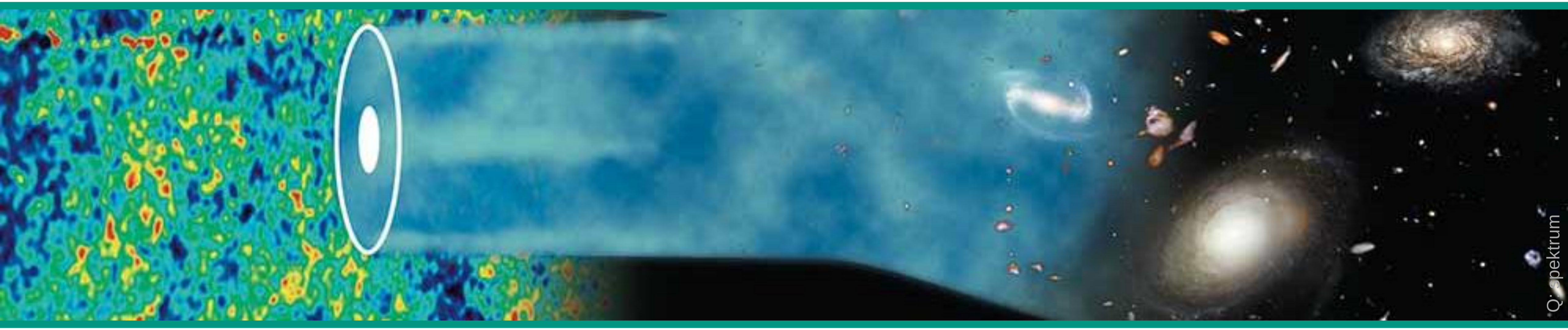
Introduction to Cosmology

Winter term 23/24 Lecture 3 Nov. 7, 2023



Recap of Lecture 2

- **Cosmology 101 (basic concepts)**

  - cosmological **building blocks**: galaxy – **groups**, – **clusters**, – **superclusters**

  - **3** pillars of the Big Bang: Hubble expansion, $\boldsymbol{BBN}$*, **3** $\boldsymbol{K}$ radiation ($\boldsymbol{CMB}$)

  - **Hubble expansion**: **systematic effects** due to proper motion of galaxies,...

  - **Hubble ´tension´**: different values for **Hubble parameter** $\boldsymbol{H_0}$

  - **dimensionless Hubble parameter** $\boldsymbol{h}$: $\boldsymbol{h^2 \sim 0.5}$

  - **cosmological ´mis–understandings´**: ⇨ expansion of **space–time**!

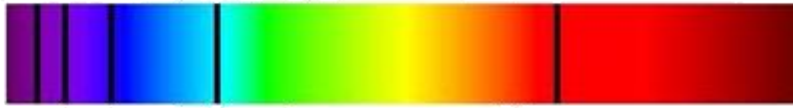
*$\boldsymbol{B}$ig $\boldsymbol{B}$ang $\boldsymbol{N}$ucleosynthesis

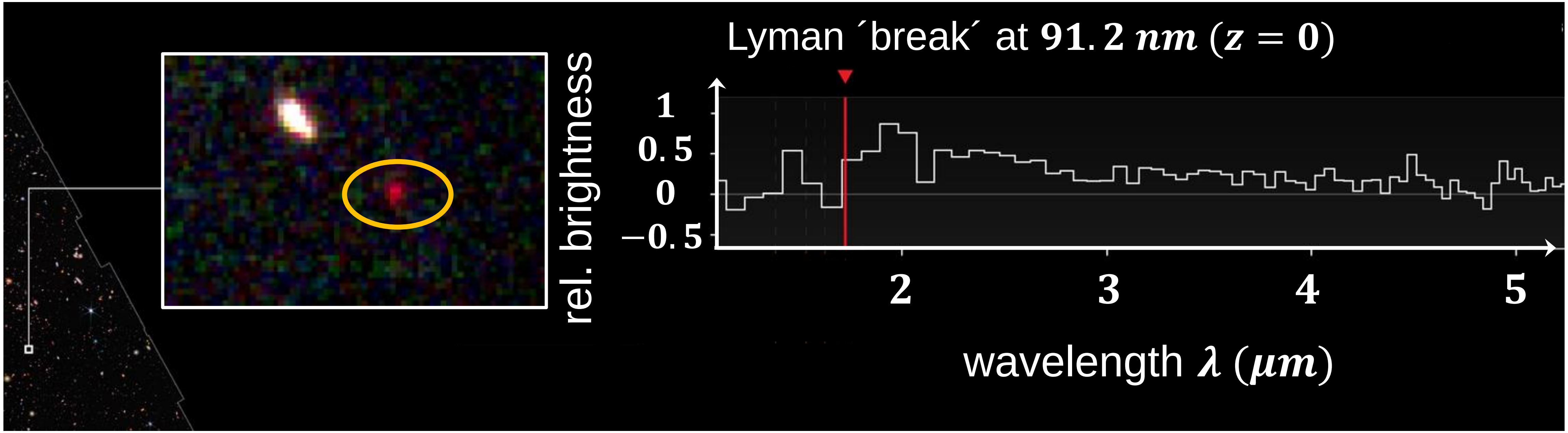
Cosmological redshift z: current record holder

- $JADES - GS - z13 - 0$: galaxy at measured cosmological redhift $z = 13.2$

  - James Webb Space Telescope ($JWST$) $10/2022$:
    galaxy–spectroscopy with $z = 13.2$

  - distance $D = 10.3\ Gpc$ $(33.6 \cdot 10^9\ ly)$

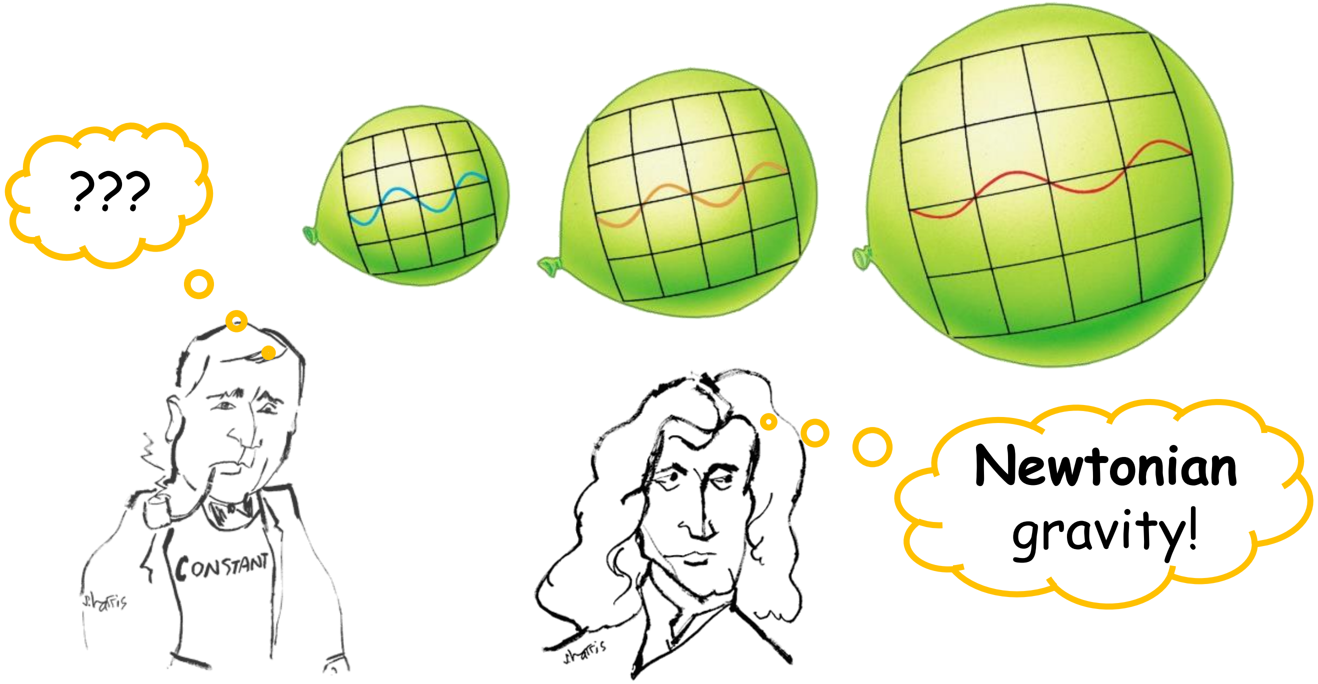




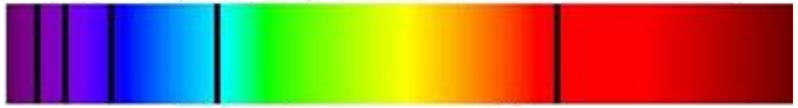
Cosmological redshift z: current record holder

- $JADES - GS - z13 - 0$: **galaxy at measured cosmological redhift** $z = 13.2$

  - for an object with $z = 13.2$, how do you calculate its **distance** D & the **time** t that it is observed after the ***Big Bang***?



Edwin Hubble *Sir Isaac Newton*



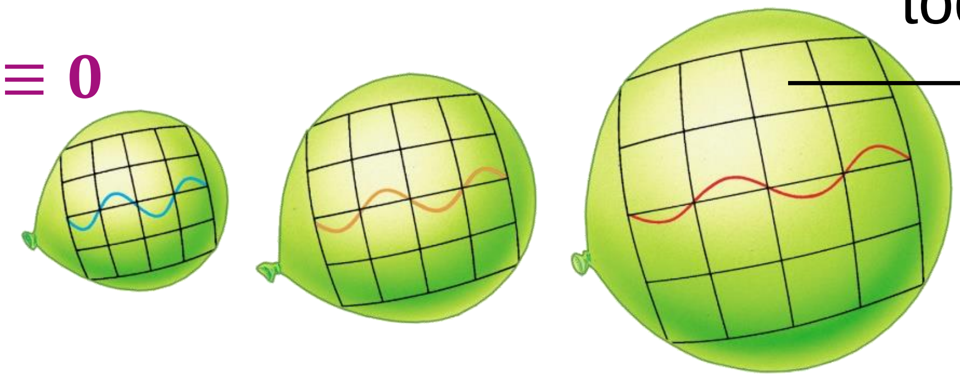
Hubble expansion: the scale factor $a(t)$

- **A dimensionless parameter for the size/scale of the universe**

  - **definition of the scale factor $a(t)$**

$a(t=0) \equiv 0$

Big Bang



today

present universe: time $t = t_0$

scale factor $a(t_0) \equiv 1$

$$a(t) = [0, 1]$$

  - fundamental assumption: **isotropic** & **homogeneous** universe

*F*riedmann – *L*emaître – *R*obertson - *W*alker (*FLRW*) metrics

Hubble expansion: scale factor $a(t)$ & redshift z

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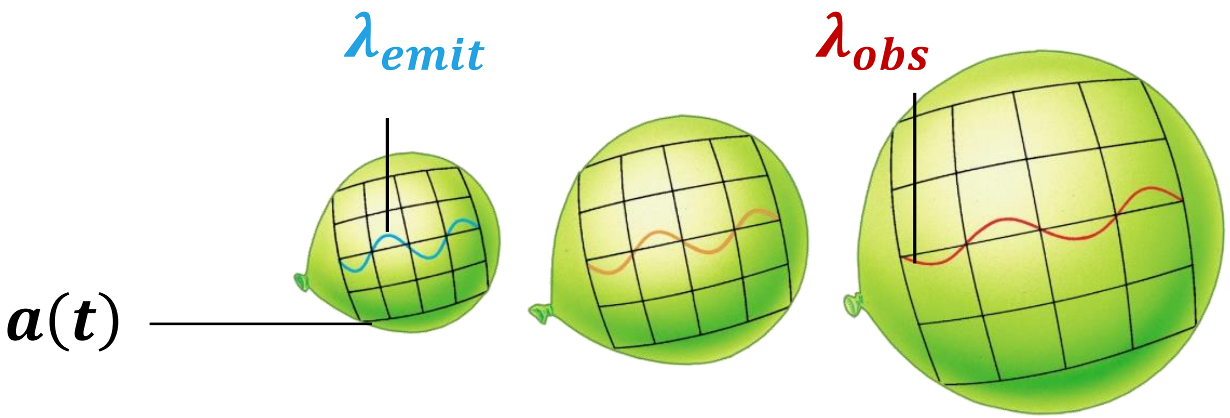
- **Relation between scale parameter $a(t)$ & photon wavelength λ**

  - **key: scale factor $a(t)$** also describes the **increase of λ of a (CMB) photon**



present universe:

scale factor $a(t_0) = 1$

$$z(t) = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}}$$

- cosmological **Hubble expansion**:

