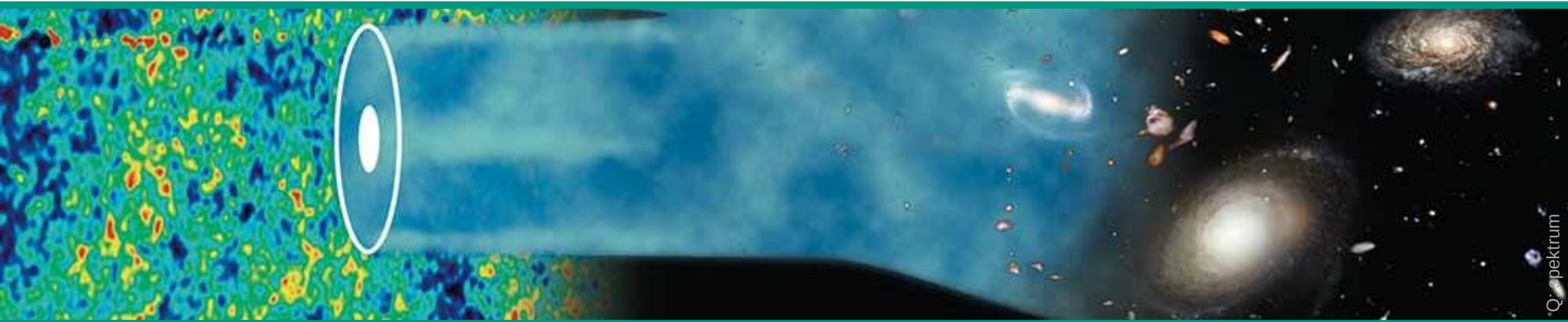


Introduction to Cosmology

Winter term 23/24 Lecture #5

Nov. 21, 2023



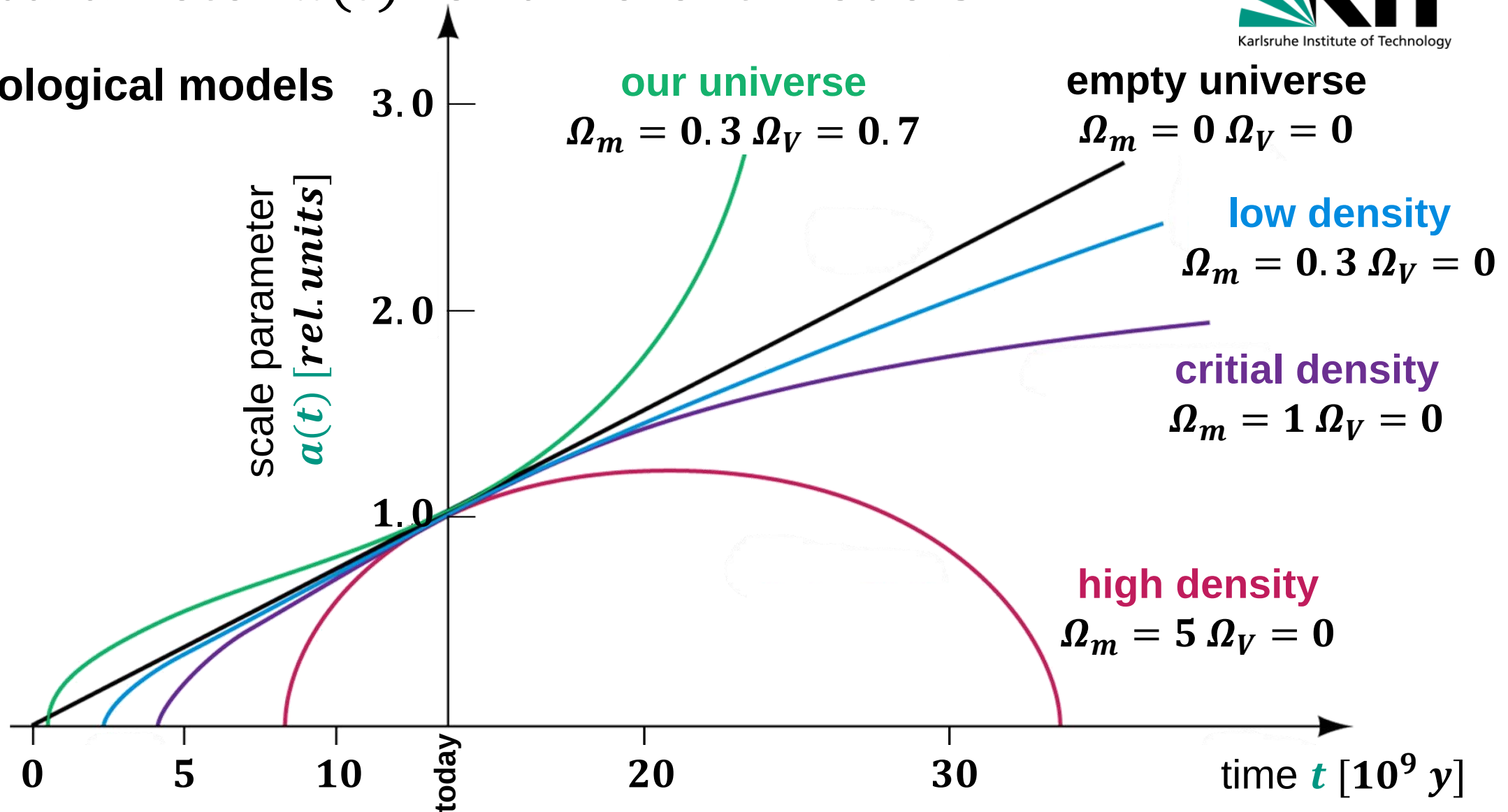
Recap of Lecture 4

■ Friedmann equation for expanding universe

- cosmological constant $\Lambda \neq 0$ (**3.6 GeV/m³** vs. 10^{121} GeV/m^3)
- integration of acceleration eq.:
$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8}{3} \cdot \pi \cdot G \cdot \rho_{m,\gamma}(t) + \frac{\Lambda c^2}{3}$$
- universe appears to be **flat**: **curvature** $k = 0$ (brief inflationary epoch?)
- three cosmological epochs: **radiation** / **matter** / **vacuum**
- total energy density $\Omega_{tot} = \Omega_\gamma + \Omega_m + \Omega_V + \Omega_k (\equiv 1, \text{ if critical density})$
- definition of **Hubble time** $t_H = (H_0)^{-1} = 13.8 \cdot 10^9 \text{ yr}$ (uniform expansion)

Scale parameter $a(t)$ for different models

■ Cosmological models



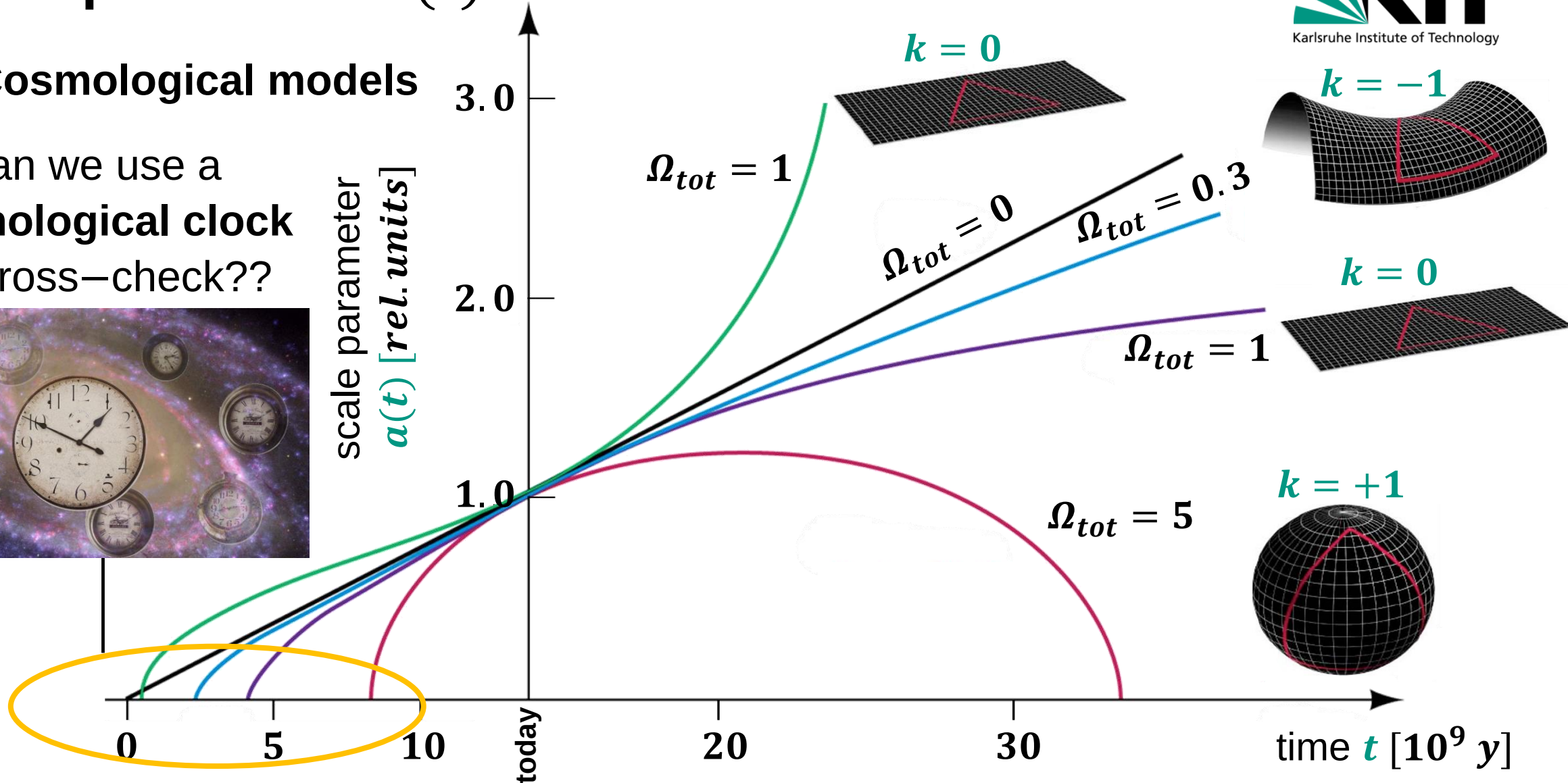
Scale parameter $a(t)$ for different models

■ Cosmological models

can we use a
cosmological clock
to cross-check??



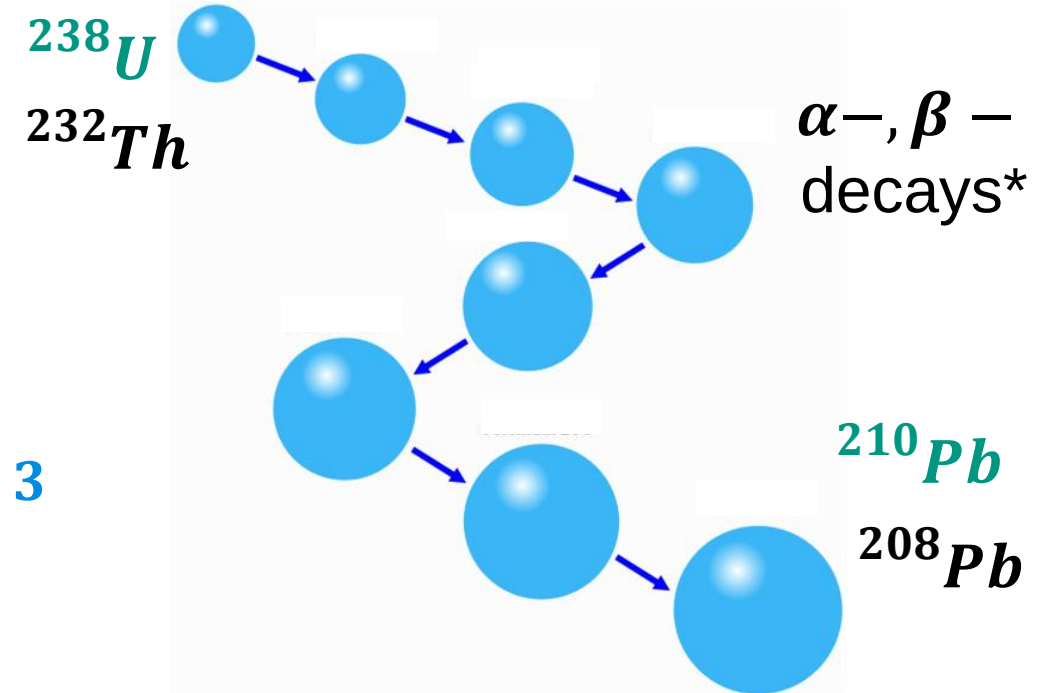
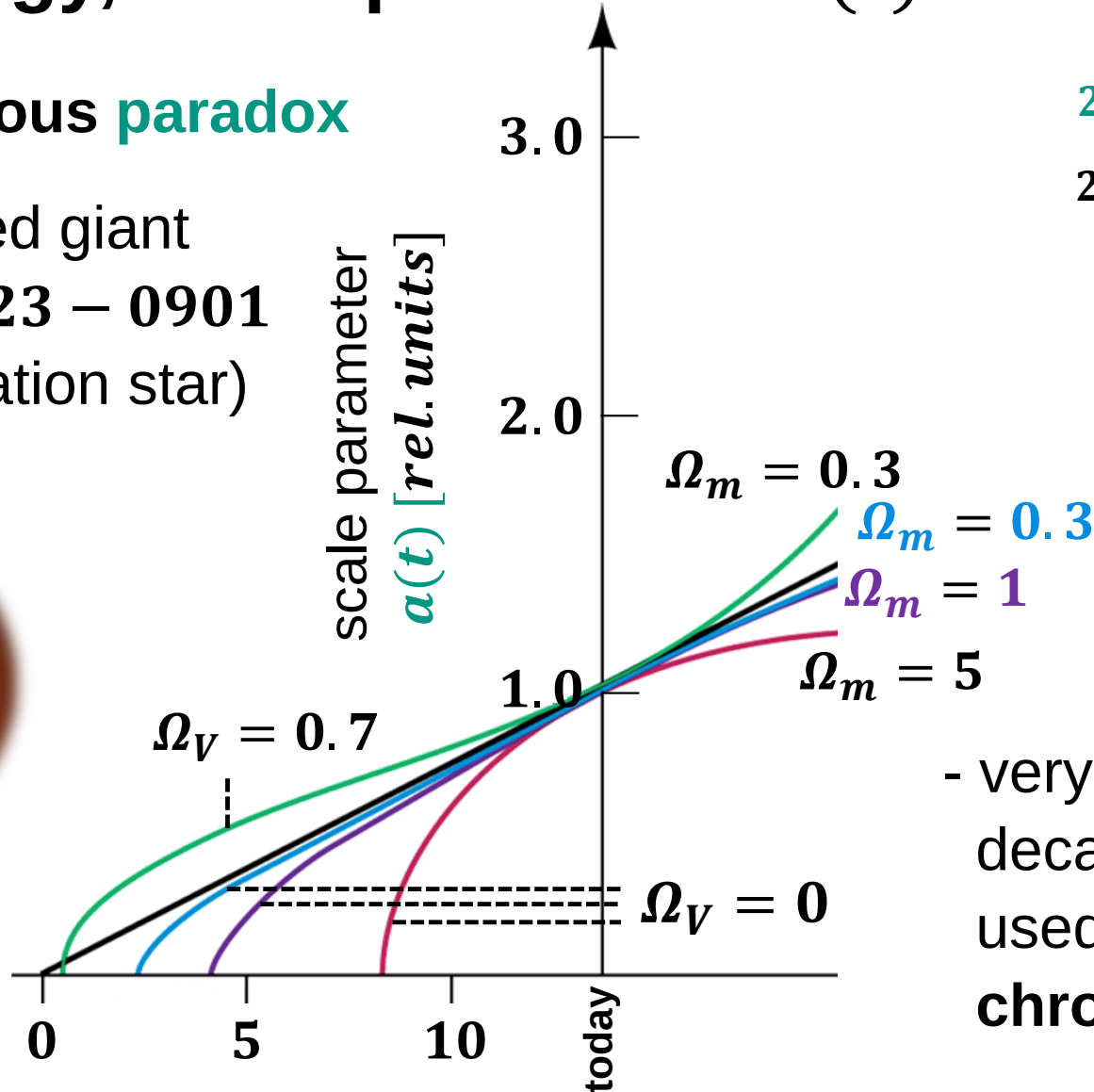
scale parameter
 $a(t)$ [rel. units]



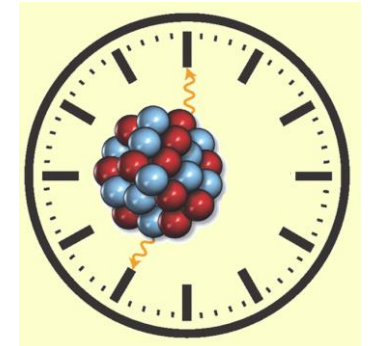
Topology, scale parameter $a(t)$ & the oldest stars

■ A famous paradox

example: red giant
star **HE 1523 – 0901**
(2nd generation star)

