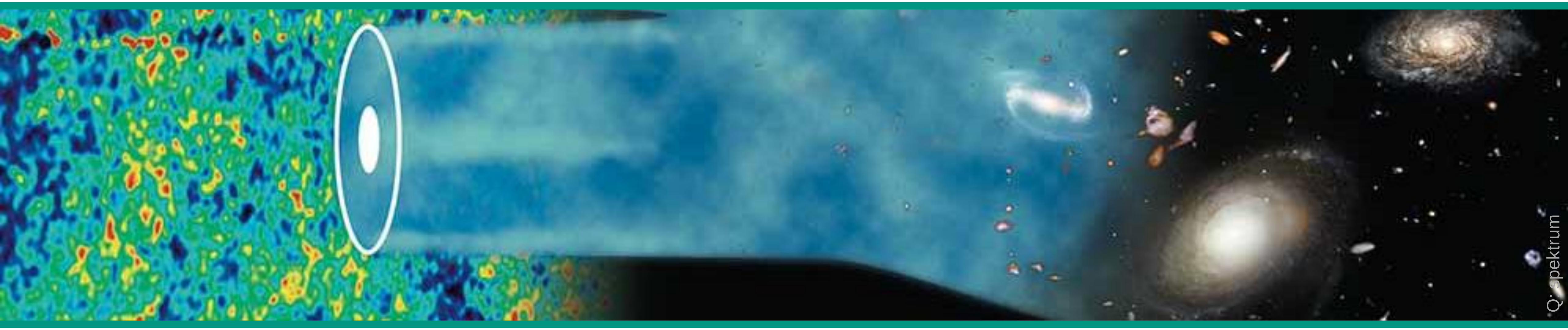
Introduction to Cosmology

Winter term 23/24 Lecture 4 Nov. 14, 2023


Q: spektrum

Recap of Lecture 3

- **Friedmann (–Lemaître) equation: braking *vs.* acceleration**

$$\boxed{\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3} \cdot \pi \cdot G \cdot \left(\rho(t) + \frac{3 \cdot P}{c^2} \right)}$$

3 cosmological epochs:

$$\rho(t) = \rho_r(t) + \rho_m(t) + \rho_V(t)$$

pressure P: important for $a(t), \dot{a}(t), \ddot{a}(t)$

- equation–of–state of **vacuum**: $P_V(t_0) = -1 \cdot \rho_V(t_0) \cdot c^2$ **(anti–gravity)**

- **Topology & geometry of the universe**

  - curvature parameter $k = -1, 0, +1$ **(hyperbolic, flat, spherical)**

  - open questions: isotropy, limited/unlimited size, complex topologies,…

Friedmann eq. with cosmological constant Λ

- **Properties ρ_V and P_V of the vacuum ´merged´ into one parameter: Λ**

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\cdot\pi\cdot G\cdot \underbrace{\left(\rho_{r,m,V}(t) + \frac{3\cdot P_{r,m,V}(t)}{c^2}\right)}_{\text{matter, radiation \& }\textbf{vacuum}}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\cdot\pi\cdot G\cdot \underbrace{\left(\rho_{r,m}(t) + \frac{3\cdot P_{r,m}(t)}{c^2}\right)}_{\text{matter \& radiation}} + \underbrace{\frac{\Lambda\cdot c^2}{3}}_{\textbf{vacuum}}$$

Λ

Cosmological constant Λ

- **vacuum: a very important, key player in cosmology**

$$\boxed{\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\cdot\pi\cdot G\cdot\left(\rho_V(t) + \frac{3\cdot P_V(t)}{c^2}\right)}$$

using vacuum
equation–of–state:

$$\rho_V(t) = -1\cdot P_V(t)$$

$$\Big\downarrow$$

$$\frac{\Lambda\cdot c^2}{3}$$

- **time–independent, constant** parameter

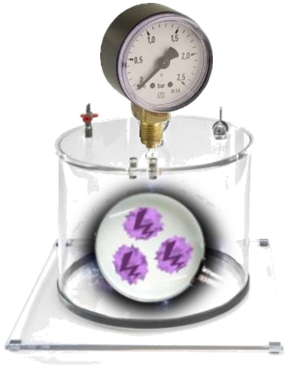
$$\Lambda = +\frac{8\pi\cdot G}{c^2}\cdot\rho_V$$

Cosmological constant Λ

- **Recap: properties of the vacuum**

$3x$

as pressure $P_V < 0$
we have $\ddot{a}(t) > 0$



$1x$

as density $\rho_V > 0$
we have $\ddot{a}(t) < 0$

we keep this relation **constant**,
⇨ **cosmological constant**

$$\rho_V(\cancel{t}) = -1 \cdot P_V(\cancel{t})$$

Cosmological constant Λ

- **A very important constant in cosmology**

$$\boxed{\Lambda = +\frac{8\pi \cdot G}{c^2} \cdot \rho_V}$$

- positive sign, thus it causes an ***accelerated*** expansion of the universe

- best experimental value at present:

$$\Lambda = [\,(2.14 \pm 0.13) \times 10^{-3} eV\,]^4$$

$$\approx 1.1 \cdot 10^{-52}\ m^2 \approx 10^{-29}\ g/cm^3$$

Cosmological constant Λ

- **A very important constant in cosmology**

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SCIENCE

Einstein's 'Biggest Blunder' Turns Out to Be Right

By · Space.com

Published November 24, 2010 3:25pm EST | Updated January 13, 2015 1:59pm EST

A young Albert Einstein lectures in Vienna in 1921. (Ferdinand Schmutzer)

Cosmological constant Λ

- **Experimental value & theoretical estimate: a ´small´ discrepancy**...

observed: $\rho_V = 3.6 \quad GeV/m^3$

estimate: $\rho_V = 10^{121} \; GeV/m^3$

zero−point−energy of a quantum field?

- **biggest discrepancy in all of science!**

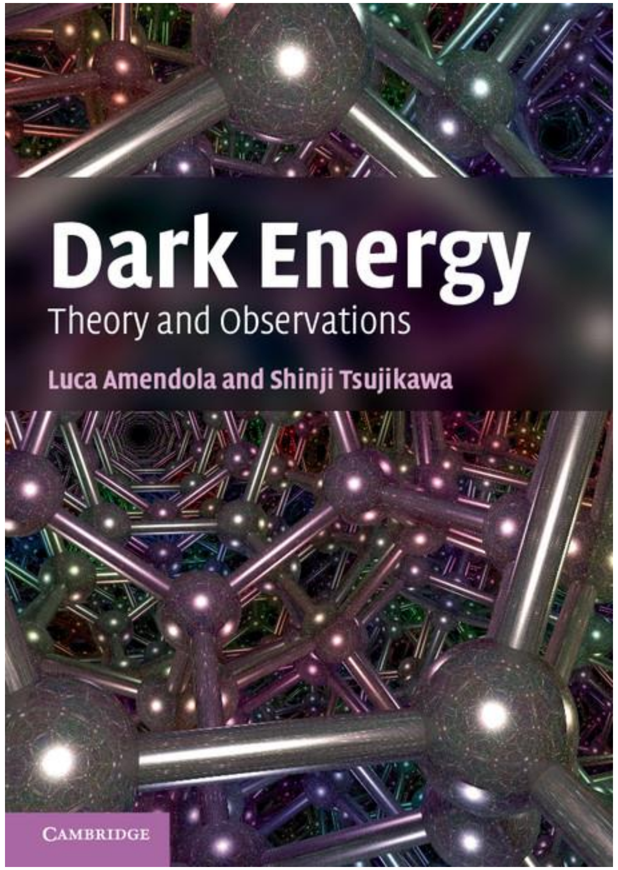
- reduced to ´only´ $60 \; orders \; of \; magnitude$
  in extended models of particle physics*

- literature tip: ***S. Weinberg*** et al. Likely Values of the Cosmological Constant

*supersymmetry ($SUSY$)

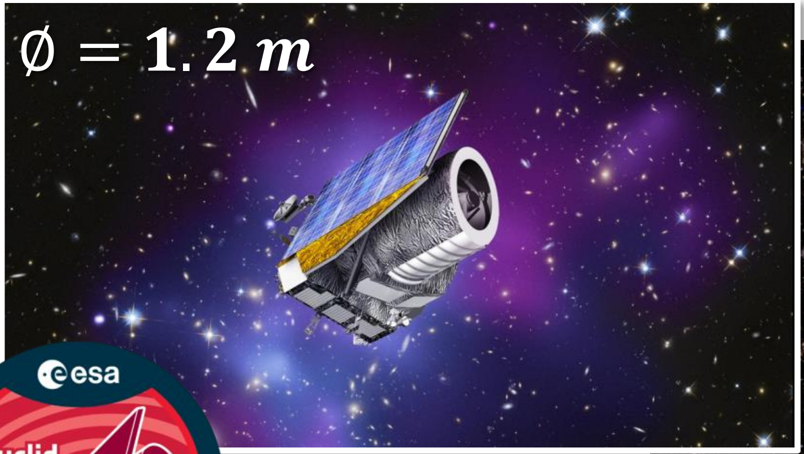
Cosmological constant Λ

- **Popular science: vacuum energy is in central focus of interest (many articles, books,…)**



First pictures of the $EUCLID$ mission: 1 week ago

- Determining the properties of the dark universe by $3D$ – galaxy surveys



- nature of dark energy?
- nature of dark matter?
- history: $a(t), \dot{a}(t), \ddot{a}(t)$

Euclid telescope: First images revealed from 'dark Universe' mission

Perseus cluster

Euclid's first images: the dazzling edge of darkness

07/11/2023 303200 VIEWS 401 LIKES

RELATED TOPIC: THE CASIMIR EFFECT

- **An experimental investigation into the strange properties of the vacuum**

- vacuum is filled with **virtual, short–lived particles** (Heisenberg uncertainty relation)

- two parallel **metal plates** separated by few nm: **boundary conditions** at the plate surfaces

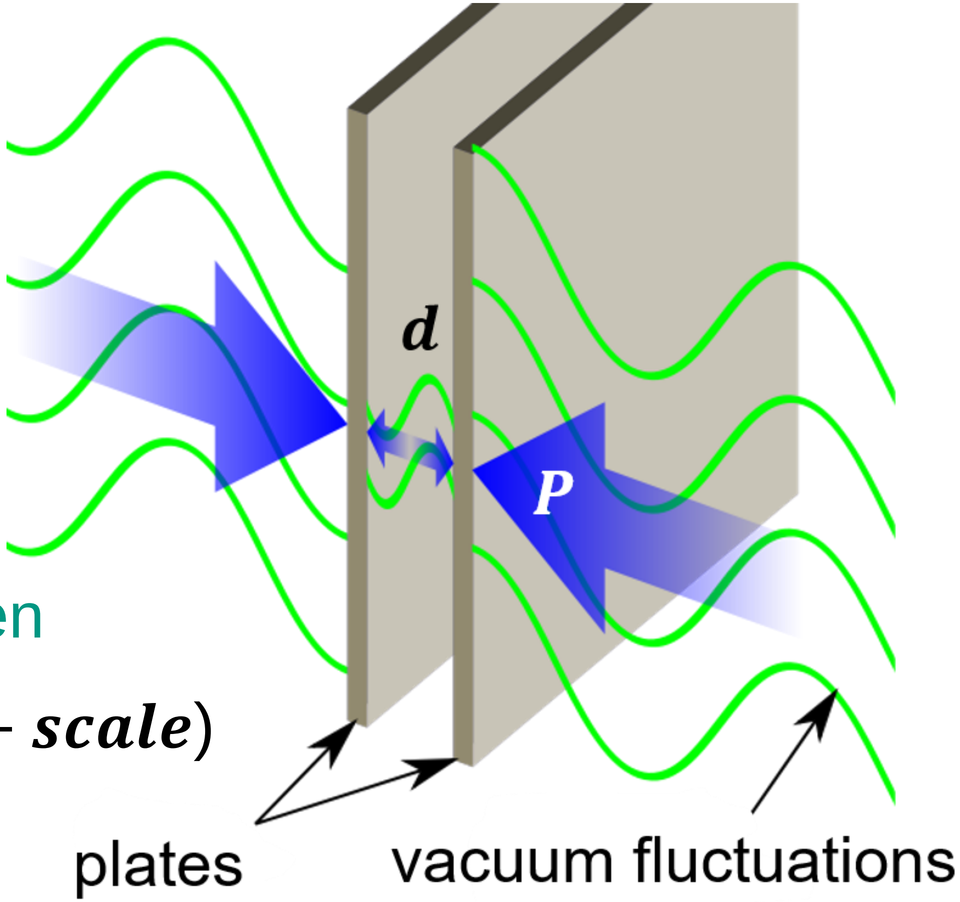
⇨ impact on electro–magnetic field (**virtual photons**)

⇨ different **zero–point energy** inbetween

⇨ net force $F \sim 1/d^3$ (dominant at $nm - scale$)

⇨ first experimental observation in 2001

Hendrik Casimir



RELATED TOPIC: THE CASIMIR EFFECT

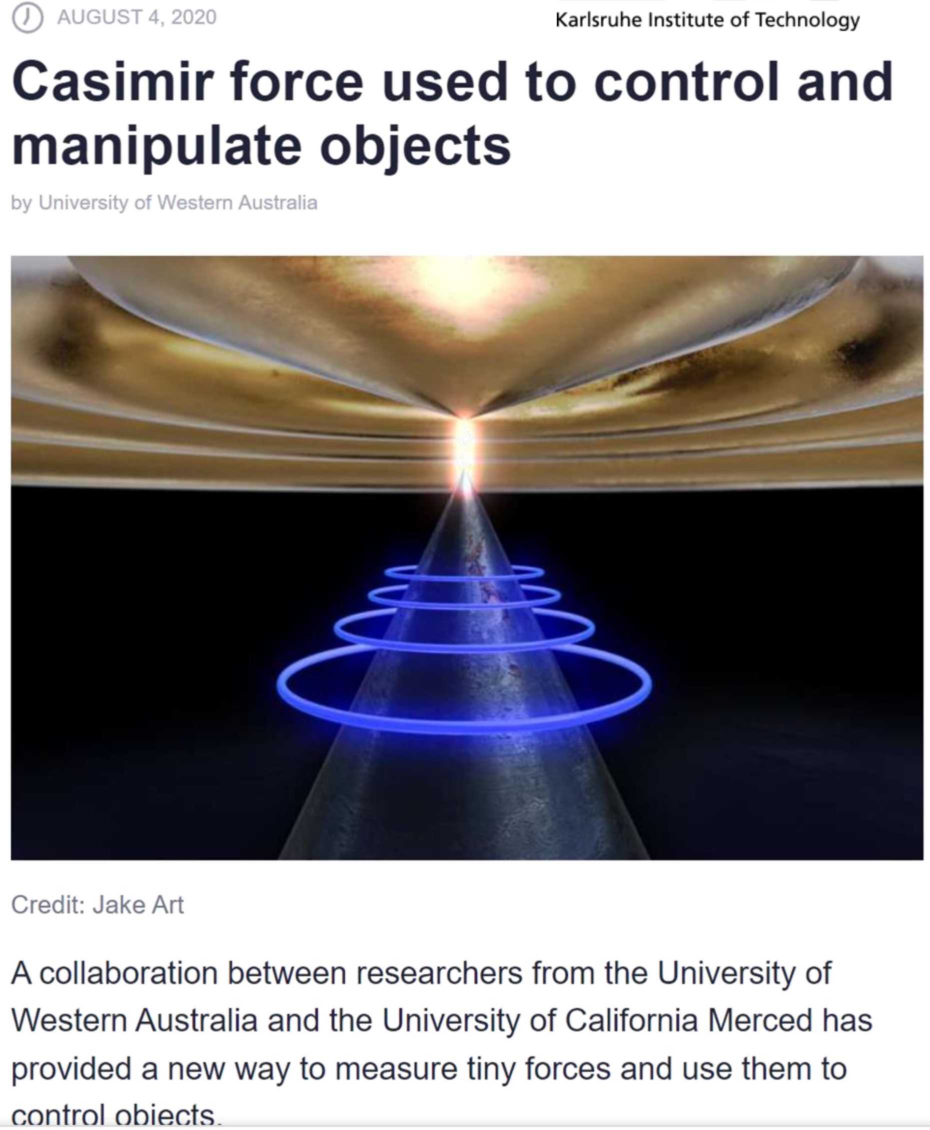
- **Successful experimental investigations**