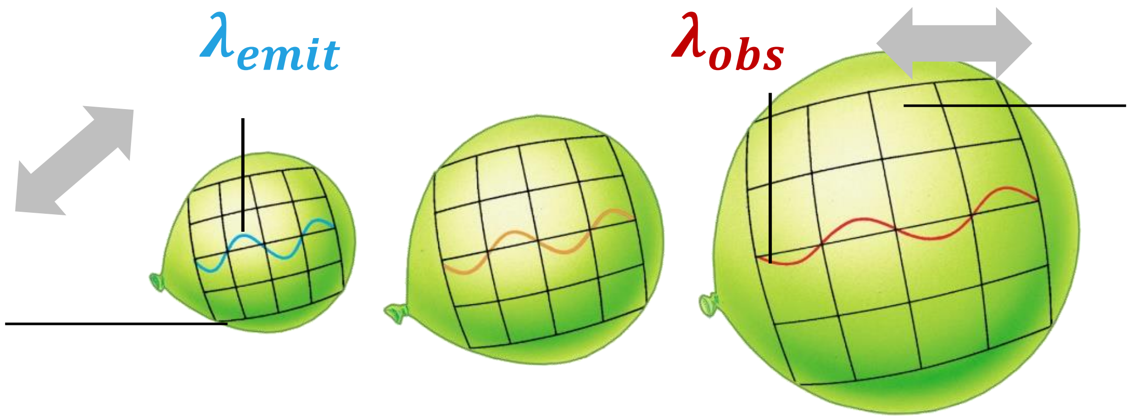
a) expansion of space–time: $a(t)$

b) cosmological redshift: $z(t)$

Hubble expansion: scale factor $a(t)$ & redshift z

- **Relation between scale parameter $a(t)$ & photon wavelength λ**

  - key: measurement of the redhsift $z(t)$ allows to map evolution of $a(t)$



redshift z: an observational means to determine scale $a(t)$

$$z(t) = \frac{\lambda_{obs}}{\lambda_{emit}} - 1 \quad \Longleftrightarrow \quad z(t) = \frac{1}{a(t)} - 1$$

$$\boxed{a(t) = \frac{1}{z(t) + 1}}$$

Redshift $z(t)$ and cosmological time t

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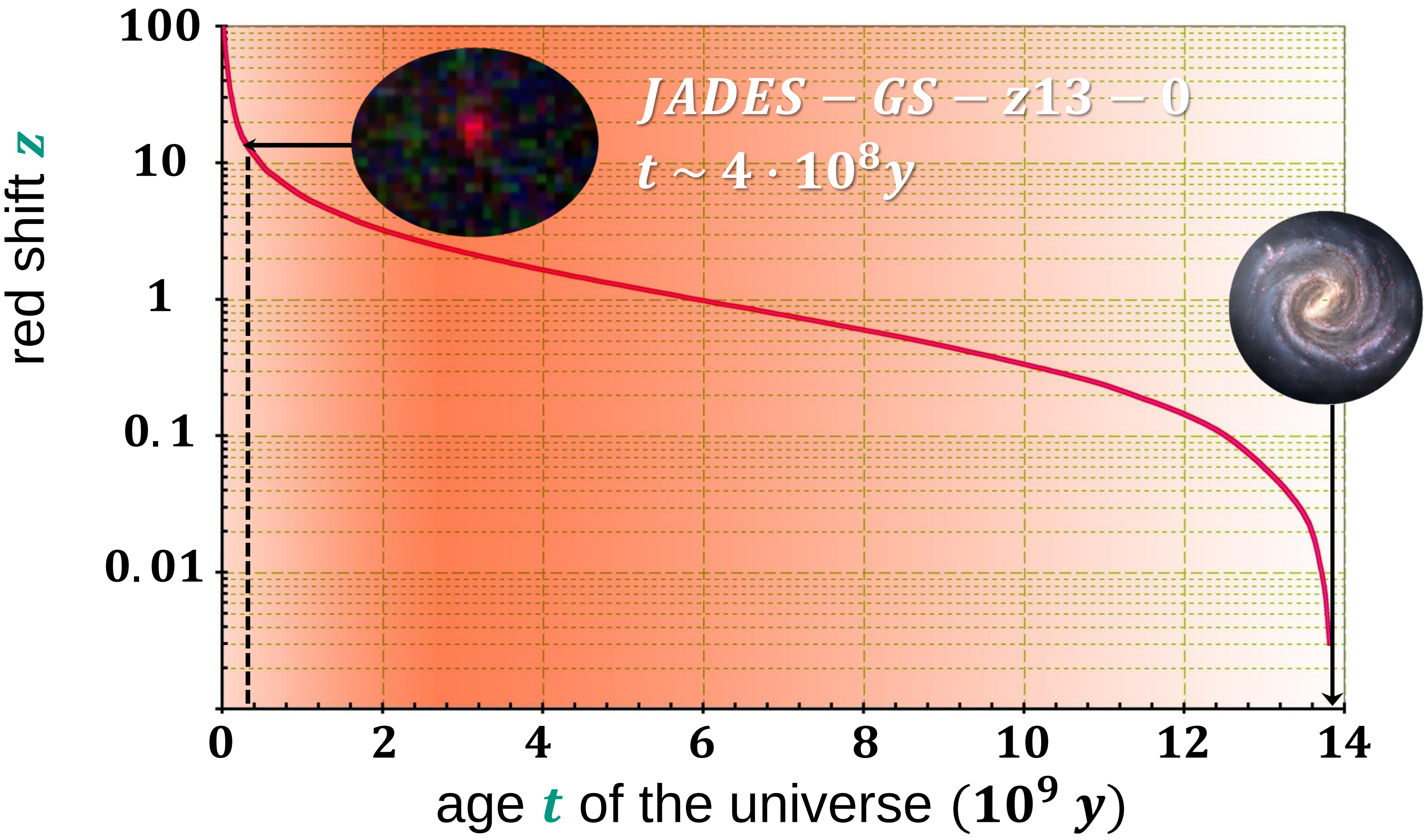
■ **What redshift do we expect to measure over cosmological time t ?**

- *calculated* redshift $z(t)$ for a specific cosmological model

- boundaries:

  $z(t = 0) = \infty$
  infinite redshift for processes during the Big Bang

  $z(t = t_0) = 0$
  no redshift for present universe



Redshift $z(t)$ and cosmological time t

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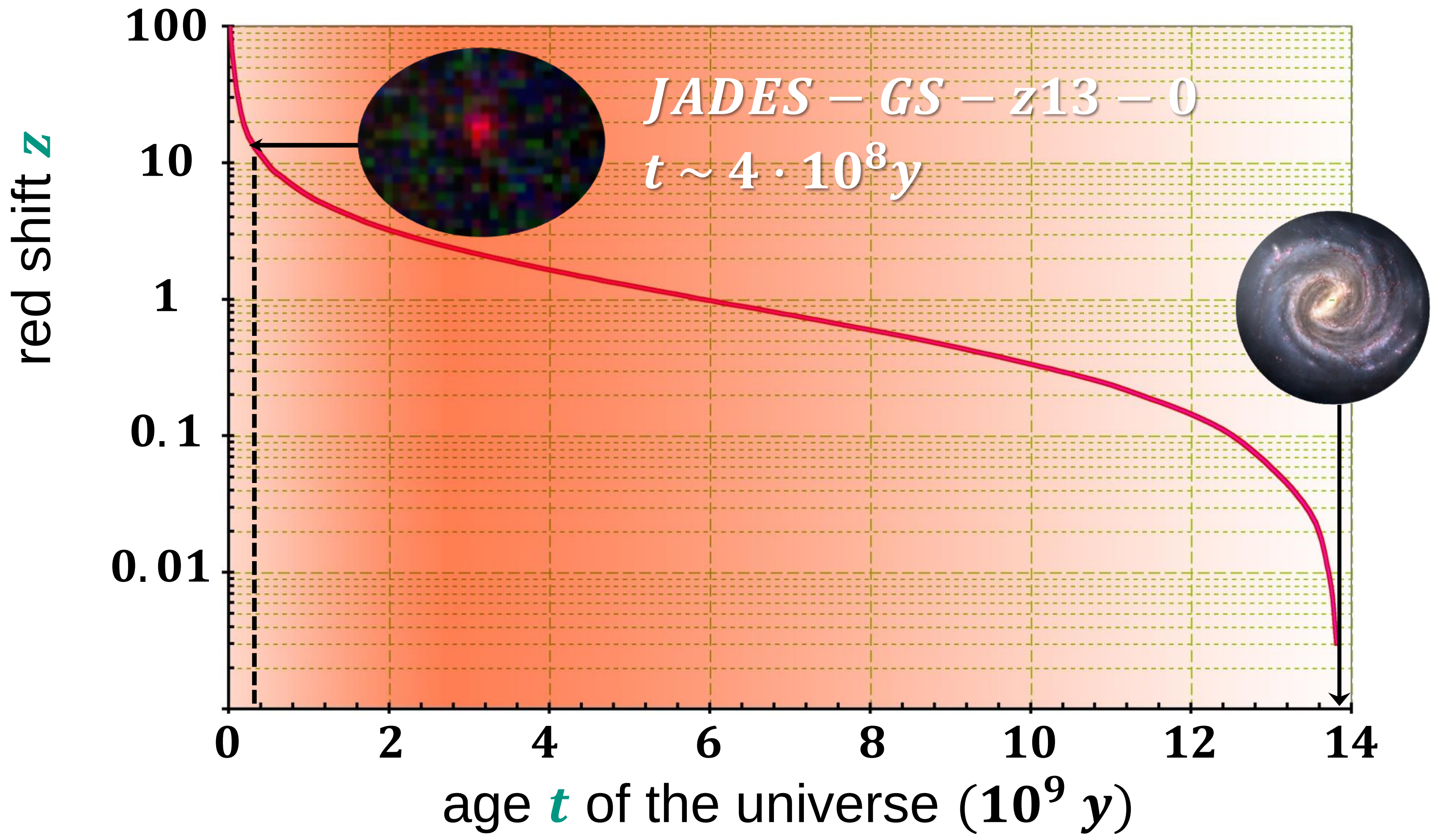
- **What scale factor do we expect to measure over cosmological time t ?**

- ***calculated*** scale **factor** $\boldsymbol{a}(\boldsymbol{t})$ for a specific cosmological model

- boundaries:

  $\boldsymbol{a}(\boldsymbol{t}=\mathbf{0})=\mathbf{0}$
  infinite small size for processes during the Big Bang

  $\boldsymbol{a}(\boldsymbol{t}=\boldsymbol{t}_{\mathbf{0}})=\mathbf{1}$
  actual size for present universe



Redshift $z(t)$ and cosmological time t

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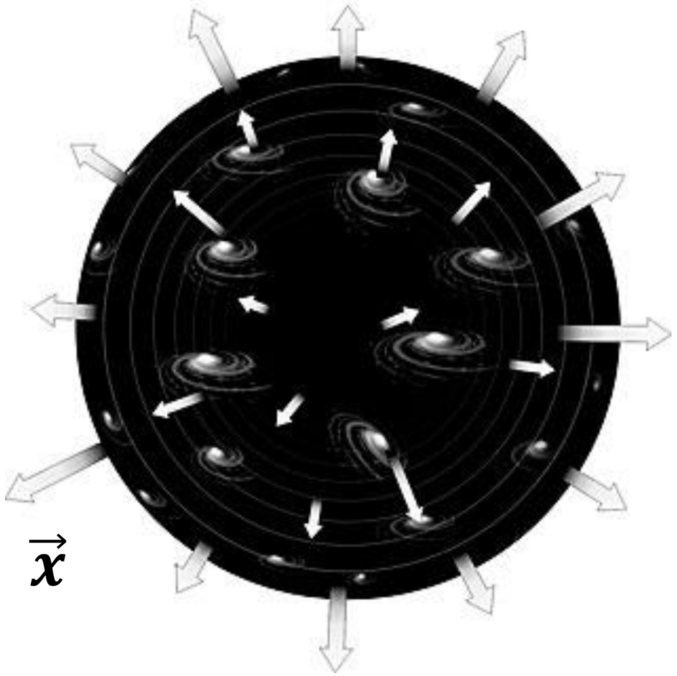
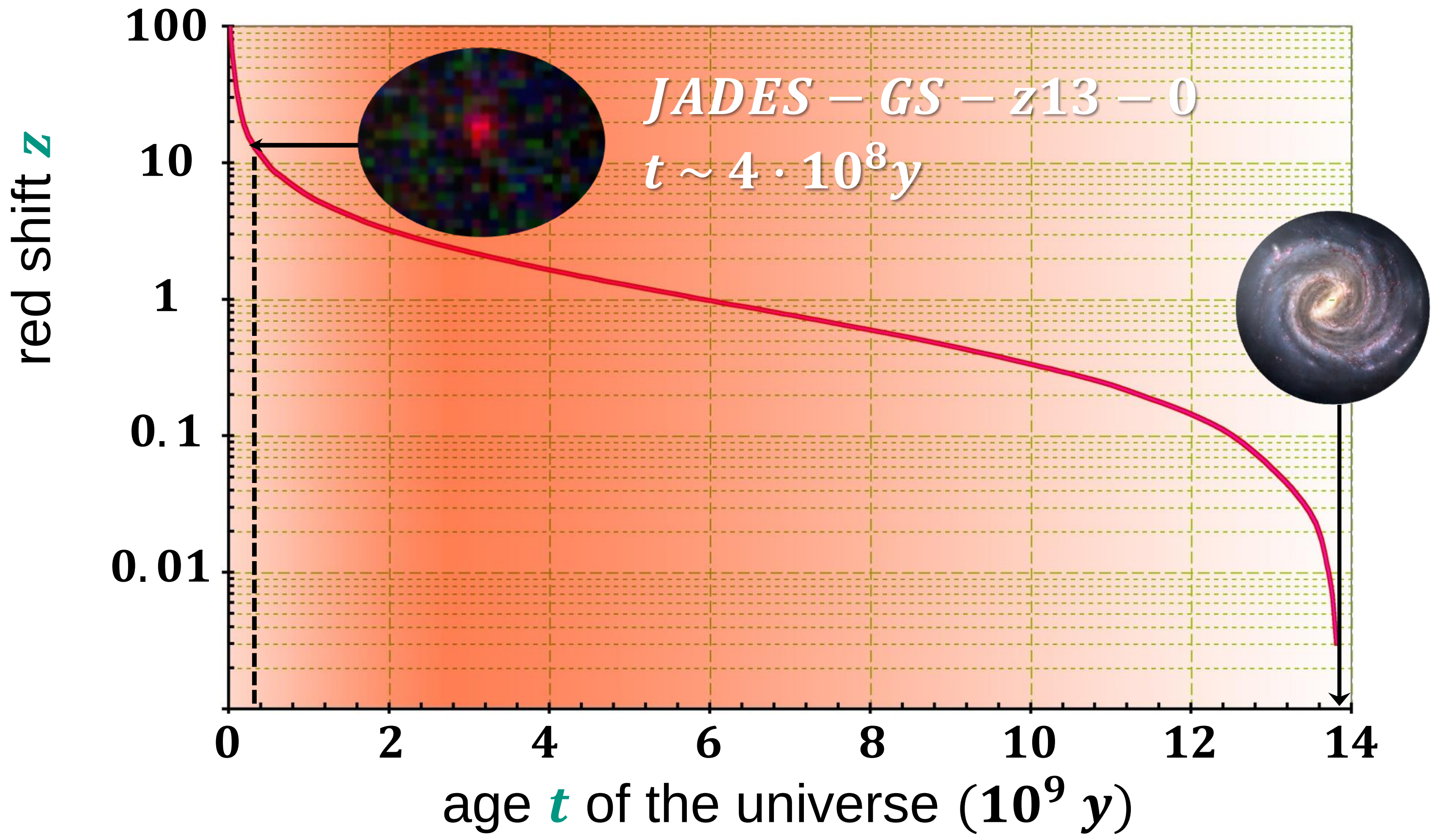
- **What cosmological model has been used here for $z(t)$ & $a(t)$?**