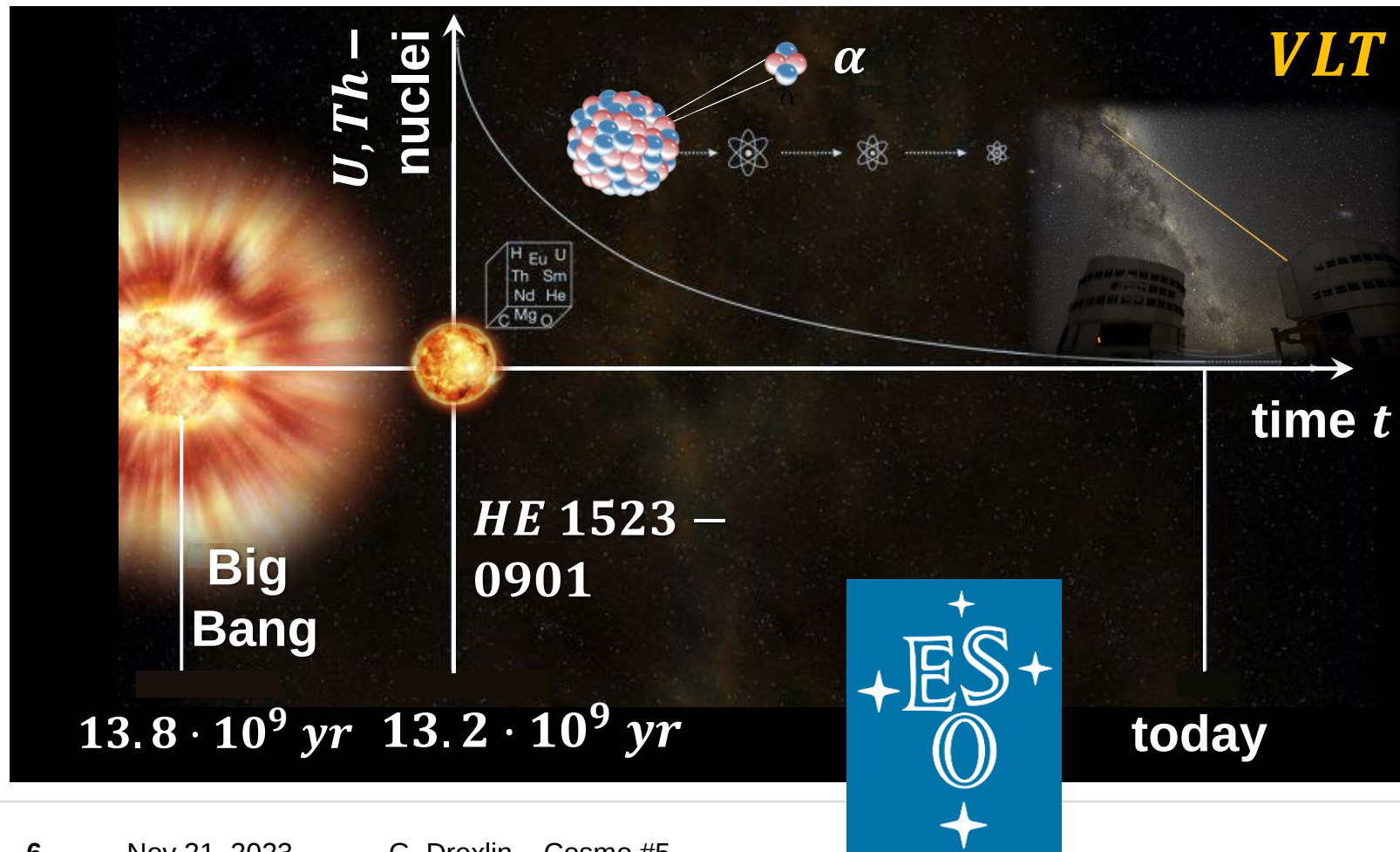


- very long-lived nuclear
decays (U, Th) can be
used as **cosmo-**
chronometers

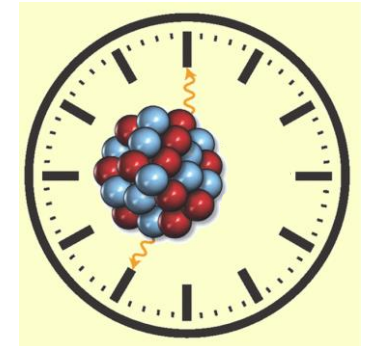


Topology, scale parameter $a(t)$ & the oldest stars

- A famous **paradox**: nuclear clocks observed in star **HE 1523 – 0901**



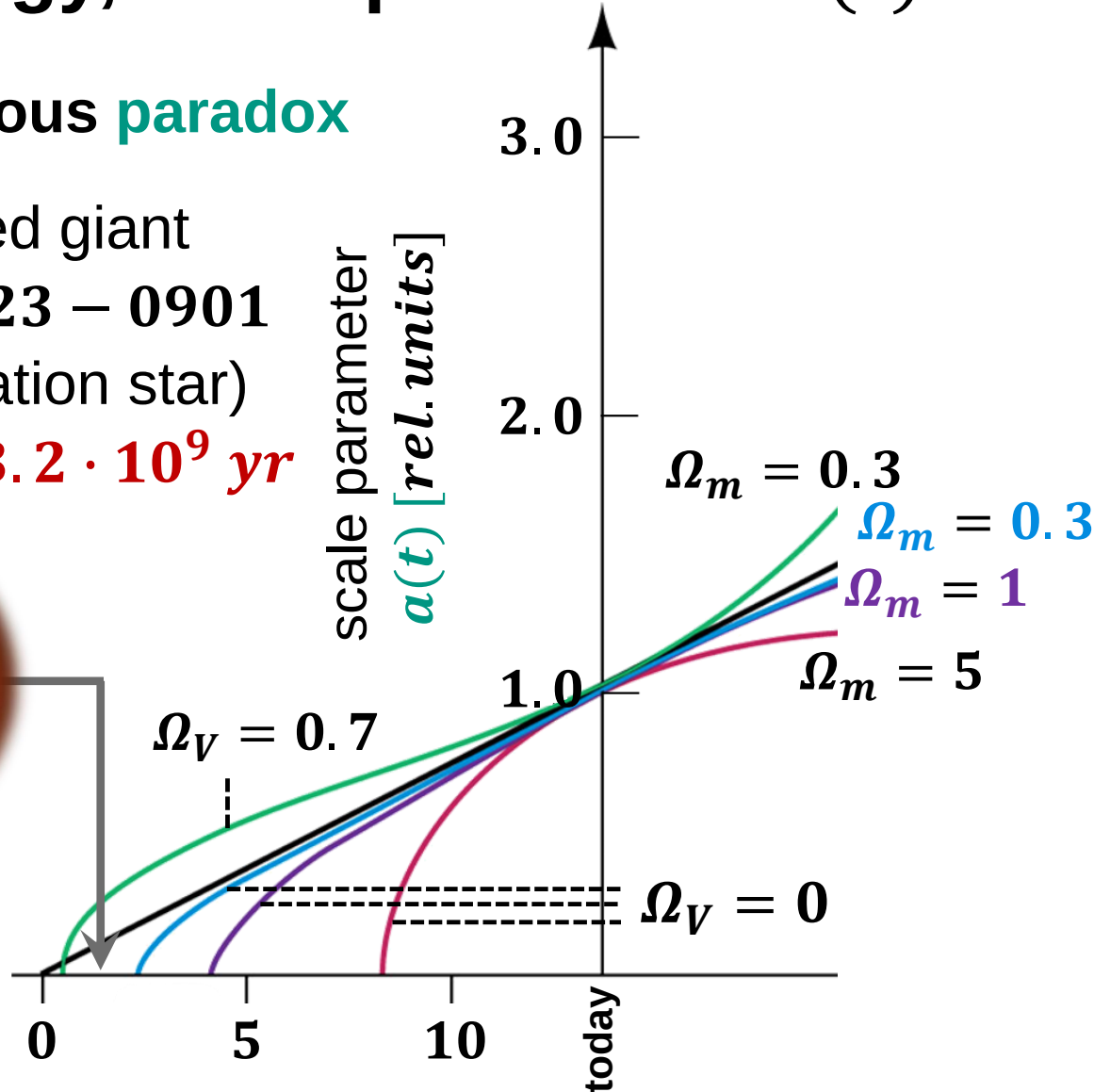
- **nuclear & particle physics**: radioactive dating is a very important method for cosmology



Topology, scale parameter $a(t)$ & the oldest stars

■ A famous paradox

example: red giant
star **HE 1523 – 0901**
(2nd generation star)
age: $t = 13.2 \cdot 10^9 \text{ yr}$



- a universe with $\Omega_m = 1$
(or with $\Omega_m = 0.3$) only:
it would be younger than the
oldest stars
(age by 'nuclear clocks')



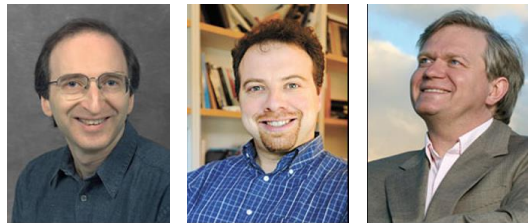
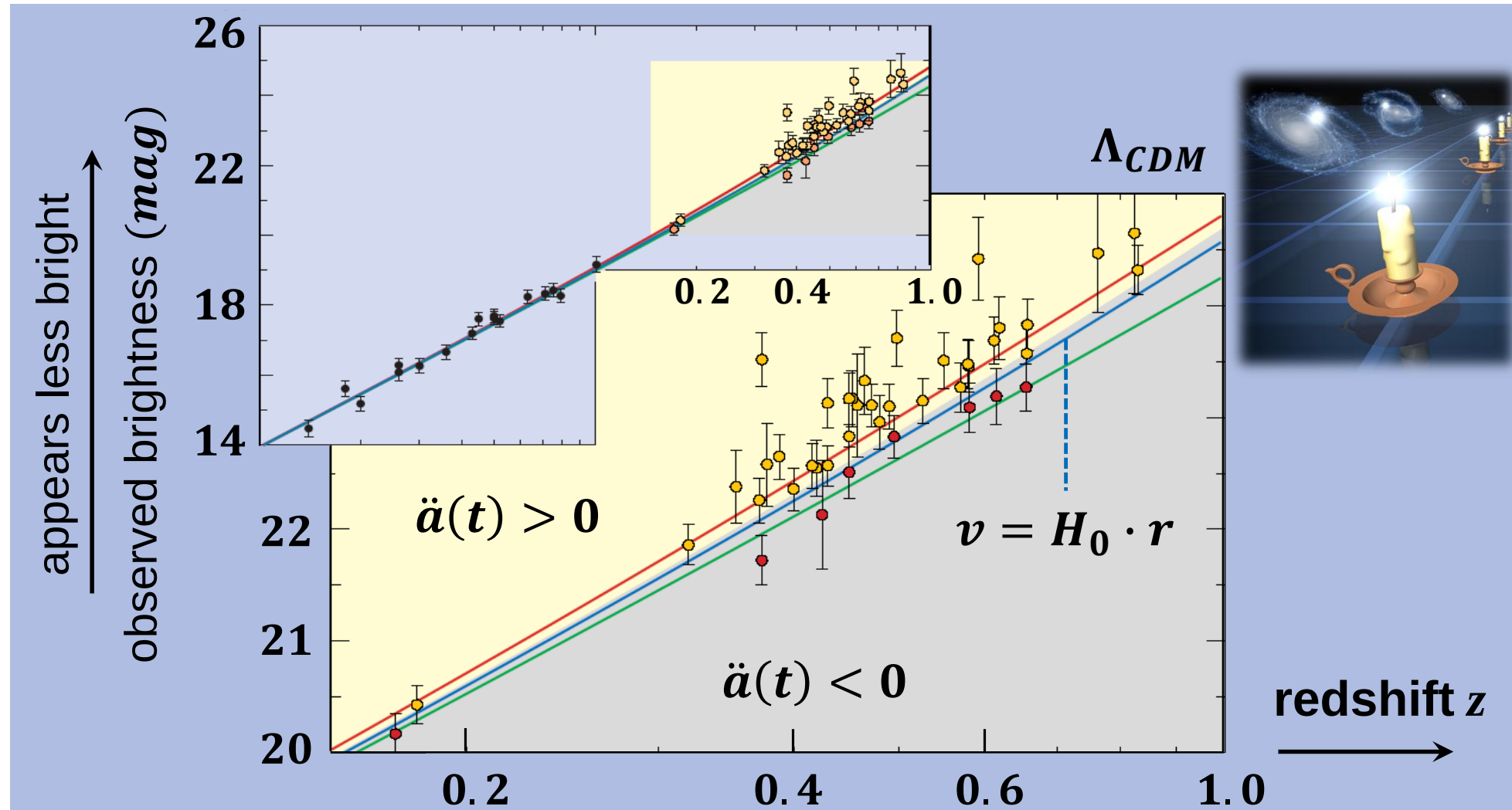
apparently a paradox



further hint for $\Lambda \neq 0$

Evidences for $\Lambda \neq 0$ & expansion with $\ddot{a}(t) > 0$

- Observed brightness of distant *SNa*e: **dimmer than one expects** (thus $\Lambda \neq 0$)



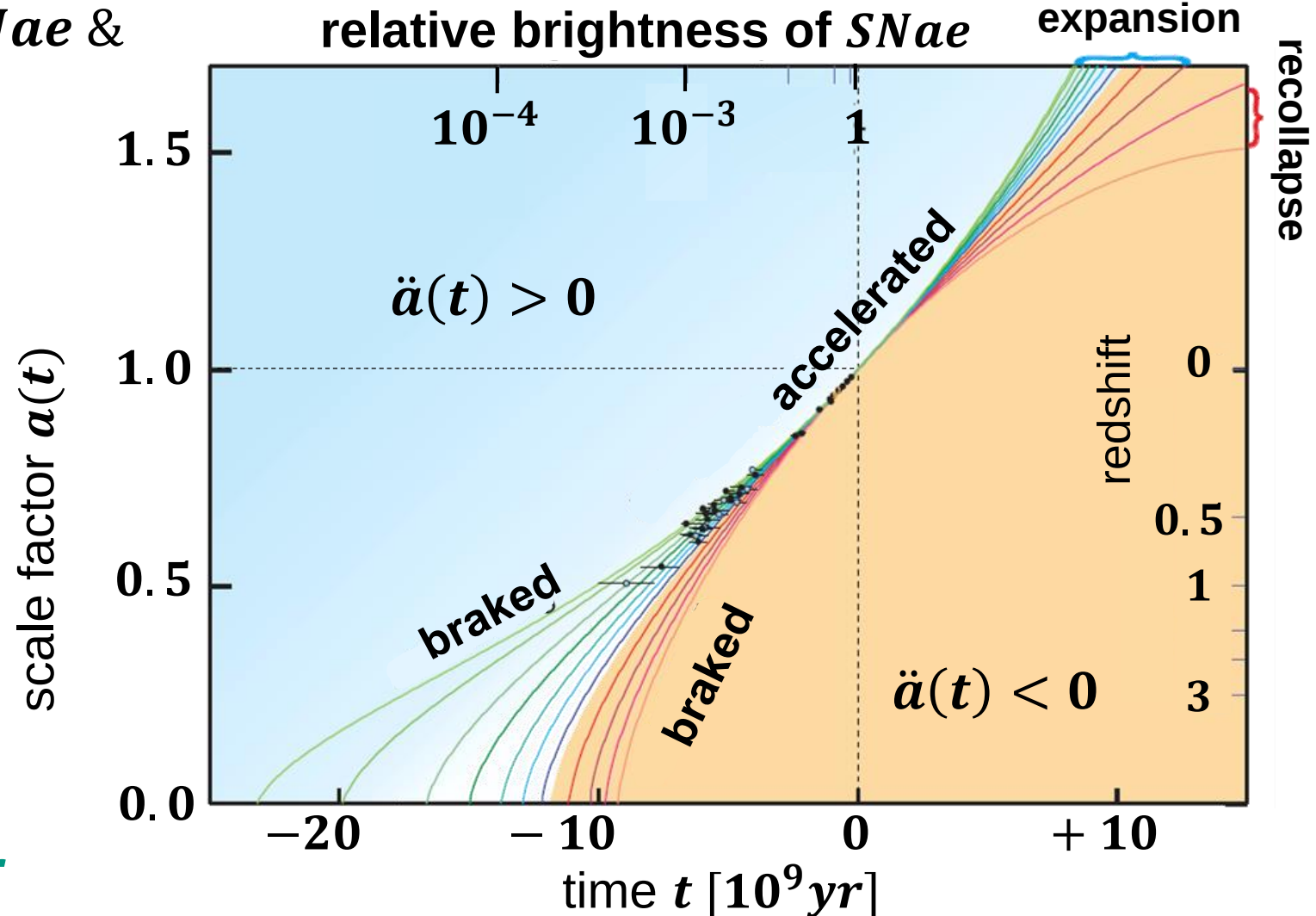
Perlmutter Reiss Schmidt

Nobel prize 2011

Evidences for $\Lambda \neq 0$ & existence of dark energy

■ Observed brightness of $SNae$ & evolution of the scale parameter $a(t)$

- $SNae$ type Ia: observed up to large **redshifts** $z = 1$ [$a(t) \sim 0.5$] via precision spectroscopy of atomic lines
- comparison of the absolute SN – brightness M with apparent brightness m
 - $\Rightarrow$ distance modulus $m - M$
 - $\Rightarrow$ sensitivity to SN – distance r



Global data set for Ω_m & Ω_v

■ Actual best fit values

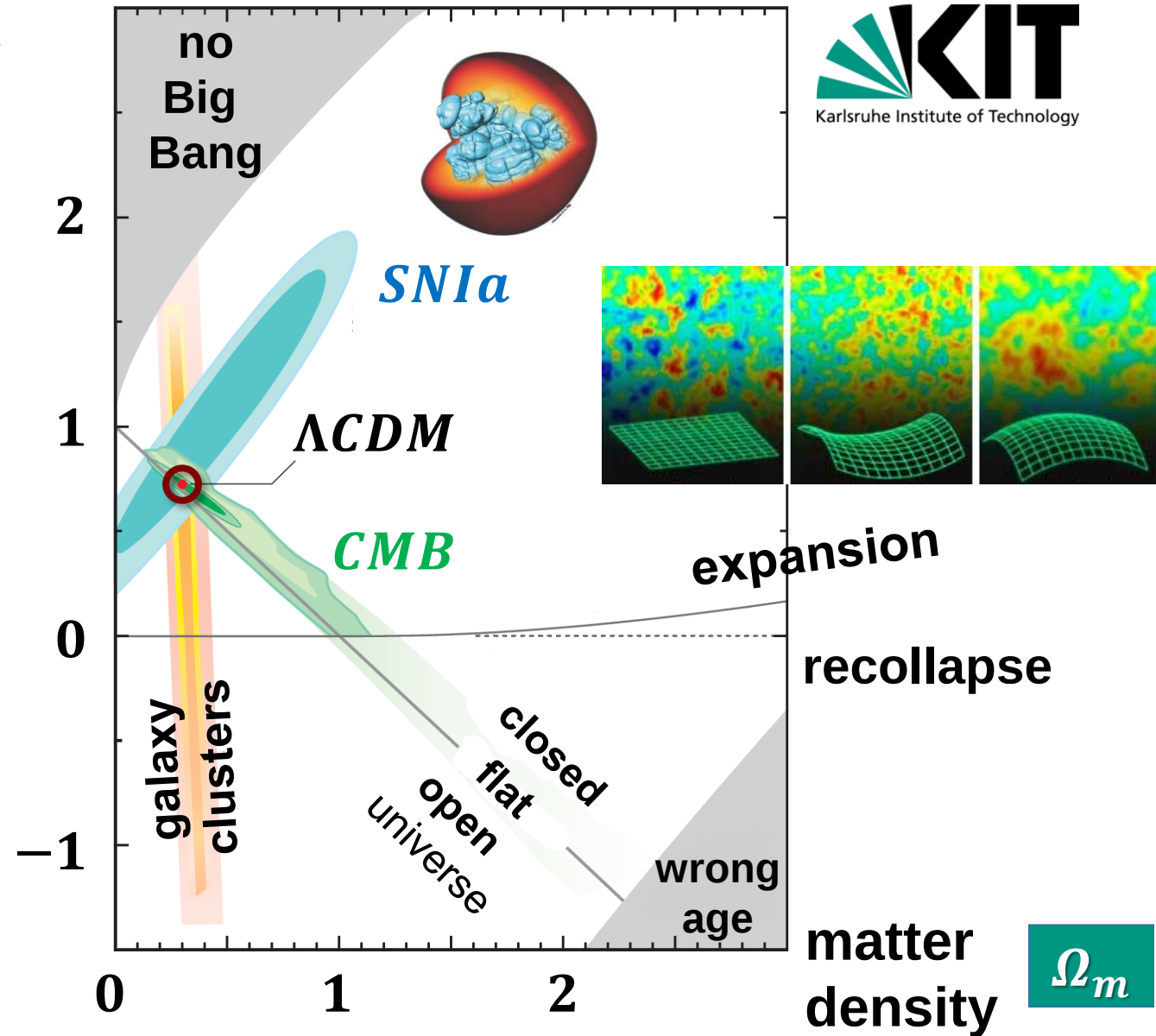
- combining data sets using:

- *SN Ia* brightness data
- *CMB* analyses
- galaxy clusters

- 'orthogonal' methods:

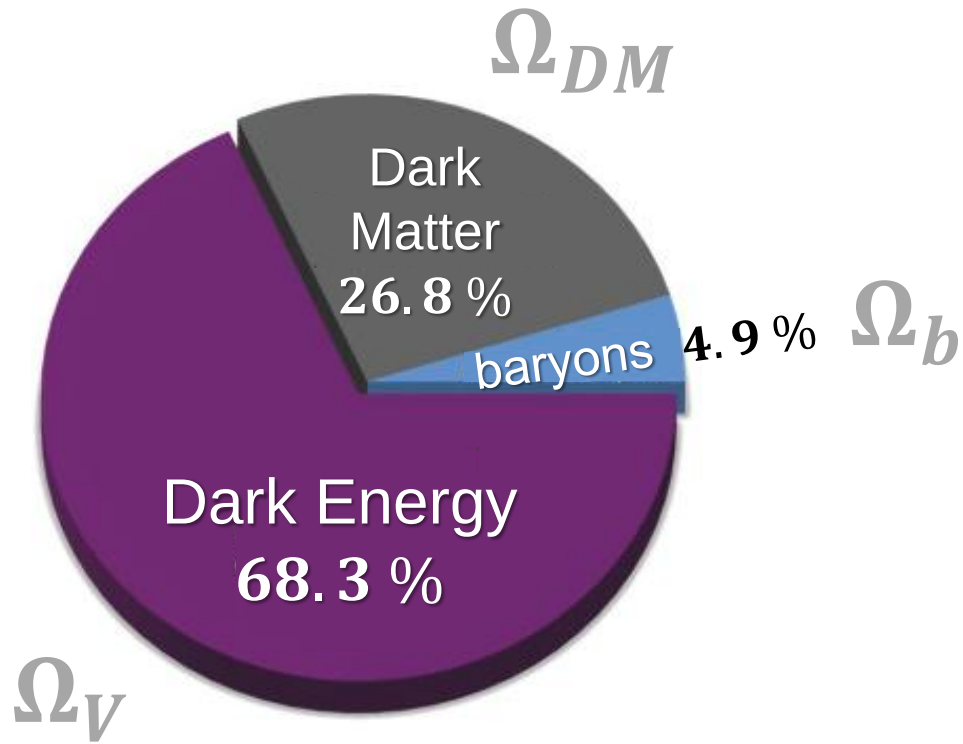
- *CMB* analyses (3 K):
sensitive to $\Omega_{tot} = \Omega_v + \Omega_m$
- *SNae* – brightness:
sensitive to value $\Omega_v - \Omega_m$