  - vacuum is filled with **virtual, short–lived particles** (Heisenberg uncertainty relation)

  - Casimir force can now be **measured** by integrated silicon chips (US–Chinese team)

at $\boldsymbol{d = 11\ nm}$

⇨ $\boldsymbol{P = 1\ bar}$



AUGUST 4, 2020

Casimir force used to control and manipulate objects

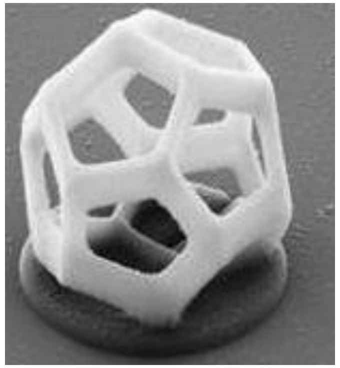
by University of Western Australia

Credit: Jake Art

A collaboration between researchers from the University of Western Australia and the University of California Merced has provided a new way to measure tiny forces and use them to control objects.

nano scribe *

(*KIT* spin–off)



*objects manufactured at $\boldsymbol{\mu m}$ – scale

SIDE–TOPIC: PRESSURE AND GRAVITY

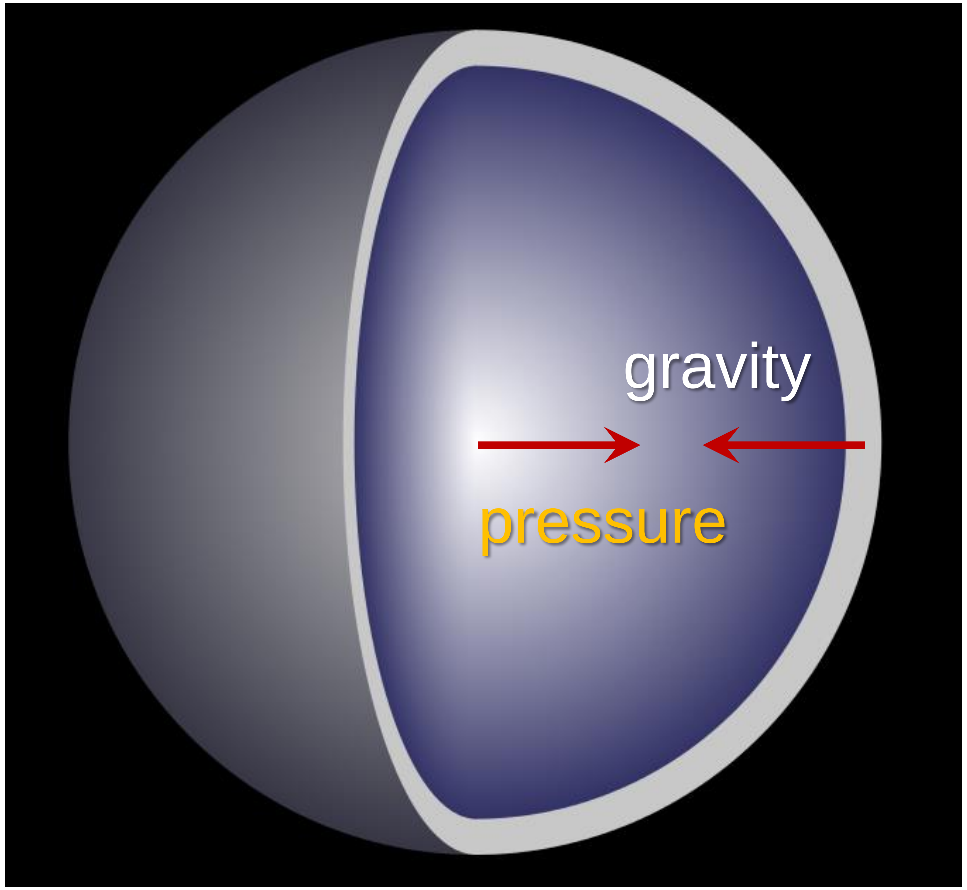
- **The other end: extreme pressure inside a compact object (neutron star)**

  - **neutron stars***: extremely compact objects
  - radius $R \sim 10 \ldots 20\ km$, mass $M < 2 \ldots 3\ M_\odot$
  - very high density $\rho \sim (6 \ldots 8) \times 10^{17}\ kg/m^3$

J. Robert Oppenheimer

- ´**degeneracy**´ pressure of neutrons counteracts gravity, but it is itself acting as a source of the objects´ super–strong gravitational field

⇨ **limited masses** of neutron stars



*s. $ATP - II$ (Summer Term 2024)

Friedmann–Lemaître Equations

- **2 fundamental equations to describe dynamics of cosmological expansion**

  - expansion rates governed by: **matter**, **radiation**, **vacuum**

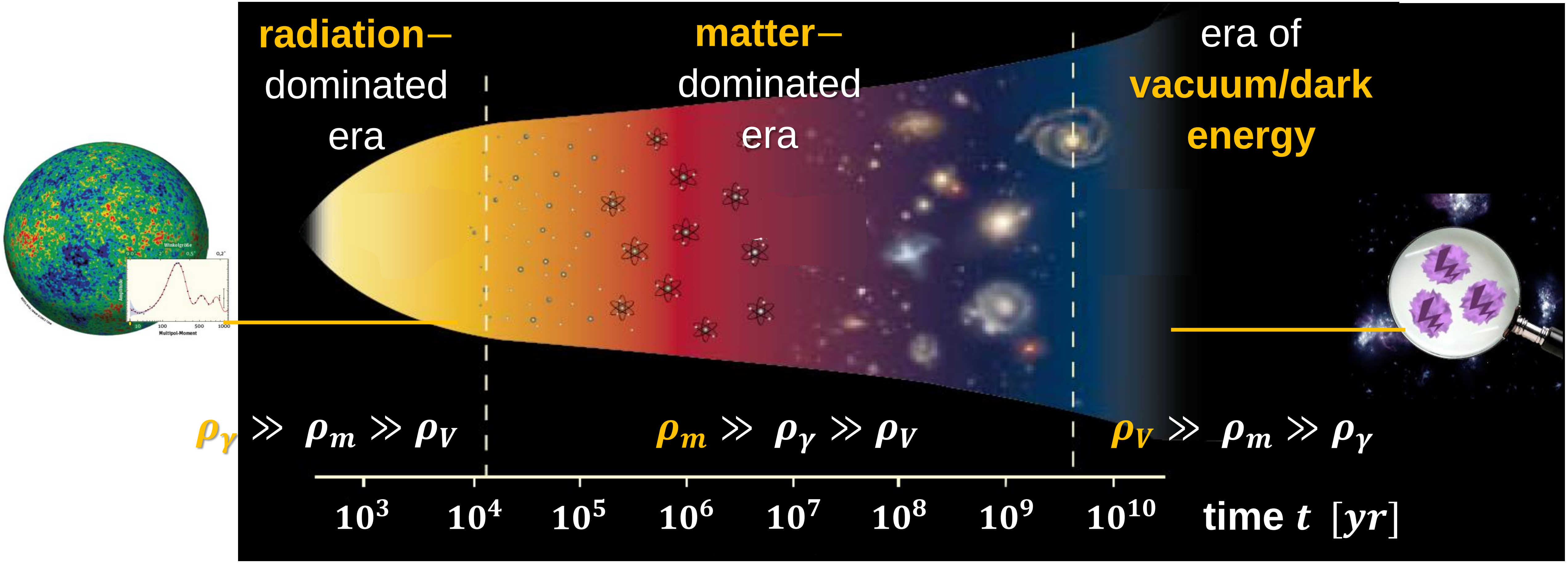
  ⇨ **total energy density & topology of the universe**

Aleksandr Friedmann
(1888 – 1925)

Georges Lemaître
(1894 – 1966)

Different cosmological epochs & $a(t)$

- **Radiation / matter / vacuum energy – dominated cosmological epochs**



radiation– dominated era

matter– dominated era

era of **vacuum/dark energy**

$\rho_\gamma \gg \rho_m \gg \rho_V$

$\rho_m \gg \rho_\gamma \gg \rho_V$

$\rho_V \gg \rho_m \gg \rho_\gamma$

10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10} time t $[yr]$

Friedmann–Equations for ΛCDM

- **Full picture of the evolution of $a(t), \dot{a}(t), \ddot{a}(t)$ for all 3 cosmological epochs**

  - we will now* start to **integrate** our well–known **acceleration** equation to obtain a relation for the ´**velocity**´ parameter $\dot{a}(t)$

$\boxed{\dot{a}(t)}$

$\int$

energy densities   pressure values   time–dependent

$$\frac{\boxed{\ddot{a}(t)}}{a(t)} = -\frac{4}{3}\cdot\pi\cdot G\cdot\left(\rho_{r,m,V}(t)+\frac{3\cdot P_{r,m,V}(t)}{c^2}\right)$$

cosmological constant

$$= -\frac{4}{3}\cdot\pi\cdot G\cdot\left(\rho_{r,m}(t)+\frac{3\cdot P_{r}(t)}{c^2}\right)+\frac{\Lambda\cdot c^2}{3}$$

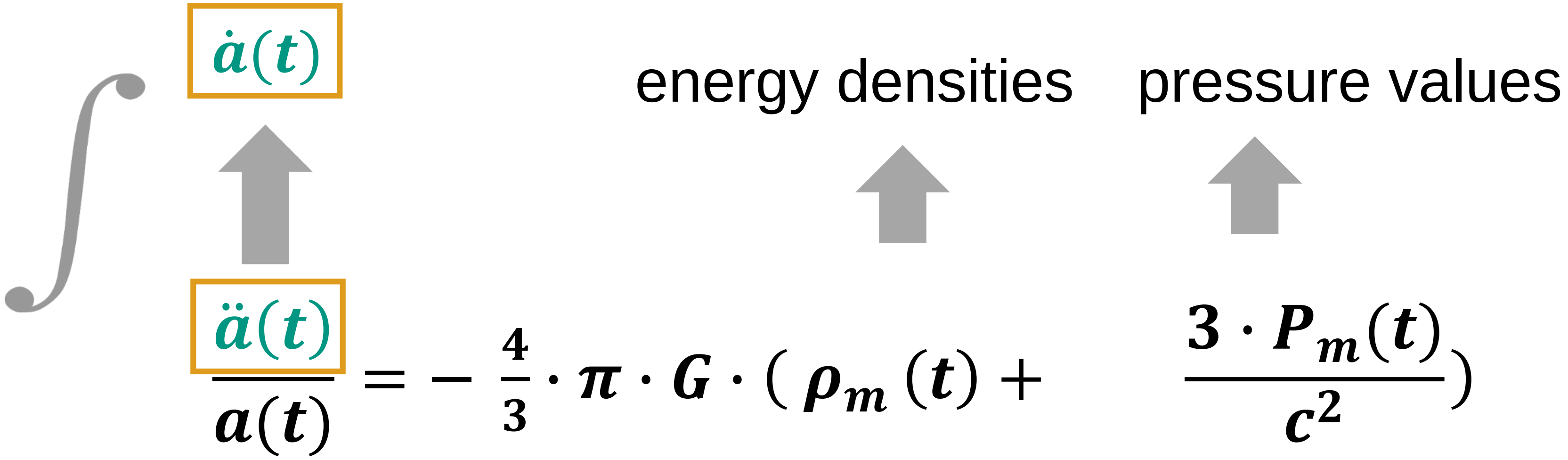
---

* a full treatment requires Einstein´s General Relativity

Friedmann–Equations for CDM

- **picture of the evolution of $a(t), \dot{a}(t), \ddot{a}(t)$ for epoch of matter dominance**

  - we now focus on the second epoch, where **pressure–less** matter is dominant at around $t \approx 10^4\ yr$ after the Big Bang



$\boxed{\dot{a}(t)}$

$\int$

energy densities

pressure values

$$\frac{\boxed{\ddot{a}(t)}}{a(t)} = -\frac{4}{3}\cdot\pi\cdot G\cdot\left(\rho_m(t) + \frac{3\cdot P_m(t)}{c^2}\right)$$

- **pressure–free**: as $P_m \ll \rho_m \cdot c^2$ (´dust´) with thermal velocities only $v_m \ll c$

$$P_m = 0$$

Friedmann–Equations for CDM

- **Let´s do some maths: integrate to obtain the second expansion equation**

$$\ddot{a}(t) = -\frac{4}{3}\cdot\pi\cdot G\cdot\rho(t)\cdot a(t)$$

$$\rho(t) = \frac{\rho_0}{a^3(t)}$$

$$\ddot{a}(t) = -\frac{4}{3}\cdot\pi\cdot G\cdot\rho_0\cdot\frac{1}{a^2(t)} \quad |\times 2\cdot\dot{a}(t)$$

$$\ddot{a}(t)\cdot 2\cdot\dot{a}(t) = -\frac{2\cdot 4}{3}\cdot\pi\cdot G\cdot\rho_0\cdot\frac{\dot{a}(t)}{a^2(t)}$$

$\int$ integration

k: integration constant

$$\dot{a}^2(t) = -\frac{8}{3}\cdot\pi\cdot G\cdot\rho_0\cdot\left(\frac{-1}{a(t)}\right) - kc^2$$

$$\rho(t) = \rho_m(t)$$

Friedmann–Equations: we´re (almost) done…

- **Second Friedmann Expansion Equation**

$$\dot{a}^2(t) = -\frac{8}{3}\cdot\pi\cdot G\cdot\rho_0\cdot\left(\frac{-1}{a(t)}\right) - kc^2$$

re–use: $\rho_0 = \rho(t)\cdot a^3(t)$

$$\dot{a}^2(t) = \frac{8}{3}\cdot\pi\cdot G\cdot\rho(t)\cdot a^2(t) - kc^2 \qquad |\ :a^2(t)$$

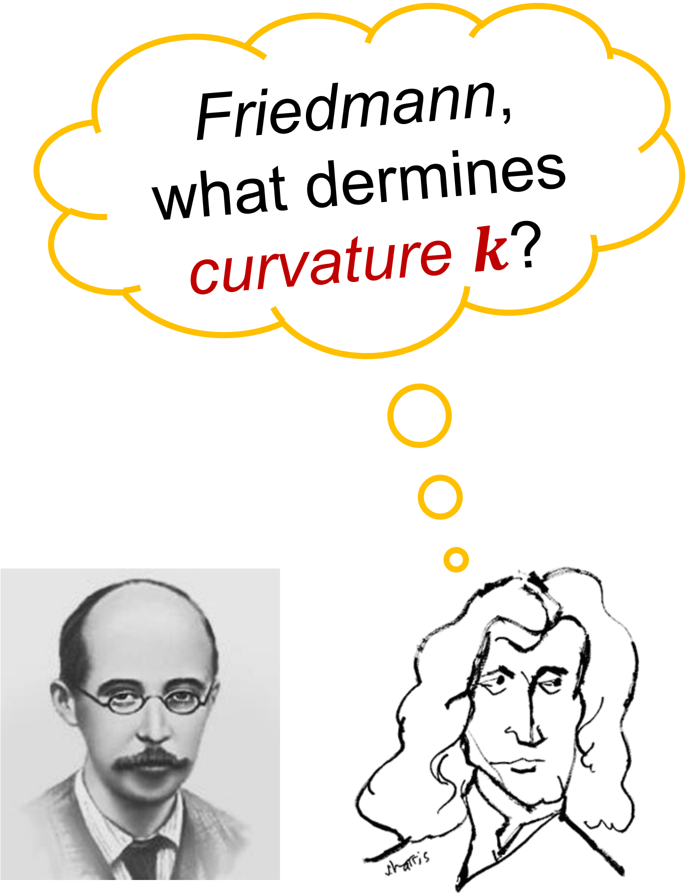
$$\boxed{H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8}{3}\cdot\pi\cdot G\cdot\rho(t) - \frac{kc^2}{a^2(t)}}$$