- **cosmological pillar**:
  universe exands
  isotropically in all $\vec{x}$

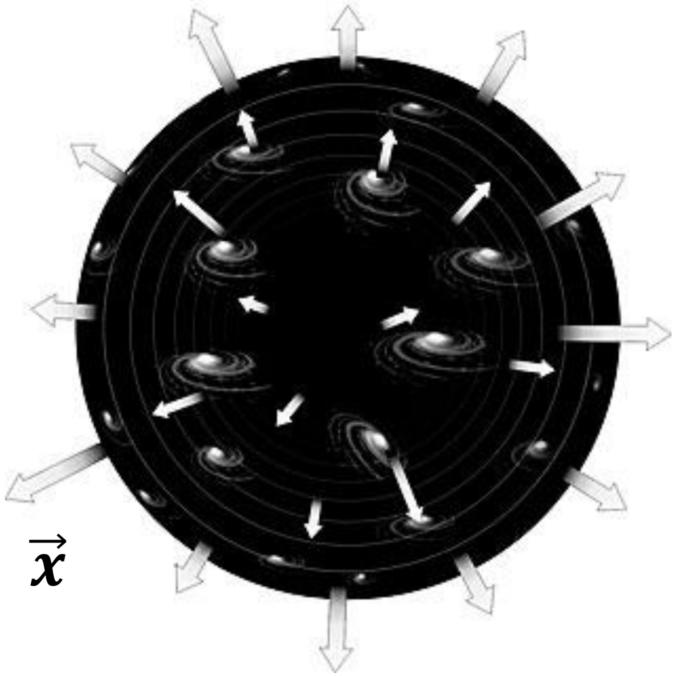




scale factor $a(t)$ and reference system

- There are 2 spatial reference systems to describe cosmolgical expansion

  - **co−moving observer: in motion with the ´Hubble flow´**



$\vec{x}$: co−moving coordinate

time−independent coordinate system:

- the coordinate $\vec{x}$ of an object does not change over cosmological time scales t

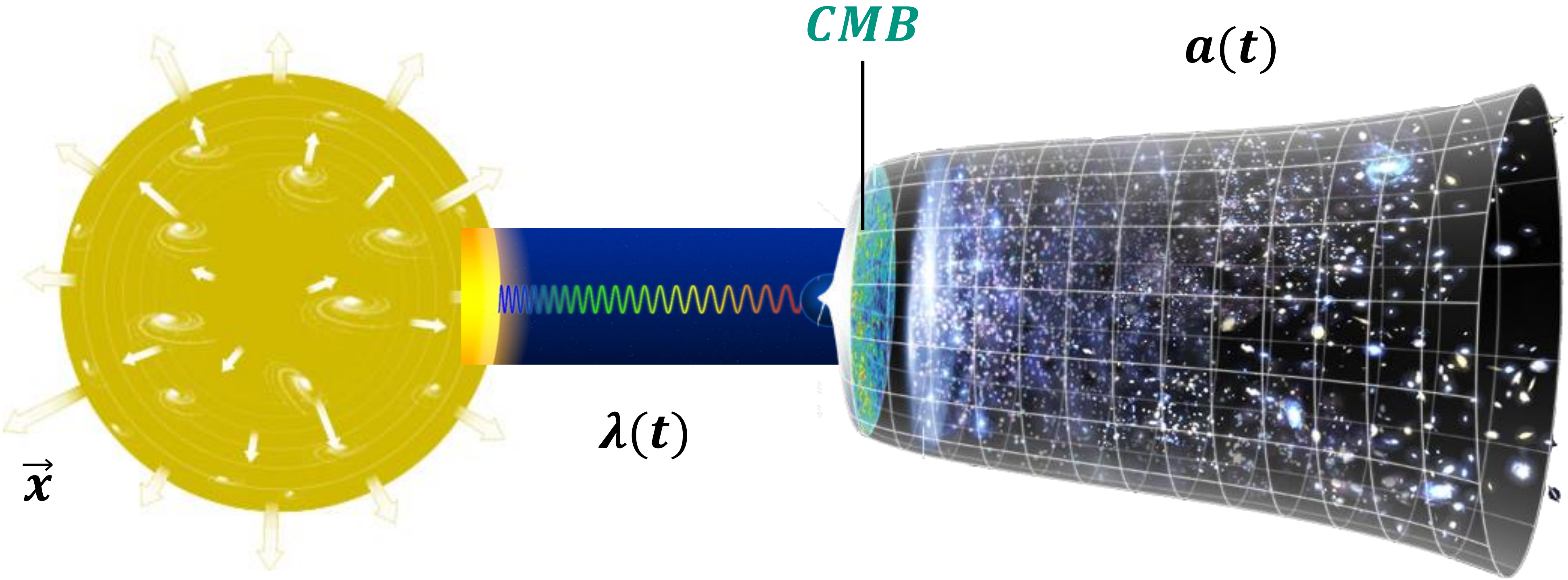
- distance between two objects does not change

- **universe appears to be static** & isotropic

scale factor $a(t)$ and reference system

- There are 2 spatial reference systems to describe cosmolgical expansion

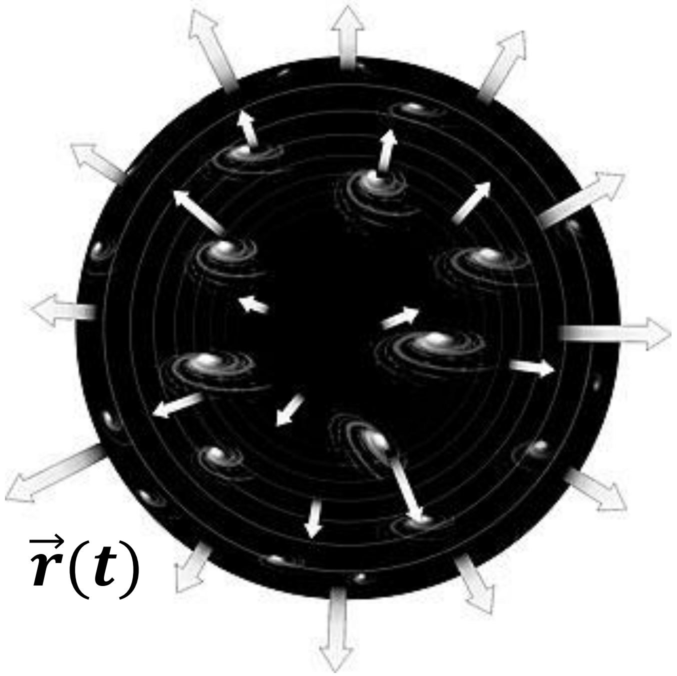
  - co$-$moving system: example Cosmic Microwave Background (CMB)



scale factor $a(t)$ and reference system

- **There are 2 spatial reference systems to describe cosmolgical expansion**

  - **´general´ observer: NOT in motion with the ´Hubble flow´**



$\vec{r}(t)$**:** non–comoving coordinate

time–dependent coordinate system:

- the coordinate $\vec{r}(t)$ of an object **does change** over cosmological time scales t

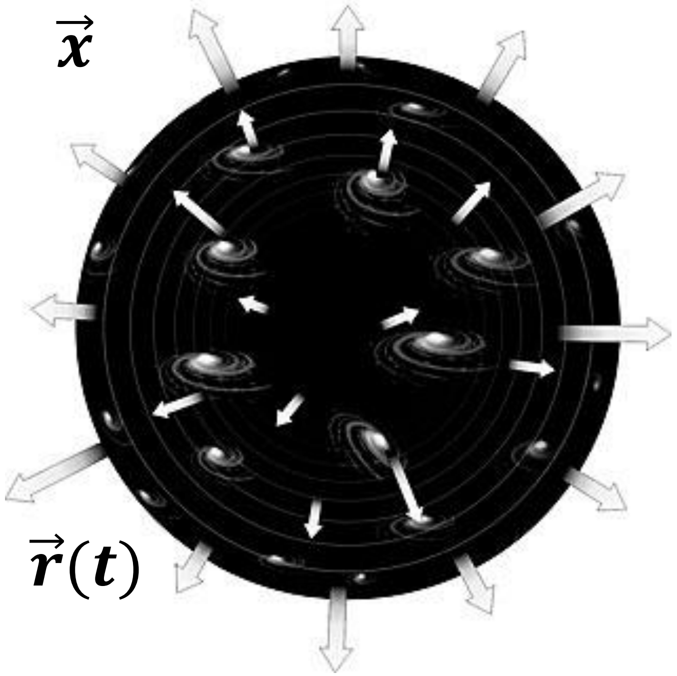
- distances between two objects will change

- universe is dynamic & expanding: **scale factor $a(t)$**

scale factor $a(t)$ and reference system

- **Switching between the 2 spatial reference systems**

  - **two observers: how do they transform their coordinates?**



$$\vec{r}(t) = a(t) \cdot \vec{x}$$

co–moving coordinate – **static**

scale factor – **time dependent**

- cosmological studies are usually performed in **co–moving coordinates** in view of the Hubble expansion

Cosmological expansion: Hubble parameter $H(t)$

Definition of the time–dependent Hubble parameter $H(t)$

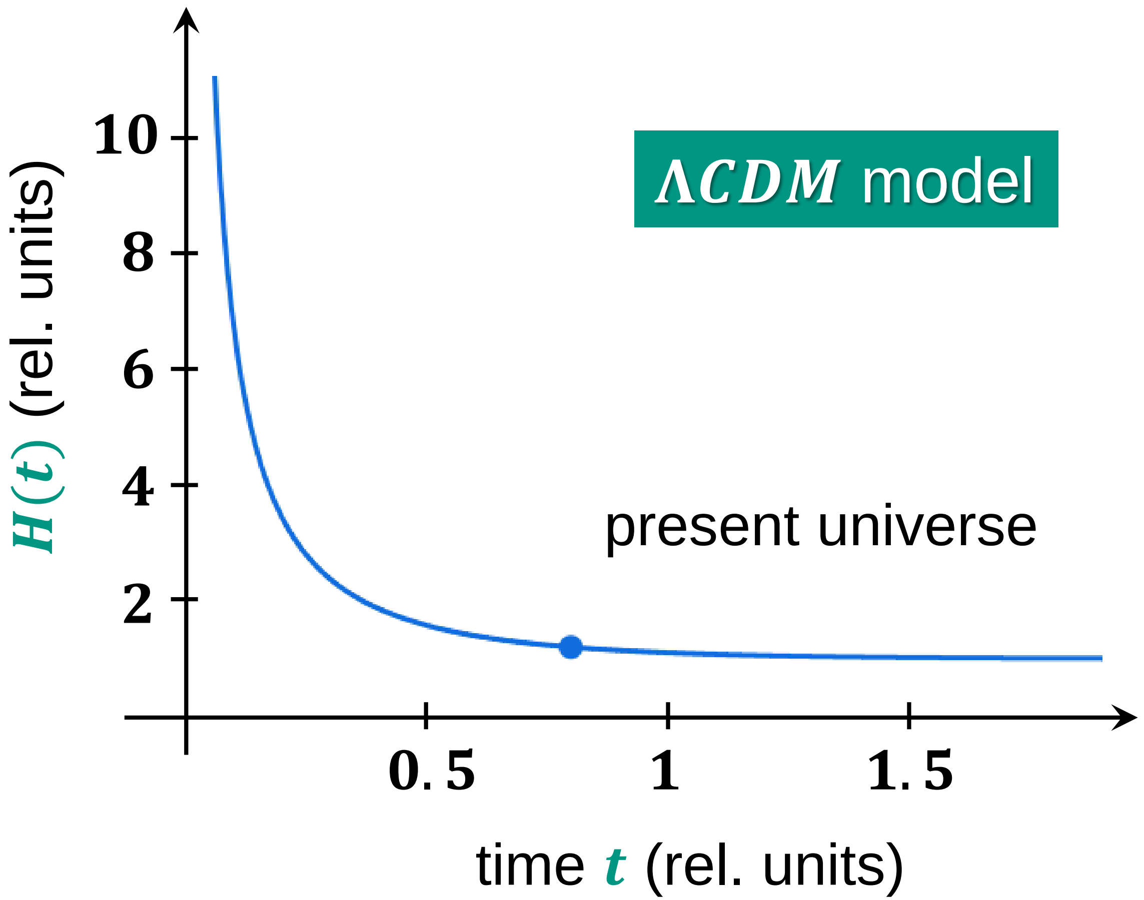
- $H(t)$: **time–dependent Hubble value**
  it is given by the **rate of change** $\dot{a}(t)$
  of the scale parameter divided
  by its **value** $a(t)$.

- $H(t)$ measures how fast $a(t)$ changes
  over cosmological times t

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

$\dot{a}(t)$ – rate of change of $a(t)$

$a(t)$ – scale factor



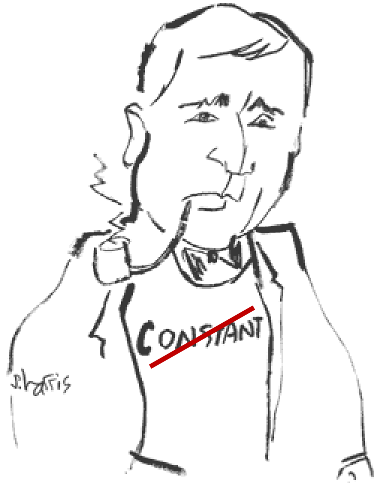
Cosmological expansion: Hubble law

■ **The Hubble law using Hubble parameter $H(t)$**

- $H(t)$: **time−dependent Hubble value & Hubble´s law in a** consistent way

- the observed ´escape velocity´ $\vec{v}(t)$ of objects at a cosmologcial distance $\vec{r}(t)$ is time−dependent

- for $t = t_0$ (**present universe**) we obtain $H(t_0) = H_0$



$$\vec{v}(t) = H(t) \cdot \vec{r}(t)$$

$$\dot{\vec{r}}(t) = \dot{a}(t) \cdot \vec{x} \qquad \vec{r}(t) = a(t) \cdot \vec{x}$$

SPECIAL TOPIC: metrics of the universe

- **Space–time structure of the universe: described by Minkowski metrics**

  - flat space–time: described by Einstein´s **Special Relativity**

  - example: **light–cone propagation** in an isotropic & homogeneous universe

  - metrics of **flat–space time**: metric tensor or **line element ds^2**

line element ds^2:

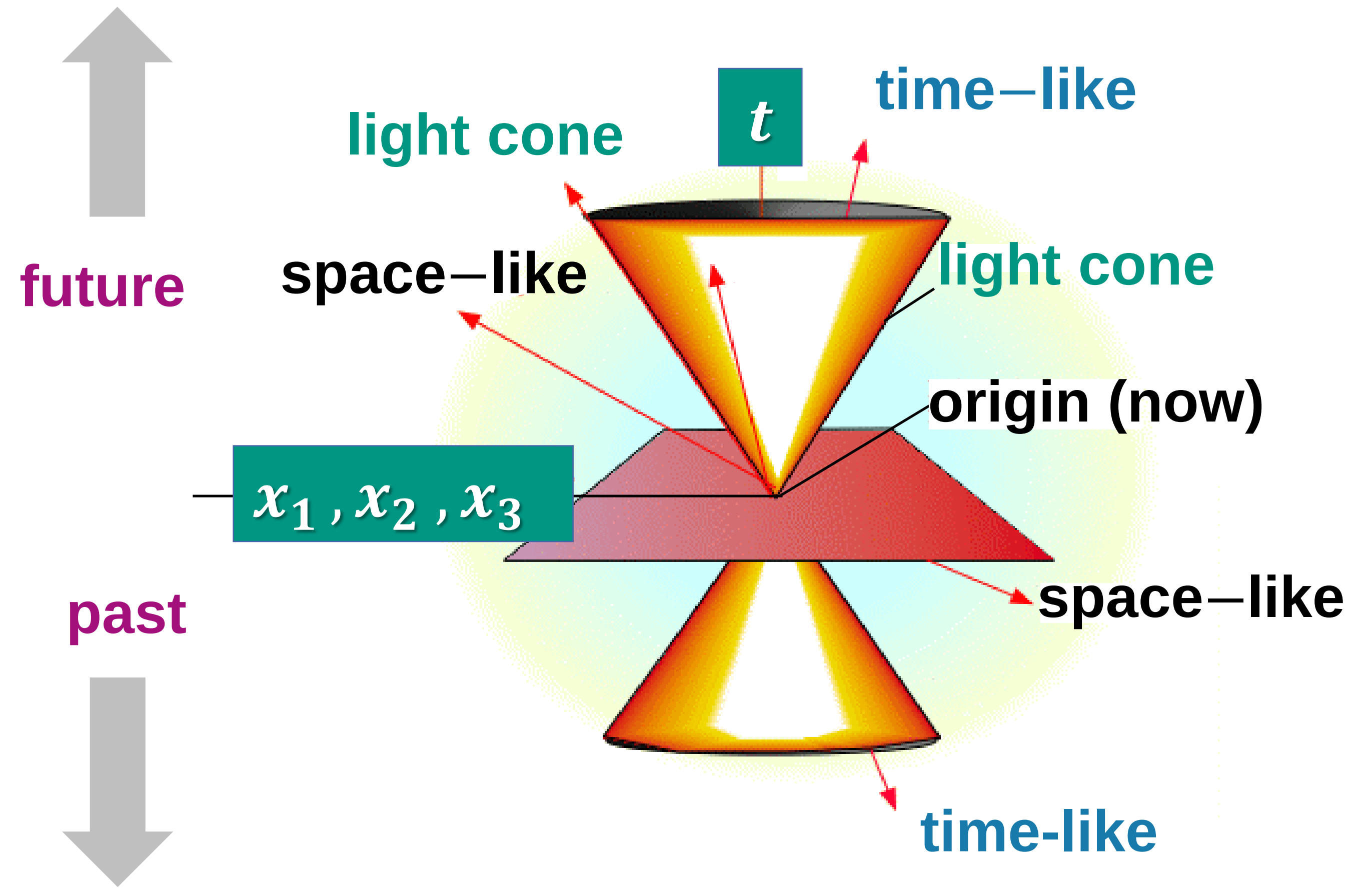
H. Minkowski

3 –dimensional space: $$ds^2 = dx_1^2 + dx_2^2 + dx_3^2$$

4 –dimensional space–time: $$ds^2 = c^2 \cdot dt^2 - (dx_1^2 + dx_2^2 + dx_3^2)$$

Minkowski metrics for flat space–time

- Minkowski diagram to visualize the ´relativity of synchronous events´



geodetic line	line element ds^2
time–like	$ds^2 > 0$
light cone	$ds^2 = 0$
space–like	$ds^2 < 0$

Friedmann–Lemaître–Robertson–Walker* metrics

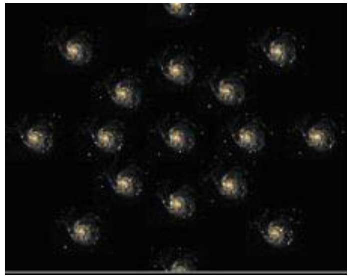
- **Relevant space–time structure of a curved, expanding universe**

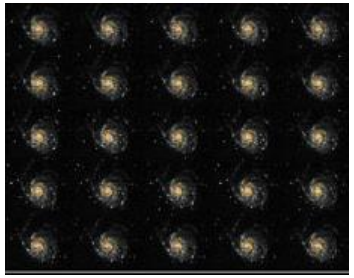
  - ***FLRW* metrics: curved space–time** of *G*eneral *R*elativity (***GR***)

  - exact solution of Einstein´s field equations for a **homogenous** and **isotropic** universe

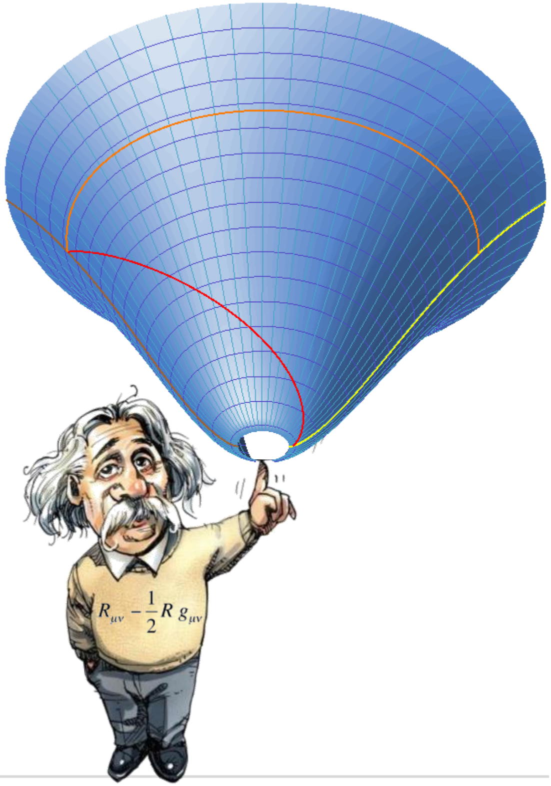
  - **cosmological principle: only look at very large scales!**

FLRW metrics only valid for **distances $D > 100\ Mpc$** due to local inhomogeneities (galaxies, galaxy clusters,…)





isotropy & homogeneity



*$FLRW$

Friedmann–Lemaître–Robertson–Walker metrics

- **Topology of the universe: flat, curved, …(i.e. more complex if non–flat)**

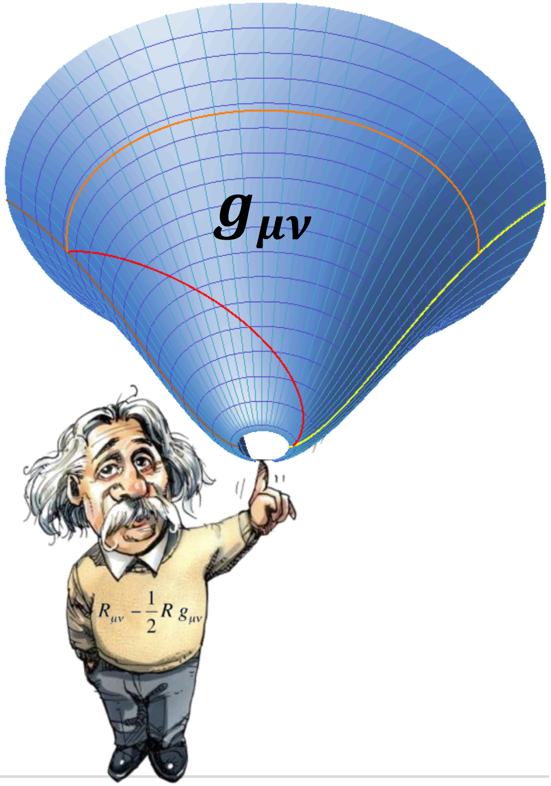
  - **key parameter: curvature constant $k = -1, 0, +1$**

parameter k	type of curvature
$k = -1$	hyperbolic
$k = 0$	flat
$k = +1$	spherical

  - **line element**: $$ds^2 = g_{\mu\nu} \cdot dx^{\mu} \cdot dx^{\nu}$$

$g_{\mu\nu}$ metric tensor

  - flat (**Euclidean**) space: $g_{\mu\nu} = (1, -1, -1, -1)$



Friedmann–Lemaître–Robertson–Walker metrics

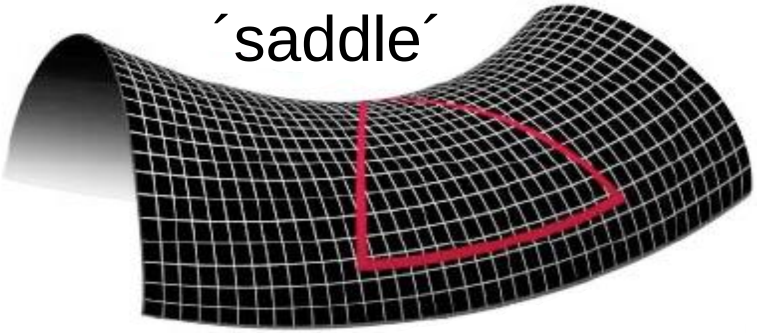
- **Relevant space–time structure of a curved, expanding universe**

  - $FLRW$: describes **flat** ($k = 0$), as well as intrinsically **curved universes** with the **two possible** curvature parameters: $k = +1, k = -1$

open, hyperbolic

$k = -1$

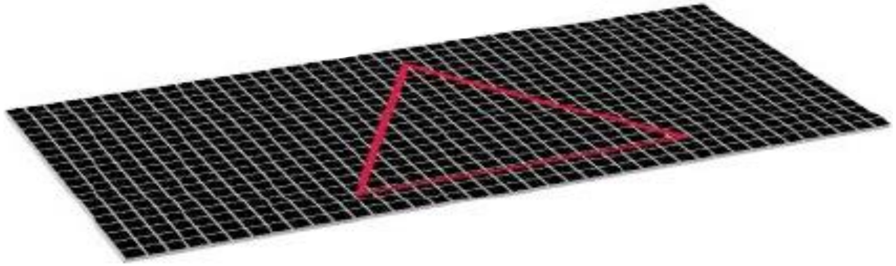
negatively curved space



flat, Euclidean

$k = 0$

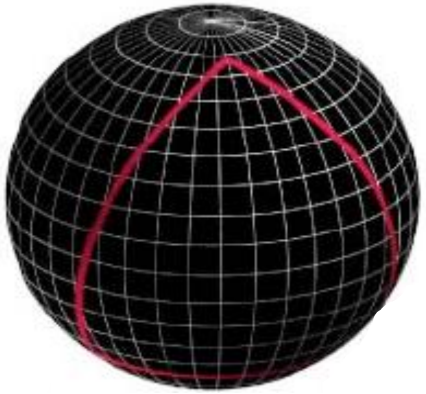
non–curved space



closed, spherical

$k = +1$

positively curved space

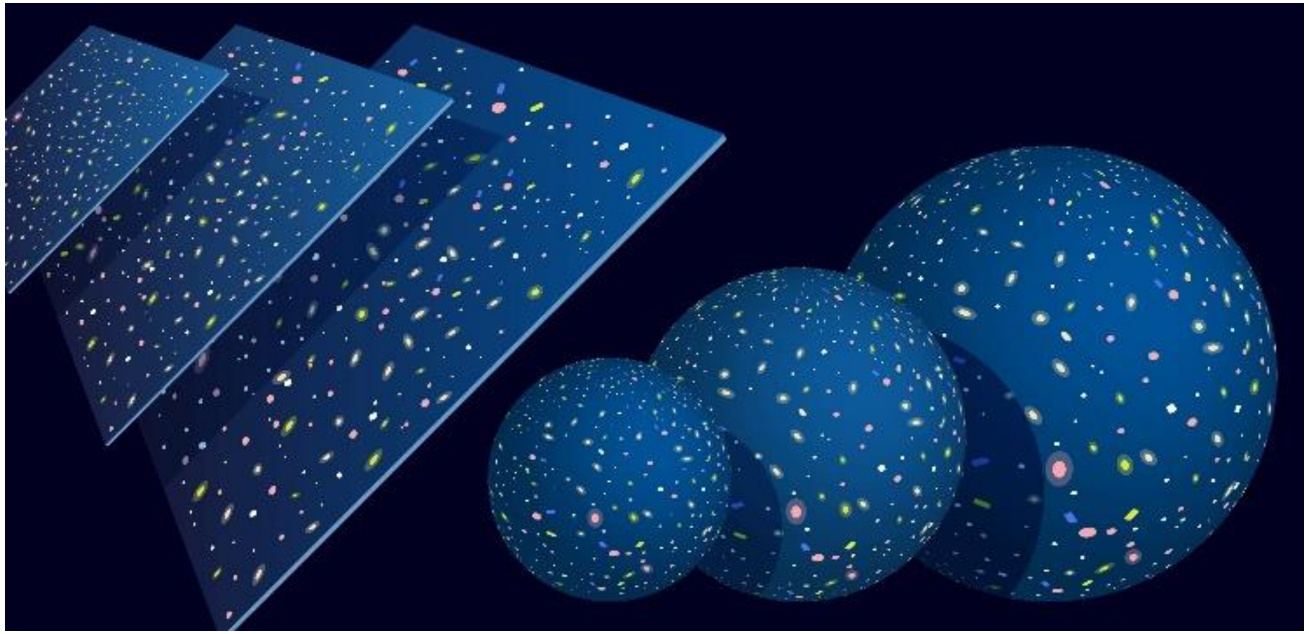


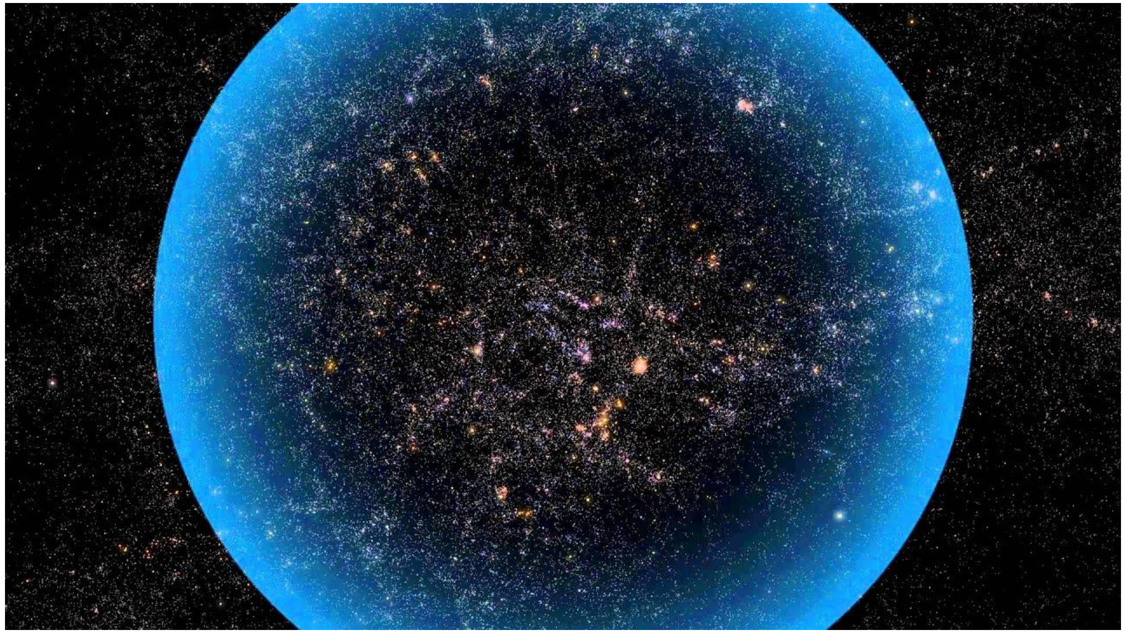
Shape & size of the universe

- $FLRW$ **metrics describes a huge variety of space−time configurations**

  - **size of the universe: can be limited or limit−less**

size of a universe with $\boldsymbol{k = +1, 0, -1}$ can be **bounded** or **unbounded**