Ω_v
vacuum energy density



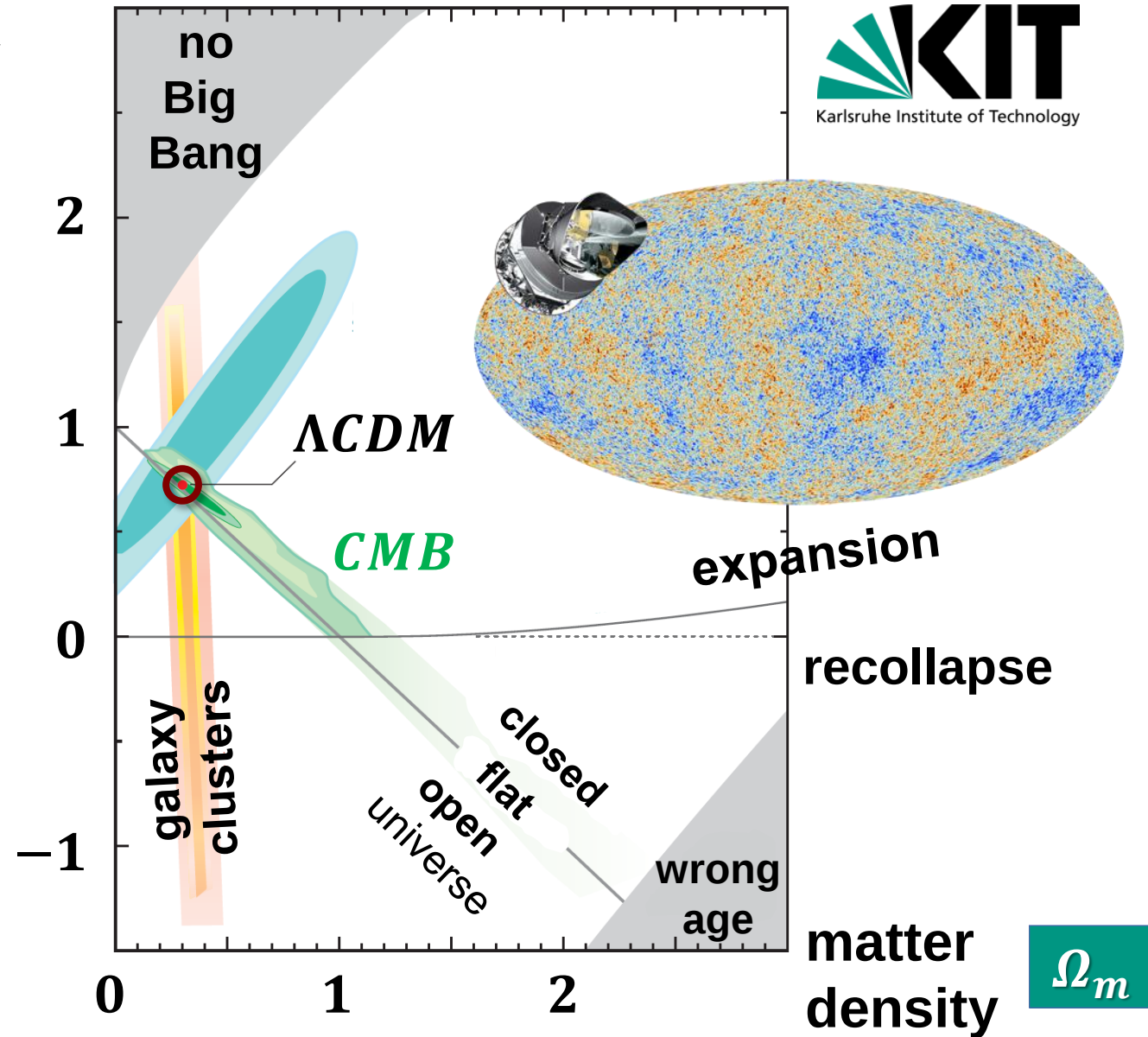
Global data set for Ω_m & Ω_v

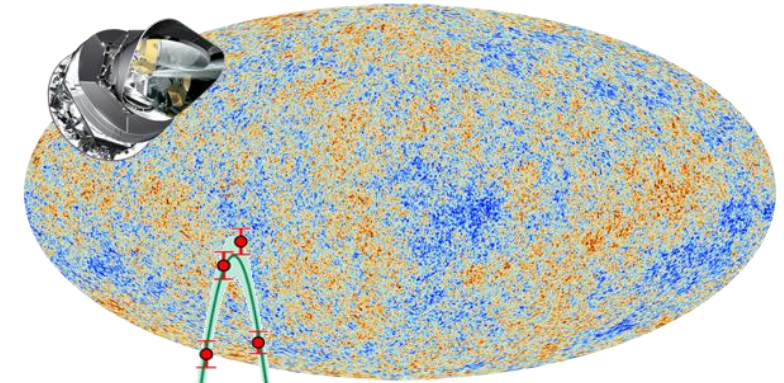
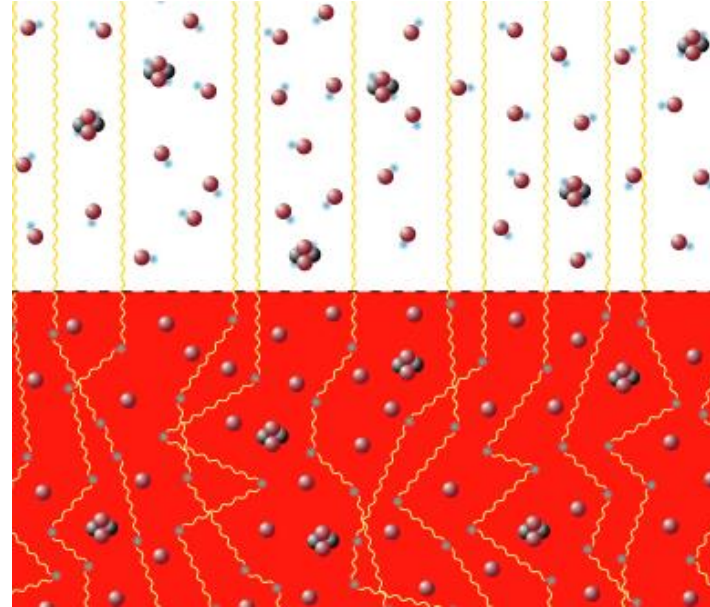
■ Λ CDM concordance model of cosmology



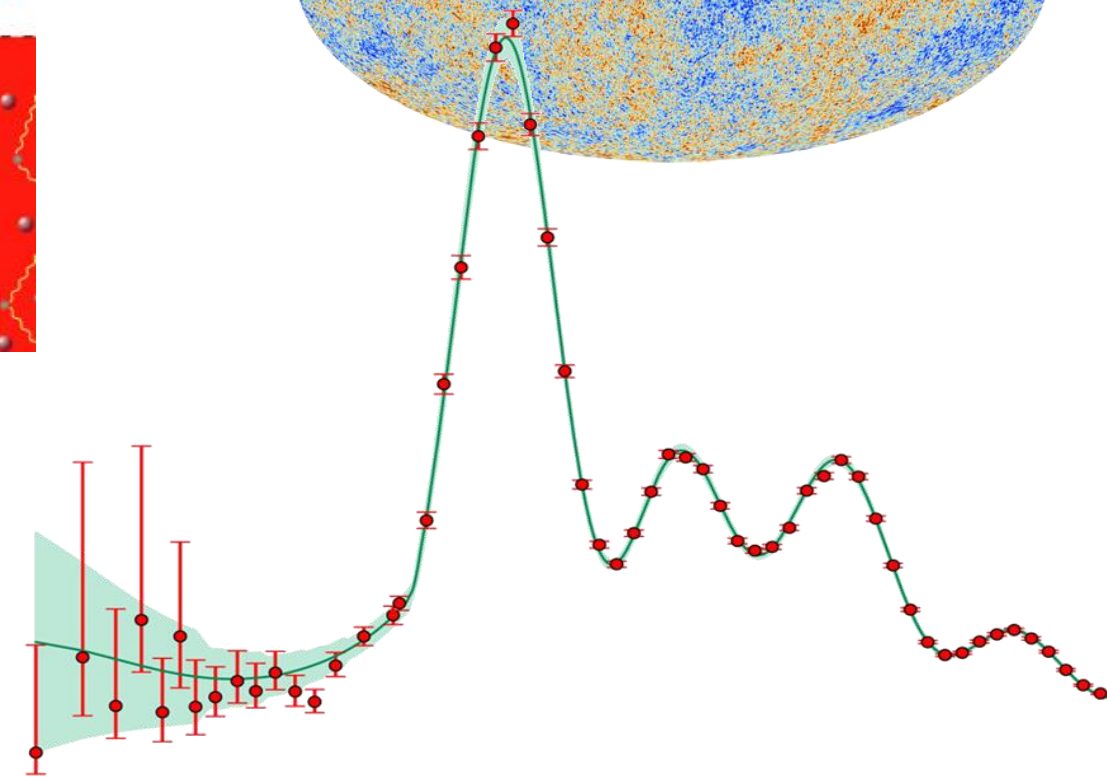
$$T_0 = 13.73 \cdot 10^9 \text{ yr}$$

Ω_v
vacuum energy density





CHAPTER 3 – THERMAL UNIVERSE



Overview: thermal history of the universe

■ An expanding universe necessarily is **cooling down**

- shortly after Big Bang:

$$t \sim 10^{-36} \text{ s}$$

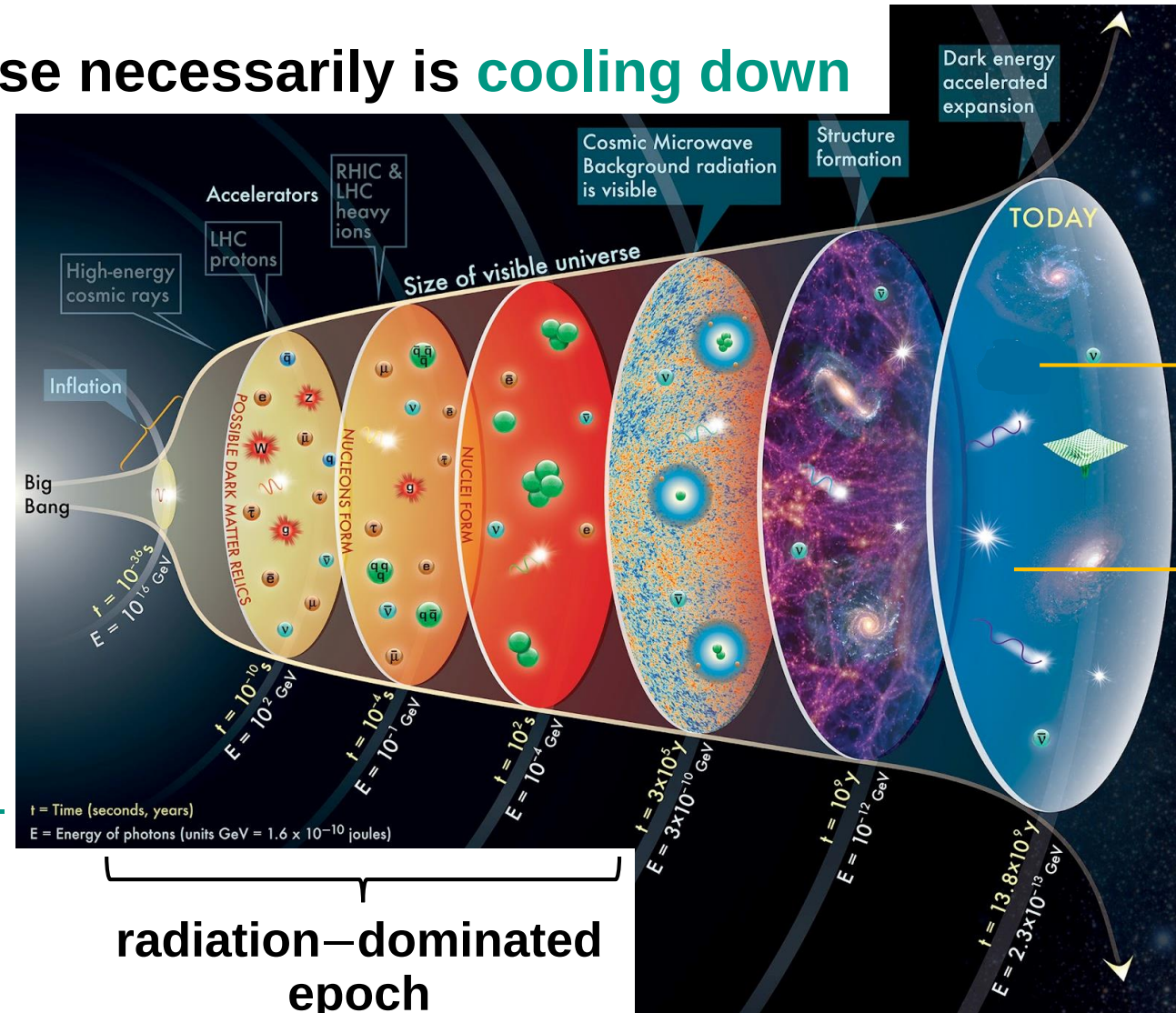
$$T \sim 10^{25} \text{ eV}$$

- present universe:

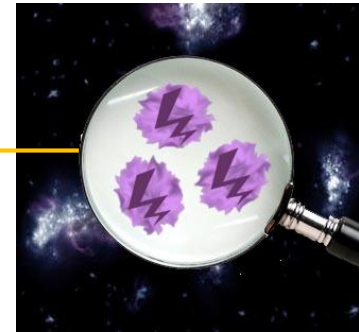
$$t \sim 13.8 \cdot 10^9 \text{ yr}$$

$$T \sim 10^{-3} \text{ eV}$$

- here: focus on **radiation–dominated universe**,
up to $t \sim 47\,000 \text{ yr}$



Dark Energy



Dark Matter
CDM

Q: LBL/PDG

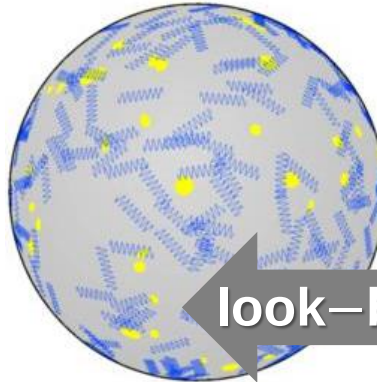
Evolution of temperature T in expanding cosmos

■ What is the relation between the scale factor $a(t)$ & temperature T ?

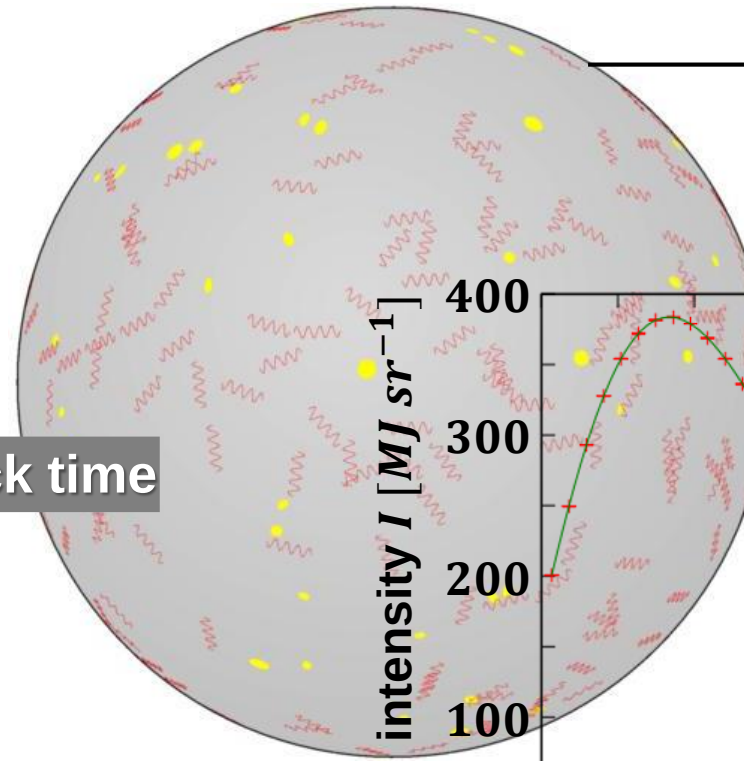
earlier (smaller & hotter) universe

$$a(t_1) = 0.5$$

$$T(t_1) = ?$$



look-back time



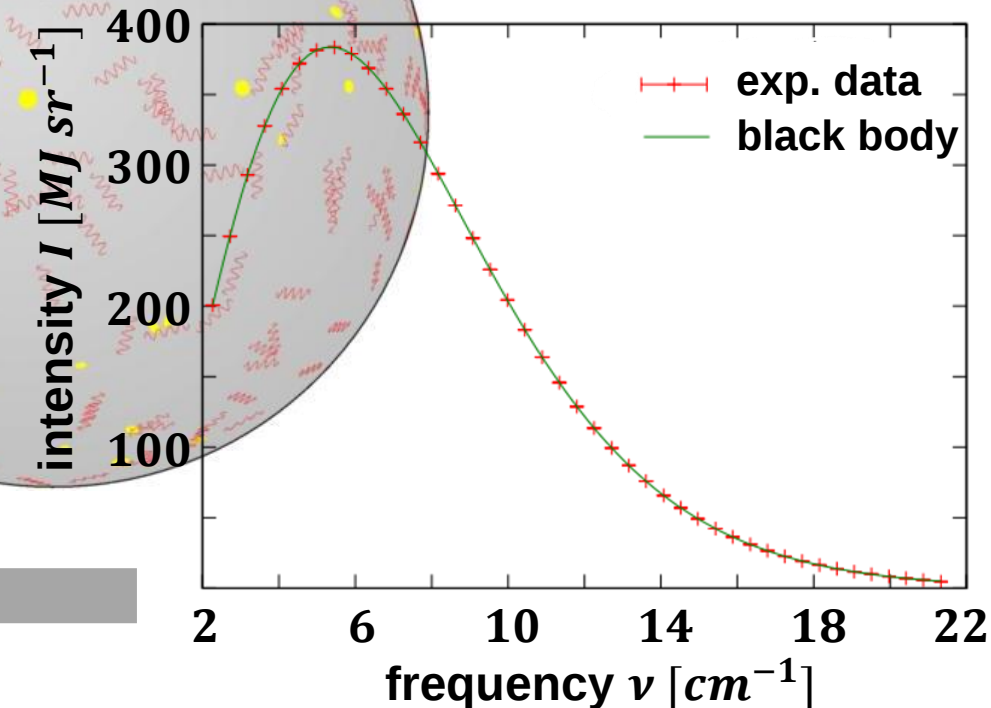
now: $t = t_0$

$$a(t_0) = 1.0$$

$$T(t_0) = 2.7 \text{ K}$$

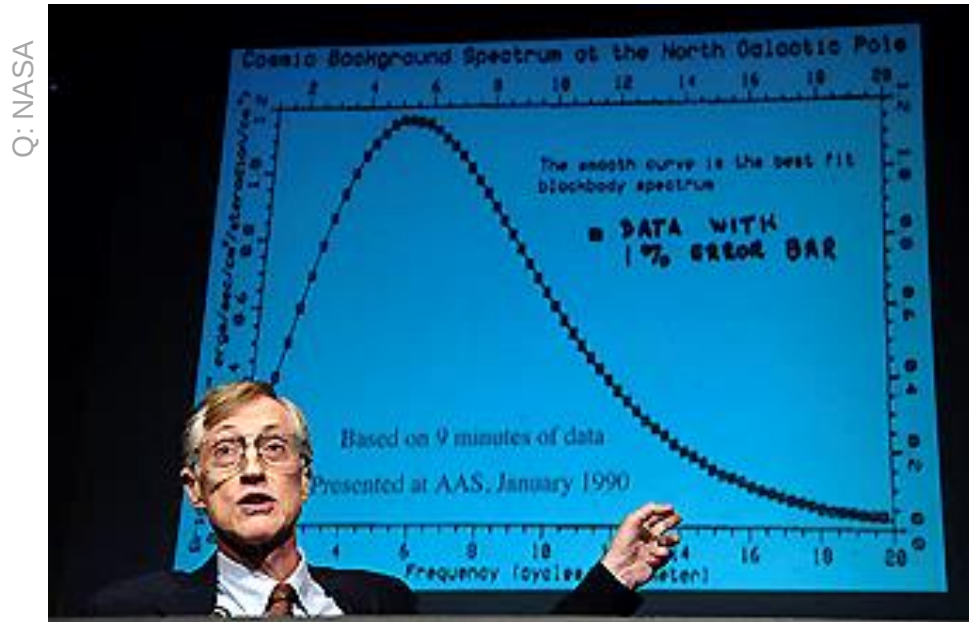
- photon wavelengths are blue-shifted for $t < t_0$