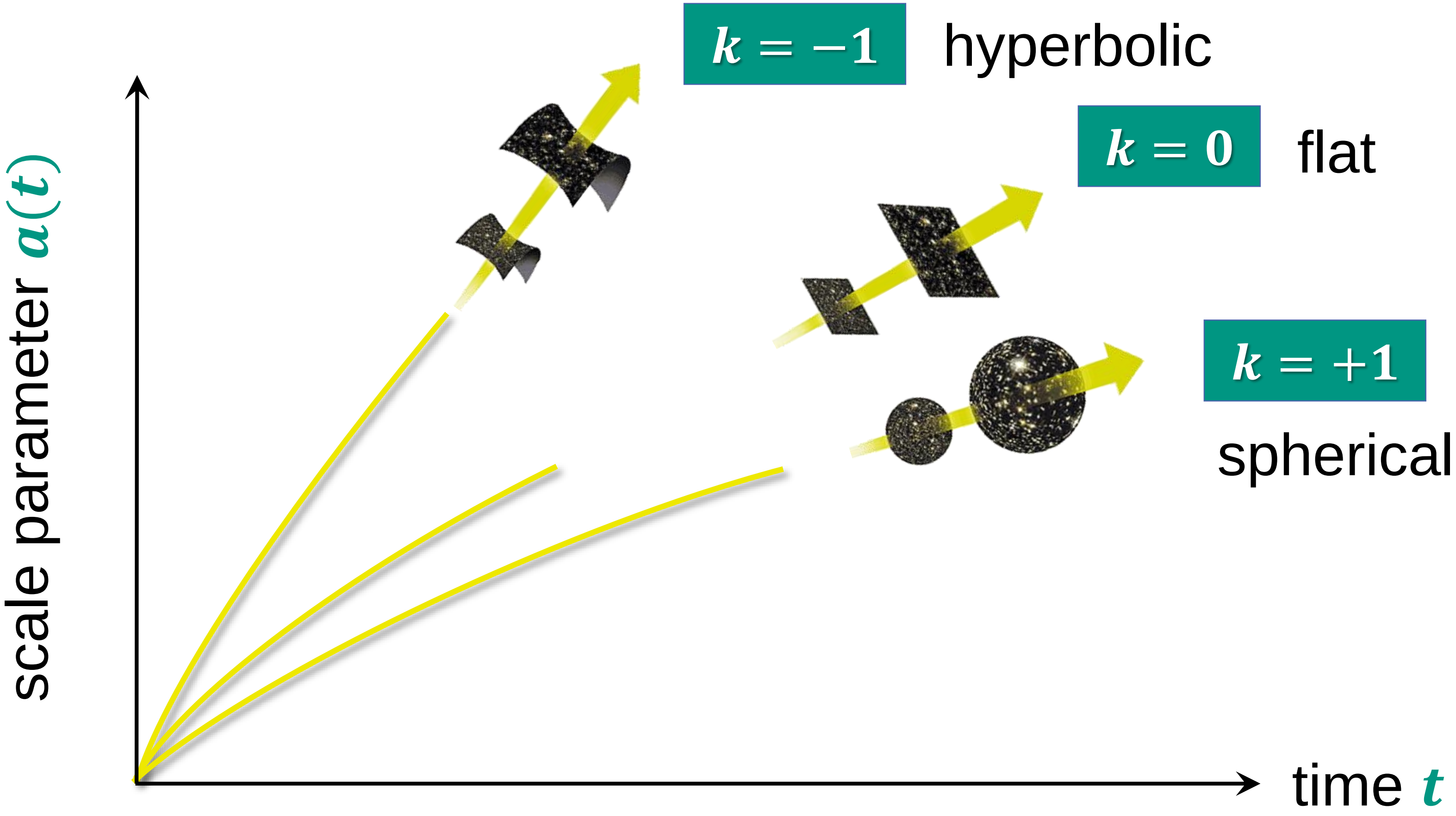
curvature k of the universe

- allows to calculate **Hubble parameter $H^2(t)$** for CDM epoch

Topology and overall energy density

- **Curvature parameter k ´from integration´: impact on scale parameter $a(t)$**





Topology and overall energy density: some math

- **To see which parameter is determining k let´s use co−moving coordinates x**

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 - \frac{8}{3}\cdot\pi\cdot G\cdot\rho(t) = -\frac{kc^2}{a^2(t)} \qquad |\cdot\frac{a(t)^2}{2}$$

$$\frac{\dot{a}(t)^2}{2} - \frac{4}{3}\cdot\pi\cdot G\cdot\rho(t)\cdot a(t)^2 = -\frac{k\cdot c^2}{2}$$

$$\frac{\dot{r}(t)^2}{2\cdot x^2} - \frac{4}{3}\cdot\pi\cdot G\cdot\rho_0\cdot\frac{x}{r(t)} = -\frac{k\cdot c^2}{2}$$

$$\rho(t) = \frac{\rho_0}{a^3(t)} = \rho_0\cdot\frac{x^3}{r^3(t)}$$

$$a(t) = \frac{r(t)}{x}$$

$$\dot{a}(t) = \frac{\dot{r}(t)}{x}$$

Topology and overall energy density: some math

- **parameter k : expression within unit sphere in co−moving coordinates x**

$$\frac{\dot{r}(t)^2}{2\cdot x^2} - \frac{4}{3}\cdot\pi\cdot G\cdot\rho_0\cdot\frac{x}{r(t)} = -\frac{k\cdot c^2}{2}$$

$$|\cdot x^2$$

$$\frac{\dot{r}(t)^2}{2} - G\cdot\frac{M(x)}{r(t)} = -\frac{k\cdot c^2}{2}\cdot x^2$$

$$M(x) = \frac{4}{3}\cdot\pi\cdot\rho_0\cdot x^3$$

for **unit** sphere

$$x \equiv 1$$

$$M = M(x)$$

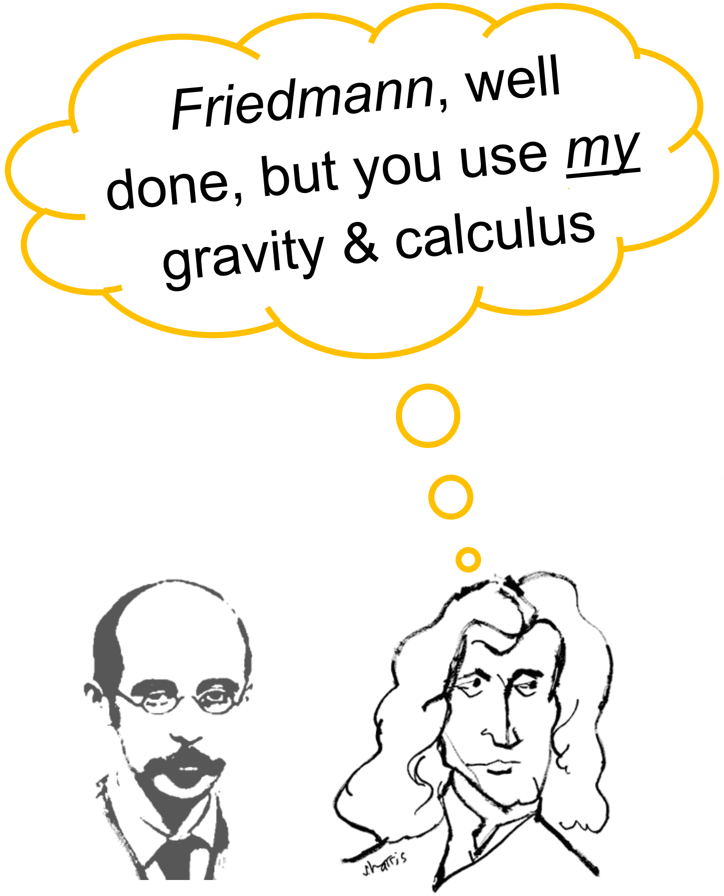
$$\underbrace{\frac{\dot{r}(t)^2}{2}} - \underbrace{G\cdot\frac{M}{r(t)}} = -\underbrace{\frac{k\cdot c^2}{2}}$$

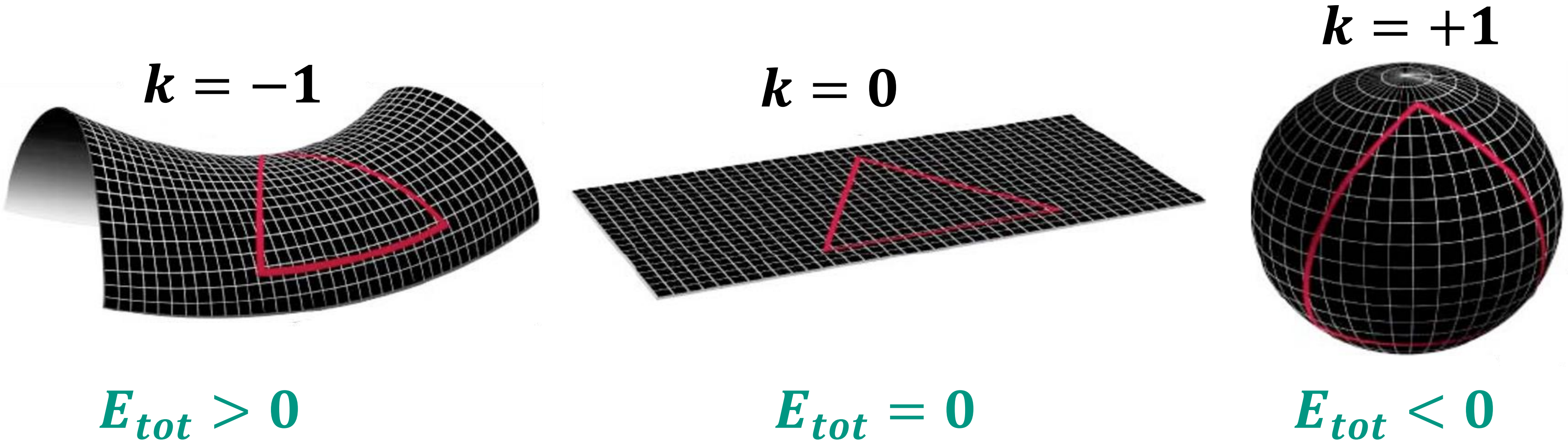
$$E_{kin} \quad + \quad E_{pot} \quad = \quad E_{tot}$$

Topology and overall energy density: curvature k

- **Curvature k of the universe is determined by its total energy E_{tot}**

constant k	curvature	topology	total energy
$k = -1$	hyperbolic	open	$E_{tot} > 0$
$k = 0$	euclidean	flat	$E_{tot} = 0$
$k = +1$	spherical	closed	$E_{tot} < 0$

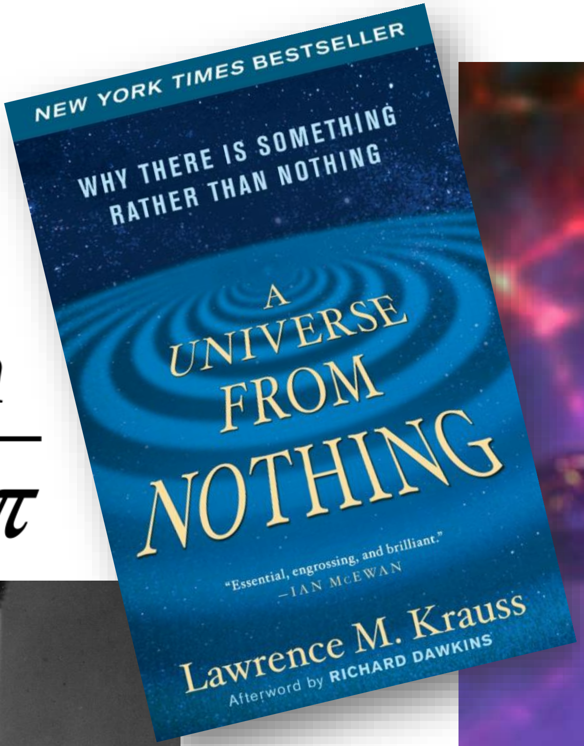




Topology and overall energy density

- **Heisenberg uncertainty relation in view of the total energy of the universe**

$$\overset{\approx 0}{\Delta E}\,\overset{\approx \infty}{\Delta t} \leq \frac{h}{2\pi}$$



NEW YORK TIMES BESTSELLER

WHY THERE IS SOMETHING RATHER THAN NOTHING

A UNIVERSE FROM NOTHING

"Essential, engrossing, and brilliant." —IAN McEWAN

Lawrence M. Krauss

Afterword by RICHARD DAWKINS

our universe: a long–lived quantum fluctuation emerging after a **huge expansion** due to inflation?