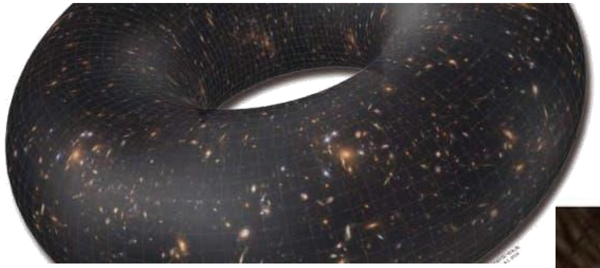


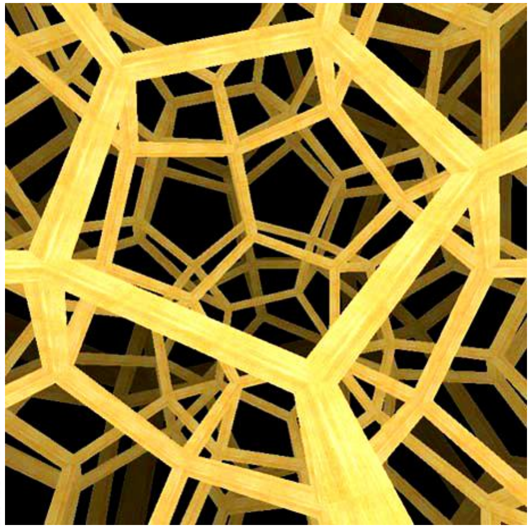


Shape & size of the universe: exotics

- **non−standard structure of more ´exotic´ universes: some examples**

  universe as torus, holographic universe, higher dimensions, topological defects…





Poincaré Dodecahedron

$FLRW$ metrics for an expanding universe

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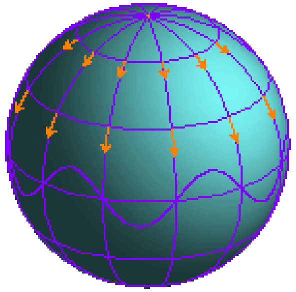
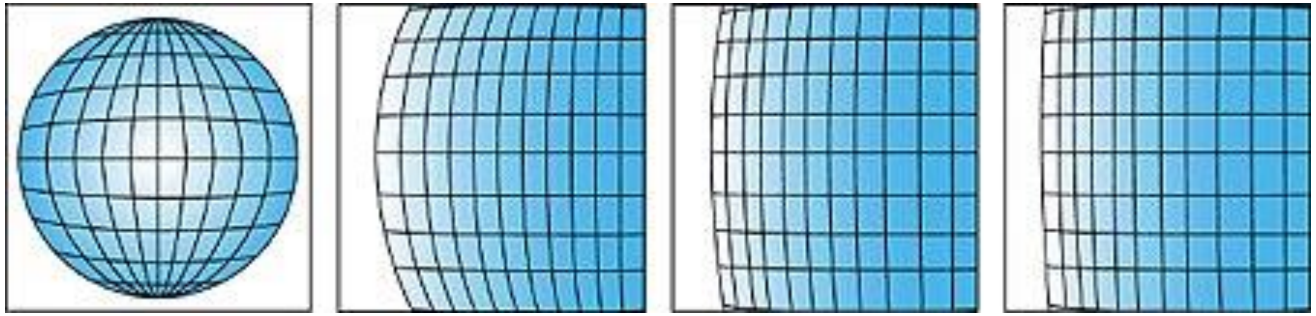
- **Line element ds^2 in spherical comoving coordinates (r, θ, φ) & curvature k**

  - describes the increase of line element ds^2 in an isotropically expanding universe



$$ds^2 = c^2 dt^2 - a^2(t) \cdot \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2 \right)$$

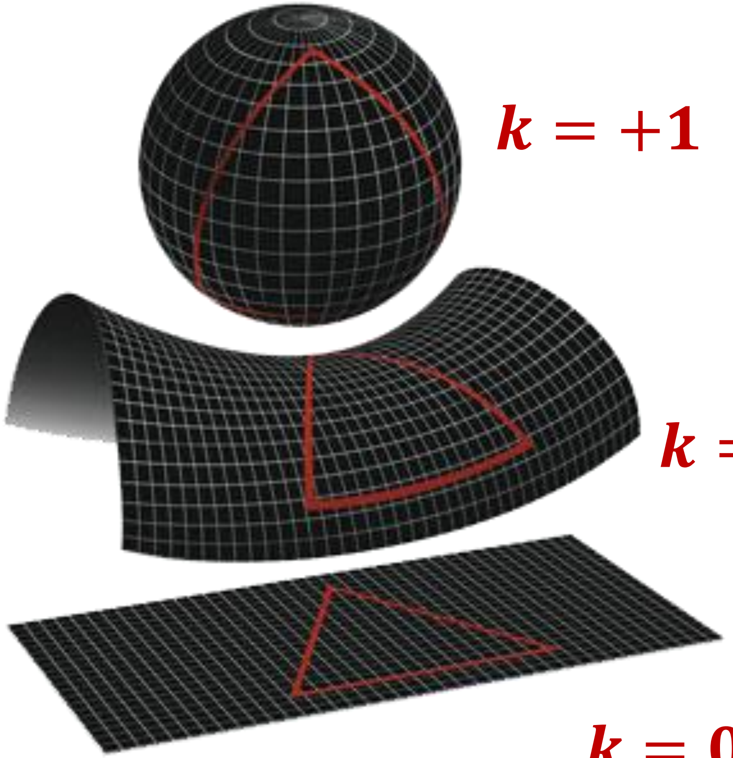


increase of the scale parameter $a(t)$

$FLRW$ metrics for an expanding universe

- **Line element ds^2 in spherical comoving coordinates (r, θ, φ) & curvature k**

  - describes the increase of line element ds^2 in an isotropically expanding universe



$$ds^2 = c^2 dt^2 - a^2(t) \cdot \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2 \right)$$

r: spatial distance of two objects

θ, φ: angular coordinates

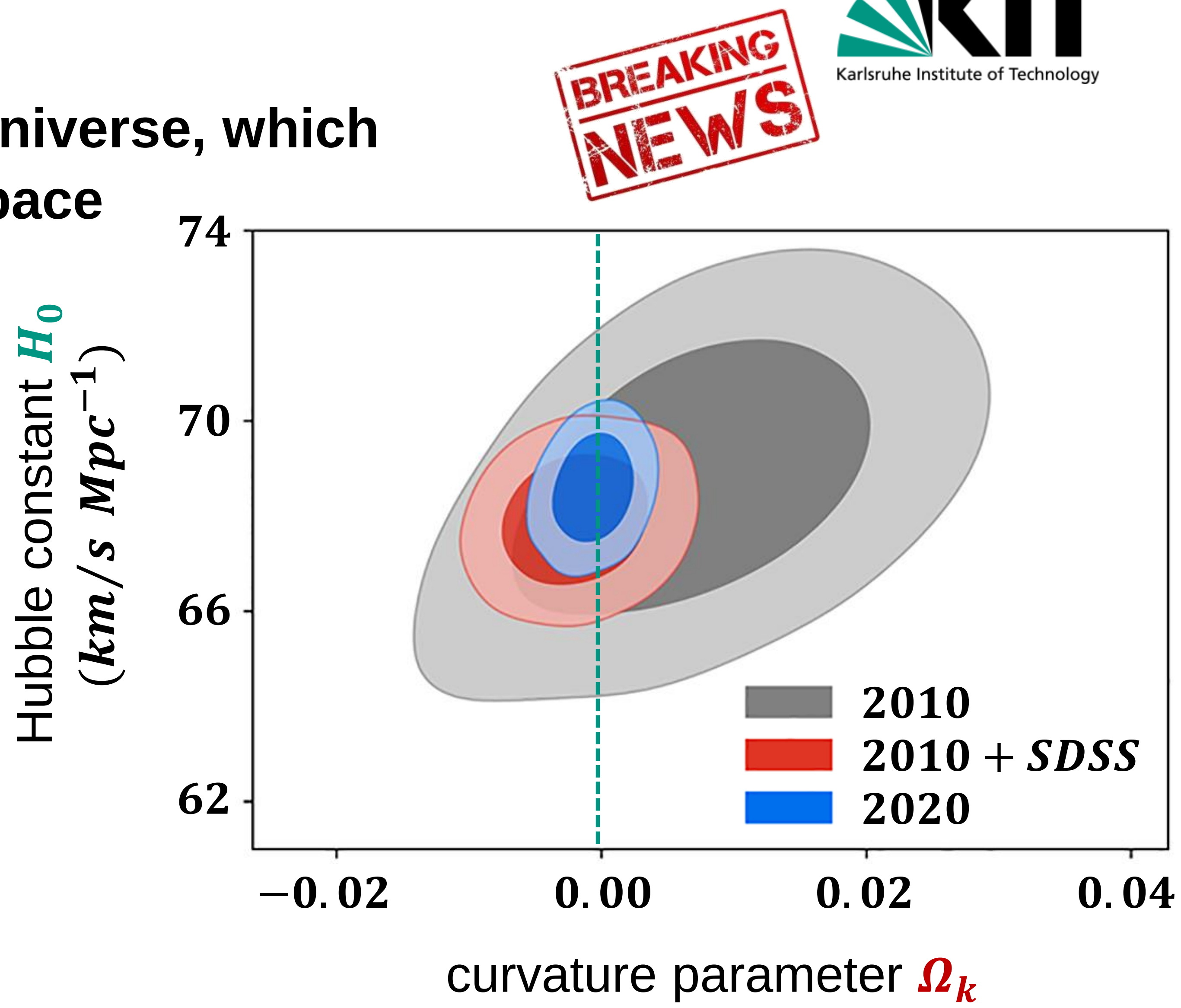
Is the universe curved?

- **Searching for an evidence of a universe, which would deviate from Euclidean space**

  - observational data prefer a **flat universe** without curvature

  - thus no contribution Ω_k of the **curvature** k of space–time to matter–energy budget Ω_{tot}

  - observational data provided by galaxy survey $SDSS$* of galaxies & quasars ($2 \cdot 10^6$ objects)

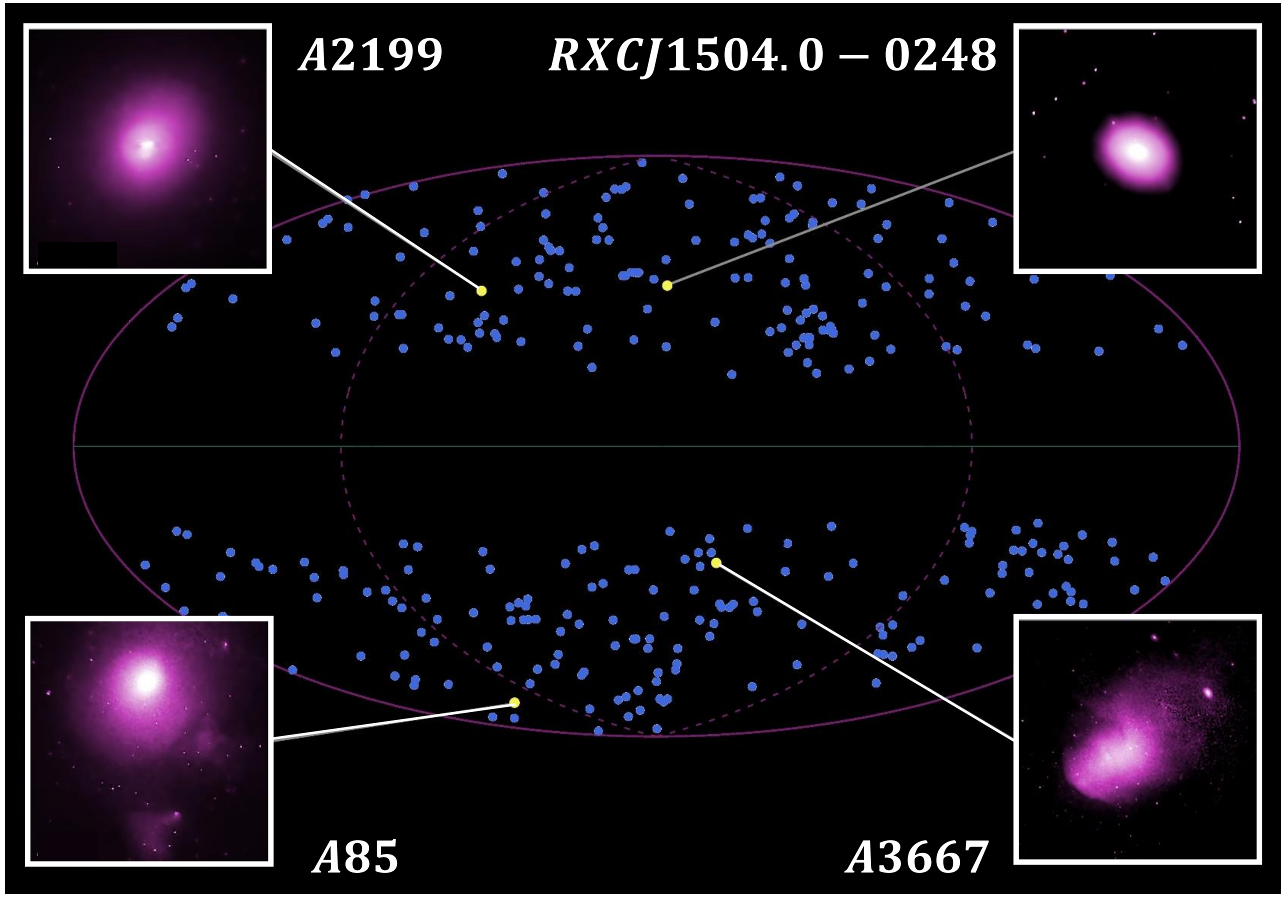


*Sloan Digital Sky Survey

Is the cosmological expansion isotropic?