- universe $\equiv$ perfect black body
 $\Rightarrow$ temperature T from Planck-distribution

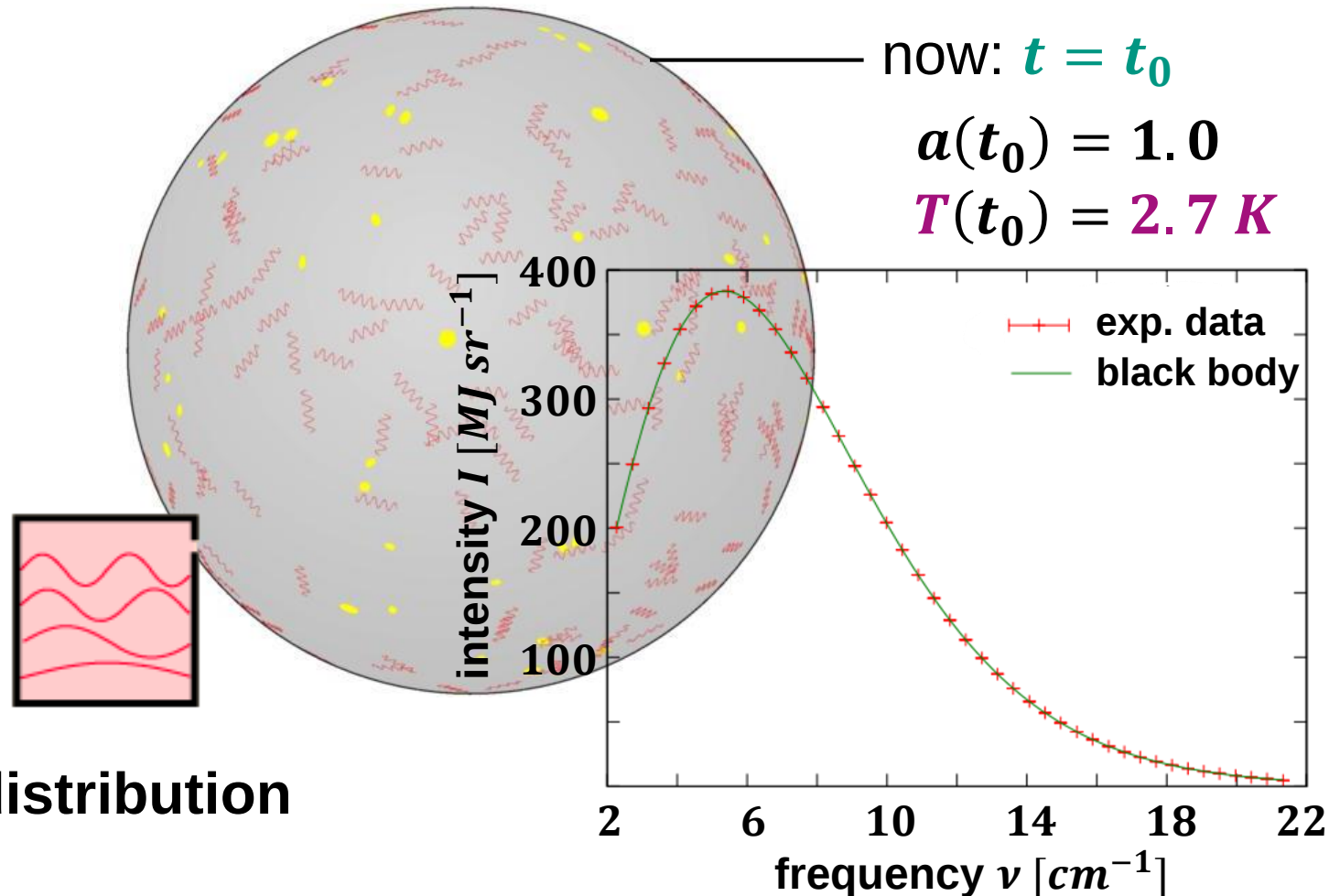


Evolution of temperature T in expanding cosmos

- What is the relation between the scale factor $a(t)$ & temperature T ?



- universe $\equiv$ perfect black body
 $\Rightarrow$ temperature T from Planck–distribution



Evolution of temperature T : Wien's law

■ Wien's displacement law and the Cosmic Microwave Background (CMB)

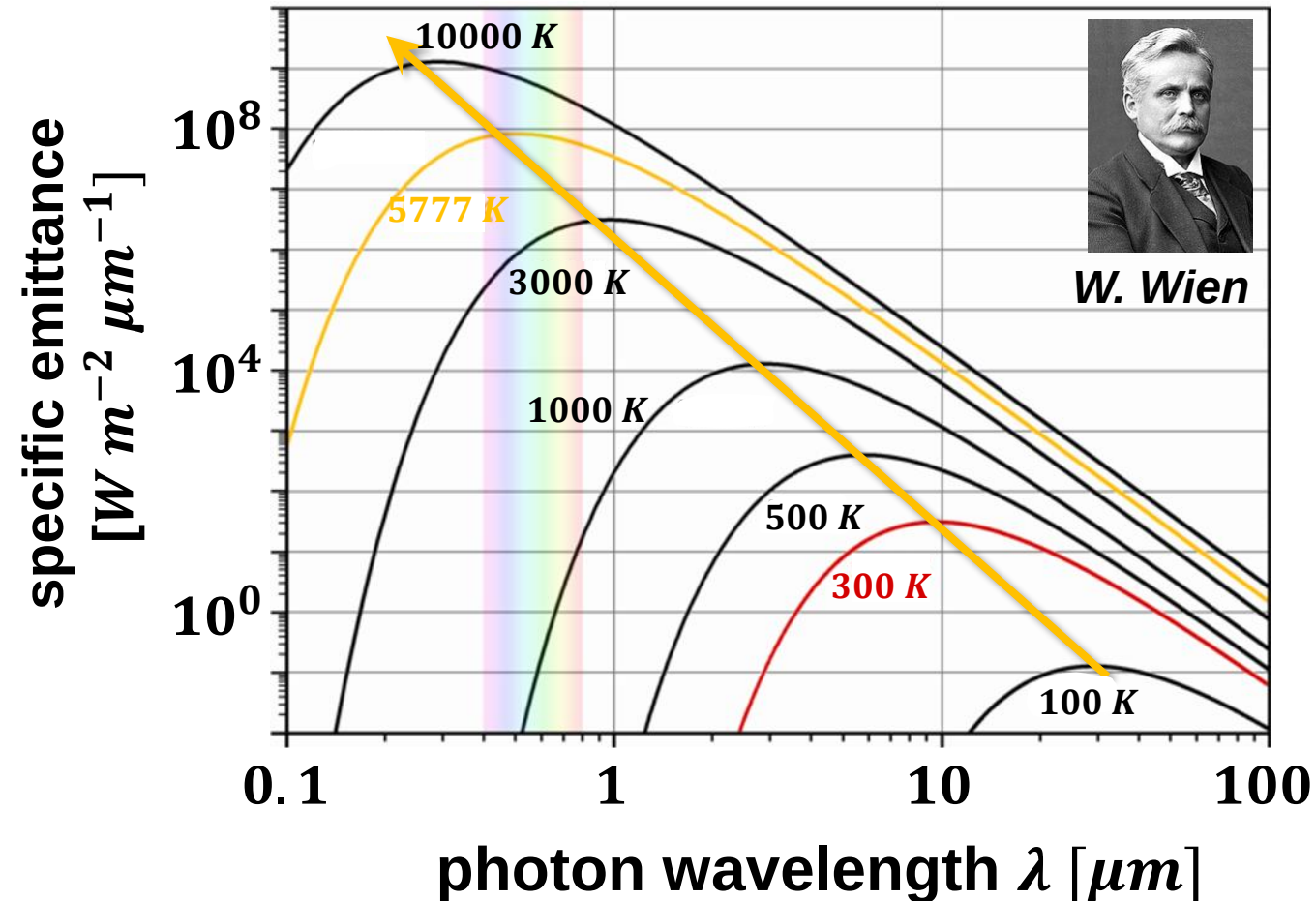
- relation between temperature T & maximum emittance λ_{max} of a thermal radiation bath (Planck)

$$\lambda_{max} \cdot T = 2.8978 \text{ mm K}$$

- adiabatic expansion of cosmos (z) & CMB – photons of frequency ν

$$T_{\gamma}(z) = T_{\gamma}(z=0) \cdot (1+z)$$

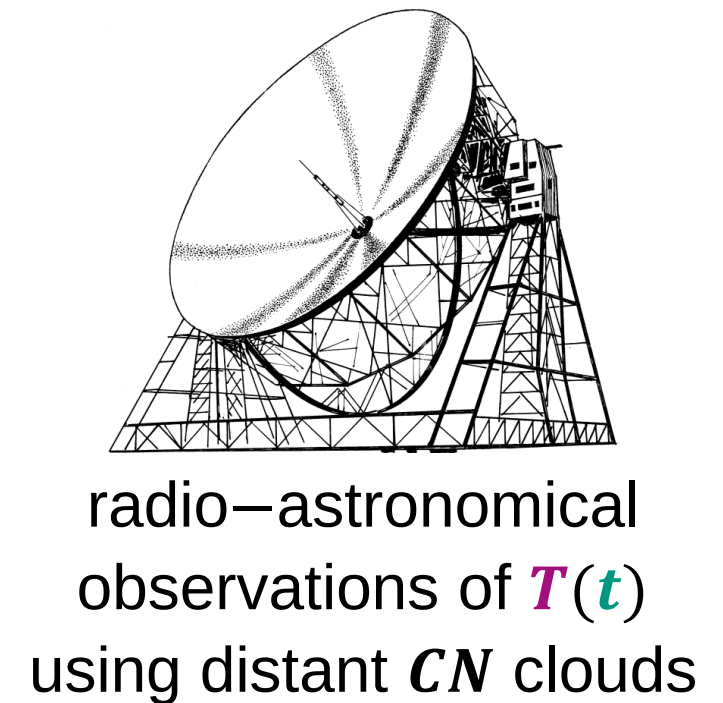
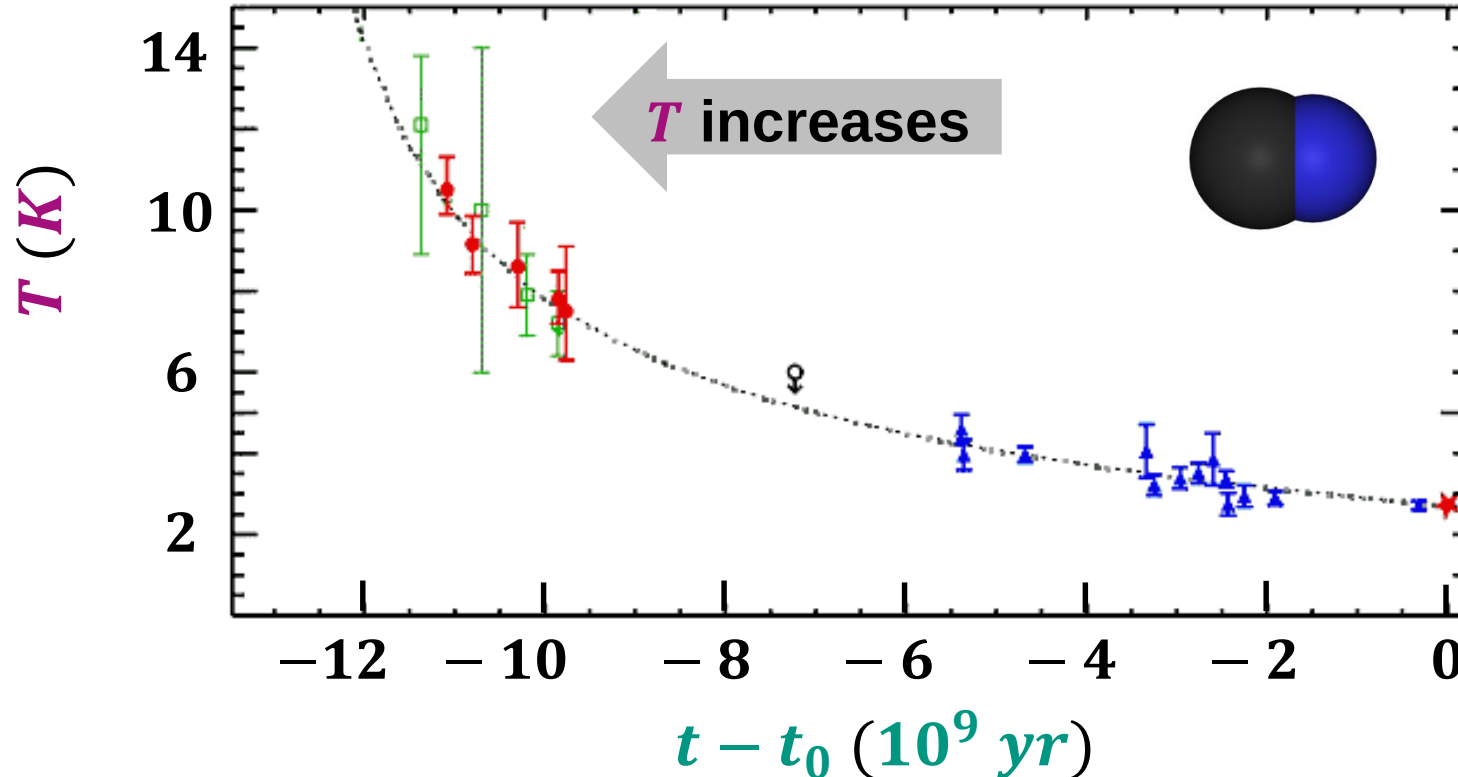
$$h\nu(z) = h\nu(z=0) \cdot (1+z)$$



Evolution of temperature T : experimental data

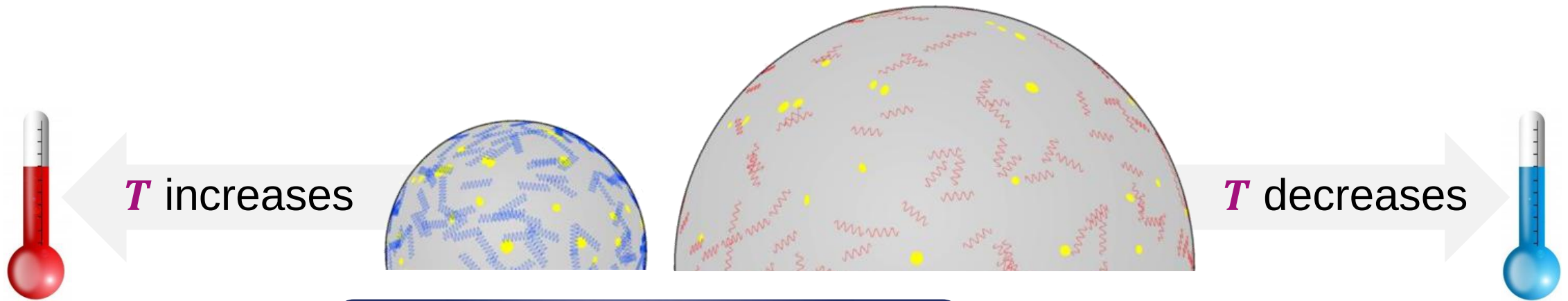
■ Observational evidence for an increase in temperature T at earlier times t

- effect: **thermal excitation of Cyan (CN) molecules** due the early, much hotter **CMB radiation** ($\Rightarrow$ more intense molecular excitation)



Evolution of temperature T & energy conservation

- A cornerstone of cosmology: **adiabatic** expansion process



A. Einstein



adiabatic cosmological expansion



J. Stefan



L. Boltzmann

Evolution of temperature T as function of t & $a(t)$

- Connecting two descriptions: **photon bath** & **thermodynamical ensemble**

time t

$$\rho_{\gamma}(t) = N_{\gamma}(t) \cdot E_{\gamma}(t)$$

temperature T

$$\rho_{\gamma}(T) = \frac{\pi^2}{15 h^3 c^3} \cdot (k_B \cdot T)^4$$

photon bath:

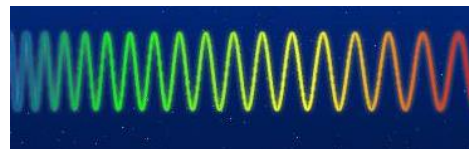
photons are
elementary
particles

ensemble:

photons act as
thermodynamical
ensemble



A. Einstein



J. Stefan



L. Boltzmann

Evolution of temperature T as function of t & $a(t)$

■ Connecting two descriptions: **photon bath** & **thermodynamical ensemble**

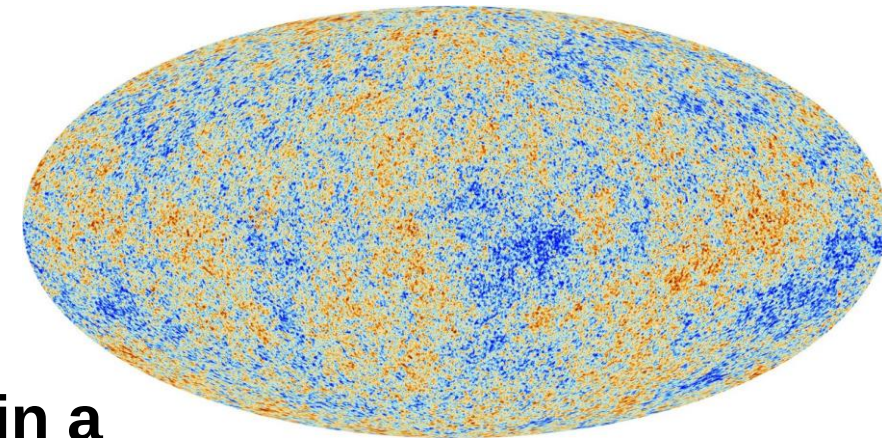
$$\rho_\gamma(t) = N_\gamma(0) \cdot a(t)^{-3} \cdot E_\gamma(0) \cdot a(t)^{-1}$$

$$\rho_\gamma(t) \sim a(t)^{-4} \longleftrightarrow \rho_\gamma(T) \sim T(t)^4$$



$$T(t) \sim \frac{1}{a(t)} \sim \frac{1}{\sqrt{t}}$$

evolution of temperature T in a
radiation–dominated universe



Properties of the radiation–dominated universe

■ Temperature scale $T(t)$ and energy scale $E(t)$ of CMB photons

- important **conversion factor**:

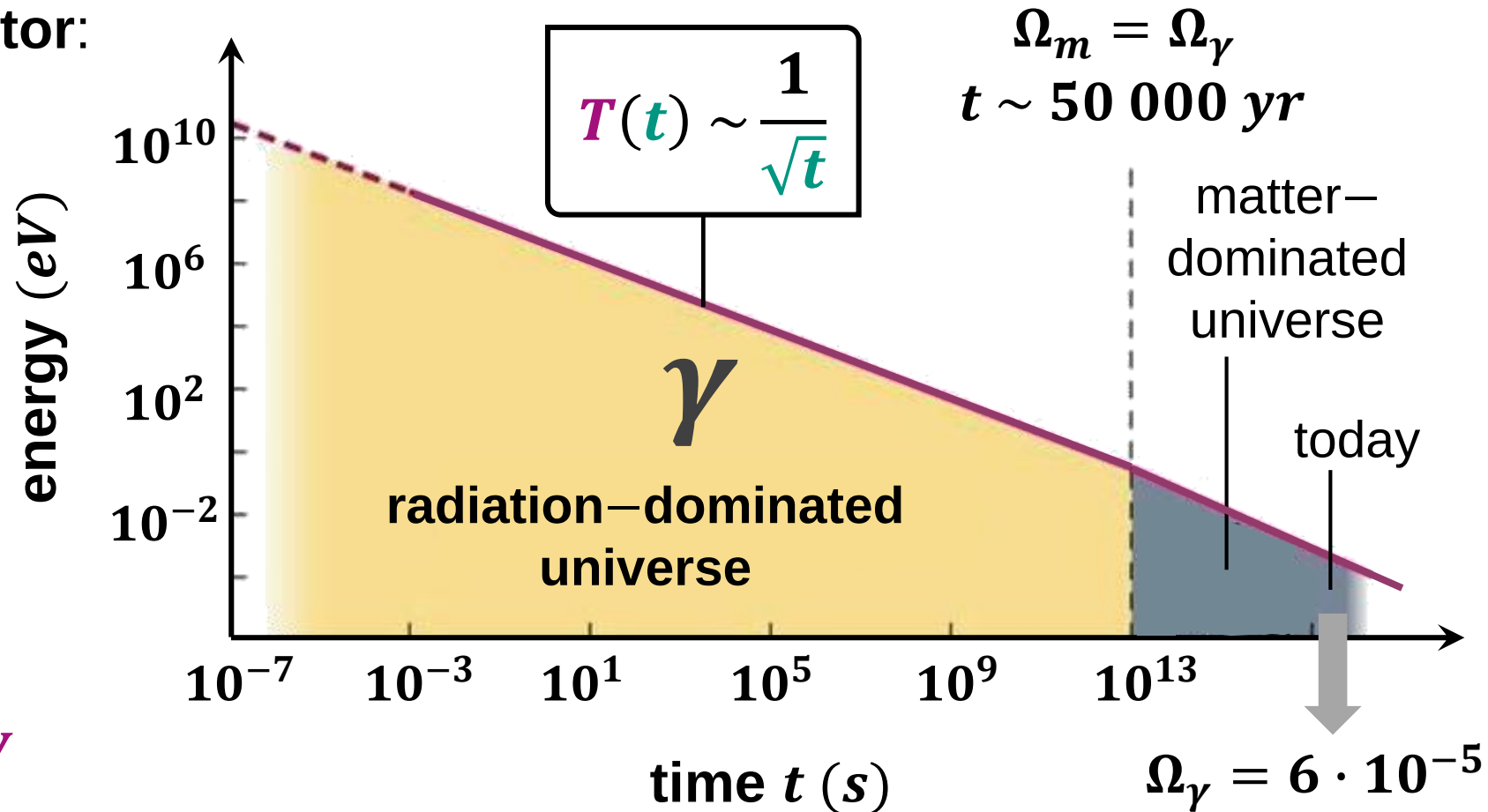
$$10^4 \text{ K} \leftrightarrow 1 \text{ eV}$$

(11 605 K)

$$T(t) \cong \sim \frac{10^{10} \text{ K}}{\sqrt{t \text{ [s]}}}$$

$$\cong \sim \frac{1 \text{ MeV}}{\sqrt{t \text{ [s]}}}$$

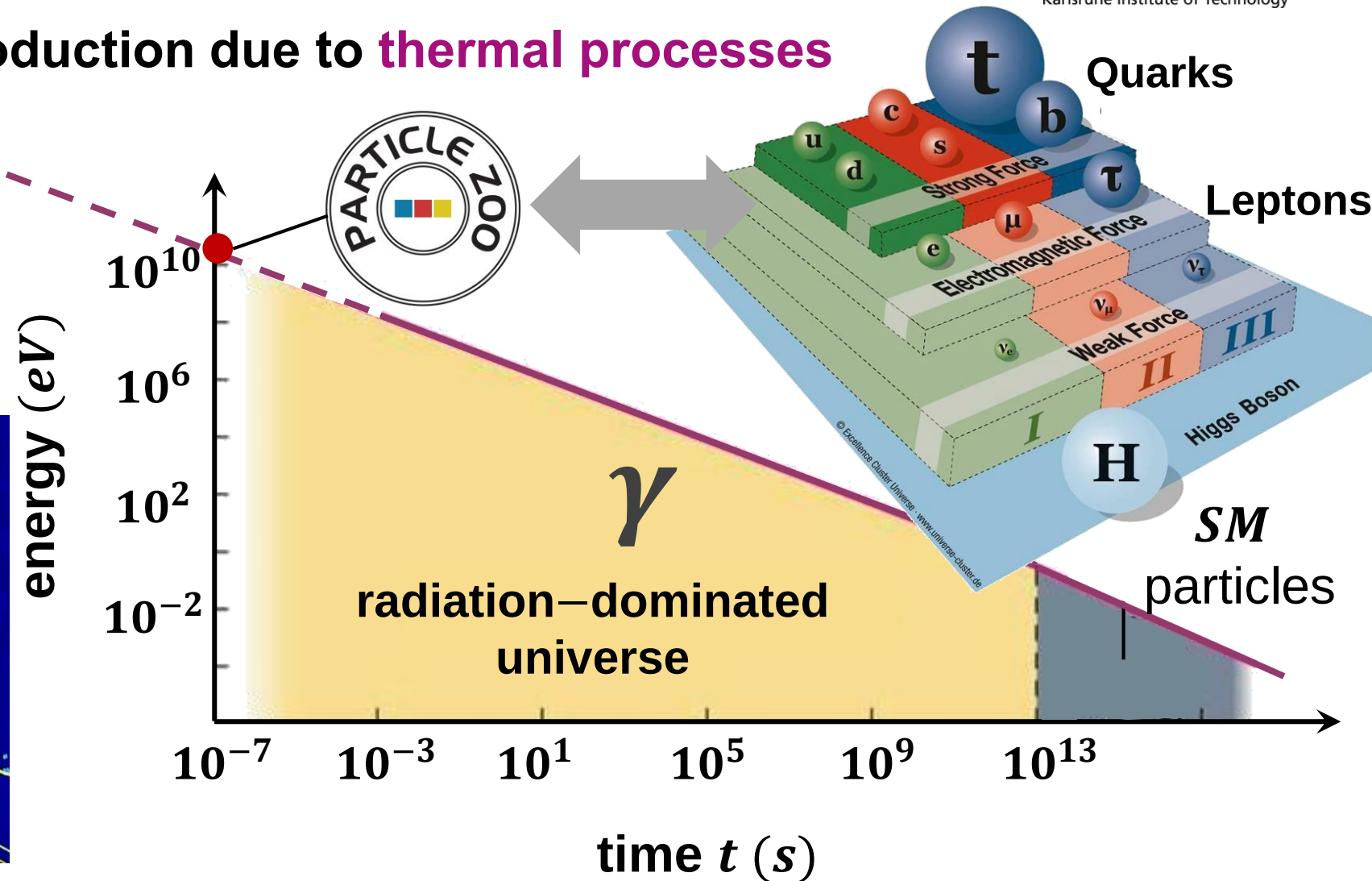
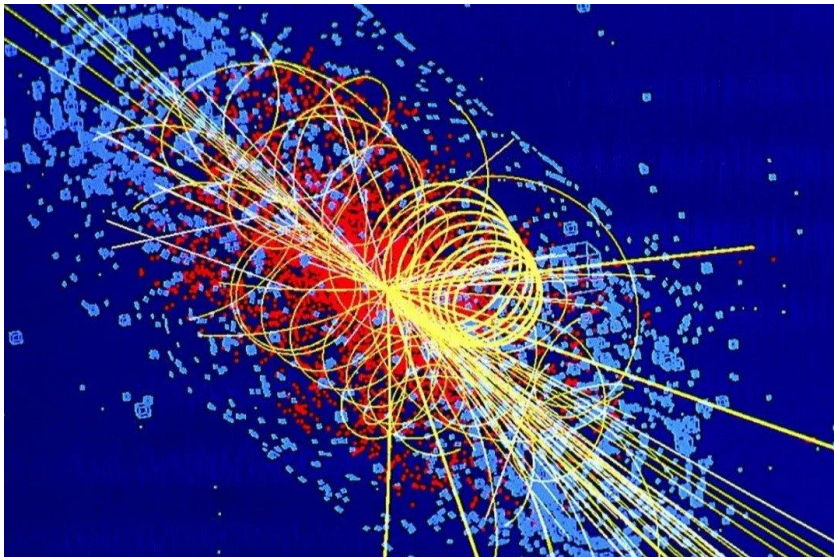
at $t = 1 \text{ s} \leftrightarrow T = 1 \text{ MeV}$



Thermal production of the 'Particle Zoo'

■ Big Bang: Particle production due to **thermal processes**

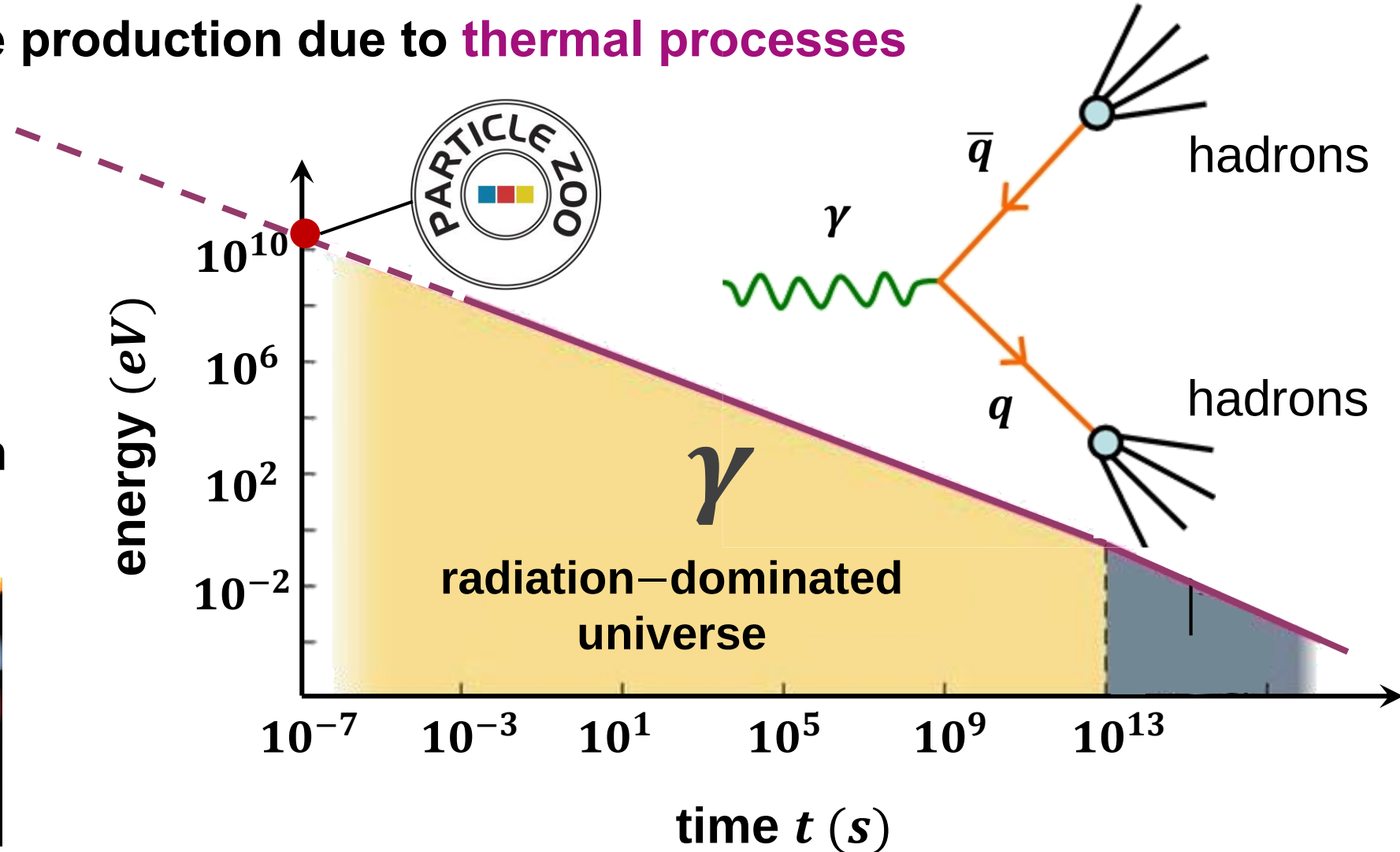
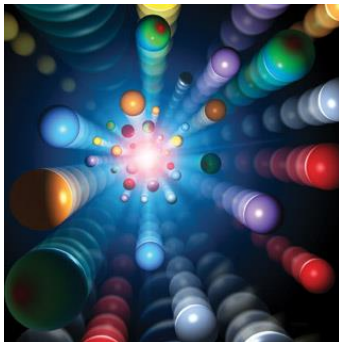
- heat bath:
particle interactions
via strong & electro-
weak interactions



Thermal production of the 'Particle Zoo'

■ Big Bang: Particle production due to **thermal processes**

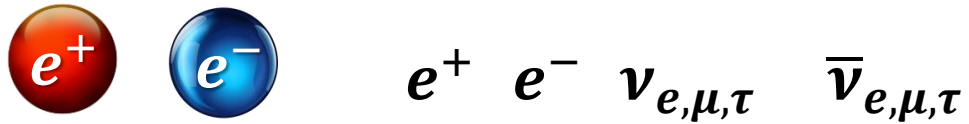
- **heat bath:**
particle interactions
via strong & electro-
weak interactions
- **thermal production**
- annihilation
- decay



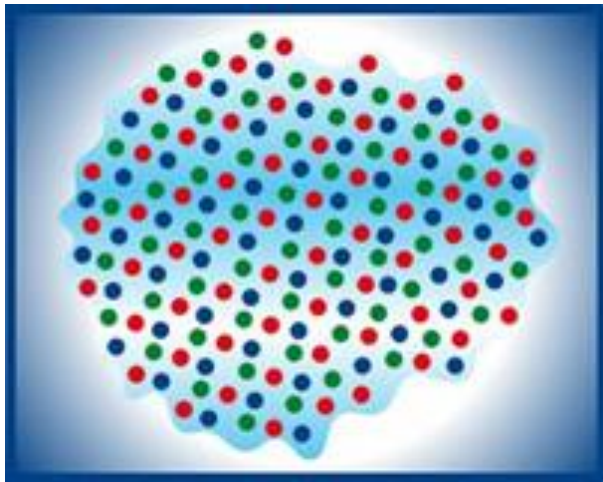
Radiation–dominated universe: Particle Zoo

Particle production during the Big Bang

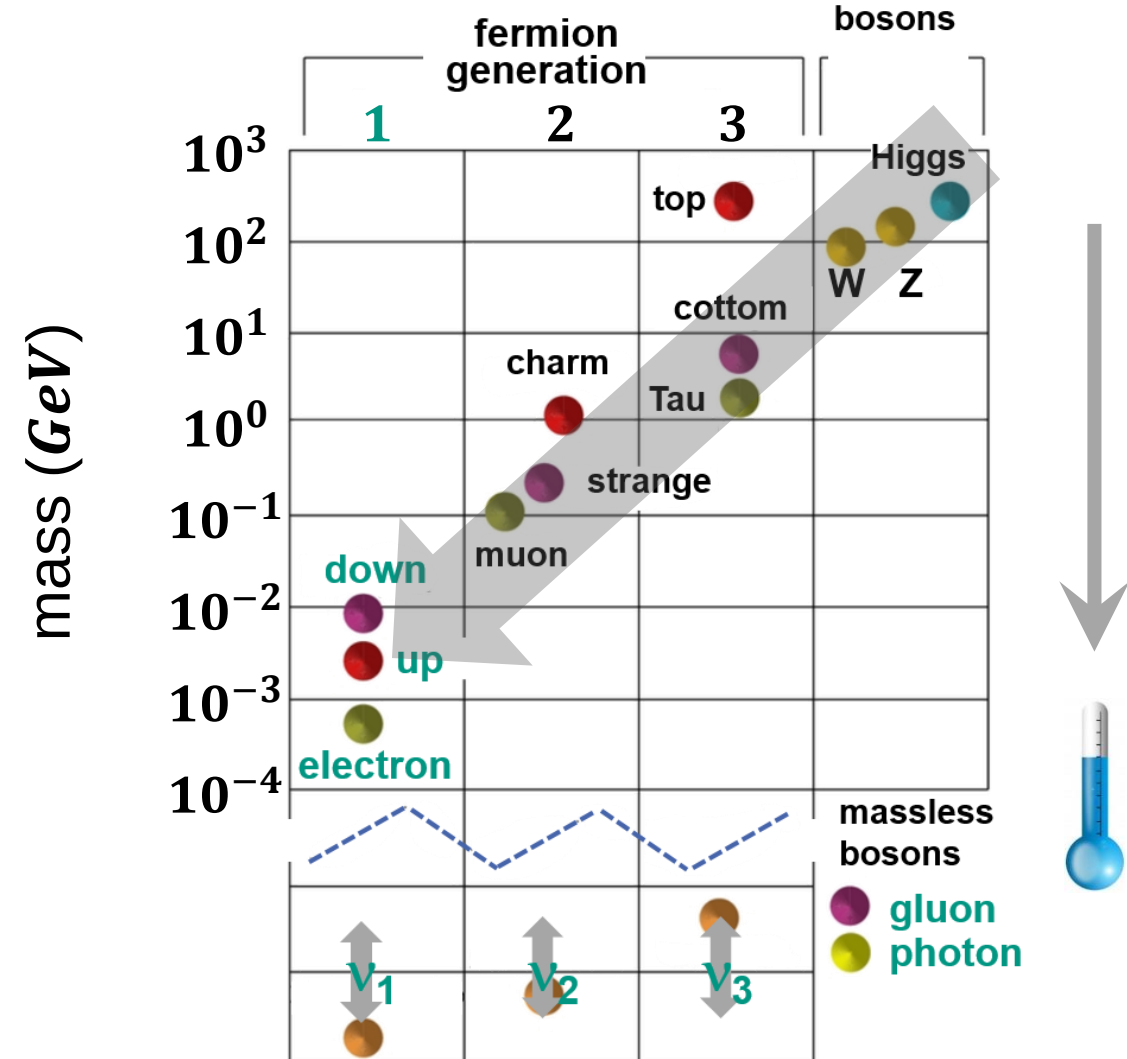
Leptons:



Quarks: u $\bar{u}$ d $\bar{d}$



Quark-
Gluon
plasma



Production of particles & phase transitions

■ During expansion & cooling of universe: **phase transitions**

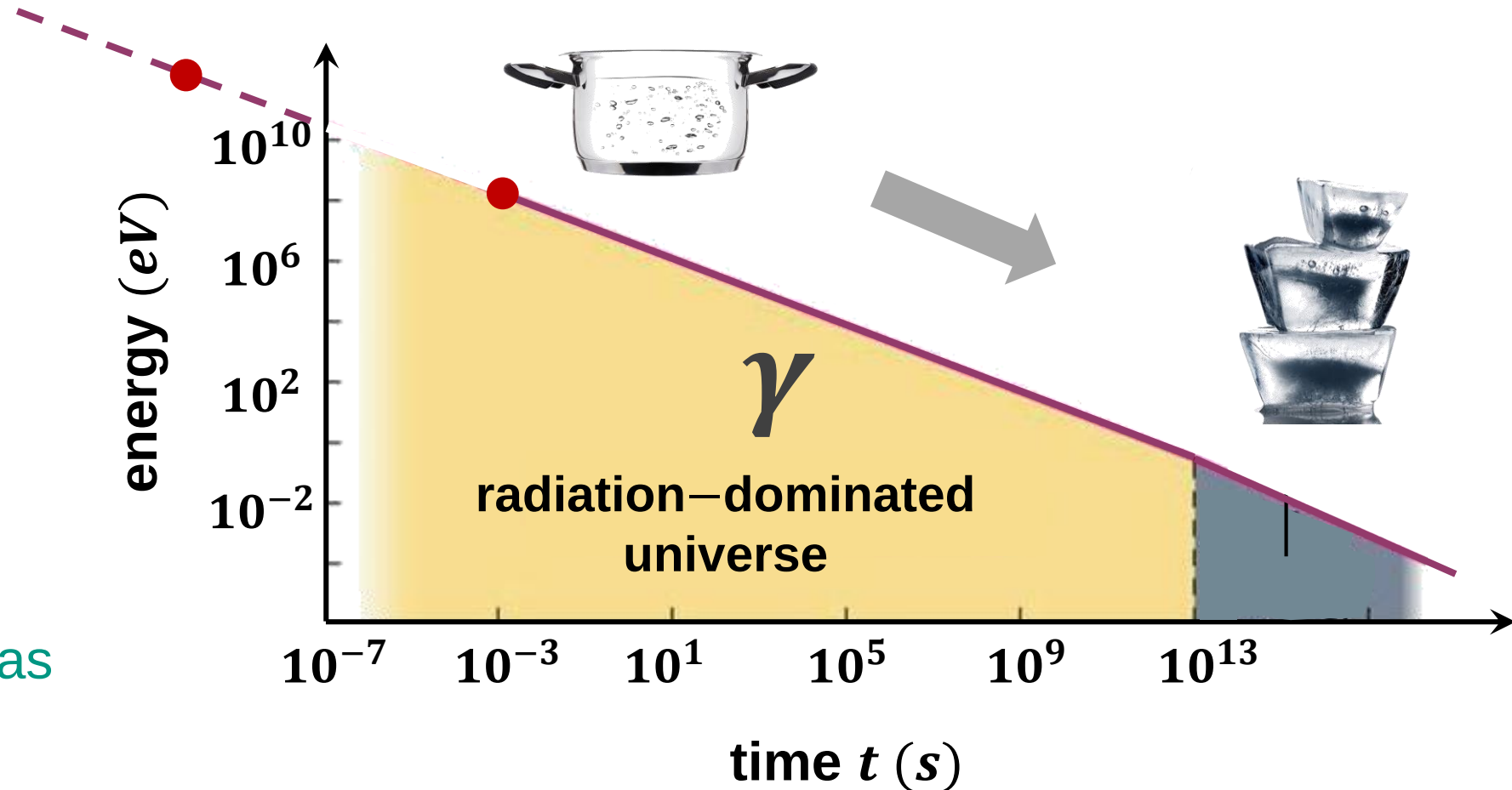
- many examples:

Higgs mechanism



QCD transition

plasma to atomic gas

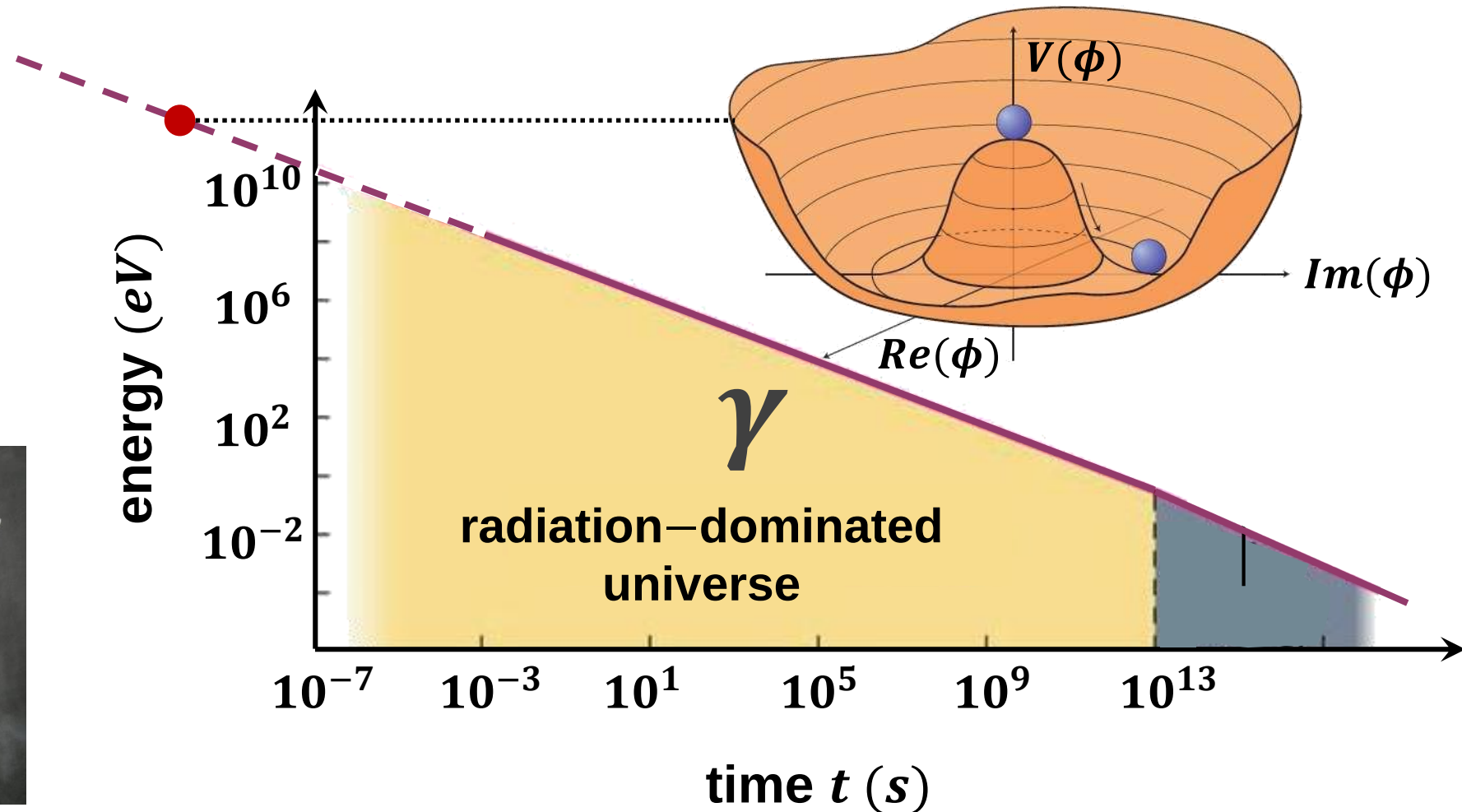
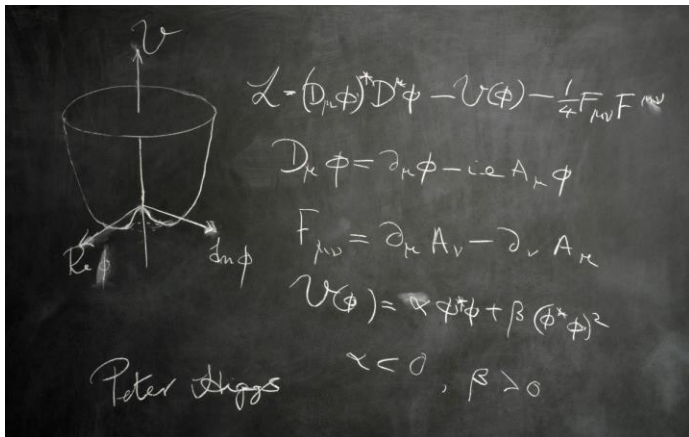


Production of particles & phase transitions

■ During expansion & cooling of universe: **electroweak phase transition**

- example:

Higgs mechanism

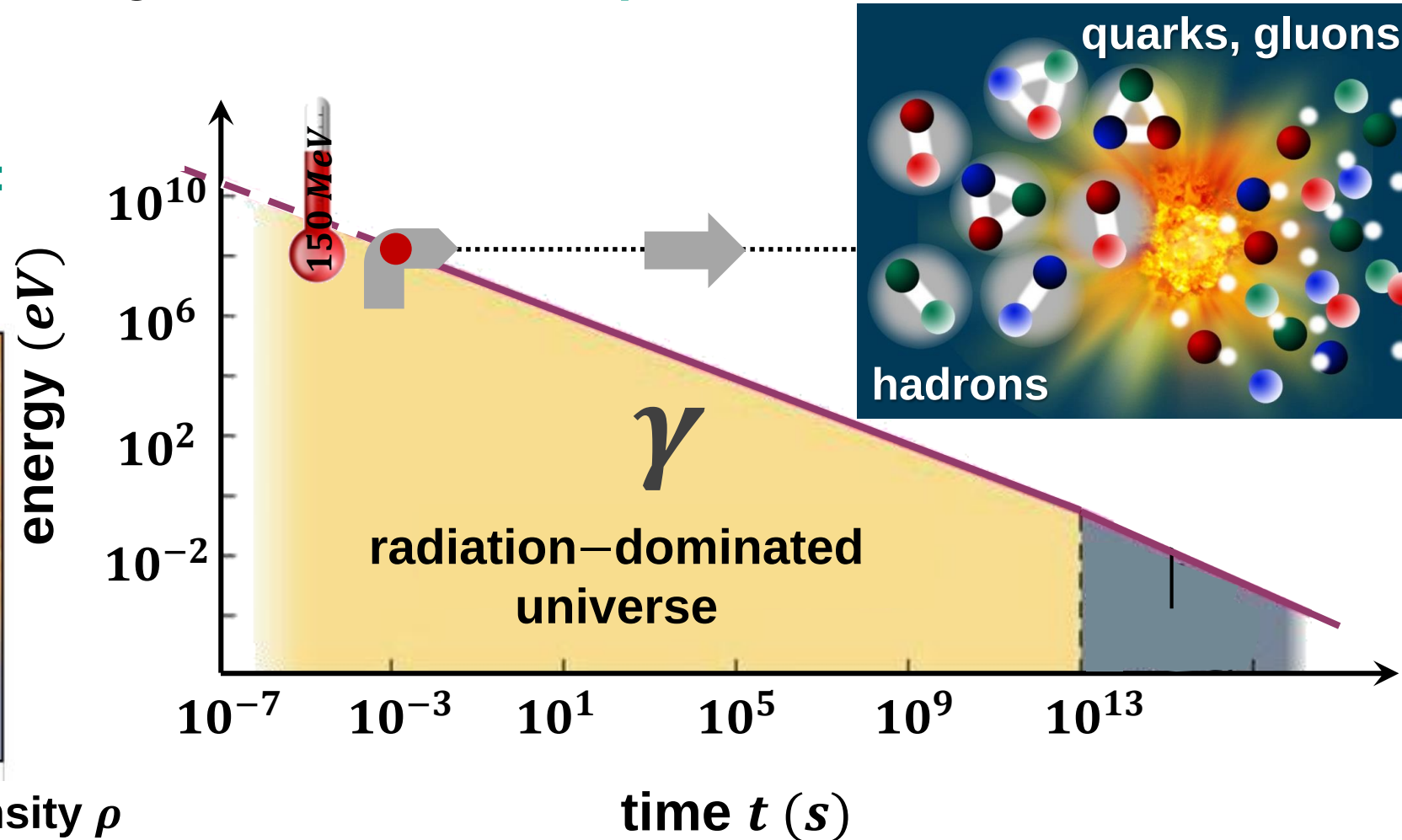
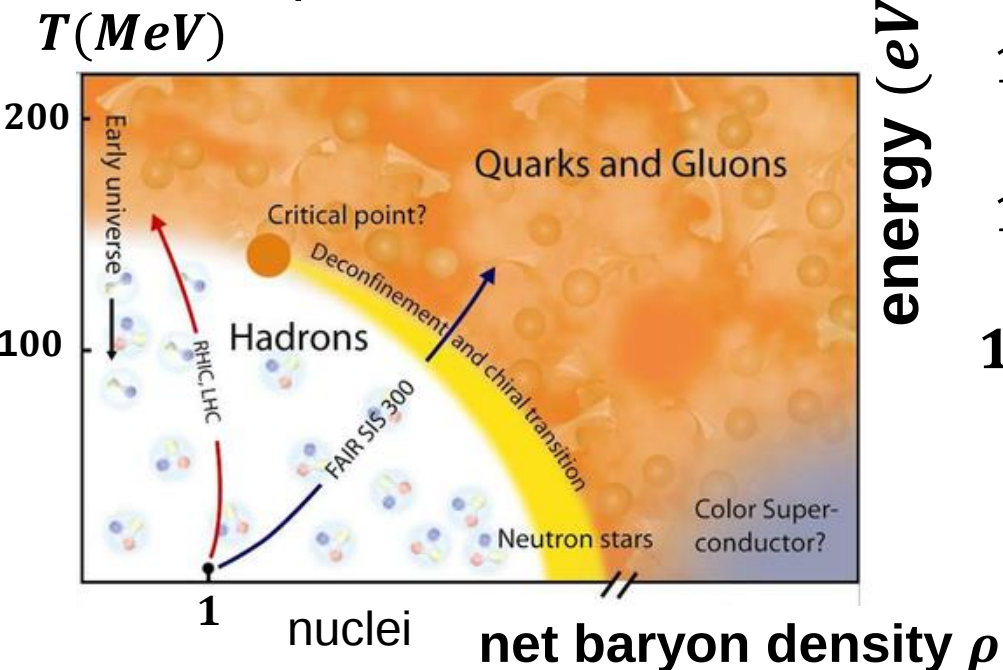


Production of particles & phase transitions

■ During expansion & cooling of universe: *QCD* phase transition

- example:

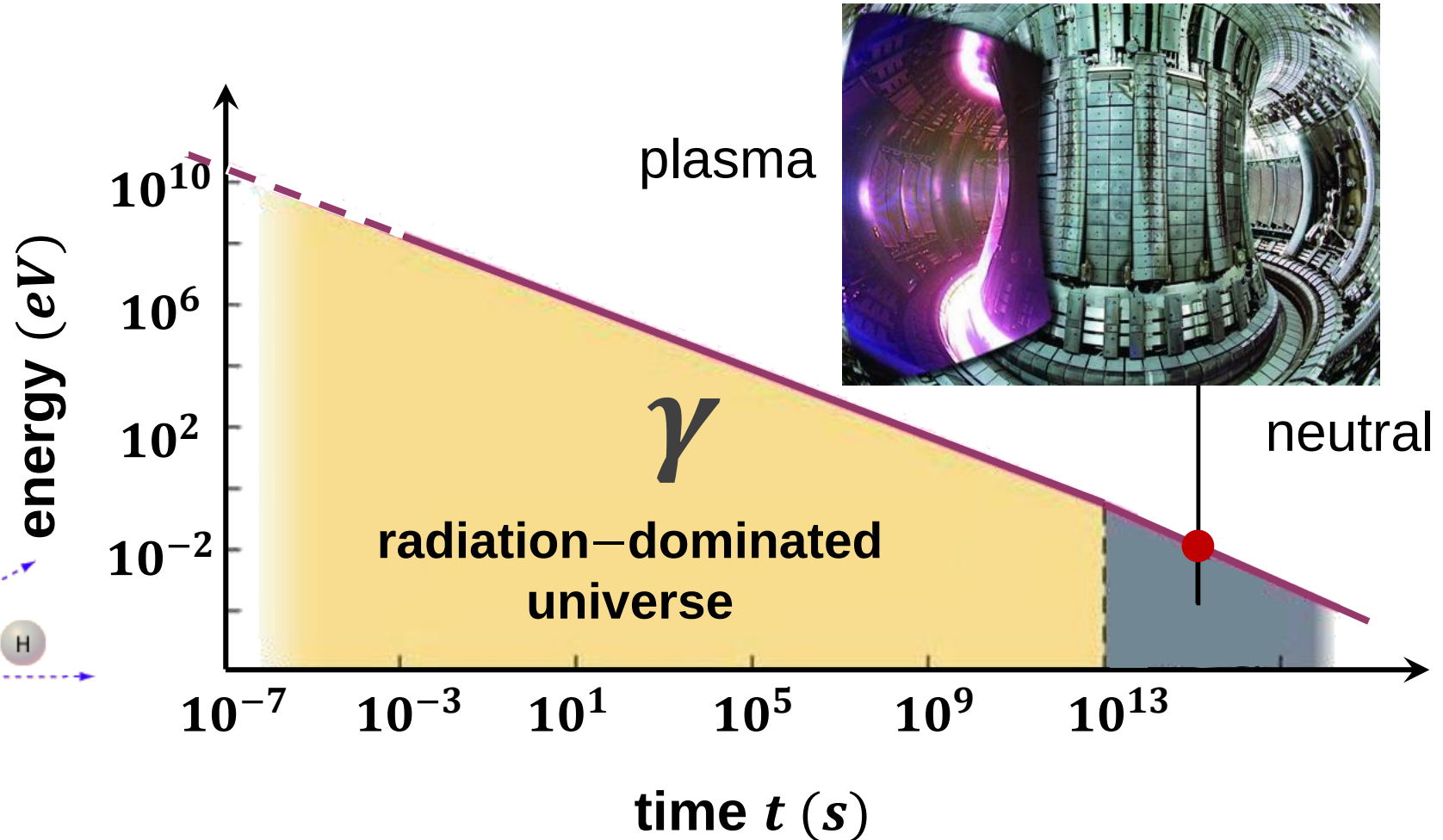
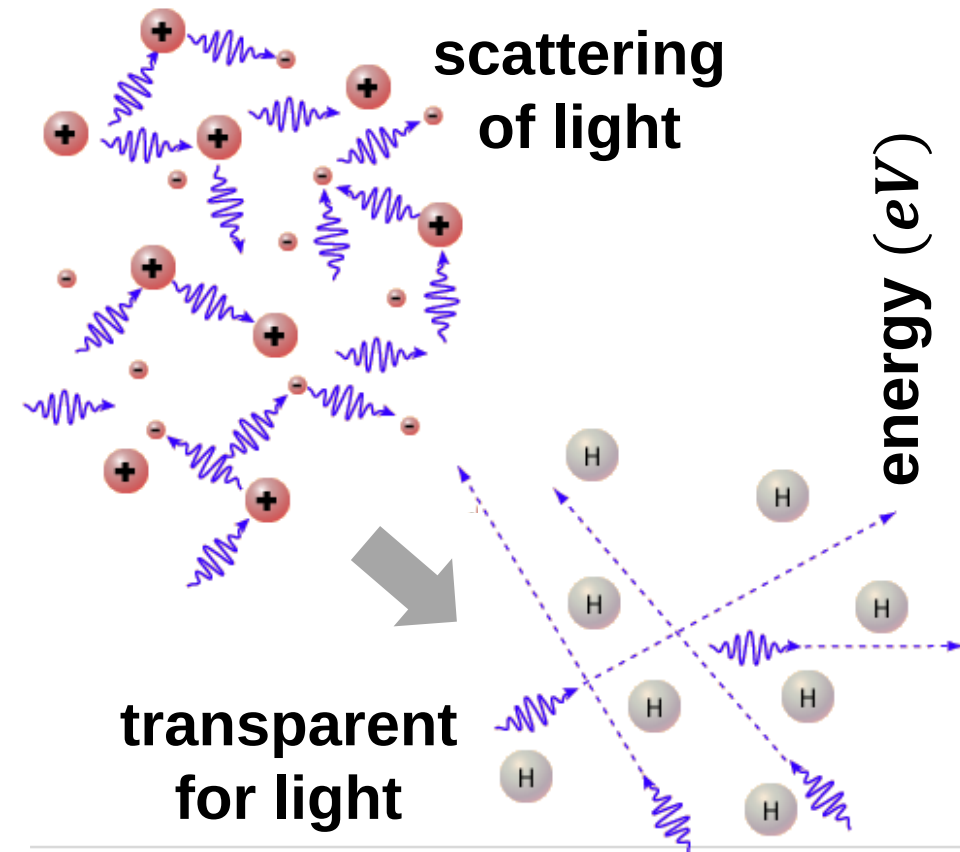
QCD at $T = 150 \text{ MeV}$:
quarks $\Rightarrow$ hadrons



Production of particles & phase transitions

■ During expansion & cooling of universe: transition **plasma** $\Rightarrow$ **neutral atoms**

