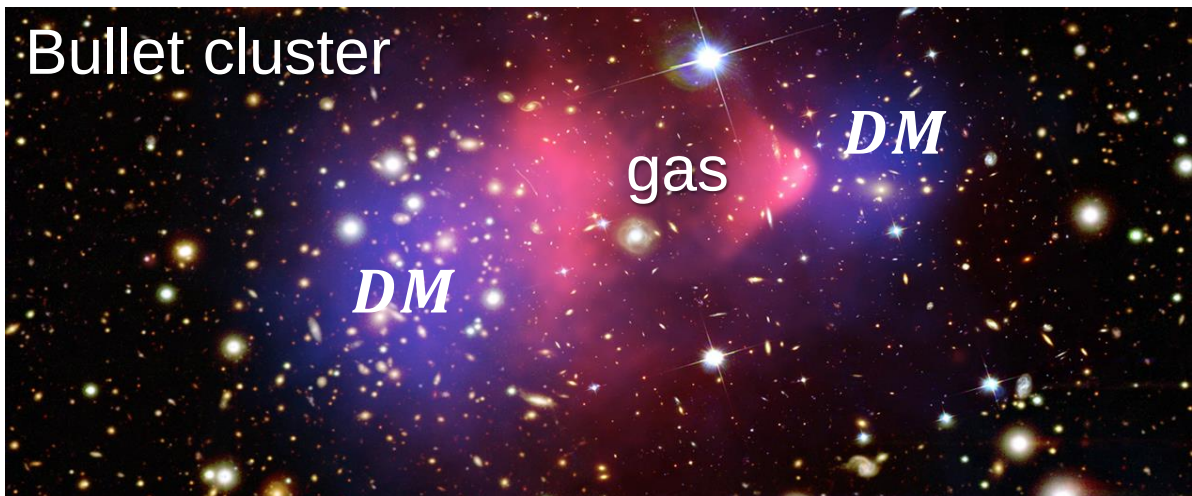
Rotational Curves & *BAO*: irrefutable proof of *DM*

- *MOND* theory: you may fit rotation curves but **fail to describe *BAO*, clusters,...**