- Is there an isotropic expansion in all directions?

- $\mathbf{2020}$: measurement of $\mathbf{313}$ distant galaxy clusters via $\boldsymbol{X}$ – ray satellites
- $\boldsymbol{X}$ – ray luminosity of clusters used to calculate their expasion speed $\boldsymbol{H(t)}$



observed distribution of galaxy clusters

Is the cosmological expansion isotropic?

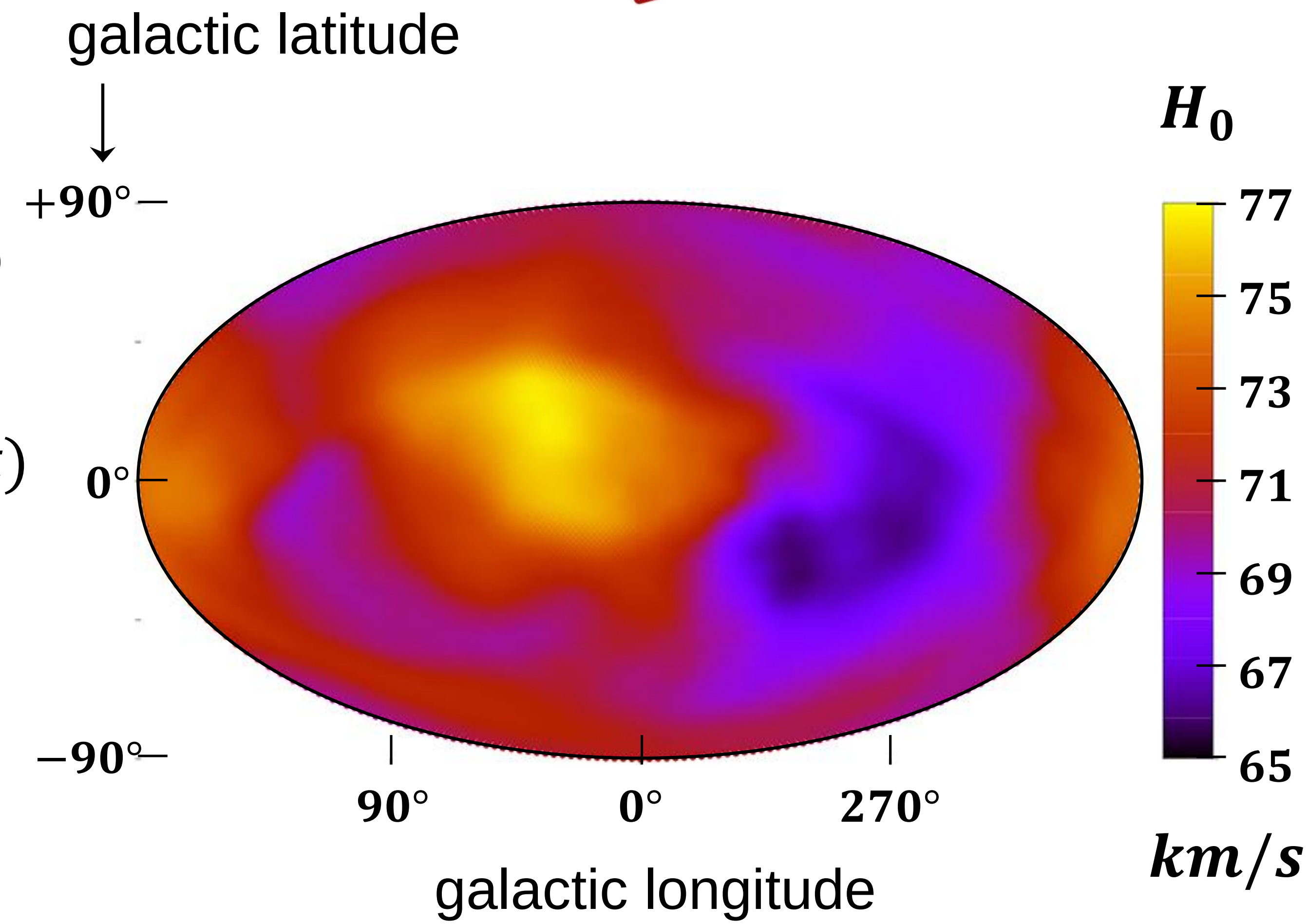
- **Is there an** **isotropic expansion** **in all directions?**

- **2020**: measurement of **313** distant galaxy clusters via $X-$ray satellites

- $X-$ray luminosity of clusters used to calculate their expasion speed $H(t)$

- **surprising result**: the measured $H(t)$ values differ for various sky patches

- **hint for a non−isotropic expansion?**



observed distribution of expansion speeds

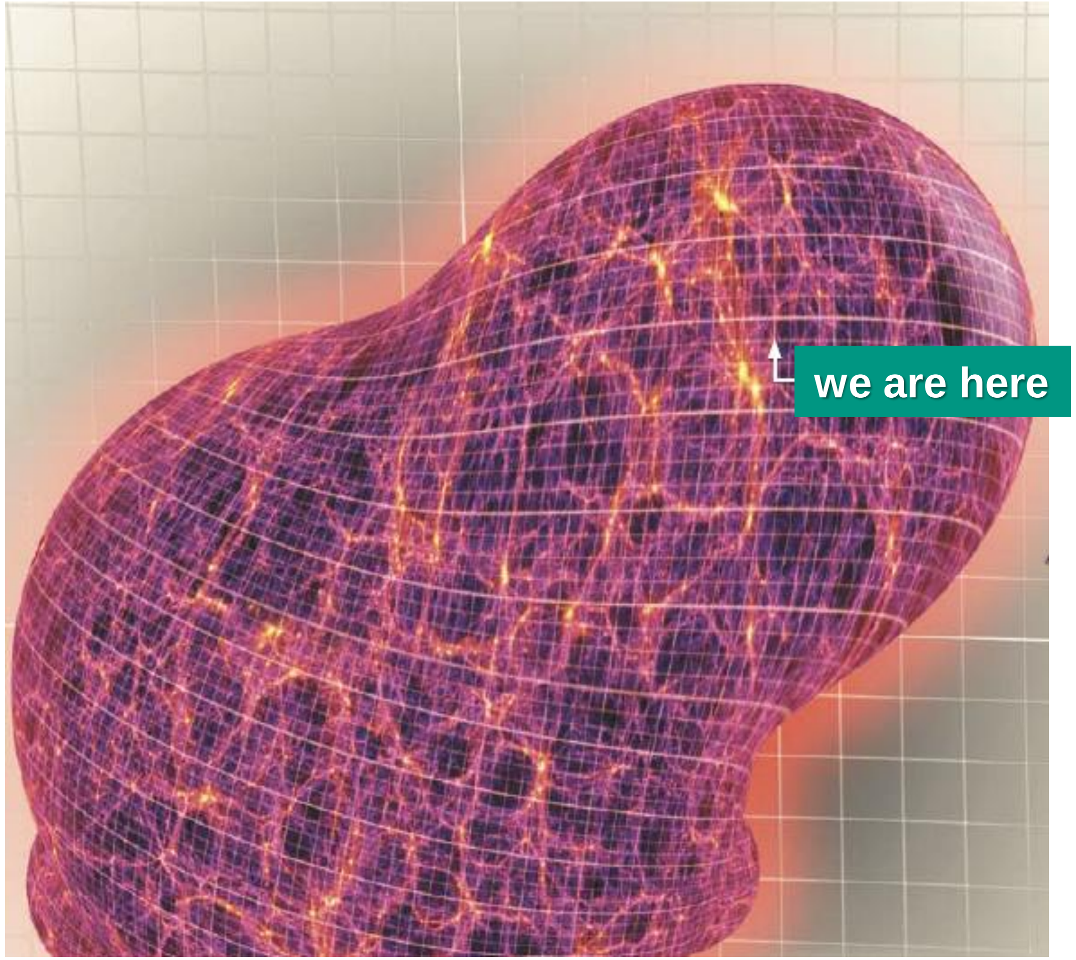
If the cosmological expansion is non-isotropic

- **a non−isotropic expansion would be a serious problem for $FLRW$ metrics**

- corresponding metrics: ***Kantowski−Sachs***

- if confirmed, **non−isotropy** would be a major probem for standard cosmological models

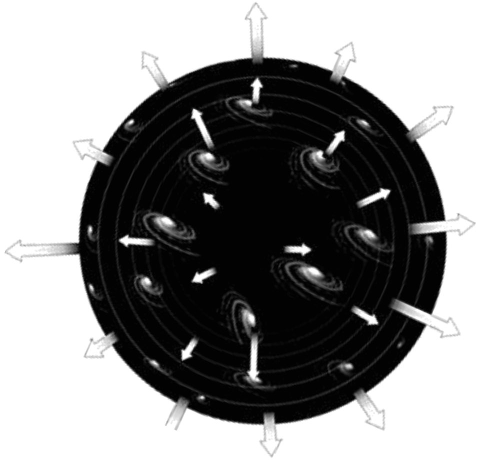
- $\mathbf{2022}$: new hints for unexplained **dipole anisotropy** of radio galaxies & quasars *(S. Sakar et al.)*

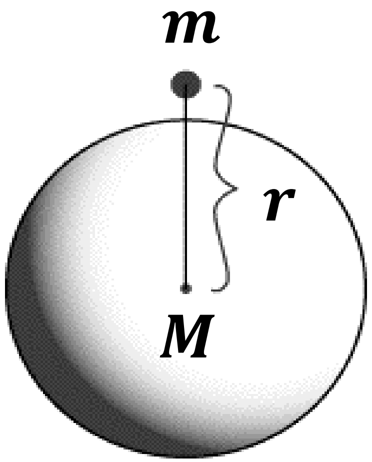


schematic view of a non−isotropic universe

How can we calculate cosmic expansion speed?

- **First goal: we want to calculate acceleration $\ddot{a}(t)$ of scale parameter $a(t)$**
  - **first actor: matter (baryonic & dark)**





matter:

gravitational attraction

⇨ is slowing down the cosmological expansion

Newtonian gravity – works on point masses, as well as the universe

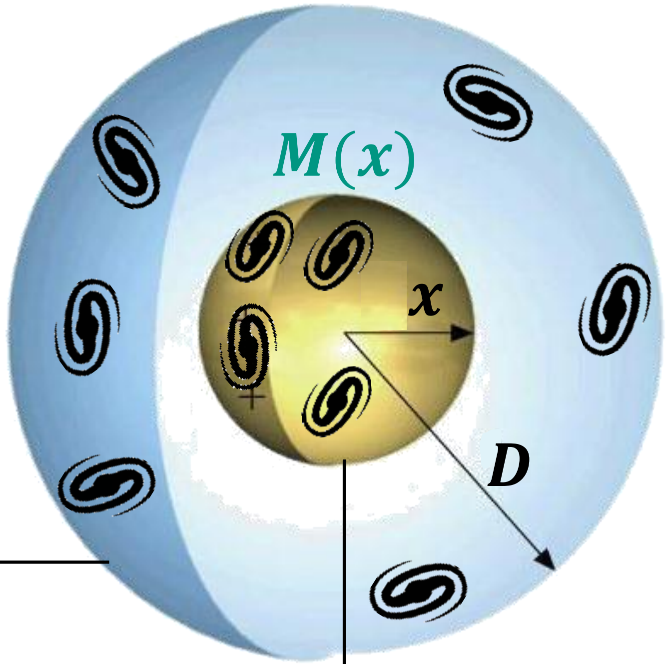
Dynamics of the cosmic expansion

- **Enclosed mass $M(x)$ in co–moving coodinates**

  - golden inner sphere: co–moving coordinate x:
    co–moving **radius x** encloses a **mass $M(x)$**
    with a given **mass density ϱ_0**

  - **mass $M(x) = const.$** (due to mass conservation)

  - blue outer region: <u>no</u> gravitational
    contribution from matter outside
    ⇨ theorem: **spherical shell**
    (see your bachelor lectures:
    Klassische Ex.–Physik I)



´co–moving´ mass $M(x)$
is participating in the
cosmological expansion

Dynamics of the cosmic expansion

- **Expansion is described best in a co–moving sphere**

  - **density** of a sphere in **2** coordinate systems:

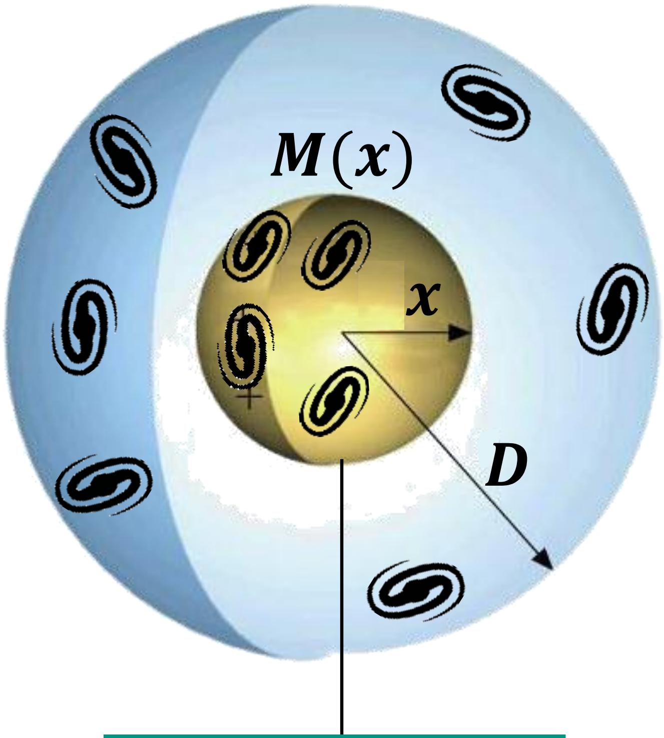
$$M(x) = 4/3 \cdot \pi \cdot \varrho_0 \cdot x^3 \quad \Rightarrow \quad \textbf{co–moving}$$

$$M(x) = 4/3 \cdot \pi \cdot \varrho(t) \cdot r(t)^3 \quad \Rightarrow \quad \textbf{general}$$