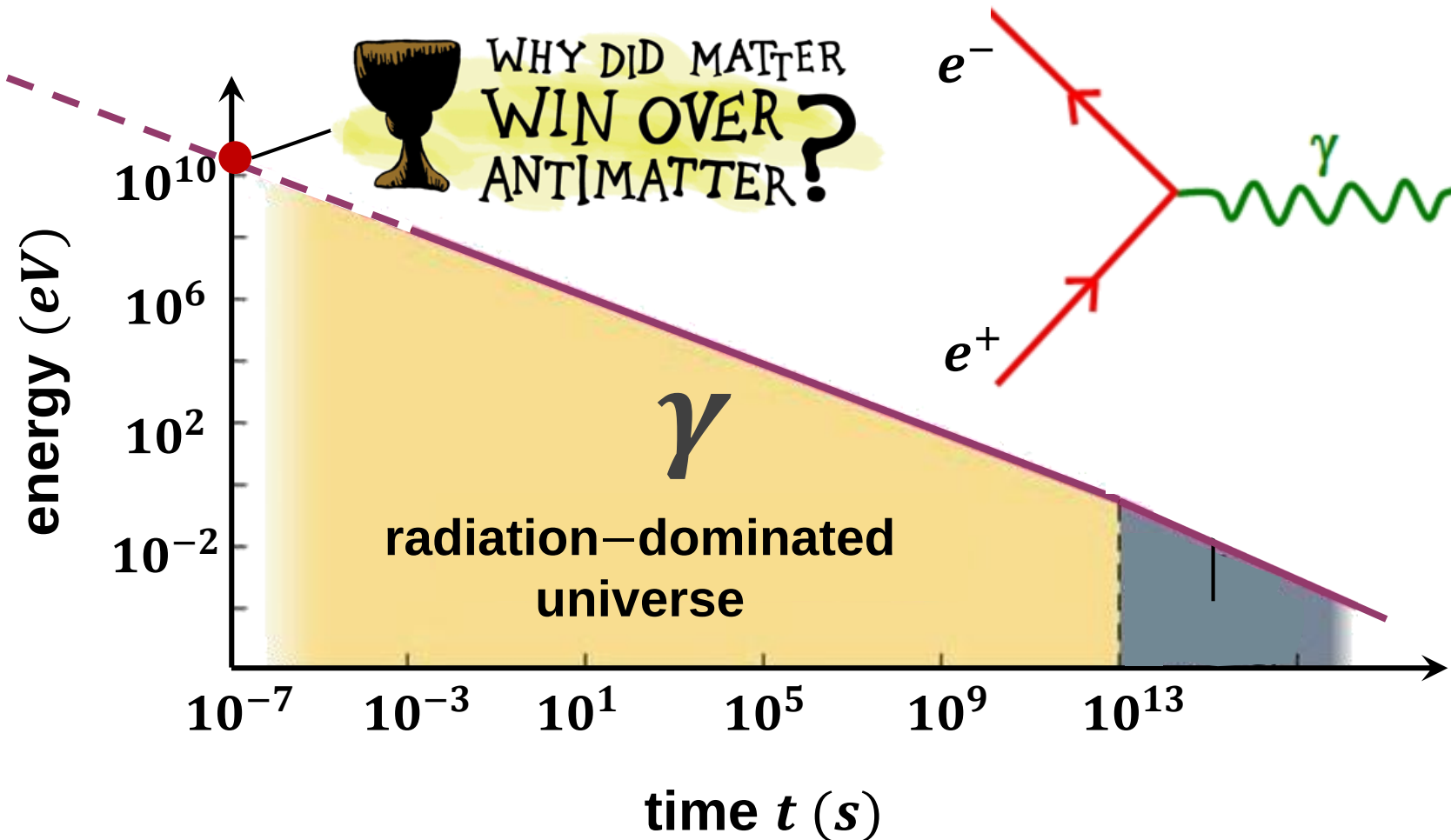
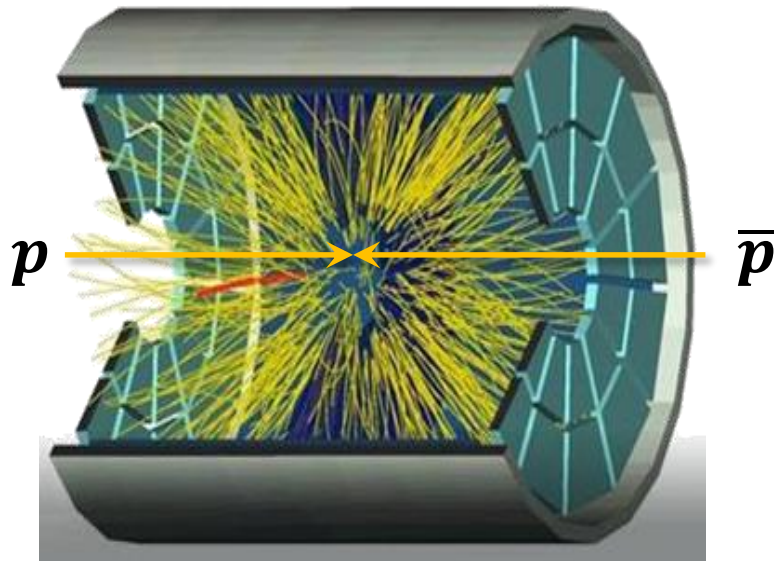
- example:



Annihilations within the 'Particle Zoo'

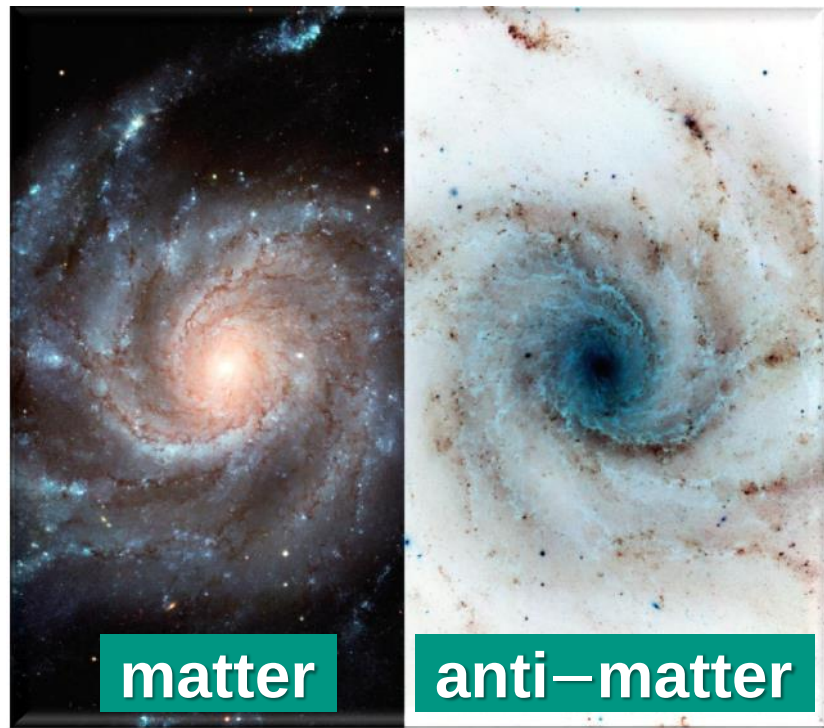
■ **Particle annihilations** will dominate in case of falling temperatures

- in case of **perfect CP symmetry**: no baryons left in the universe

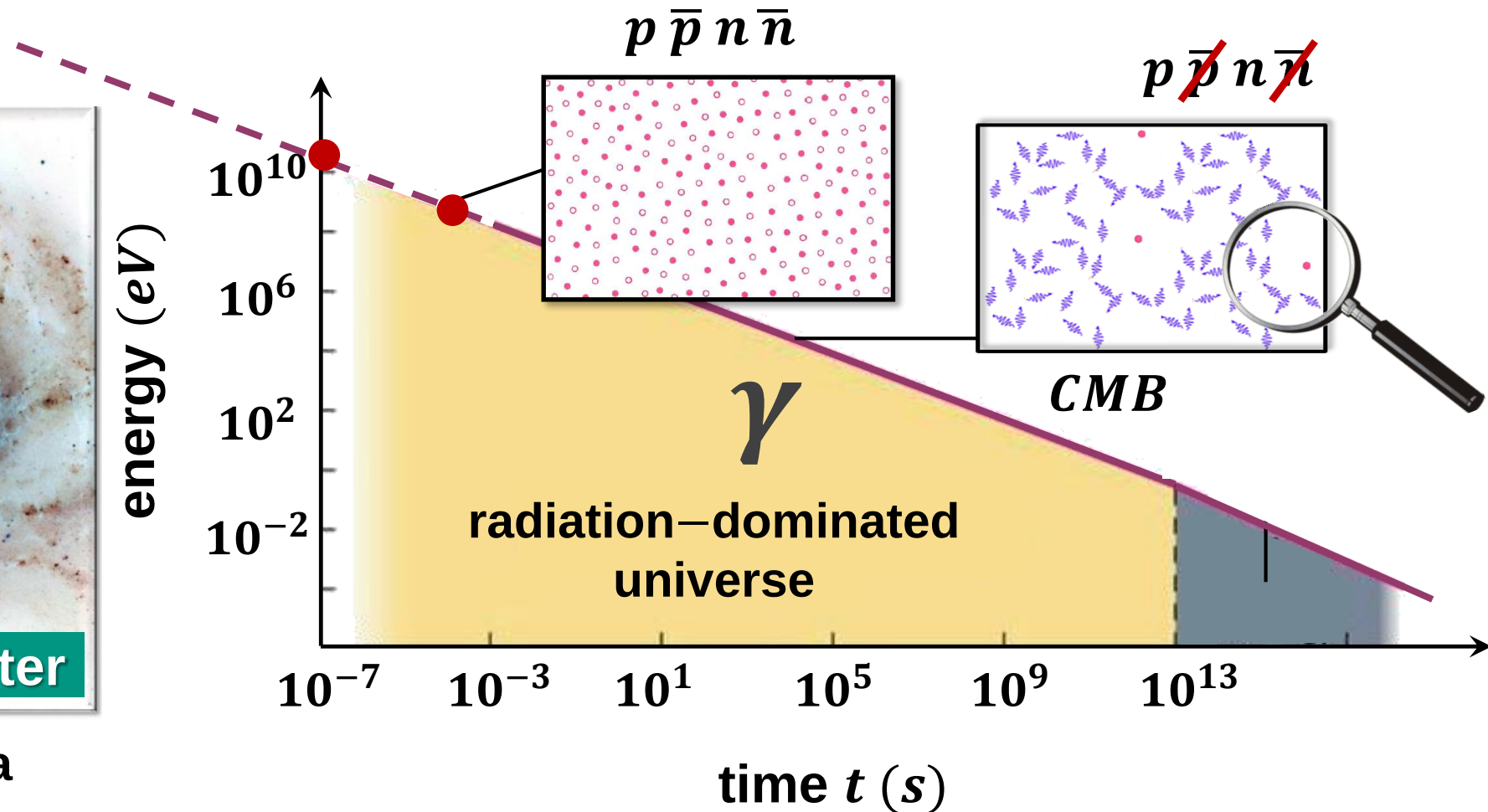


Annihilations within the 'Particle Zoo'

- Particle annihilations: tiny preference for matter particles due to $\cancel{CP}$

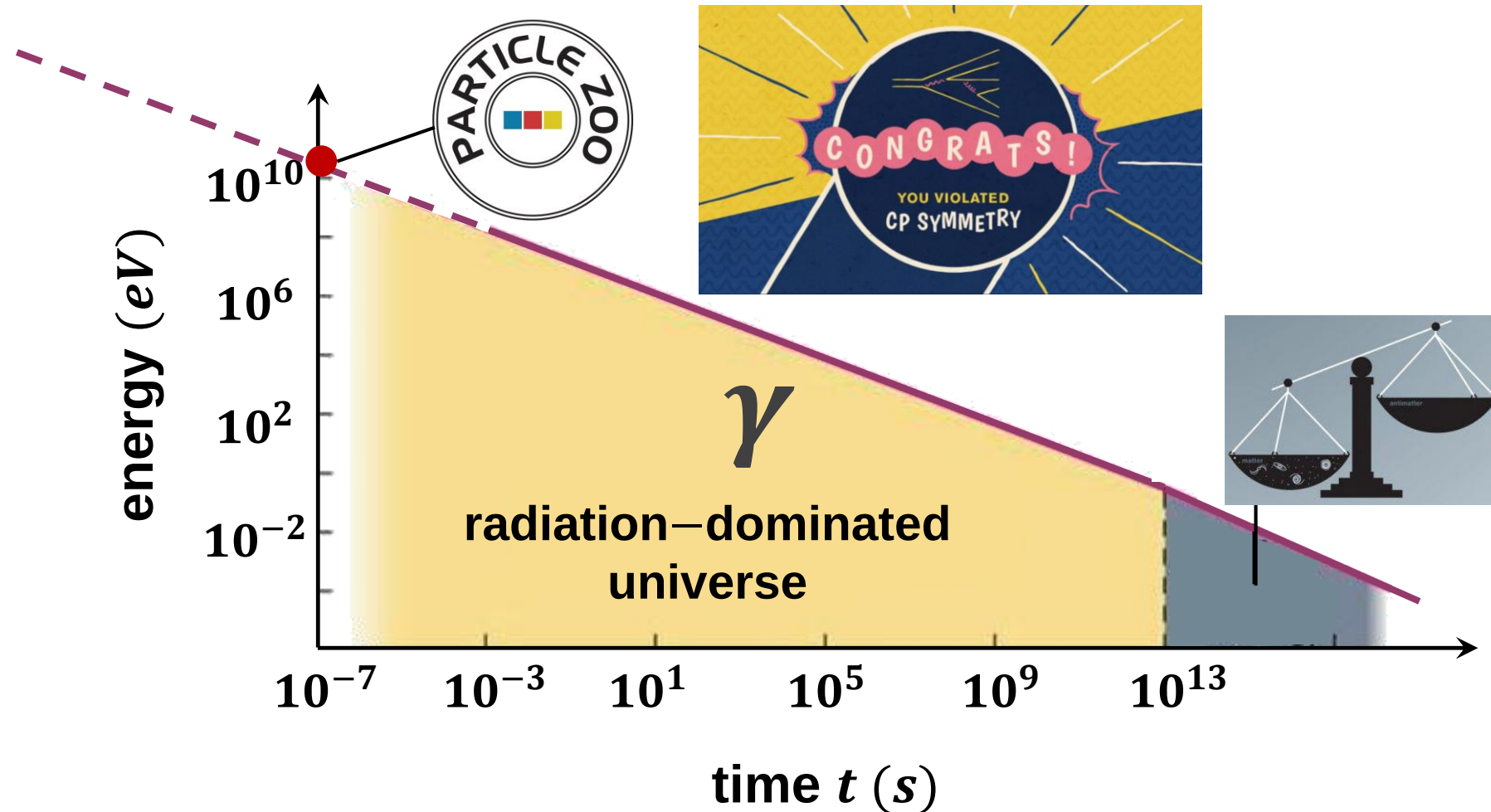
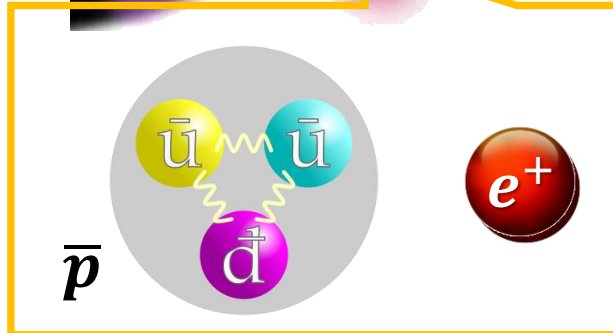
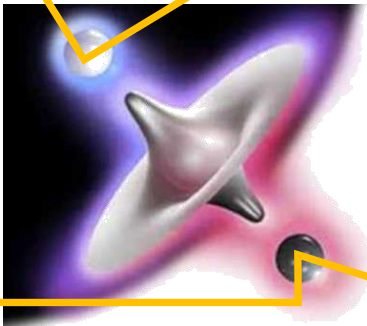
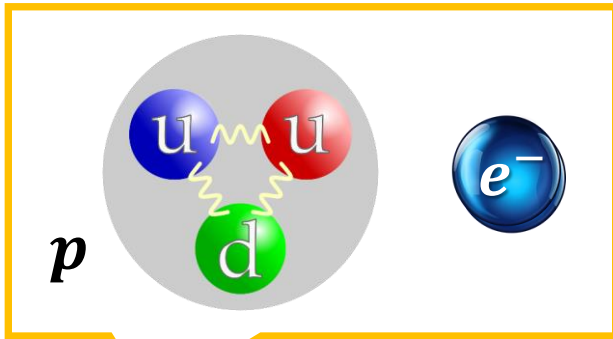


3 Sacharov-criteria



Annihilations within the 'Particle Zoo'

■ Particle annihilations: an extremely tiny preference for matter particles...

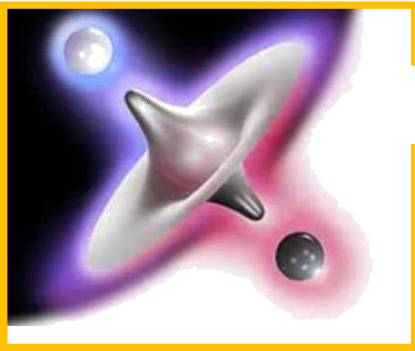


Annihilations within the 'Particle Zoo'

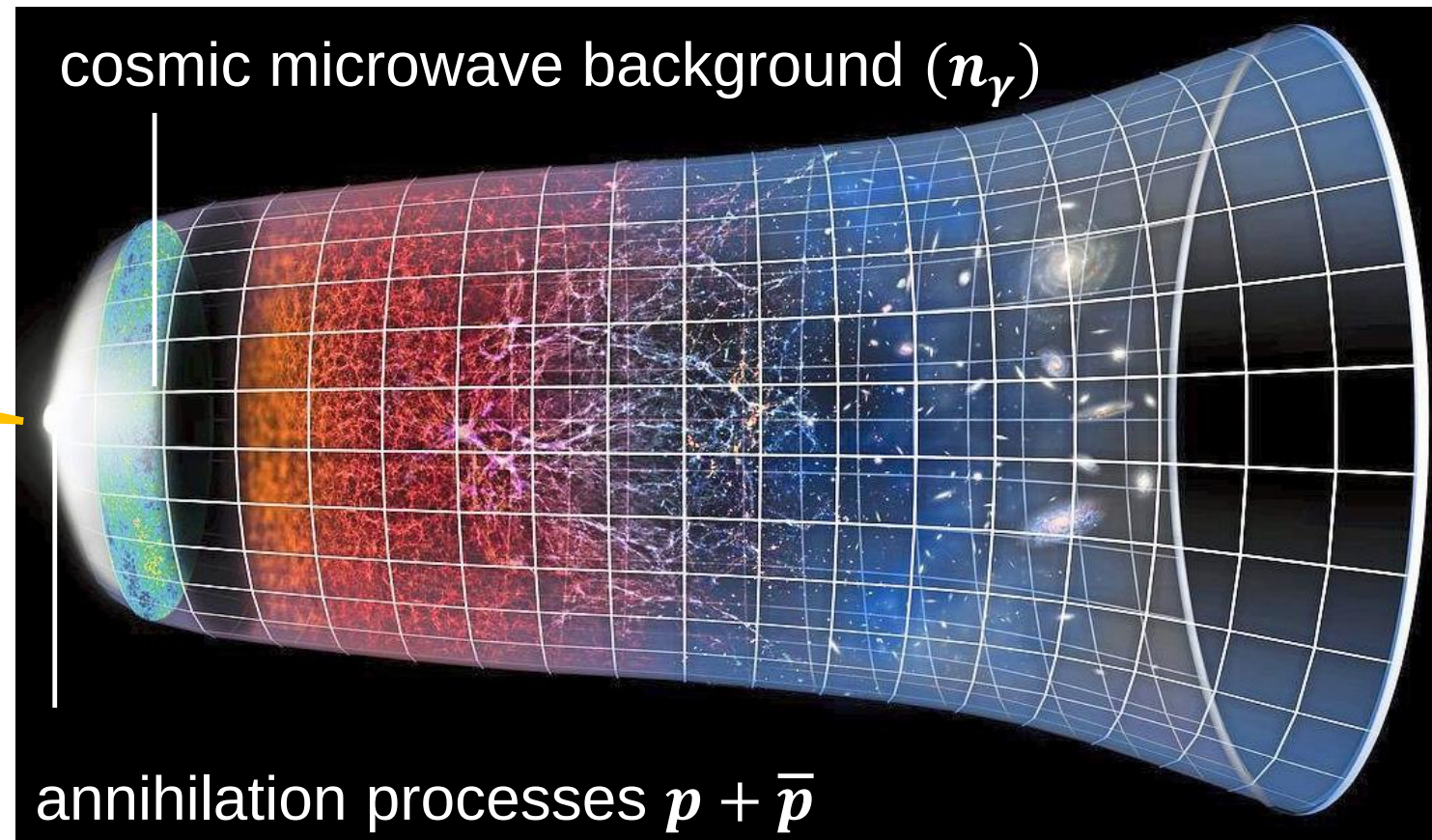
■ Particle annihilations after the Big Bang & origin of baryon number violation

- universe with a net
baryon asymmetry η

$$\eta = (6.14 \pm 0.24) \cdot 10^{-10}$$



$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$$



Further processing of baryons: nucleosynthesis!