MOND theory not compatible with

a) *Baryon Acoustic Oscillations* via gravitational potential by **Dark Matter**

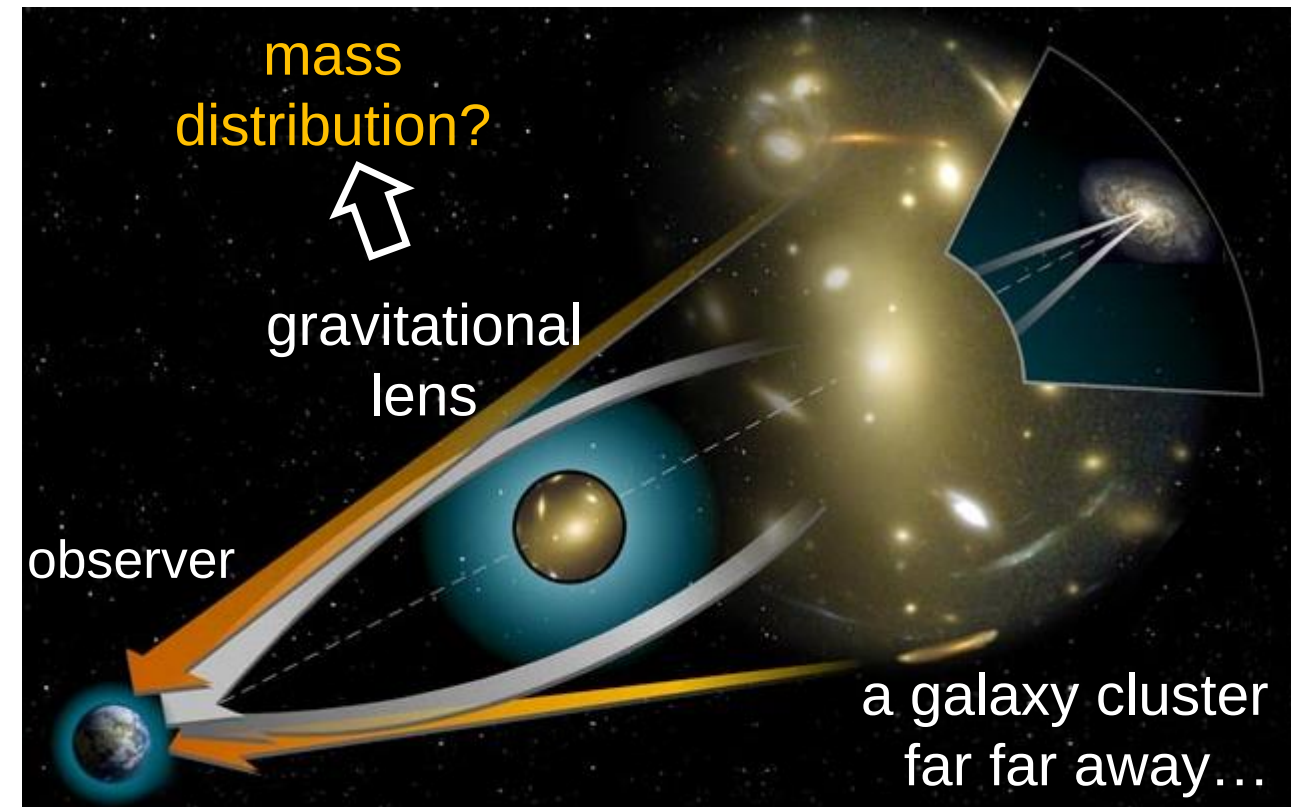


b) **Bullet cluster**
collision of **two galaxy clusters**:
separation of baryons (hot cluster gas) from **Dark Matter** (made visible by gravitational lensing)

5.2 Gravitational Lenses

■ Revealing the presence of *DM* via the process of **gravitational lensing**

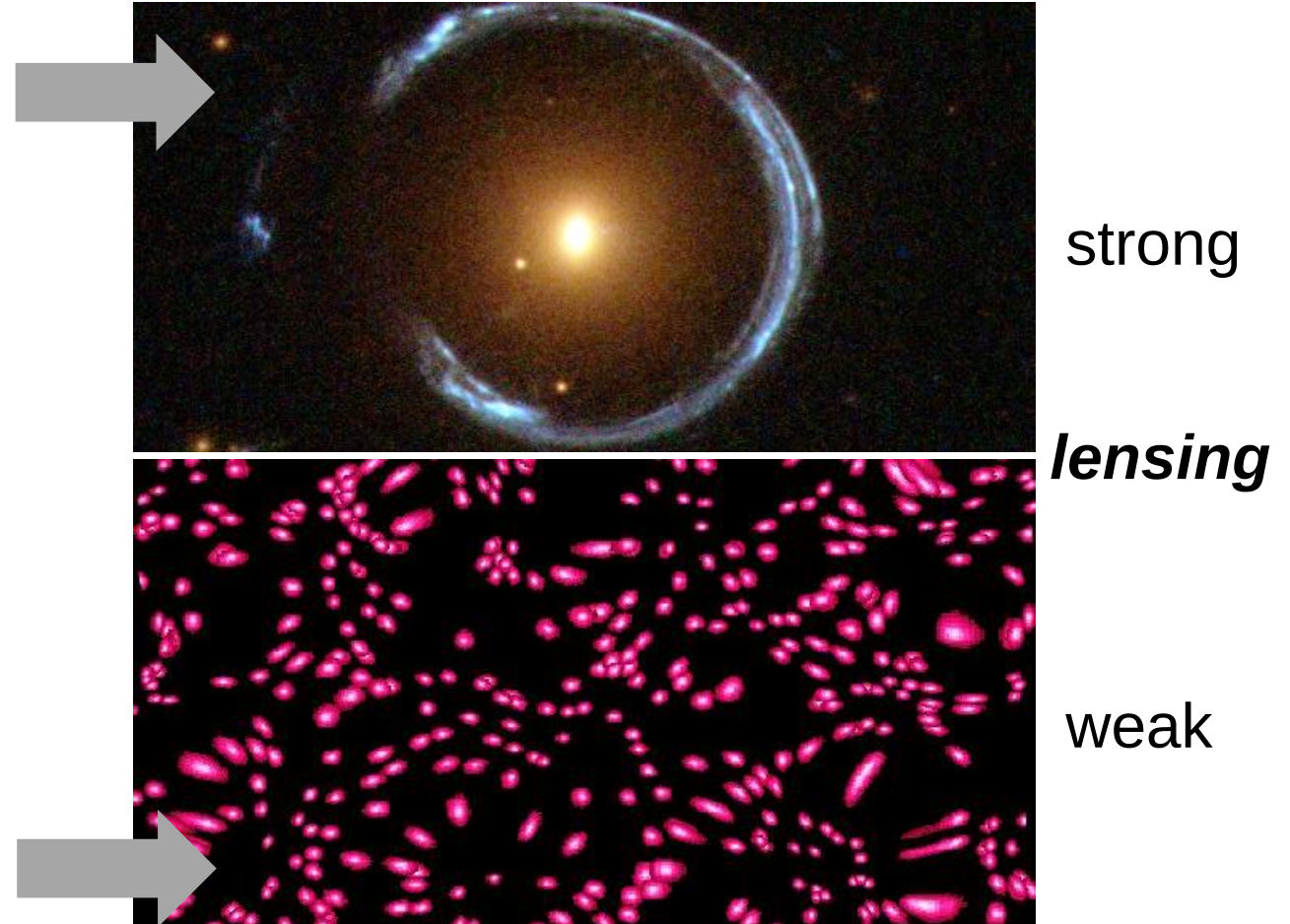
- **A. Einstein**: light is propagating along **geodesic lines**
- **gravitational lenses**:
distortion of the optical imaging due to gravitational potentials can be used to derive **mass distribution of large objects** (galaxy clusters,...)
- **strong lensing**: arcs, rings, multiple images of far-off galaxies/quasars
- **weak lensing**: **statistical** distortion of images of single galaxies