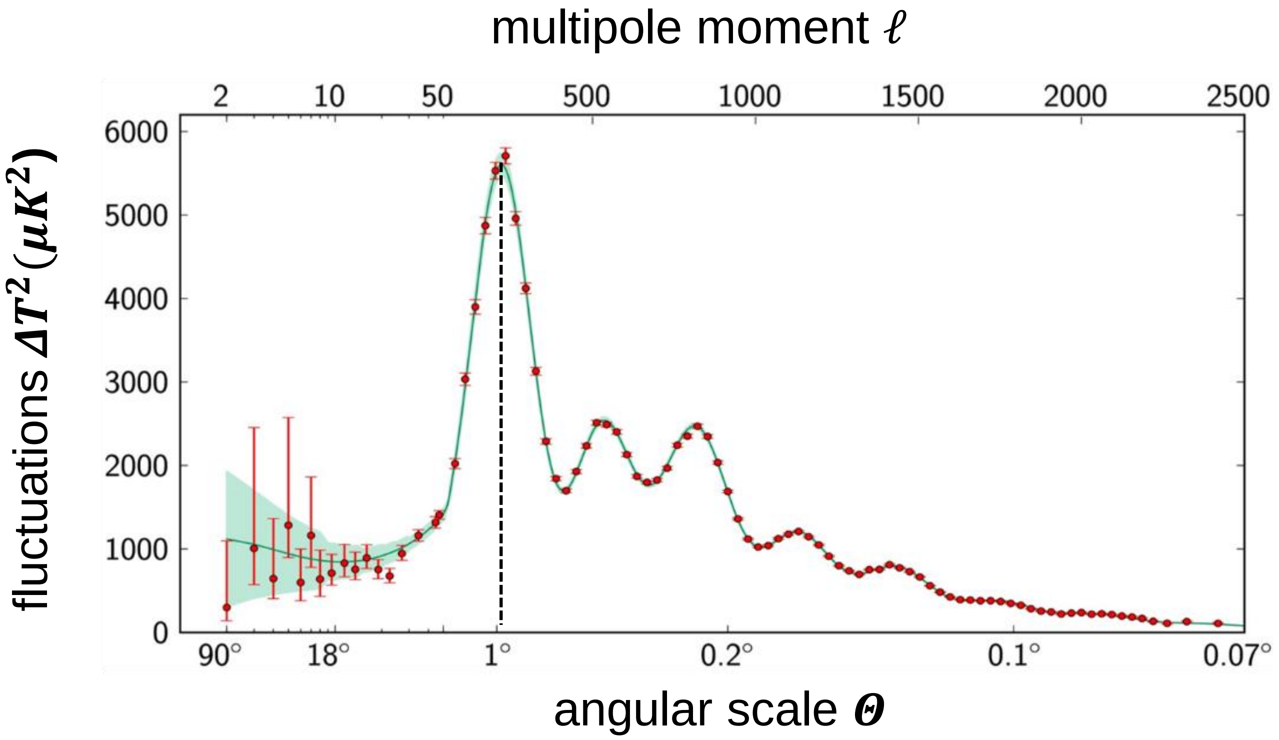
Topology and overall energy density

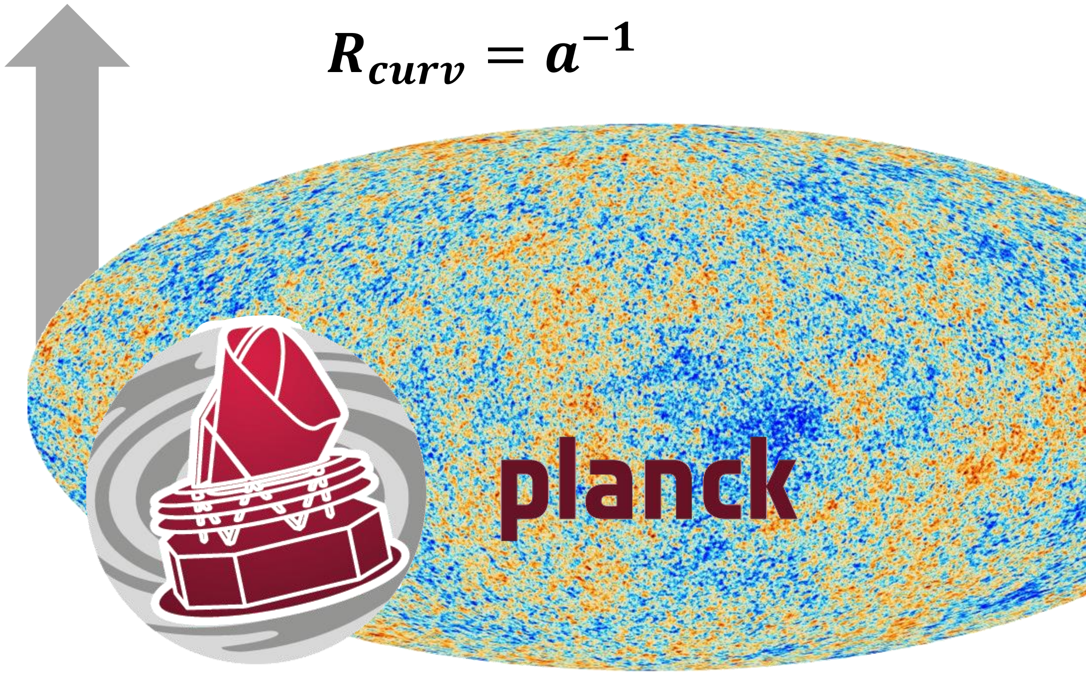
- **2015 findings of the Planck satellite mission: a universe <u>*without*</u> curvature**

\- analysis of the $\boldsymbol{CMB}$ **multipole distribution***
curvature $\boldsymbol{k = 0}$ $(\boldsymbol{\Omega_k})$ from $\mathbf{1.}$ **peak** at $\ell \sim \mathbf{200}$

$$\boldsymbol{\Omega_k = \frac{-kc^2}{H_0^2 \cdot R_{curv}^2} = 0.000 \pm 0.005}$$

$$\boldsymbol{R_{curv} = a^{-1}}$$





*see following lectures # $\mathbf{8}$ & $\mathbf{9}$

overall energy density & inflationary cosmology

- **2015 findings of the $Planck$ satellite vs. expectation from theory of inflation**

\- **inflation theory**:

exponential increase of size $a(t)$ of universe at time $t = 10^{-36} \dots 10^{-32}\ s$ due to **evolution of a scalar field**

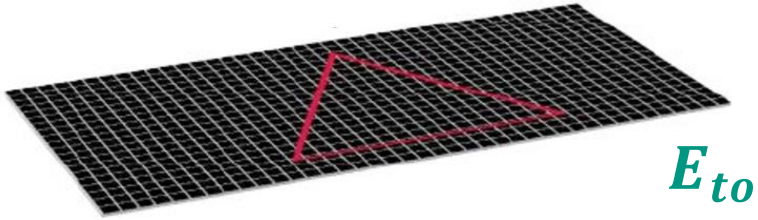
⇨ typical expansion factor $\gg 10^{26}$

⇨ **flat space, no curvature**

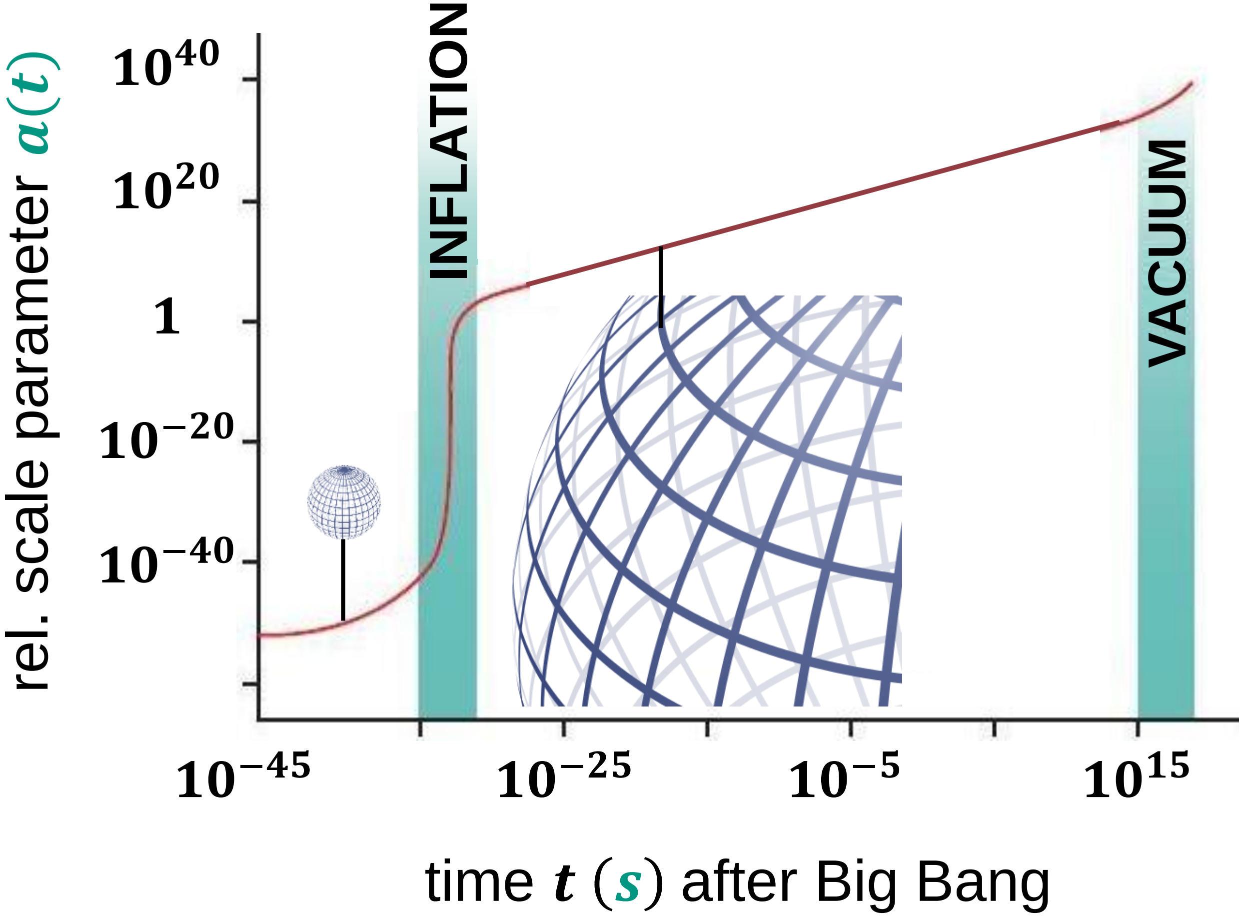
\- **observational fact** $(Planck, 2015)$:

⇨ space is **flat** to $\sim 0.5\%$

$k = 0$



$E_{tot} = 0$

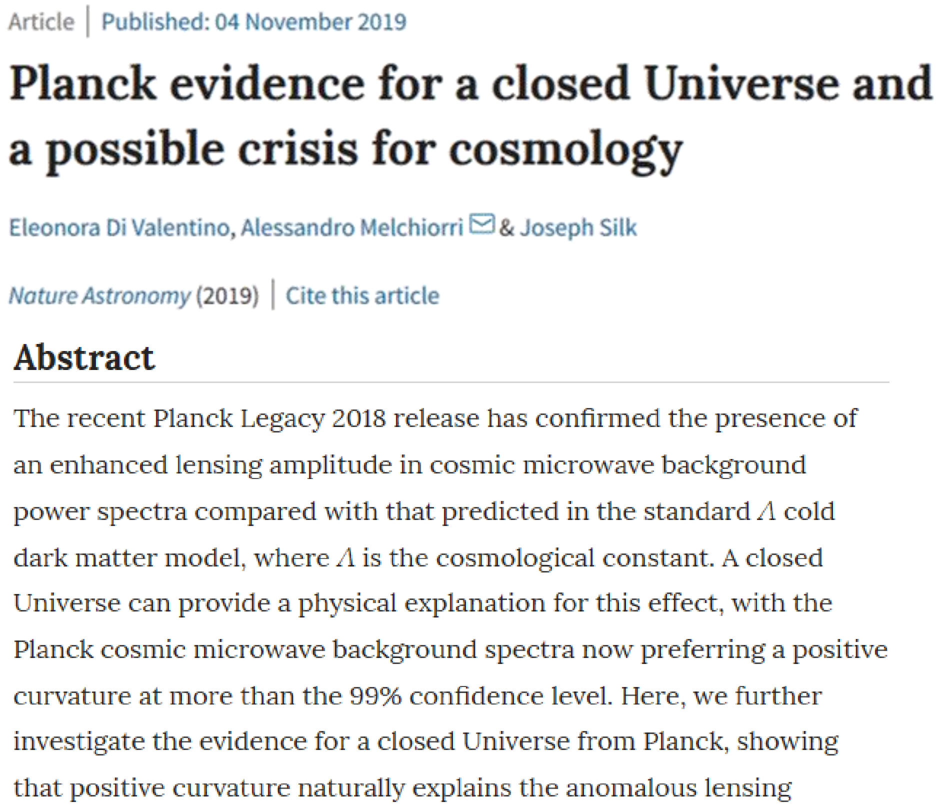


Topology and overall energy density

- **2018 findings of the $\boldsymbol{Planck}$ satellite mission: a universe _with_ curvature??**

  - analysis of $\boldsymbol{CMB}$ radiation using **lensing effect***

$$\boldsymbol{\Omega_k = -0.007 \ldots - 0.095}$$
$$\boldsymbol{(99\%\ CL)}$$



Article | Published: 04 November 2019

Planck evidence for a closed Universe and a possible crisis for cosmology

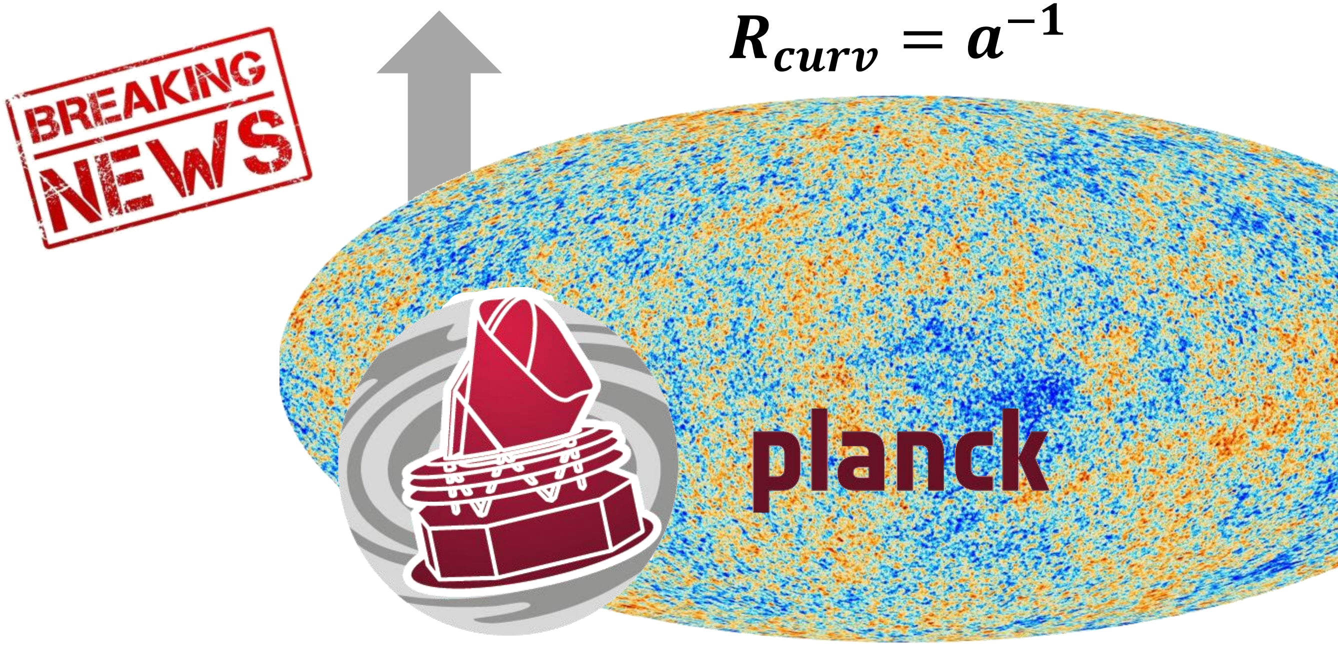
Eleonora Di Valentino, Alessandro Melchiorri & Joseph Silk

Nature Astronomy (2019) | Cite this article

Abstract

The recent Planck Legacy 2018 release has confirmed the presence of an enhanced lensing amplitude in cosmic microwave background power spectra compared with that predicted in the standard Λ cold dark matter model, where Λ is the cosmological constant. A closed Universe can provide a physical explanation for this effect, with the Planck cosmic microwave background spectra now preferring a positive curvature at more than the 99% confidence level. Here, we further investigate the evidence for a closed Universe from Planck, showing that positive curvature naturally explains the anomalous lensing

$$\boldsymbol{R_{curv} = a^{-1}}$$



*see lectures # **13** & **14**

inflationary cosmology: acclerated masses

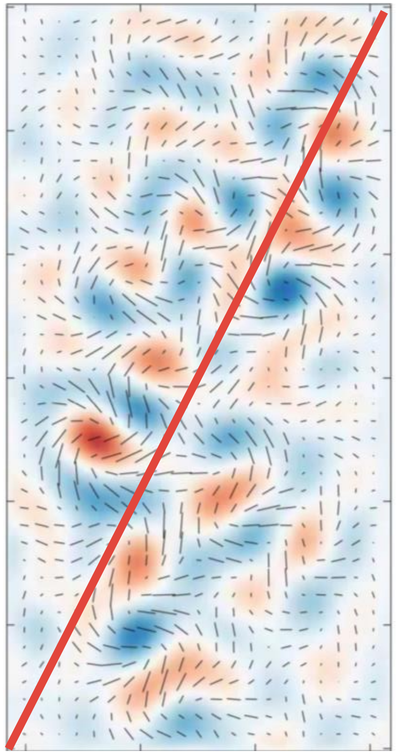
- **2021 update from $BICEP3$ vs. expectation from the theory of inflation**