  - density of **inner sphere** in **2 coordinate systems**:

    - co–moving: ϱ_0 **constant** due to selected coordinate system

    - general: $\varrho(t)$ **decreases** due to cosmic expansion



$$\rho(t) = \frac{\rho_0}{a^3(t)}$$

Dynamics of the cosmic expansion

- **Expansion described typically in an expanding sphere**

  - **density** of a sphere in **2** coordinate systems:

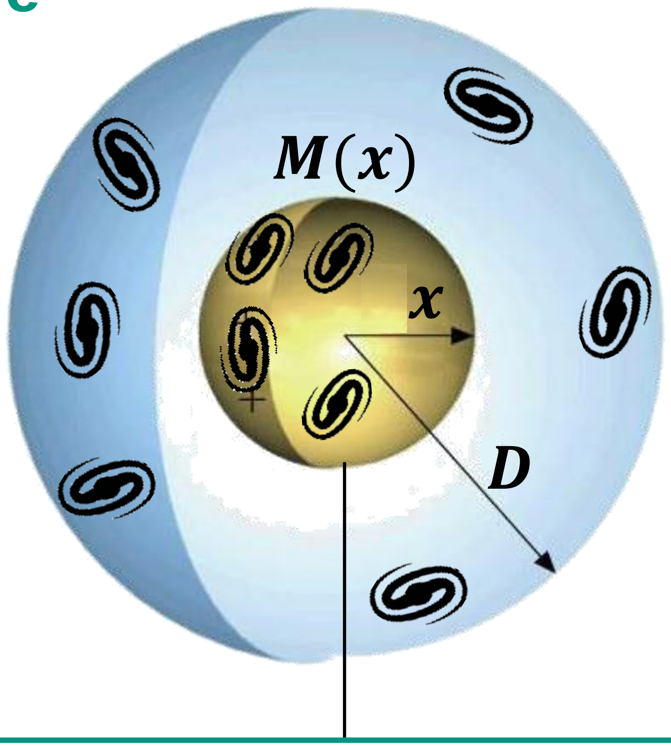
    $M(x) = 4/3 \cdot \pi \cdot \varrho_0 \cdot x^3$ ⇒ **co–moving**

    $M(x) = 4/3 \cdot \pi \cdot \varrho(t) \cdot r(t)^3$ ⇒ **general**

  - density of **inner sphere**:

    - co–moving: ϱ_0 **constant** due to selected coordinate system

    - general: $\varrho(t)$ **decreases** due to cosmic expansion



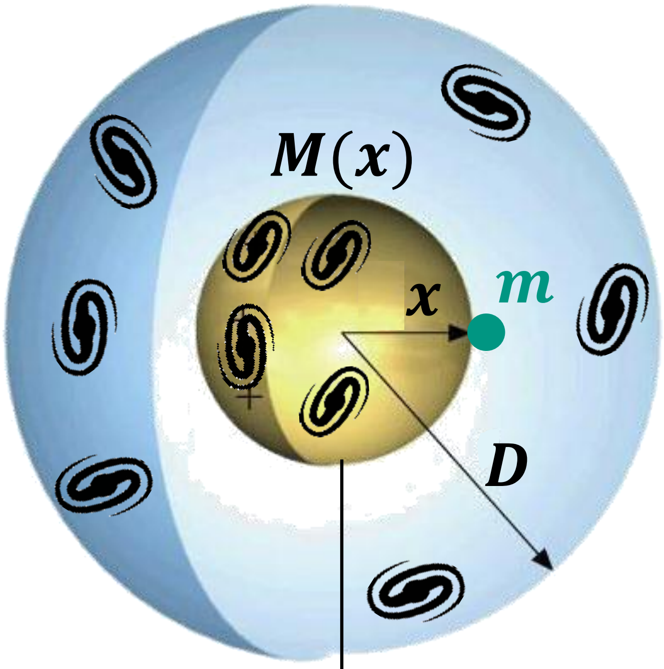
$$M(r,t) = \frac{4}{3} \cdot \pi \cdot \varrho(t) \cdot r^3(t)$$

Dynamics of the cosmic expansion

- **Acceleration $\ddot{a}(t)$ of scale parameter $a(t)$**

  - general coordinates:
    expansion rate of inner sphere is slowed down by gravity:
    ⇨ time–dependent variables: $r(t)$, $\dot{r}(t)$, $\ddot{r}(t)$

  - **graviational attraction** of $M(r,t)$ on a small test mass m will result in (negative) **acceleration $\ddot{r}(t)$**



$$M(r,t) = \frac{4}{3} \cdot \pi \cdot \varrho(t) \cdot r^3(t)$$

$$\ddot{r}(t) = -\frac{G \cdot M(r,t)}{r^2}$$

$$\ddot{r}(t) = -\frac{4}{3} \cdot \pi \cdot G \cdot \rho(t) \cdot r(t)$$

Dynamics: Friedmann–Lemaître equation

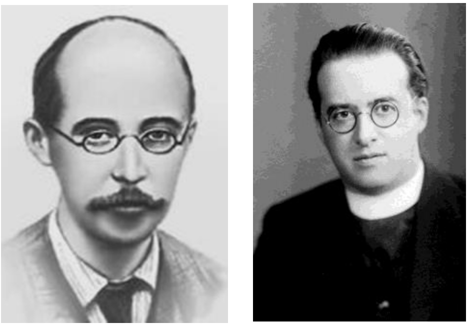
- **Acceleration $\ddot{a}(t)$ of scale parameter $a(t)$**

$$\ddot{r}(t) = -\frac{4}{3} \cdot \pi \cdot G \cdot \varrho(t) \cdot r(t) \mid : x$$

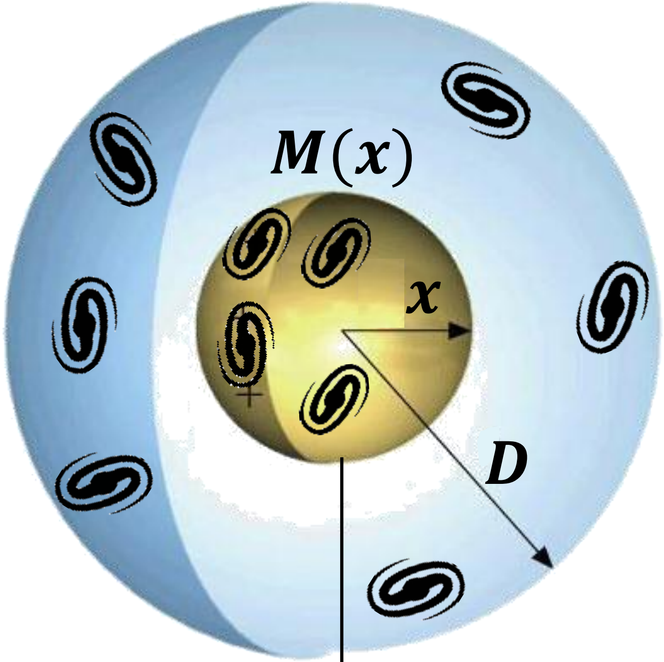
$$\ddot{a}(t) = -\frac{4}{3} \cdot \pi \cdot G \cdot \varrho(t) \cdot a(t)$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi \cdot G \cdot \varrho(t) < 0$$

⇨ gravitational attraction of matter with **density** $\varrho(t)$



Friedmann-Lemaître



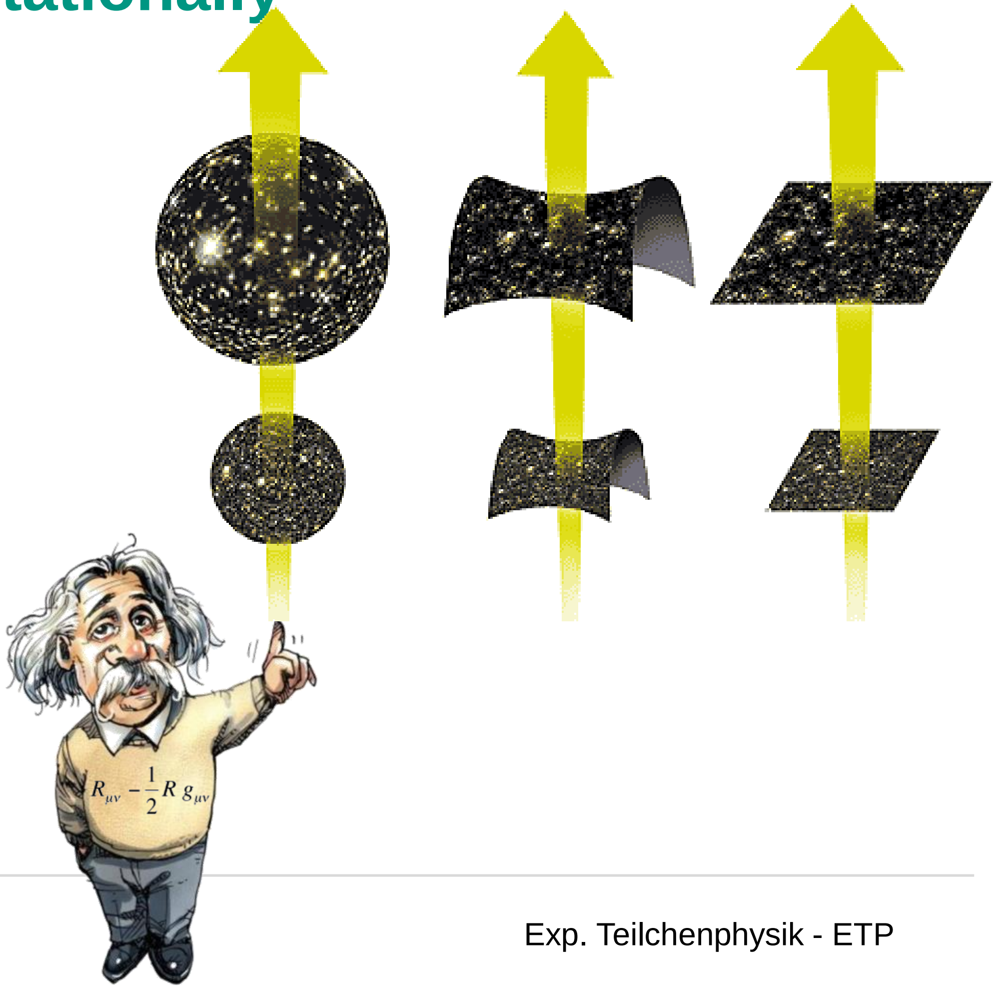
$$a(t) = \frac{r(t)}{x} \qquad \ddot{a}(t) = \frac{\ddot{r}(t)}{x}$$

Dynamics: Friedmann-Lemaître equation

- **Taking into account Einstein´s General Relativity**

  - **<u>all</u>** types of energy−density do interact **gravitationally**

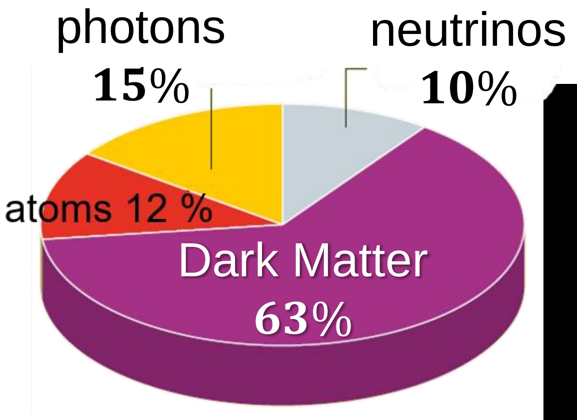
matter (baryons, $\boldsymbol{DM}$)	$\boldsymbol{\rho_m(t)}$
radiation fields ($\boldsymbol{CMB}$)	$\boldsymbol{\rho_r(t)}$
vacuum energy	$\boldsymbol{\rho_V(t)}$
pressure	$\boldsymbol{P(t)}$
curvature of space−time	$\boldsymbol{k = +1, -1}$