Strong and weak gravitational lensing

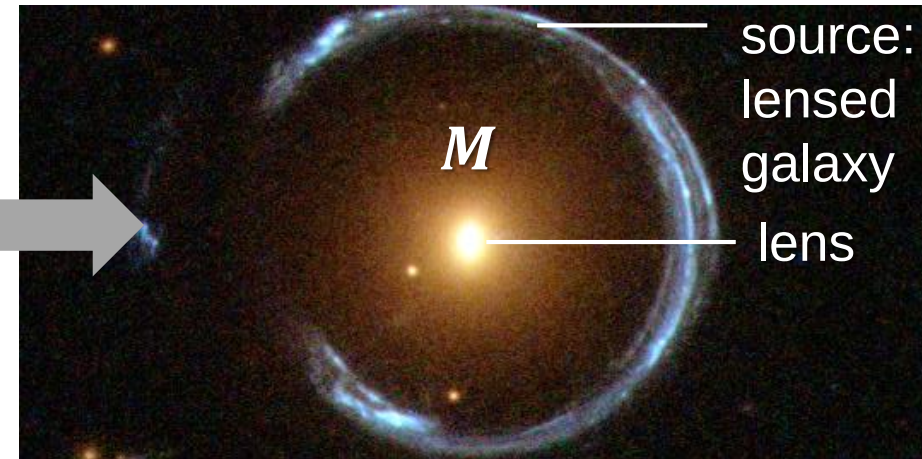
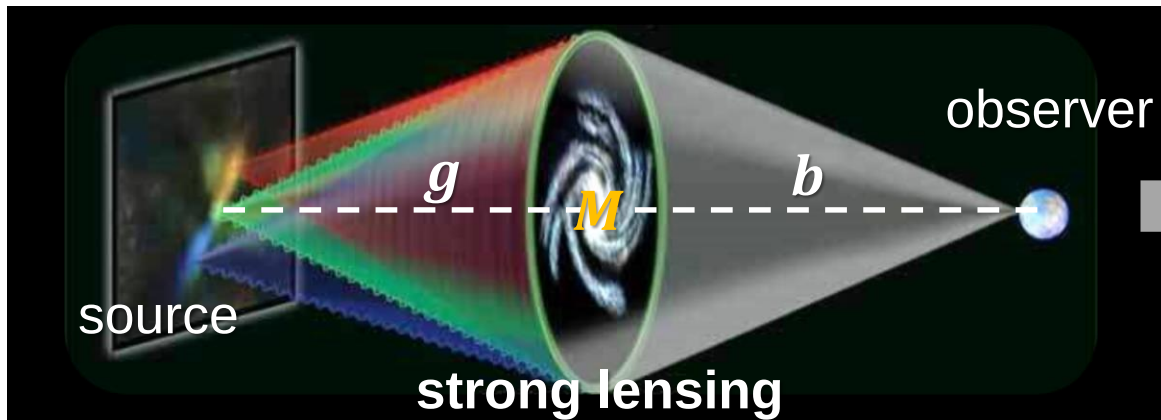
■ Important techniques to **map out regions of Dark Matter**: strong & weak