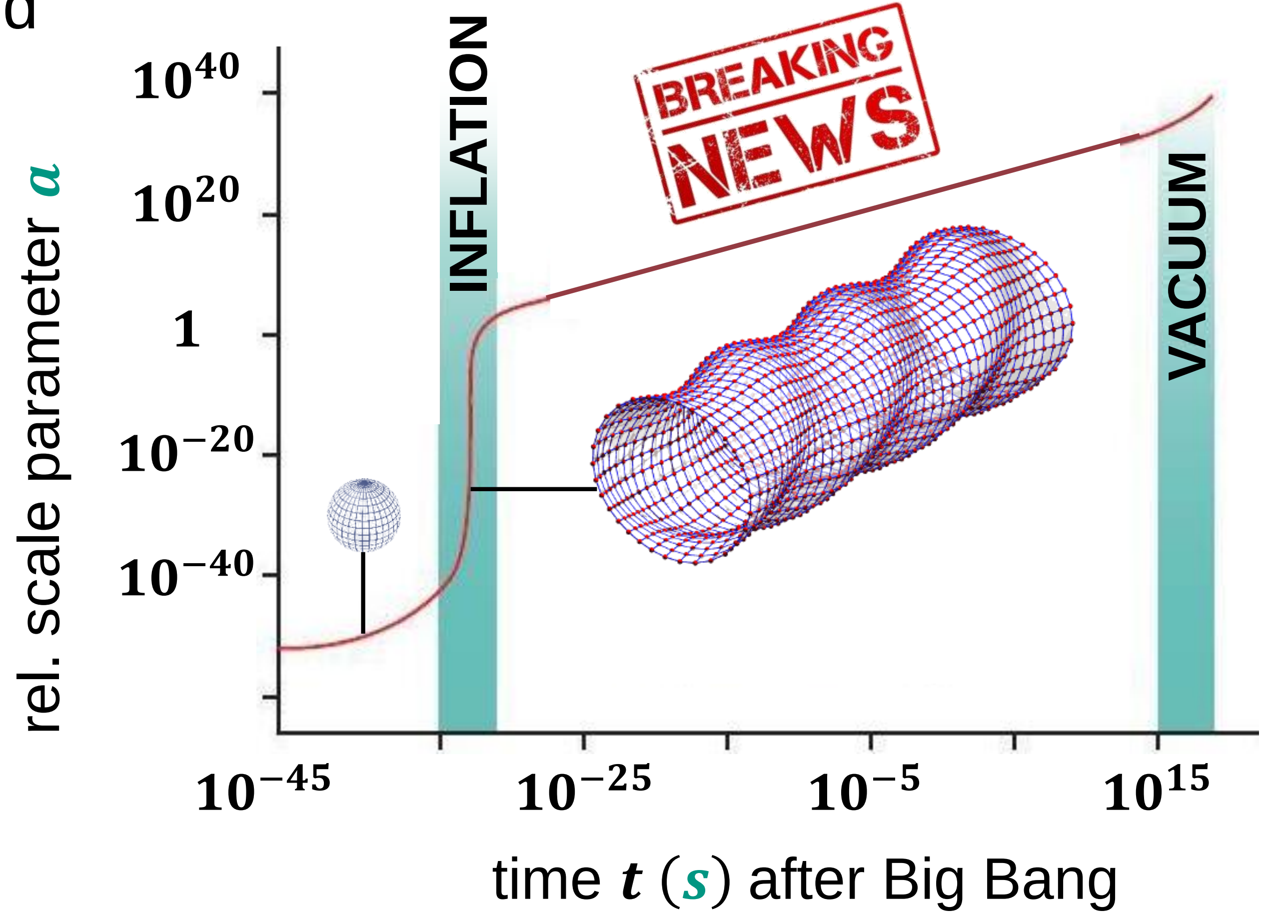
- inflationary epoch should have produced **specific GW^*** signal: but **no detection**!

BICEP3 tightens the bounds on cosmic inflation

10/26/21 | By Nathan Collins

A new analysis of the South Pole-based telescope's observations has all but ruled out several popular models of inflation.





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*Gravitational Waves, see $ATP-2$ (summer 2024)

Friedmann equations for a flat CDM universe

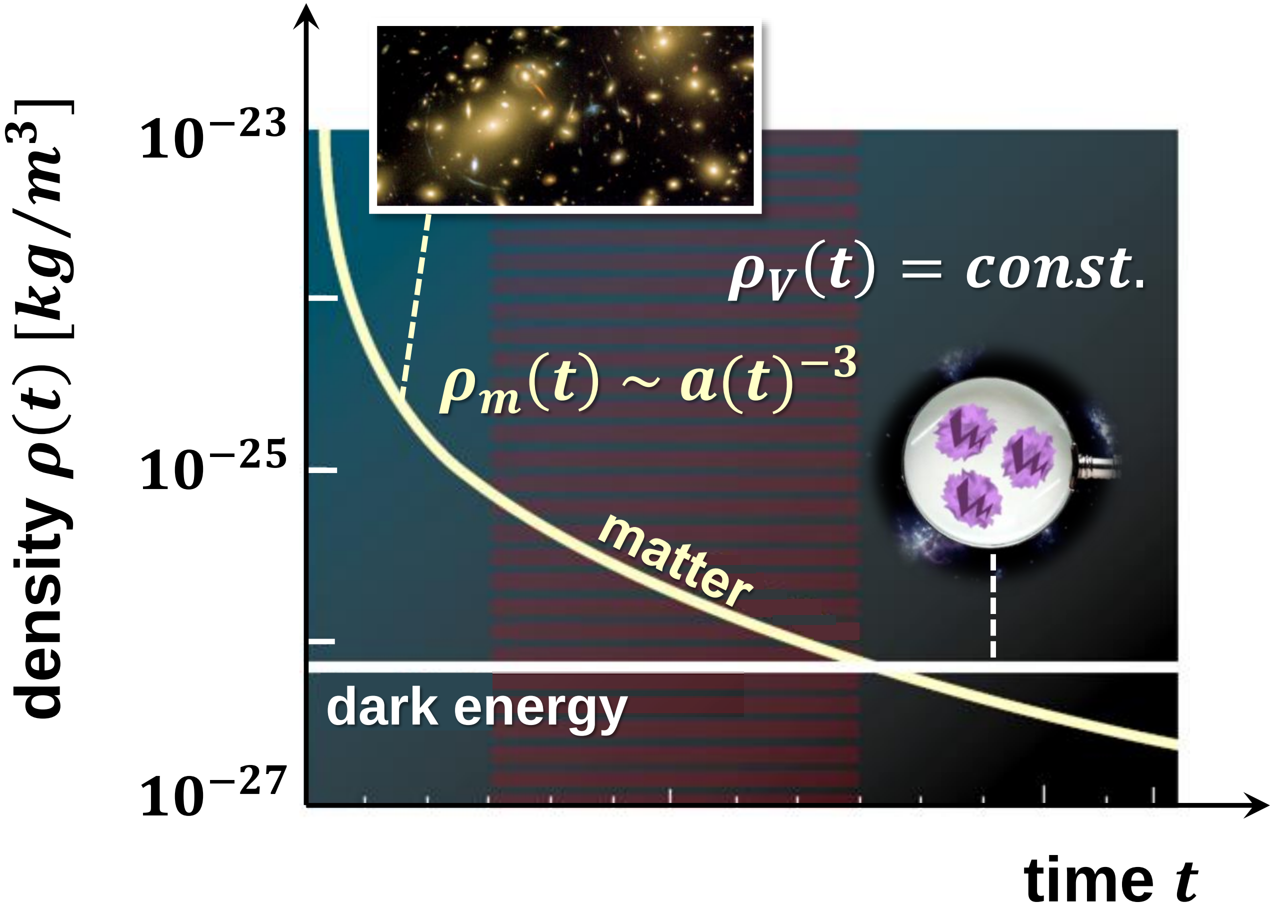
- **Second expansion equation: development of $\rho(t)$ over cosmic time scales t**

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2$$

$$= \frac{8}{3} \cdot \pi \cdot G \cdot \rho_m(t)$$

$$\boxed{H(t) \sim \sqrt{\rho_m(t)}}$$

- we now need to account for the vacuum energy (Λ)



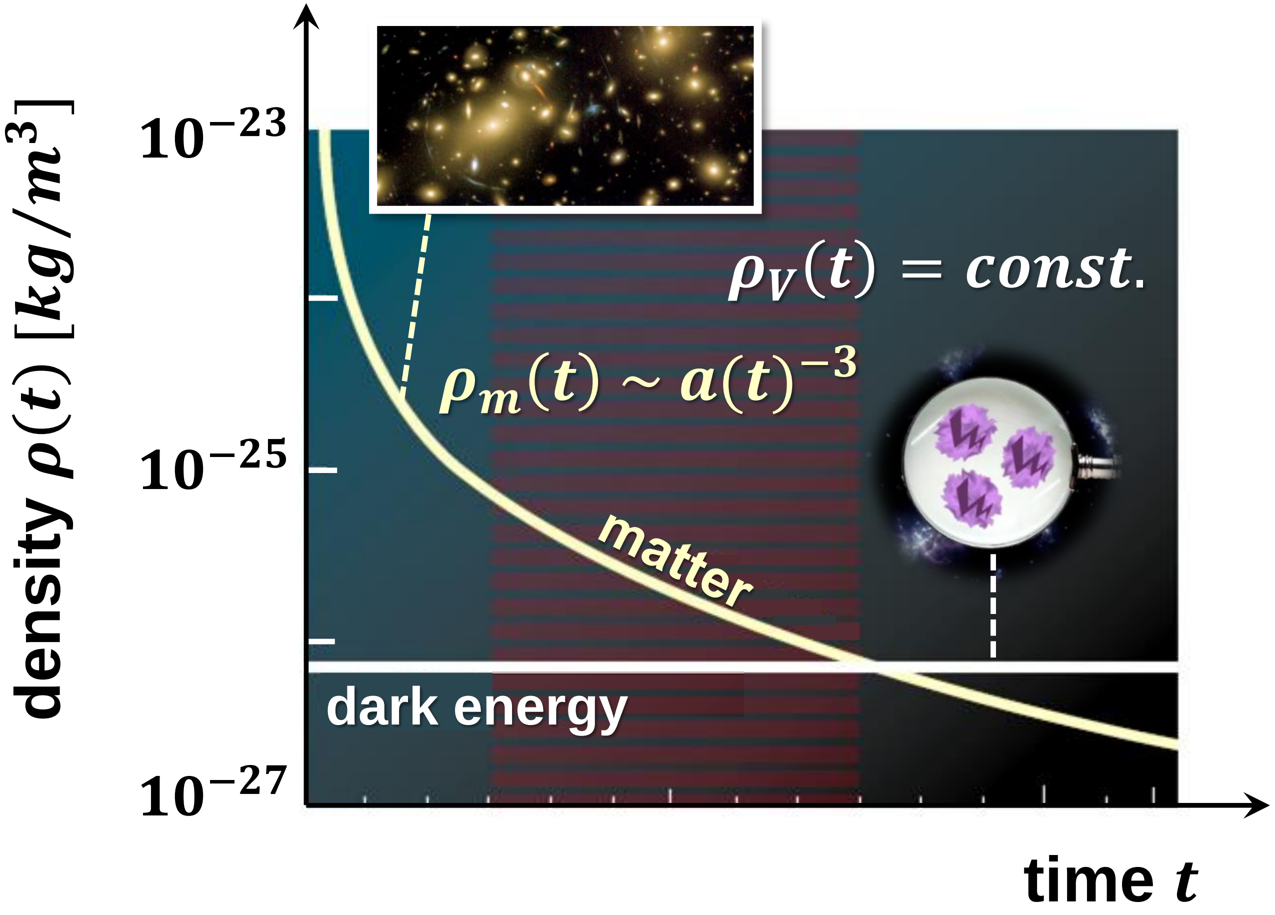
Friedmann equations for a flat ΛCDM universe

- **Second expansion equation taking into account the cosmological constant**

cosmological constant

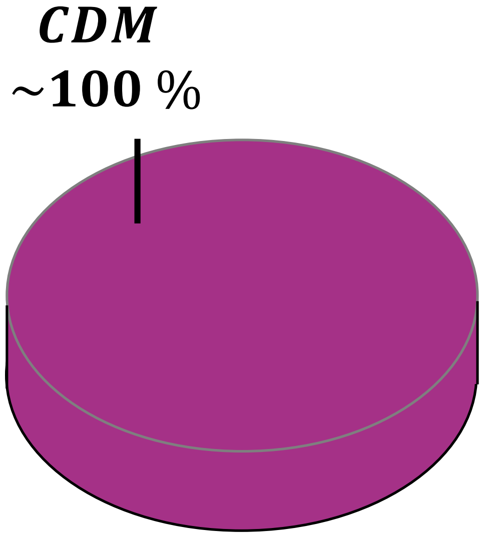
$$\frac{\boxed{\ddot{a}(t)}}{a(t)} = +\frac{\Lambda \cdot c^2}{3}$$

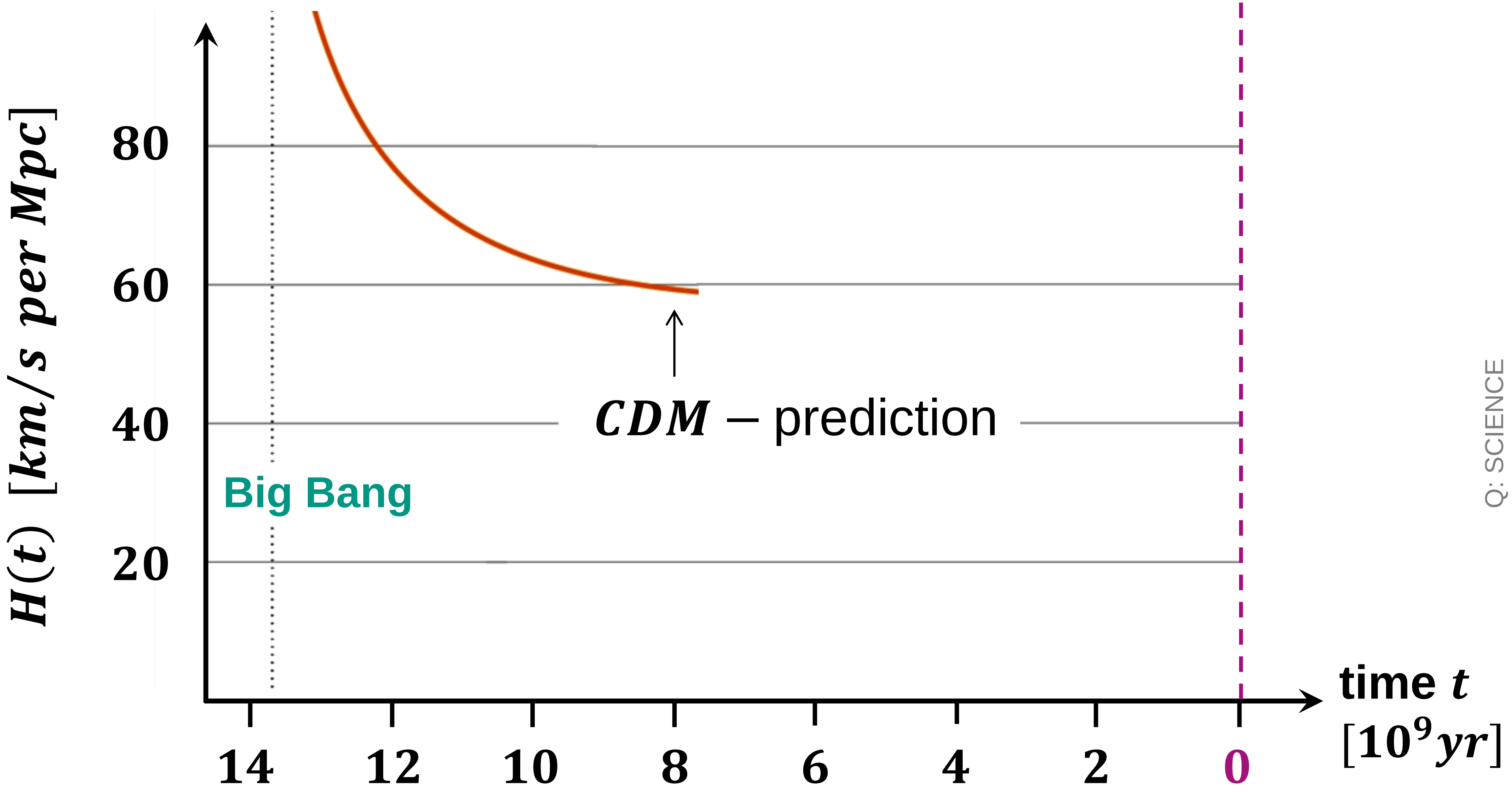
- integration introduces an additional term for $H(t)$ which is dominant at present