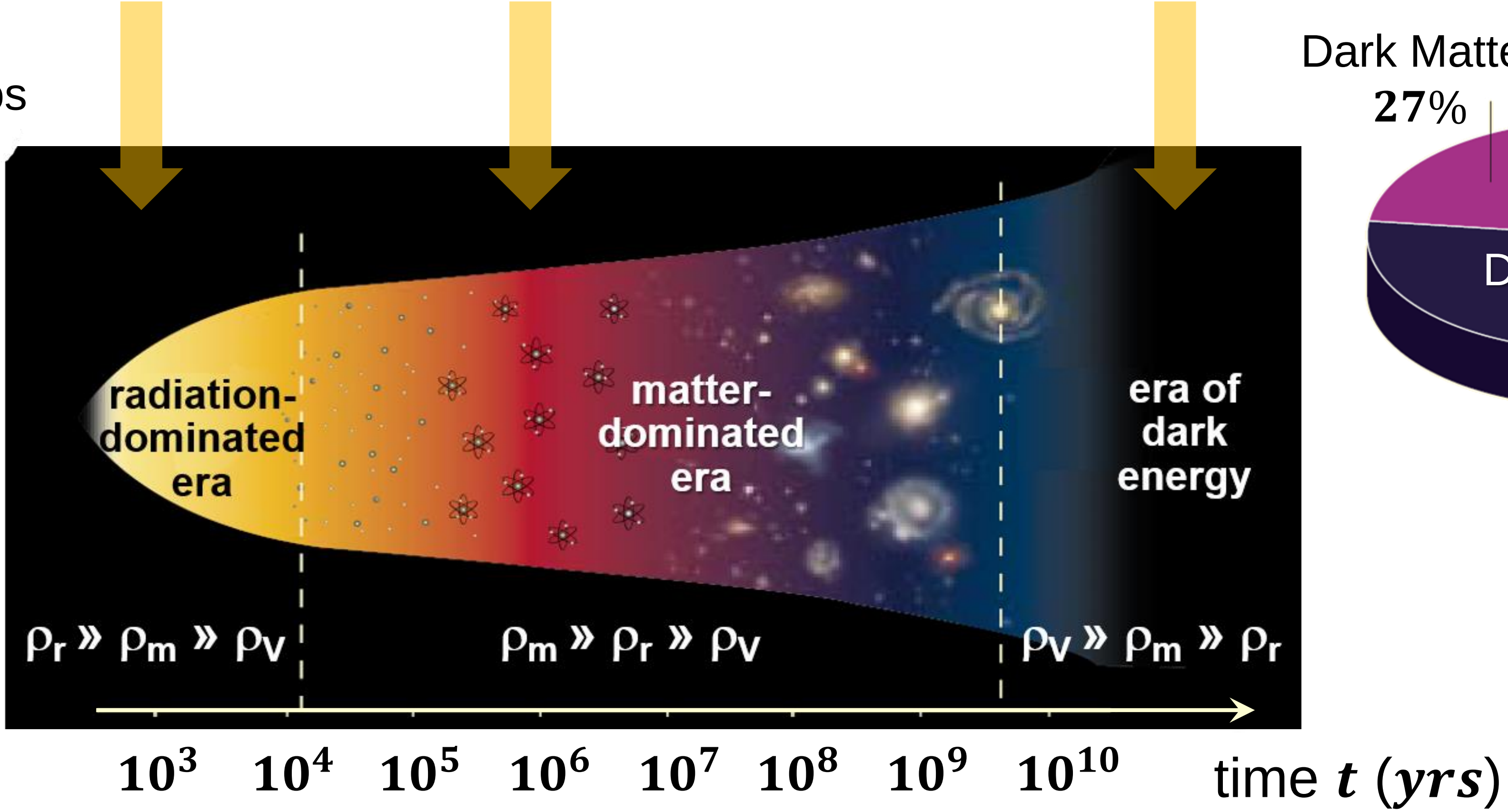


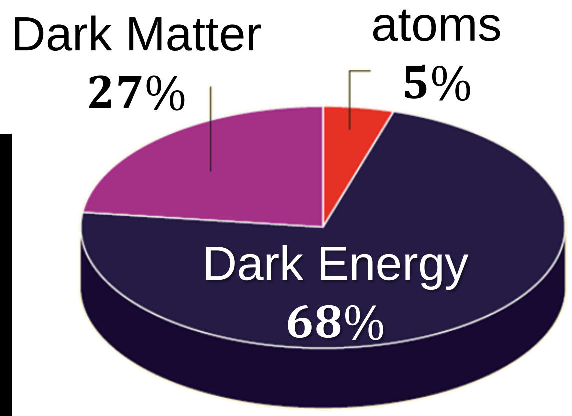
Cosmological expansion: different values of $\ddot{a}(t)$

- **Universe undergoes several phase transitions: each era is unique**

- **radiation**−dominated / **matter**−dominated / **vacuum**−**energy**−dominated universe

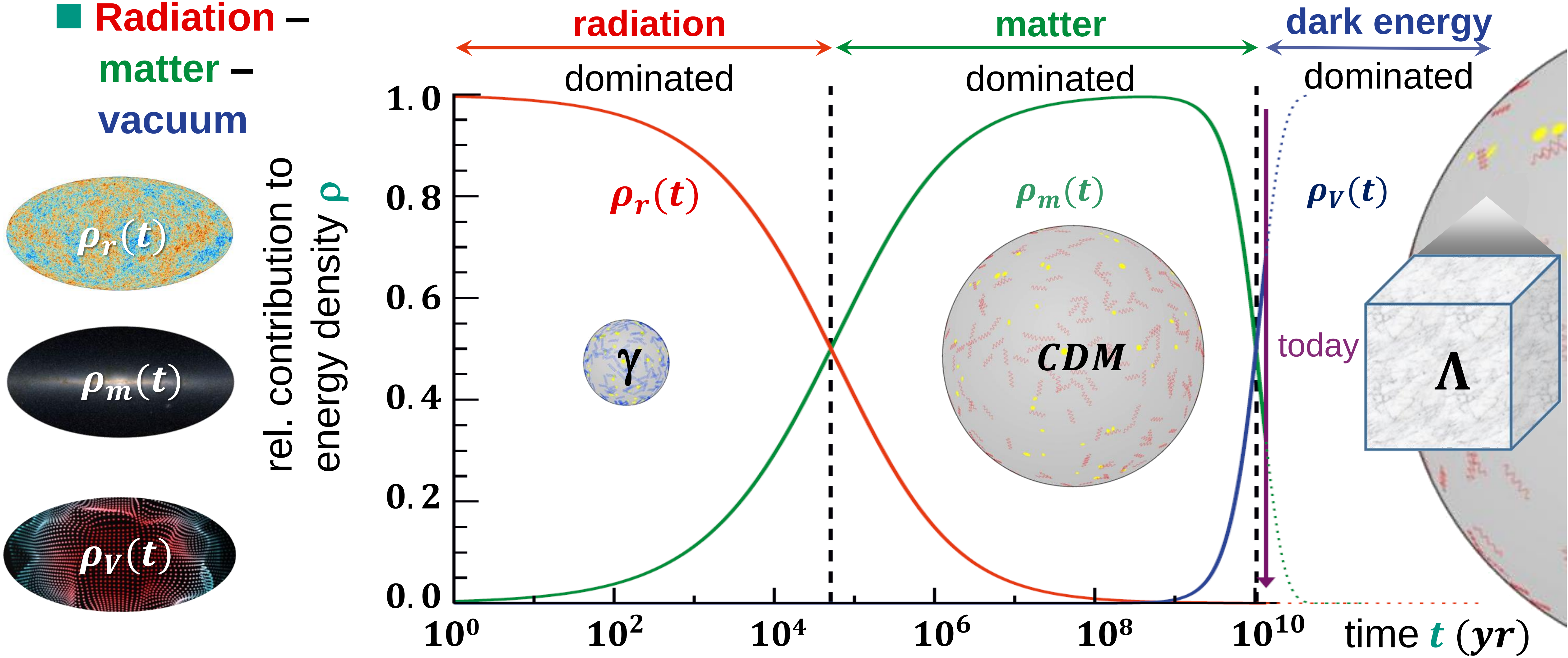






The three different cosmological epochs

- **Radiation – matter – vacuum**



Equation-of-state: the key role of ρ and P

- **Einstein´s General Relativity based on energy−momentum tensor $T_{\mu\nu}$**

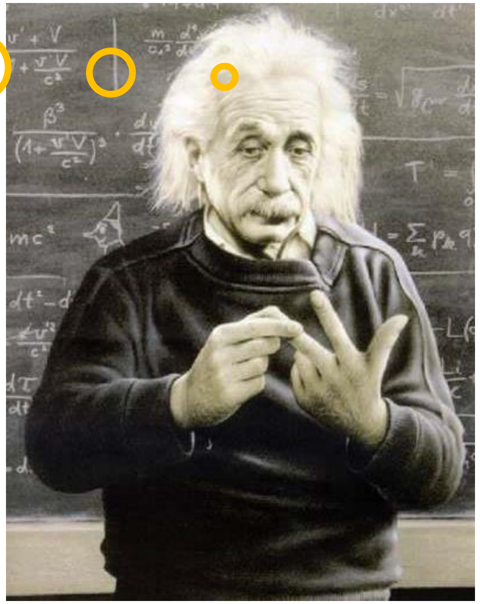
- cosmology: content of universe is treated as an **ideal fluid** using $T_{\mu\nu}$

$$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & -P & & \\ & & -P & \\ & & & -P \end{pmatrix}$$

energy density ρ pressure P

GR: pressure acts as source of gravity

pressure P in GR:
factor 3



- galaxies in a **co−moving system**
  ≡ elements of an isotropic, ideal ´**fluid**´

Equation-of-state: the key role of ρ and P

Karlsruhe Institute of Technology

- **Friedmann equation taking into account General Relativity**

  - the expansion of the **cosmological fluid** can be described by:

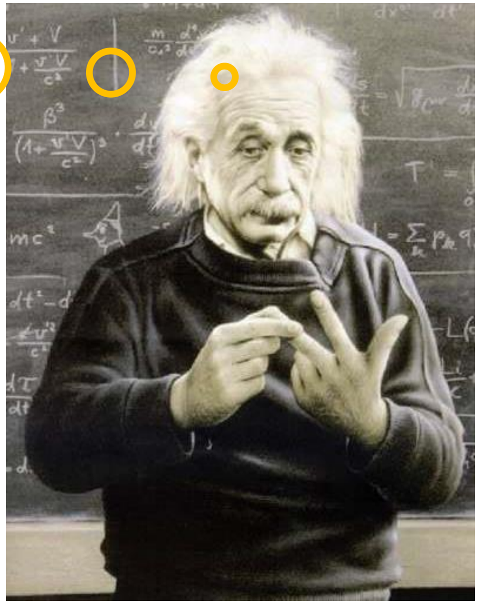
$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\cdot\pi\cdot G\cdot\left(\rho(t)+\frac{3\cdot P}{c^2}\right)$$

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_V\,(t)$$

matter radiation vacuum

GR: pressure acts as source of gravity

pressure P in GR:
factor 3

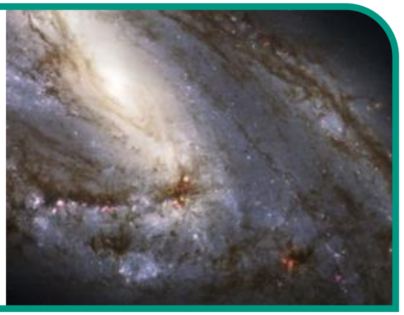


Equation–of–state: the key role of ρ and P

matter (baryonic & DM)

- **pressure–free**: as $P_m \ll \rho_m \cdot c^2$ (´dust´)
  with thermal velocities only $v_m \ll c$

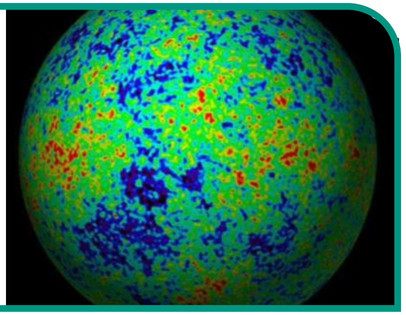
$$P_m = 0$$



radiation ($3K\ CMB$ radiation)

- **positive pressure**: as $k_B T \gg m \cdot c^2$ (´photons´)
  with relativistic velocities c

$$P_r = {}^1/_3 \cdot \rho_r \cdot c^2$$

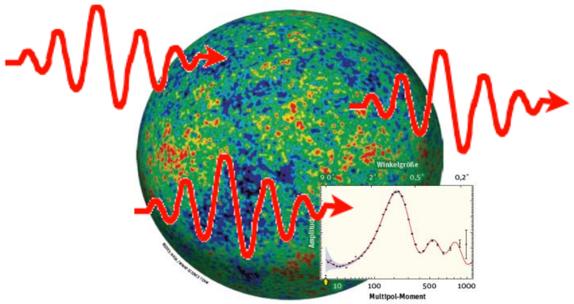
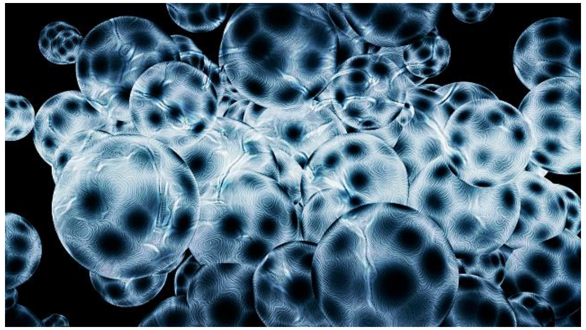


vacuum energy (cosmological constant Λ)

- **negative pressure**: as $dU = -P_V \cdot dV$ (´therm.´)
  dU: ´inner´ energy of empty space with $dU > 0$
  dV: increase of volume with $dV > 0$

$$P_V = -1 \cdot \rho_V \cdot c^2$$

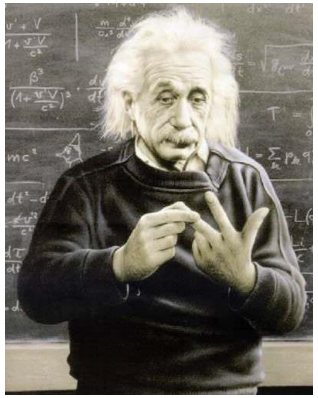
Matter, Radiation & vacuum properties

matter	radiation	vacuum
		
energy density $\rho_m > 0$	energy density $\rho_r > 0$	energy density $\rho_V > 0$
$\rho_m(t_0) = 1.4\, GeV/m^3$	$\rho_r(t_0) = 0.26\, MeV/m^3$	$\rho_V(t_0) = 3.6\, GeV/m^3$
pressure $P_m = 0$	pressure $P_r > 0$	pressure $P_V < 0$
$P_m(t_0) = 0$	$P_r(t_0) = +⅓ \cdot \rho_r(t_0) \cdot c^2$	$P_V(t_0) = -1 \cdot \rho_V\,(t_0) \cdot c^2$

Vacuum properties & cosmic evolution

- **Counteracting contributions of the vacuum as to the cosmological evolution**

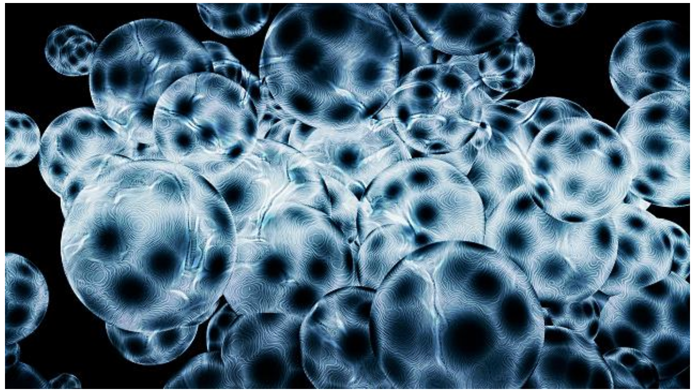
- **vacuum energy**: resulting **braking**– / **acceleration**– parameter $\ddot{a}(t)$



$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3} \cdot \pi \cdot G \cdot \left(\rho_V(t) + \frac{3 \cdot P_V}{c^2} \right)$$

$1 \times$ | $3 \times$

as $\rho_V > 0$
we have $\ddot{a}(t) < 0$



vacuum

as $P_V < 0$
we have $\ddot{a}(t) > 0$