- we can 'see' where **Dark Matter** is



Strong gravitational lensing

- An ideal method to map the spatial distribution of DM on galactic scales



thin lens
formula

$$\frac{1}{g} + \frac{1}{b} = \frac{1}{f}$$

g : distance of source

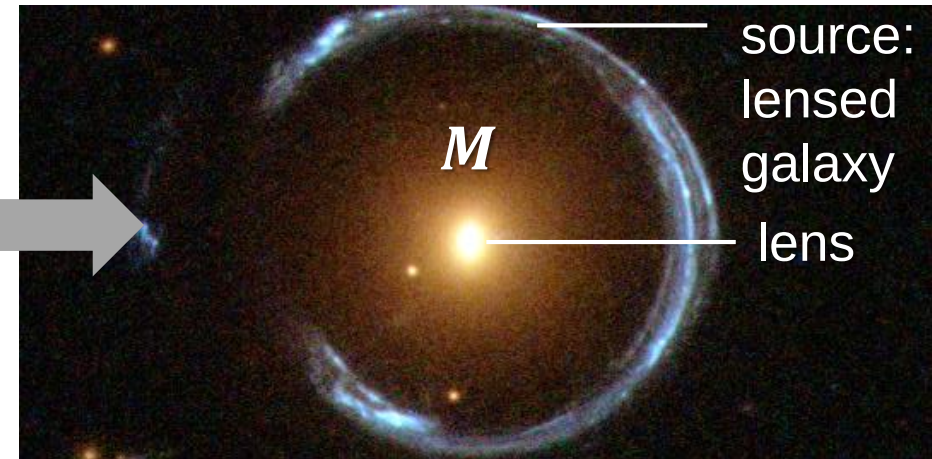
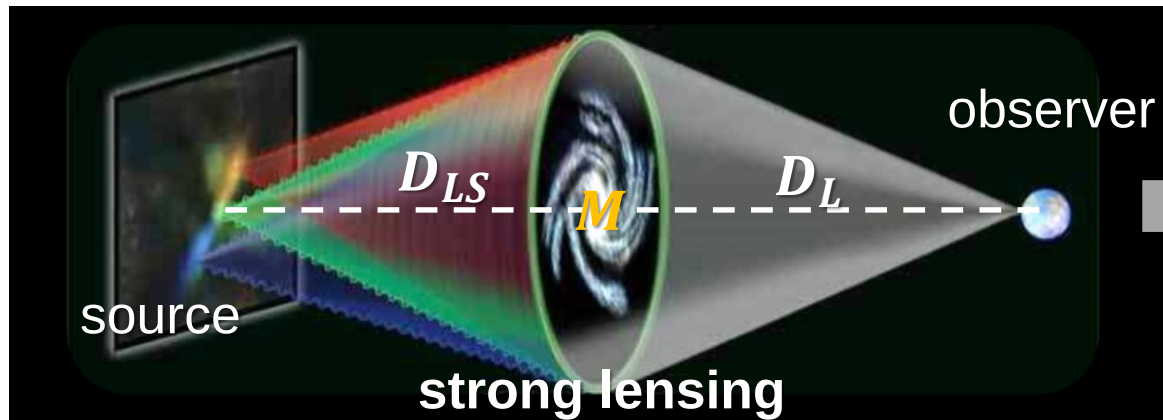
b : distance of observer

f : focal length

- perfect alignment of source, lens & observer:
we see an Einstein ring with an opening angle θ_E

Strong gravitational lensing

- An ideal method to map the spatial distribution of DM on galactic scales



lensing
formula

$$\theta_E = \sqrt{\frac{4 G \cdot M}{c^2} \cdot \frac{D_{LS}}{D_S \cdot D_L}}$$

M : Mass of the lens (shows presence of DM)

$D_S = D_{LS} + D_L$ (source – observer)

- perfect alignment of source, lens & observer
- ~ 70 Einstein rings/arcs observed:
always considerable amount of DM