Calculated values of $H(t)$

- **Expansion speed of the universe calculated for the CDM model**

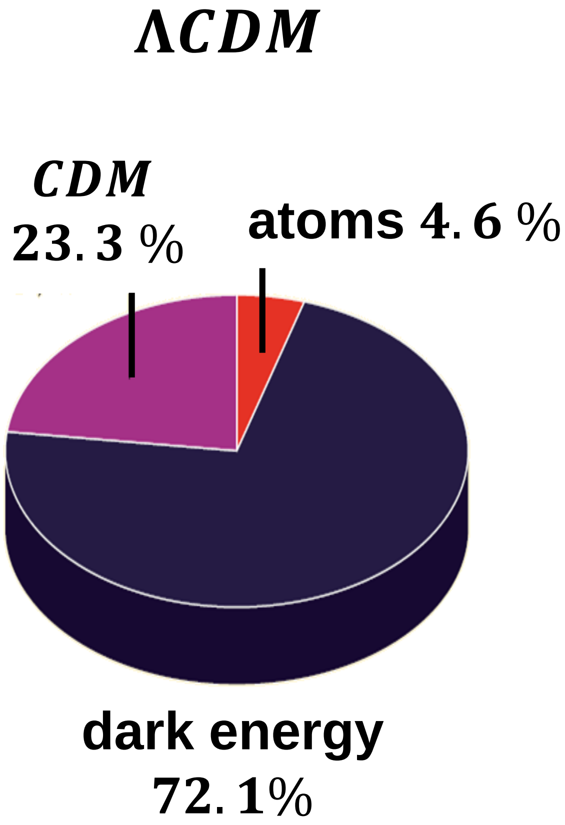


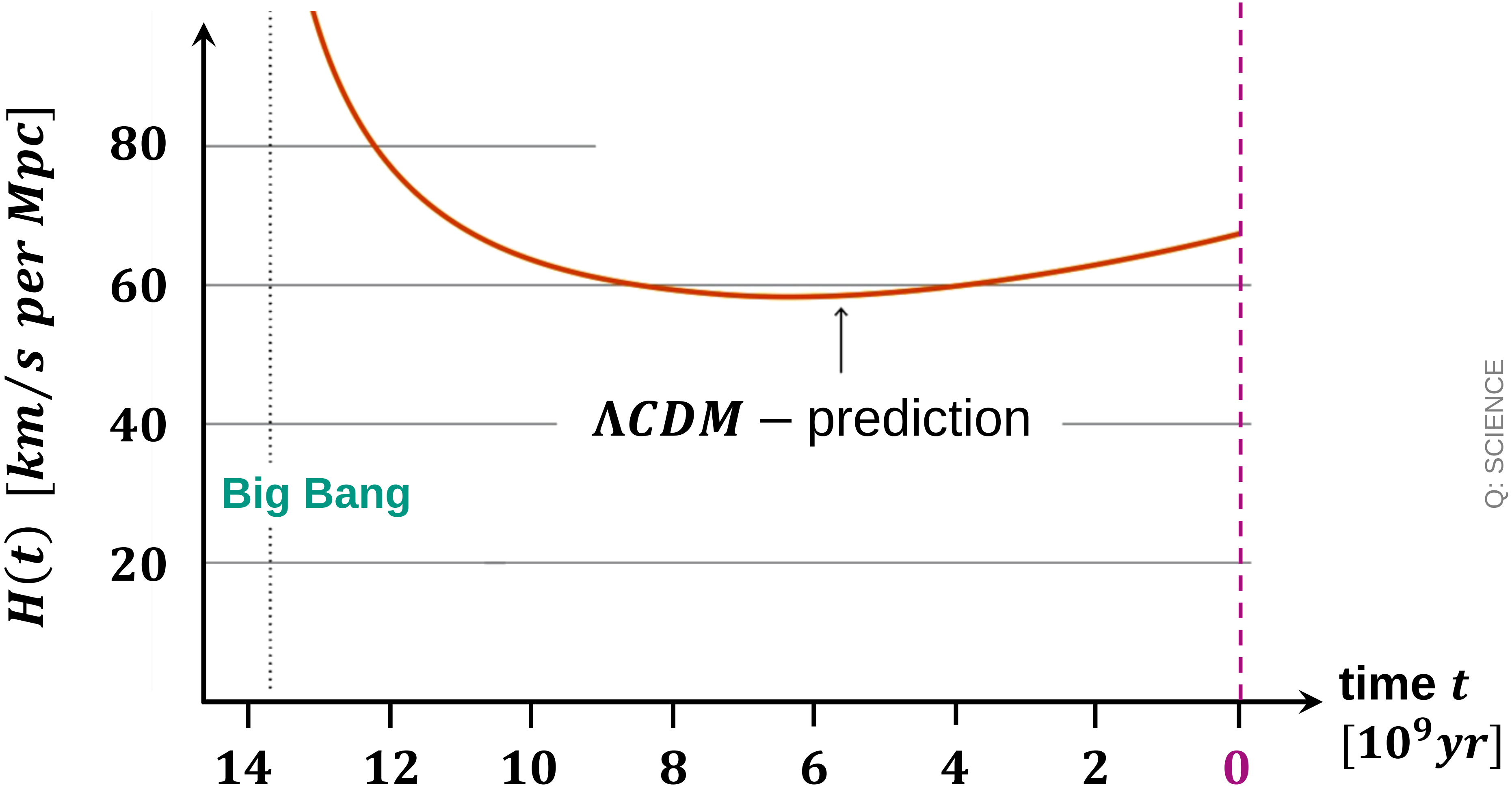


Q: SCIENCE

Calculated values of $H(t)$

- Expansion speed of the universe calculated for the ΛCDM model



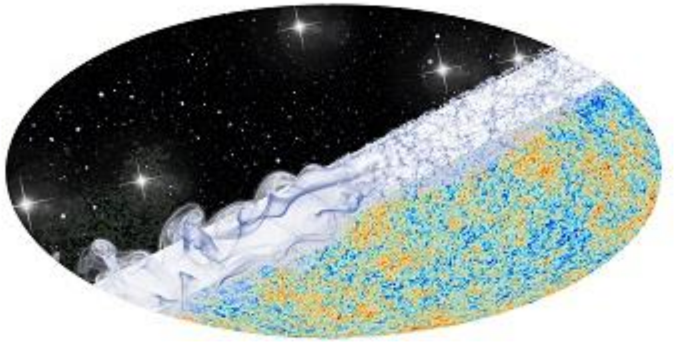


Q: SCIENCE

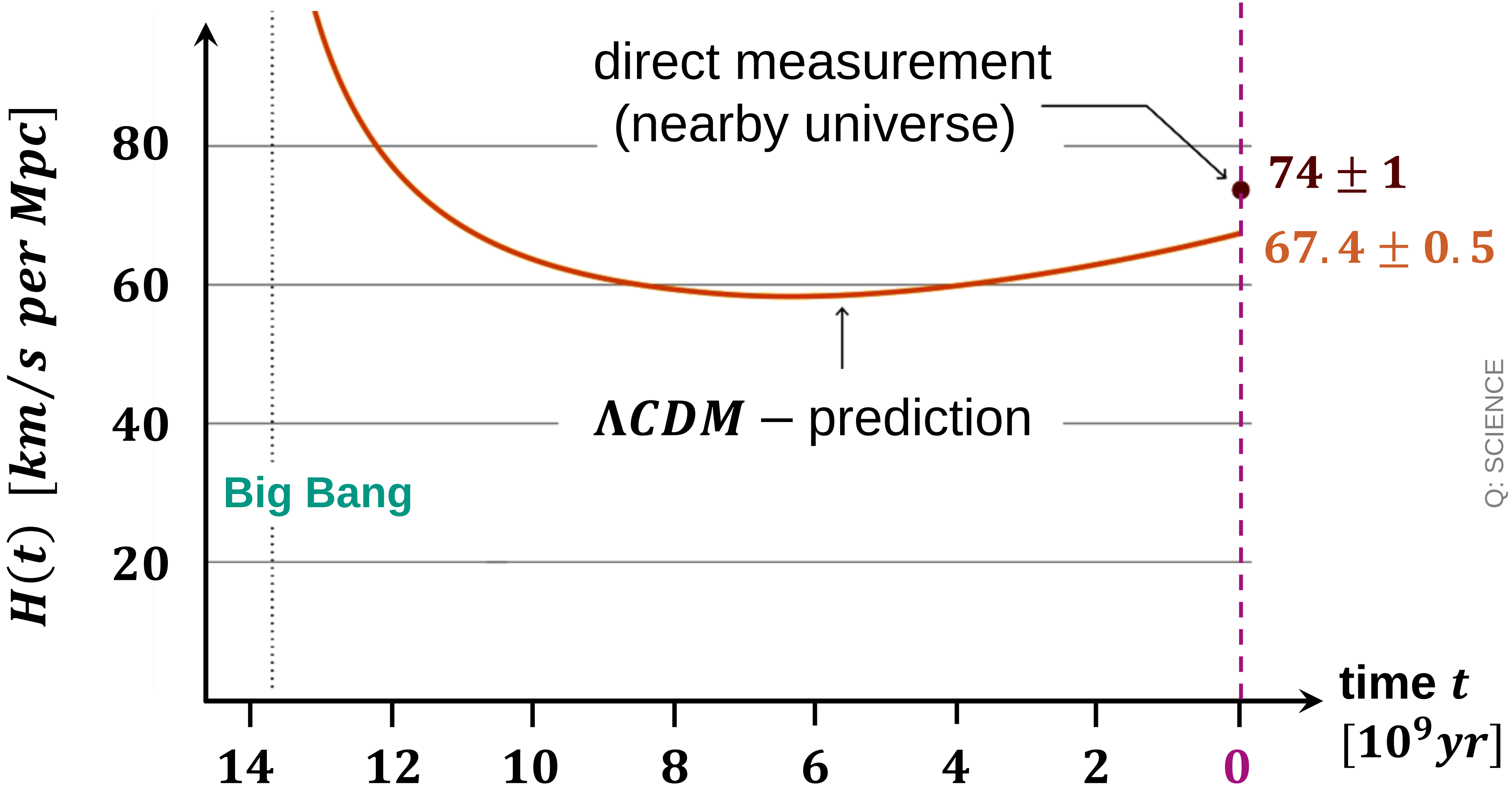
Calculated values of $H(t)$ and todays H_0

- **Expansion speed of the universe calculated for the ΛCDM model**

nearby universe



CMB data + ΛCDM

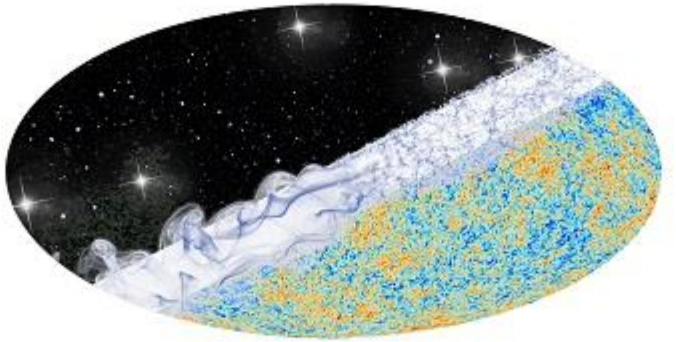


Q: SCIENCE

Hubble tension*: ´true´ value of H_0

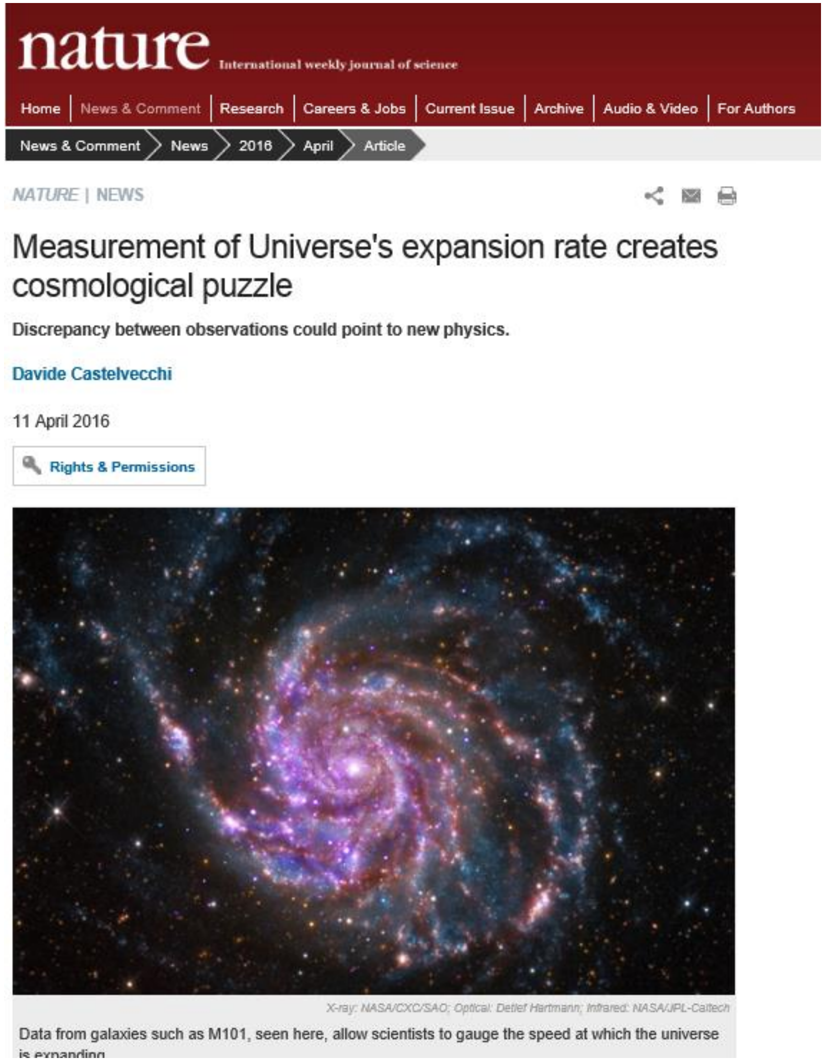
- ΛCDM model

nearby universe



CMB data + ΛCDM

is it the final answer?



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NATURE | NEWS

Measurement of Universe's expansion rate creates cosmological puzzle

Discrepancy between observations could point to new physics.

Davide Castelvecchi

11 April 2016

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X-ray: NASA/CXC/SAO; Optical: Detlef Hartmann; Infrared: NASA/JPL-Caltech

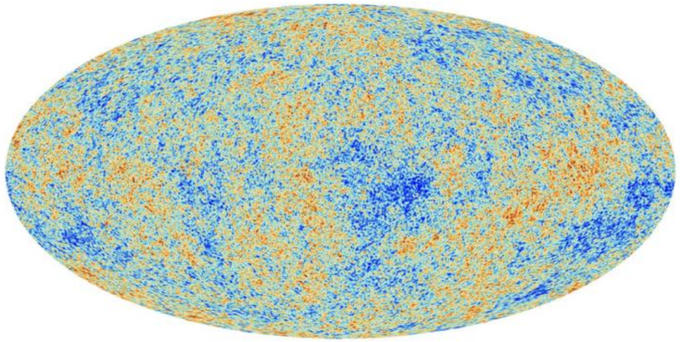
Data from galaxies such as M101, seen here, allow scientists to gauge the speed at which the universe is expanding

Science

The universe's puzzlingly fast expansion may defy explanation, cosmologists fret

The controversial “Hubble tension” promises deep insight but, like dark matter and dark energy, could remain just another mystery

1 NOV 2023 • 2:55 PM ET • BY ADRIAN CHO



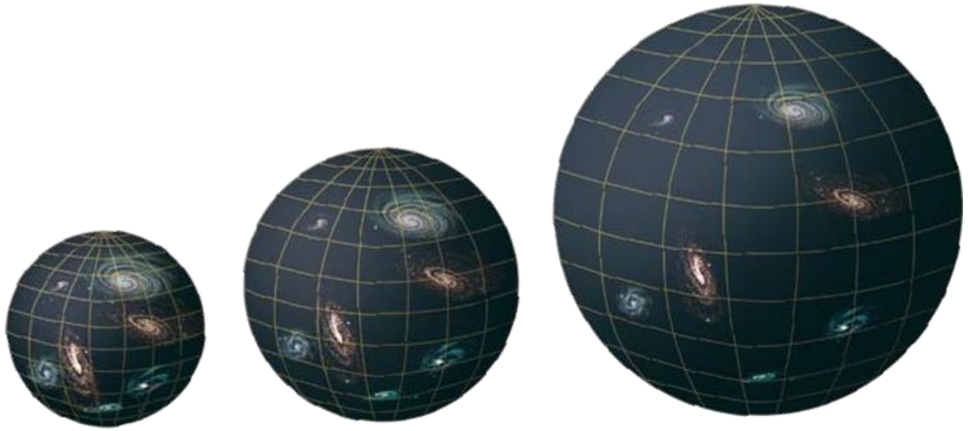
*more than 1000 publications up to now…

Friedmann Equations, cosmological constant Λ

- **The two equations governing the cosmological evolution**

expansion equation for $k = 0$

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8}{3} \cdot \pi \cdot G \cdot \rho_{m,\gamma}(t) + \frac{\Lambda c^2}{3}$$



acceleration equation for $k = 0$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3} \cdot \pi \cdot G \cdot \left(\rho_{m,\gamma}(t) + \frac{3 \cdot P_{m,\gamma}(t)}{c^2} \right) + \frac{\Lambda c^2}{3}$$

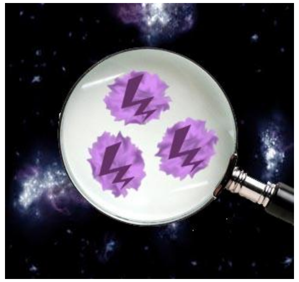
Aleksandr Friedmann

Different cosmological epochs & $a(t)$

- **Radiation / matter / vacuum energy – dominated cosmological epochs**

  - evolution of **scale parameter $a(t)$** calculated with Friedmann equations

dominant part	equation–of–state	density	scale parameter
radiation	$P_r = +1/3 \cdot \rho_r c^2$	$\rho_r \sim a^{-4}$	$a(t) \sim t^{1/2}$
matter	$P_m \cong 0$	$\rho_m \sim a^{-3}$	$a(t) \sim t^{2/3}$
vacuum energy	$P_V = -1 \cdot \rho_V c^2$	$\rho_V = const.$	$a(t) \sim e^{\alpha \cdot t}$



$$\Lambda$$

constant density

$\rho_V = 3.6\ GeV/m^3$

$\alpha = \sqrt{\Lambda/3}$

exponential increase

Afterthought #1: matter–dominated, flat universe

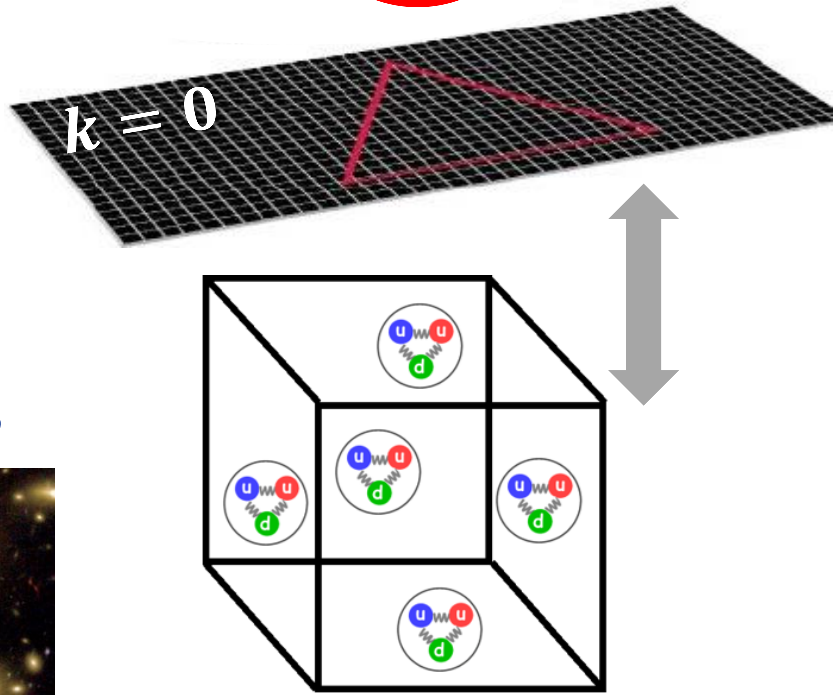
- **Hypothetical assumption: present, flat universe that contains only baryons**

  - flat universe ($\boldsymbol{k = 0}$), no vacuum energy ($\boldsymbol{\Lambda = 0}$)

  - critical energy density $\boldsymbol{\rho_c}$ for a flat universe with baryons only:

$$\boxed{\boldsymbol{\rho_c = \frac{3}{8 \cdot \pi \cdot G} \cdot H_0^2 = 9.2 \cdot 10^{-27}\, kg/m^3}}$$

$$\boldsymbol{= 5.1\ GeV/m^3\ (i.e. \sim 5\ \text{protons per}\ m^3)}$$



- **our present universe features a baryon density $\boldsymbol{\rho_b}$**

$$\boldsymbol{\rho_b = 0.2\ GeV/m^3}$$

$$(\boldsymbol{i.e.\ \rho_b < 5\%\ \text{of}\ \rho_c})$$

Afterthought #2: Hubble time t_H – definition

- Hubble time t_H is based on a scenario* with uniform expansion rate H_0

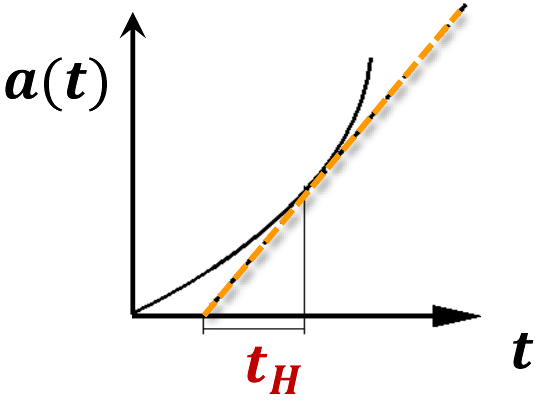
scenario: $H(t) = const. = H_0$

$$\boxed{t_H = \frac{1}{H_0}}$$

$$H_0 = \frac{72\ km/s}{Mpc} = 2.3 \cdot 10^{-18}\ s^{-1}$$

$$t_H = \frac{1}{2.3 \cdot 10^{-18}}\ s$$

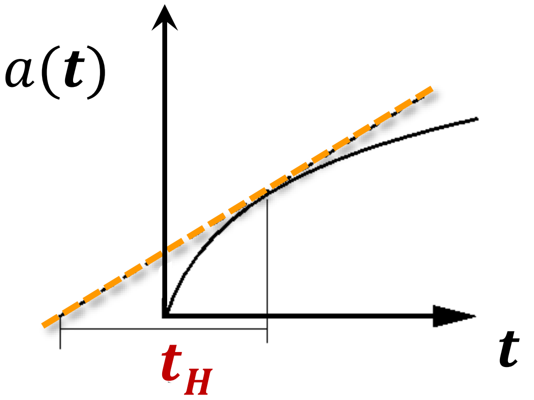
$$t_H = 13.8 \cdot 10^{9}\ yr$$



accelerated expansion

$\rho_{tot} < \rho_c$

$t_H < t_{real}$



braked expansion

$\rho_{tot} > \rho_c$

$t_H > t_{real}$

---

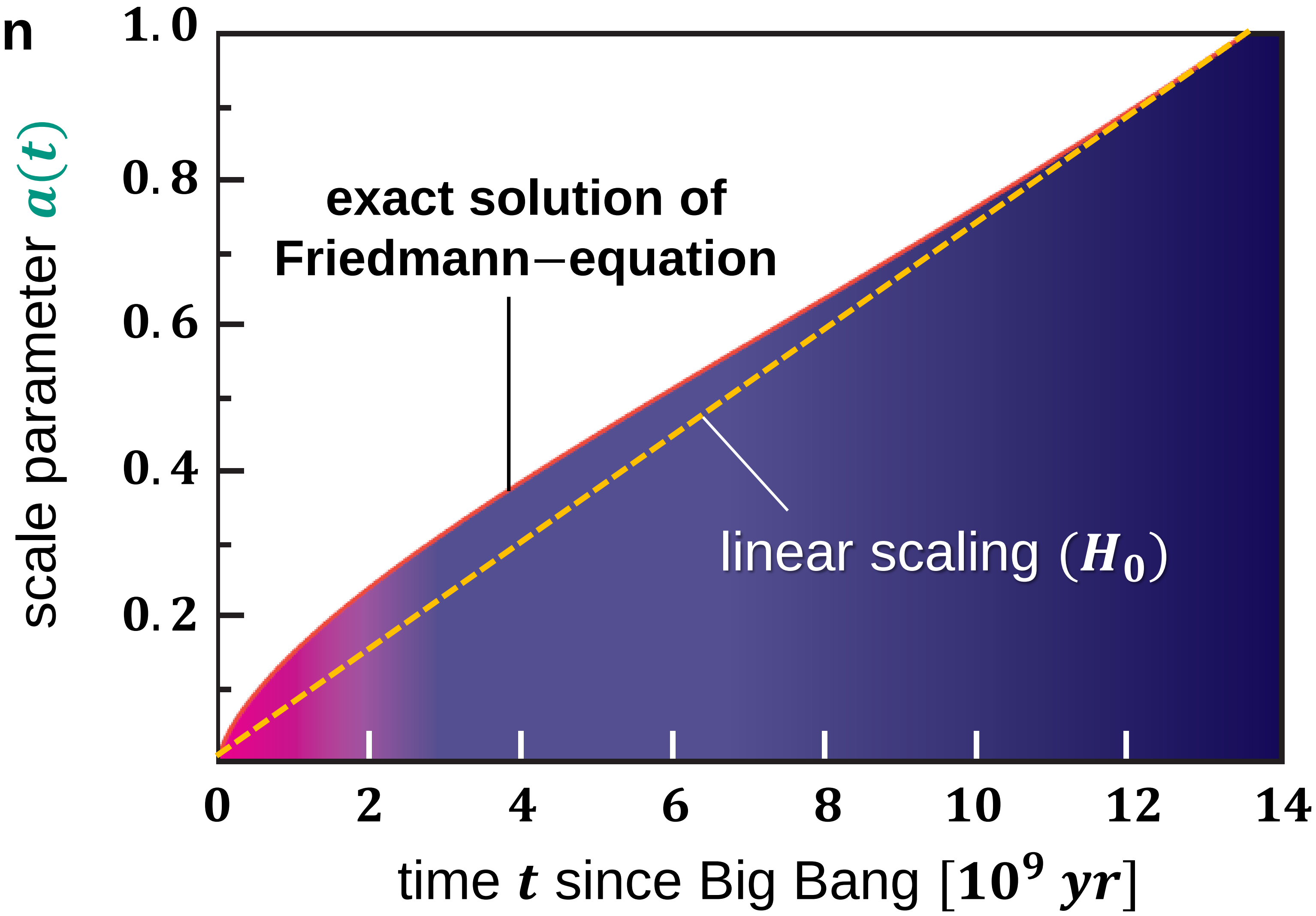
*here we use H_0 from local measurements

Afterthought #2: Hubble time t_H & H_0

- **Linear and acutal expansion rate of our universe**

\- <u>surprise</u>:
rather good approximation of $a(t)$ by a **linear increase** using *<u>present</u>* value of H_0

\- <u>exact Friedmann solution</u>:
at first **braked** expansion ($\ddot{a}(t) < 0$), now **accelerated** expansion with $\ddot{a}(t) > 0$

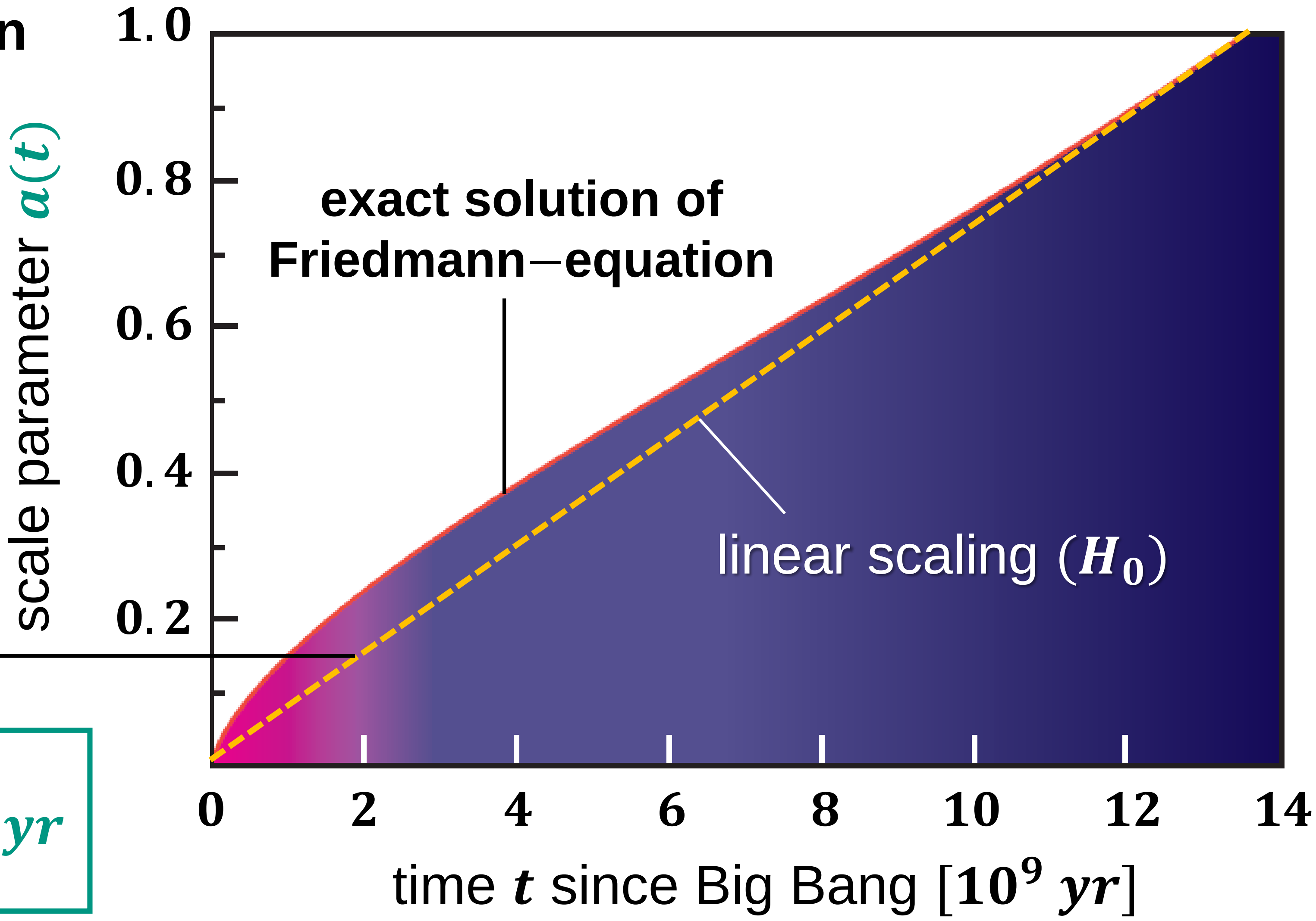


Afterthought #2: Hubble time t_H & H_0

- **Linear and acutal expansion rate of our universe**

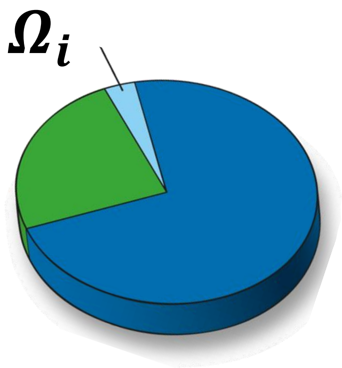
- surprise:
  rather good approximation of $a(t)$ by a **linear increase** using *present* value of H_0

$$t_H = \frac{1}{2.3 \cdot 10^{-18}}\, s = 13.8 \cdot 10^9\, yr$$



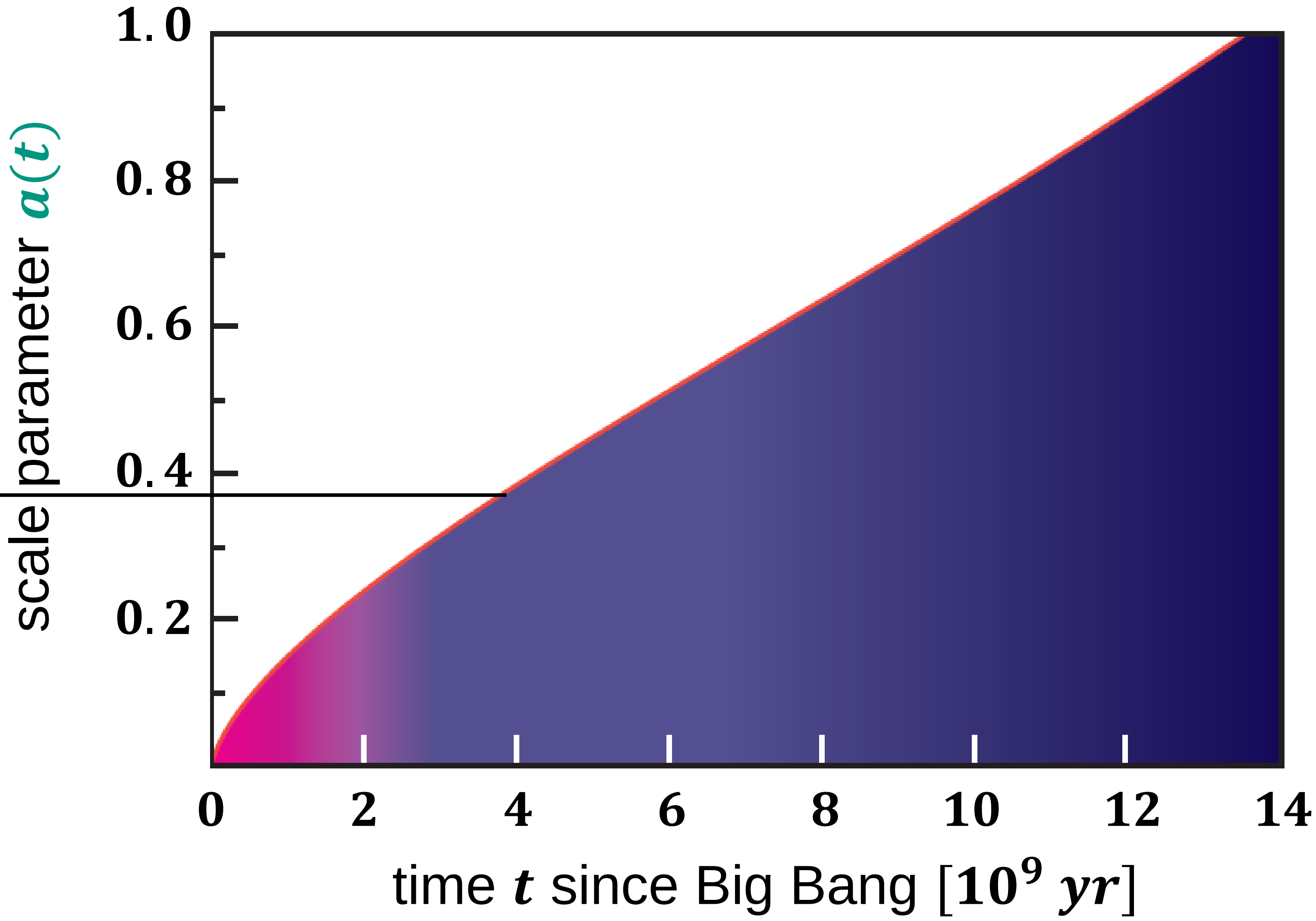
Actual expansion rate $a(t)$ & Ω_i

- **describing expansion $a(t)$**



- $\mathbf{4}$ contributions Ω_i to $\Omega_{tot} = 1$

$$\frac{H(t)^2}{H_0^2} = \Omega_r \cdot a^{-4} + \Omega_m \cdot a^{-3} + \Omega_V + \Omega_k \cdot a^{-2}$$



Building a Standard Model of cosmology

- **Dimensionless density parameters Ω_i**

  - Ω_i are a dimensionless parameters, given by ratio of **actual density ρ_i** relative to **critical critical density ρ_c** for a **flat universe**

$$\boxed{\Omega_i = \frac{\rho_i}{\rho_c} = \frac{8\pi \cdot G}{3\,H_0^2} \cdot \rho_i}$$

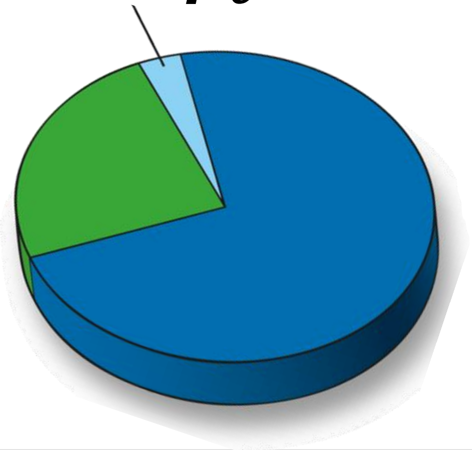
  - $\Omega_{tot} = \sum \rho_i = 1$ for a flat universe with $E_{tot} = 0$

- **Summing up all density parameters Ω_i**

  - contributions: matter – radiation – vacuum – curvature $k \neq 0$

$$\boxed{\Omega_{tot} = \Omega_m + \Omega_\gamma + \Omega_V + \Omega_k}$$

$$\Omega_i = \frac{\rho_i}{\rho_c}$$



Hubble expansion $H(t)$: $CosmoCalc$ – a very useful app

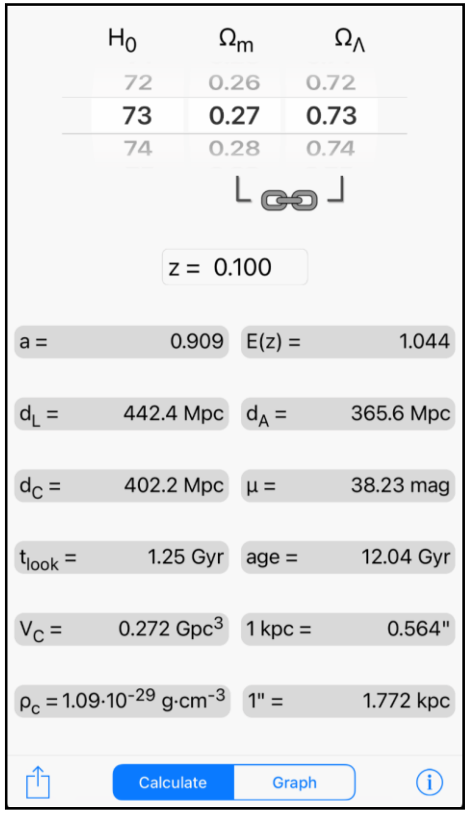
- **Cosmological parameters & their implications: an app for your smartphone**

  - select **your model universe** & see its properties

$$H(t)^2 = H_0^2 \cdot [\ (\Omega_r(t) + \Omega_m(t) + \Omega_V(t) + \Omega_k(t)\ ]$$

$$H(t)^2 = H_0^2 \cdot \left[\begin{array}{l} \quad\Omega_r(0)\cdot(1+z)^4 \\ +\,\Omega_m(0)\cdot(1+z)^3 \\ +\,\Omega_V(0) \\ +\,\Omega_k(0)\cdot(1+z)^2 \end{array}\right] \begin{array}{l} \sim 1/a^4 \\ \sim 1/a^3 \\ const. \\ \sim 1/a^2 \end{array}$$

$CosmoCalc$ App for iOS



H_0	Ω_m	Ω_Λ
72	0.26	0.72
73	0.27	0.73
74	0.28	0.74

z = 0.100

a = 0.909

$E(z)$ = 1.044

d_L = 442.4 Mpc

d_A = 365.6 Mpc

d_C = 402.2 Mpc

μ = 38.23 mag

t_{look} = 1.25 Gyr

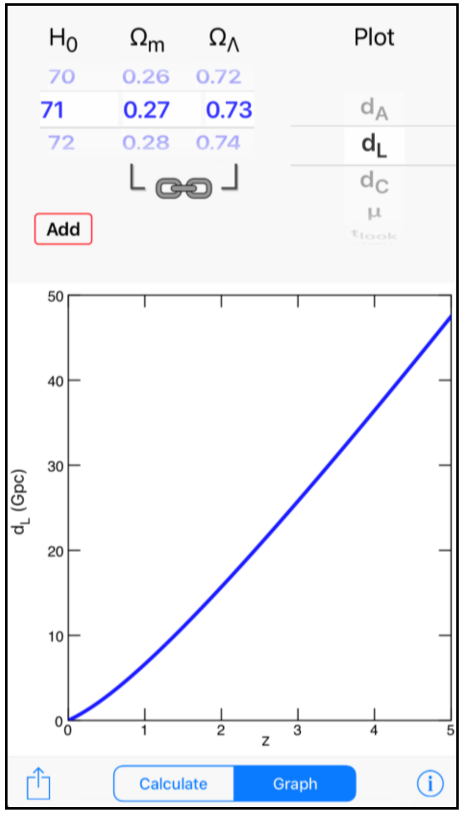
age = 12.04 Gyr

V_C = 0.272 Gpc^3

1 kpc = 0.564"

$\rho_c = 1.09\cdot10^{-29}$ g·cm^{-3}

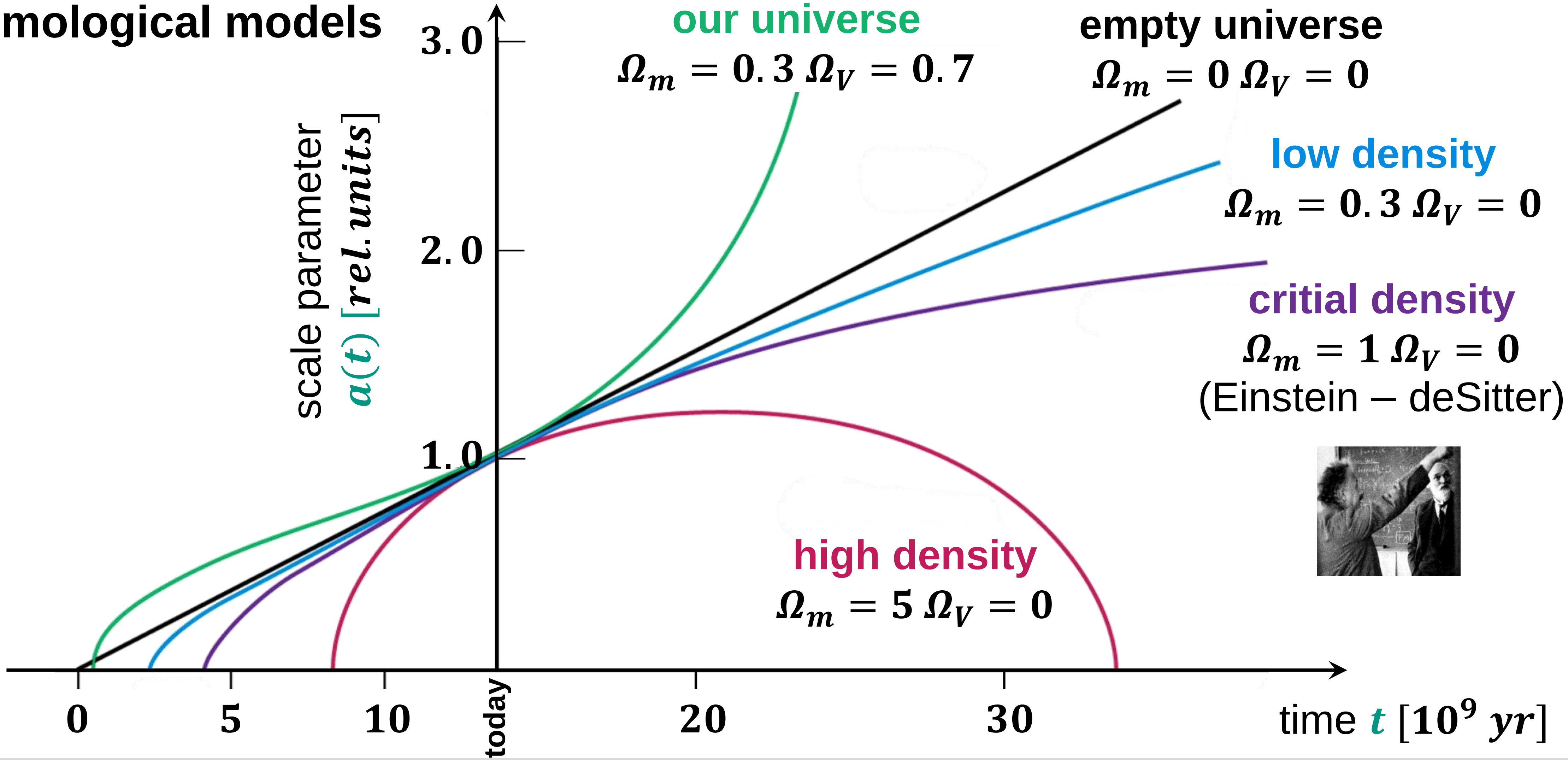
1" = 1.772 kpc



H_0	Ω_m	Ω_Λ
70	0.26	0.72
71	0.27	0.73
72	0.28	0.74

Scale parameter $a(t)$ for different models

- Cosmological models



The end (of todays´ lecture)

- **Friedmann equations revisited**

$$\frac{H(t)^2}{H_0^2} = \Omega_r \cdot a^{-4} + \Omega_m \cdot a^{-3} + \Omega_V + \Omega_k \cdot a^{-2}$$

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8}{3} \cdot \pi \cdot G \cdot \rho_{m,\gamma}(t) + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3} \cdot \pi \cdot G \cdot \left(\rho_{m,\gamma}(t) + \frac{3 \cdot P_{m,\gamma}(t)}{c^2} \right) + \frac{\Lambda c^2}{3}$$

´Is THAT it?´

Is THAT the BIG BANG?