■ Nuclear reactions produce the **light elements** in the universe

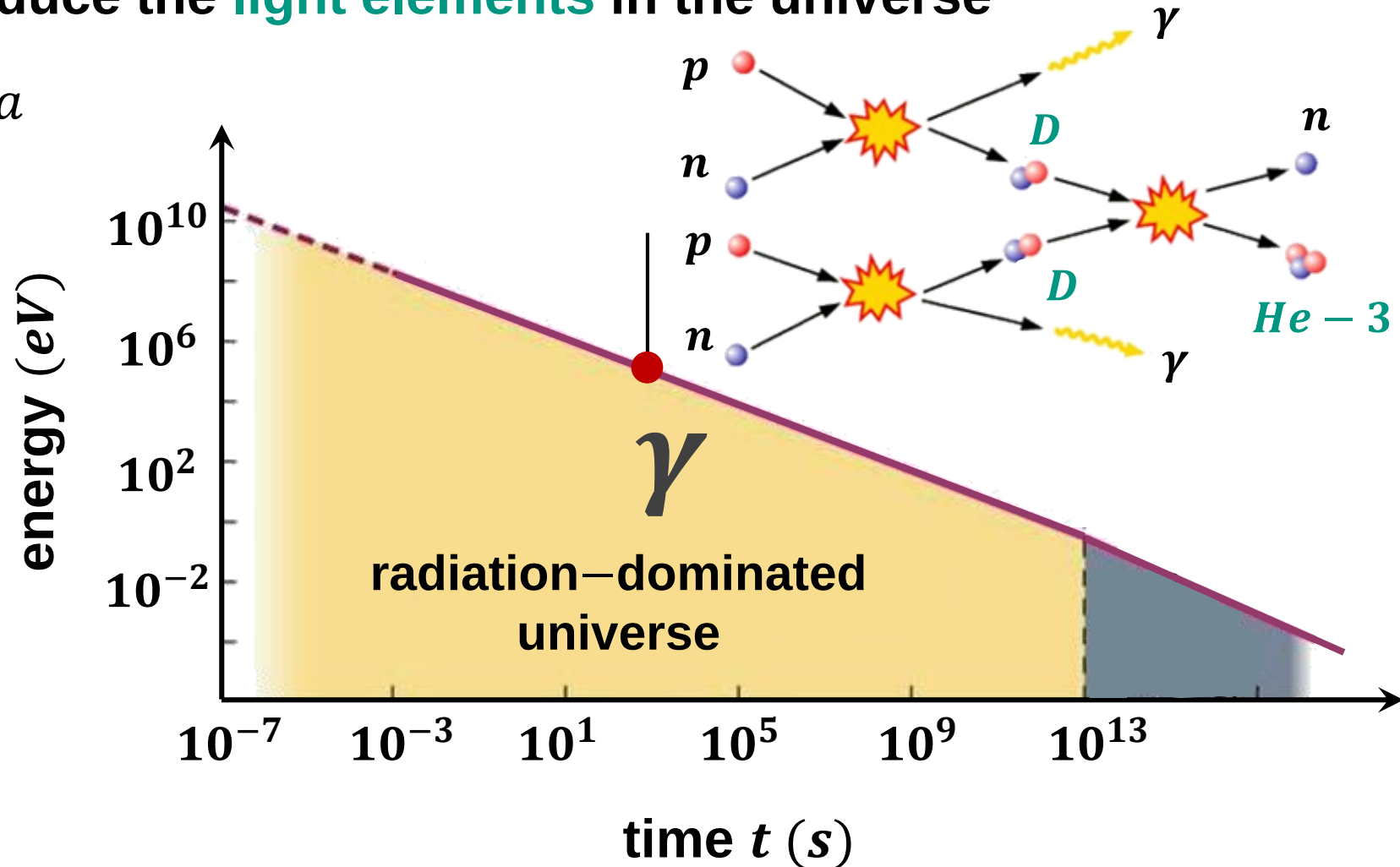
- during a short interval *aka* **‘the first 3 minutes’**

$$t = 1 \dots 3 \text{ min}$$

$$T \approx 10^8 K$$

$$E \approx \text{few keV}$$

- fusion of **light isotopes**:
 D , ${}^3\text{He}$, ${}^4\text{He}$, ${}^7\text{Li}$, ...



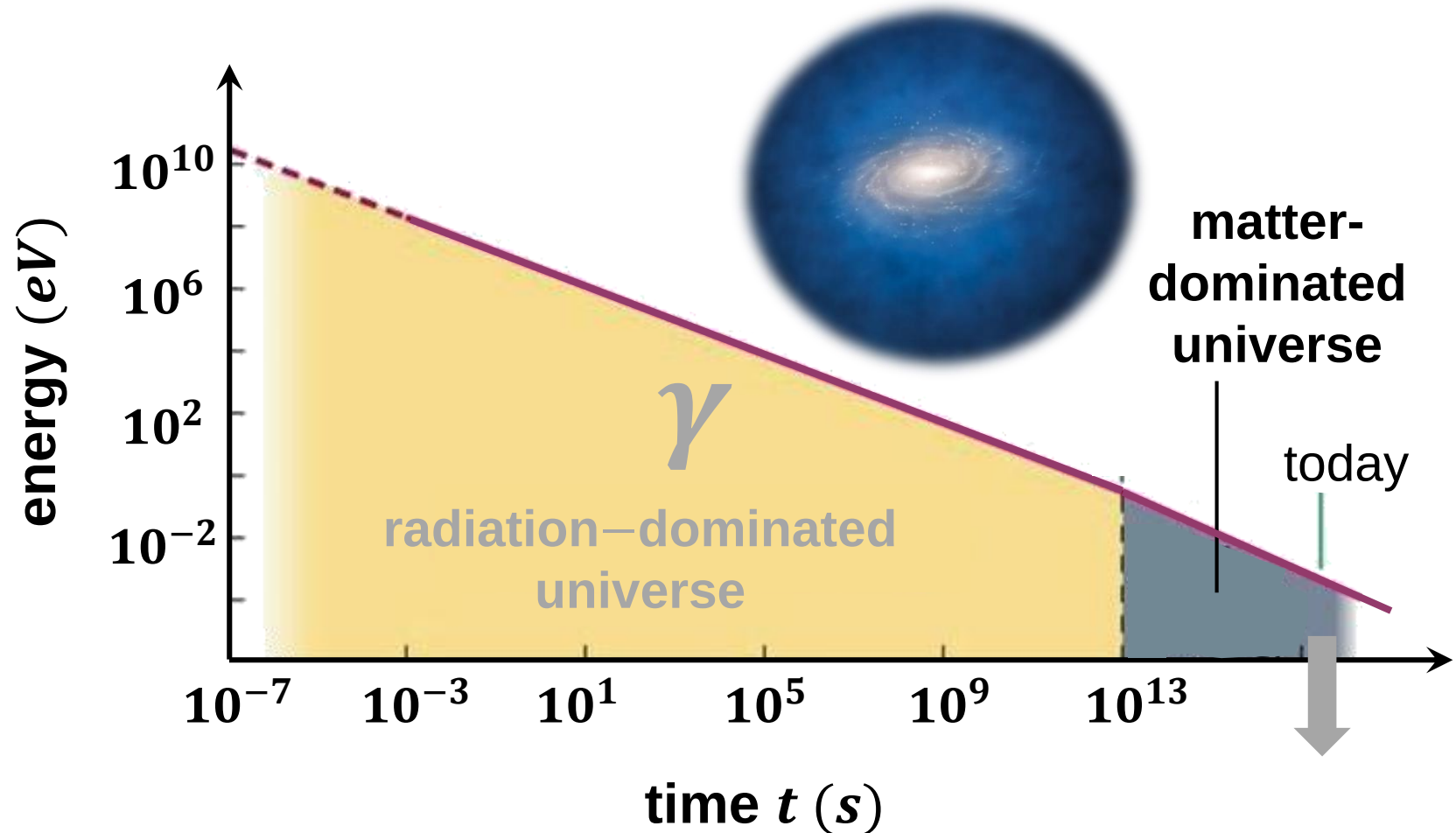
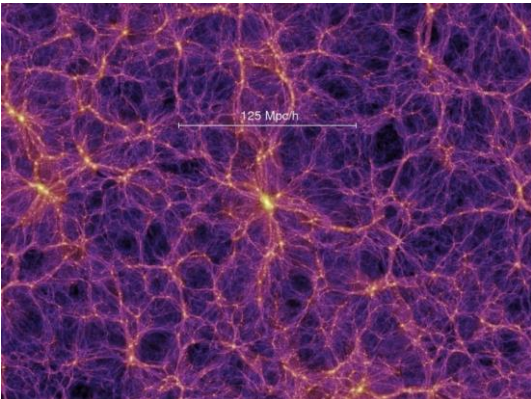
Properties of the matter–dominated universe

■ After $t \sim 50\,000\text{ yr}$: evolution is dominated by matter (dark matter, baryons)

- two key players today:

Dark Matter:
gravitational attraction

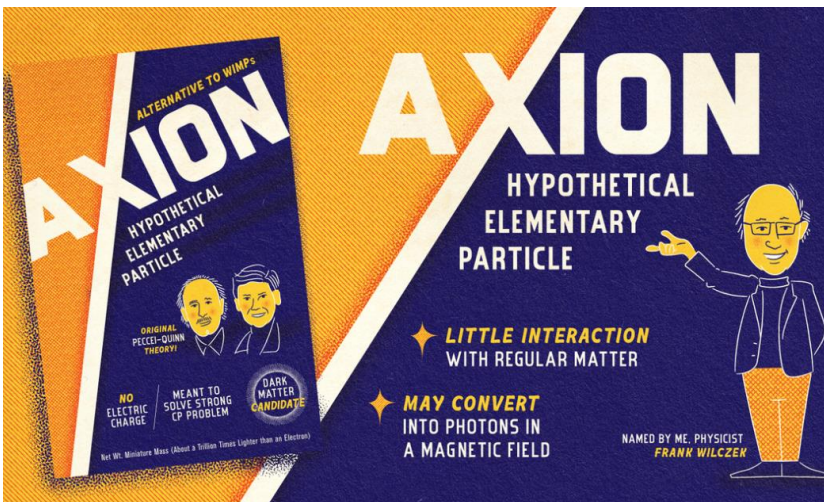
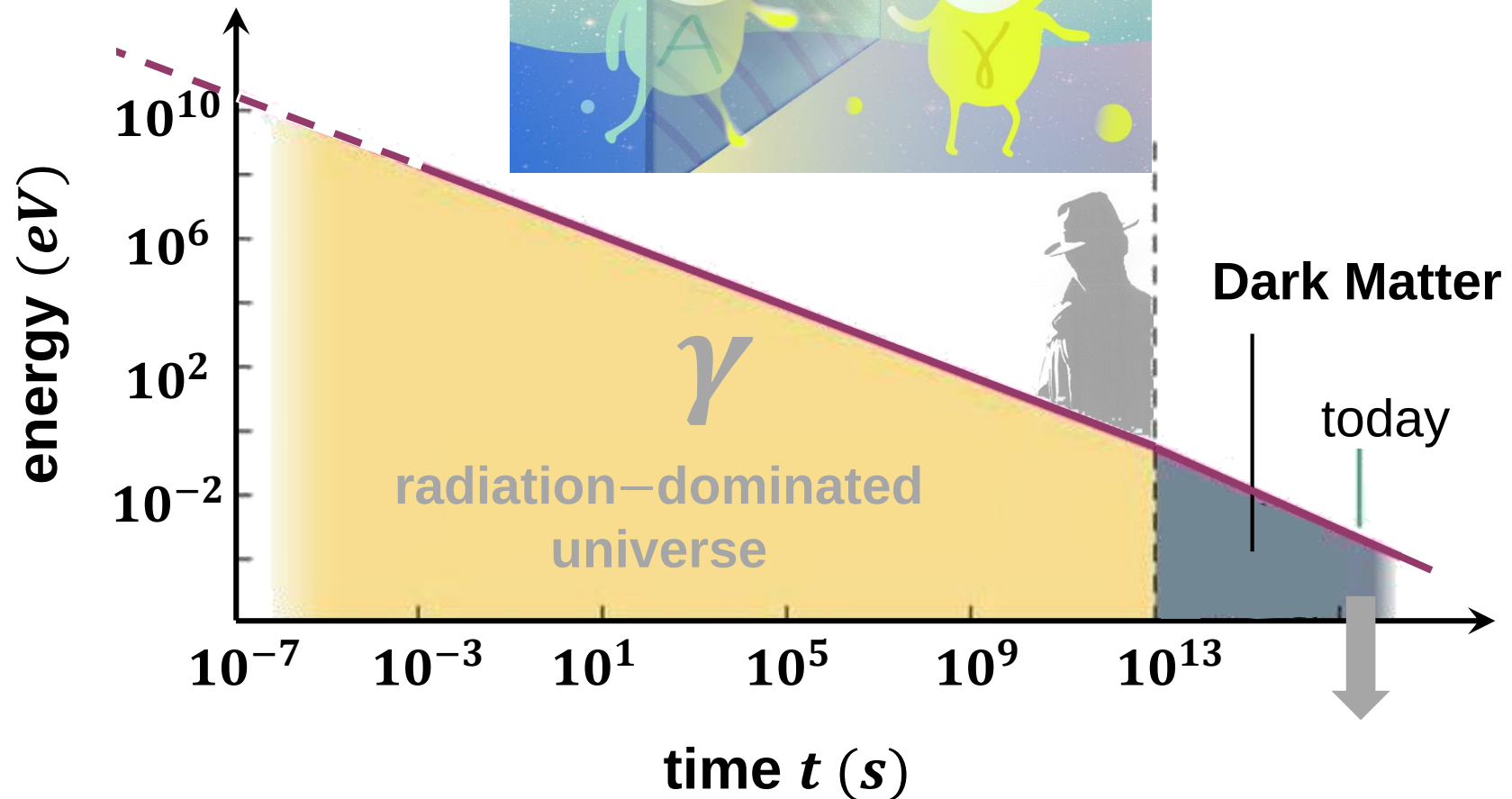
Dark Energy:
anti–gravity



Properties of the matter–dominated universe

■ Non–thermal processes: axions

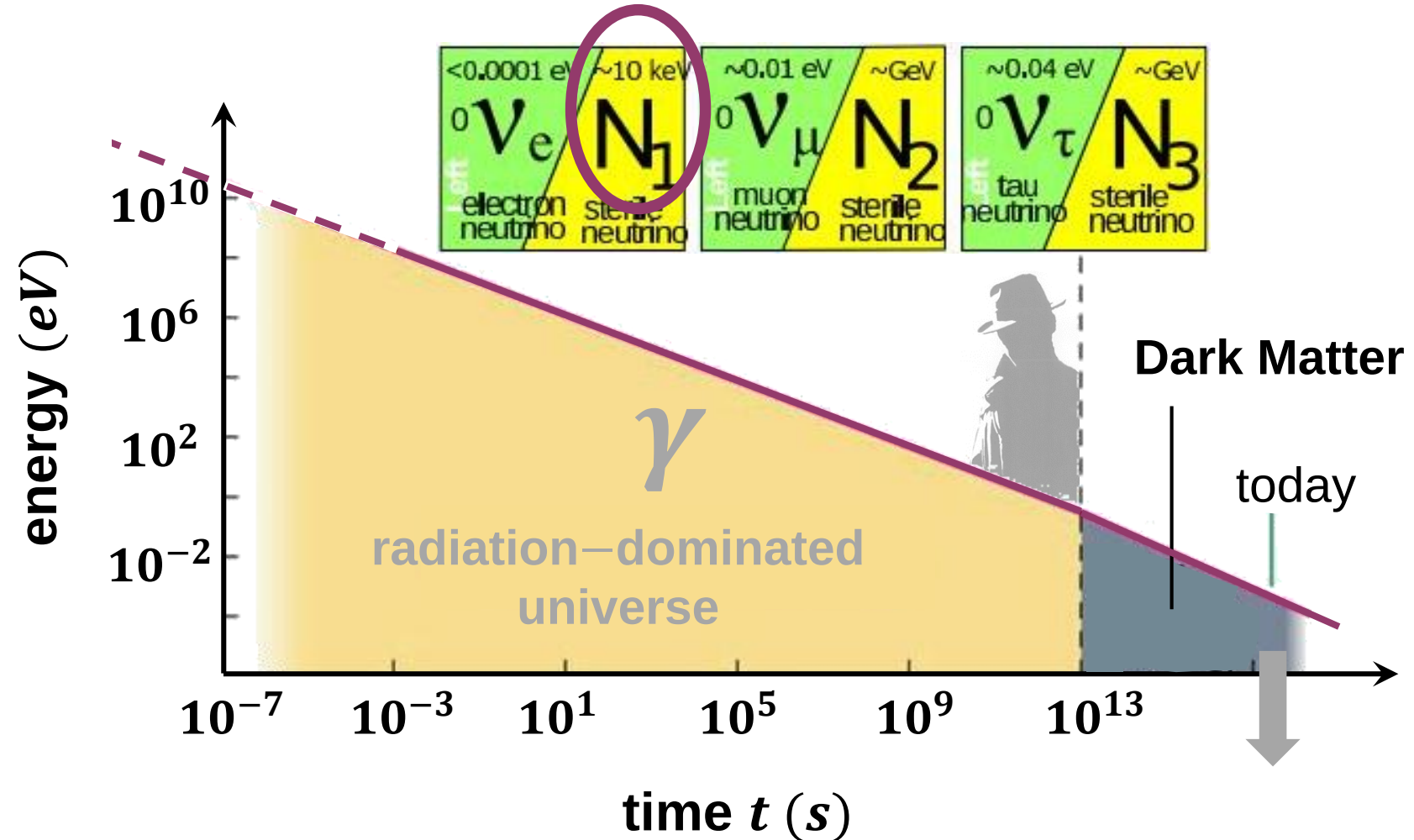
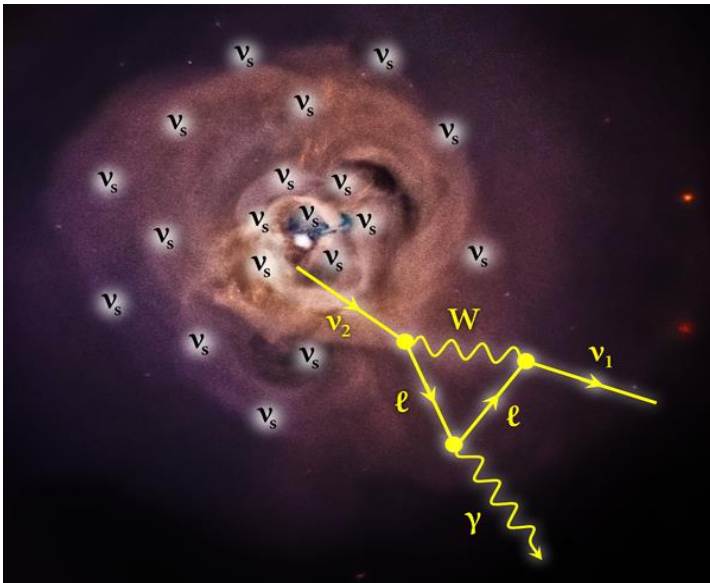
- particles **without** coupling to heat bath*: production via **breaking** of specific (Peccei–Quinn) symmetry



Properties of the matter–dominated universe

■ Non–thermal processes: **sterile neutrinos**

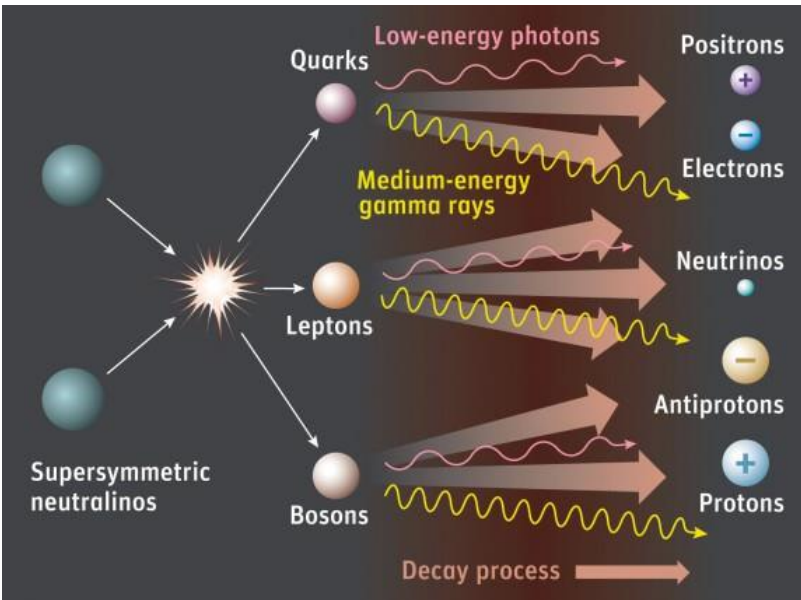
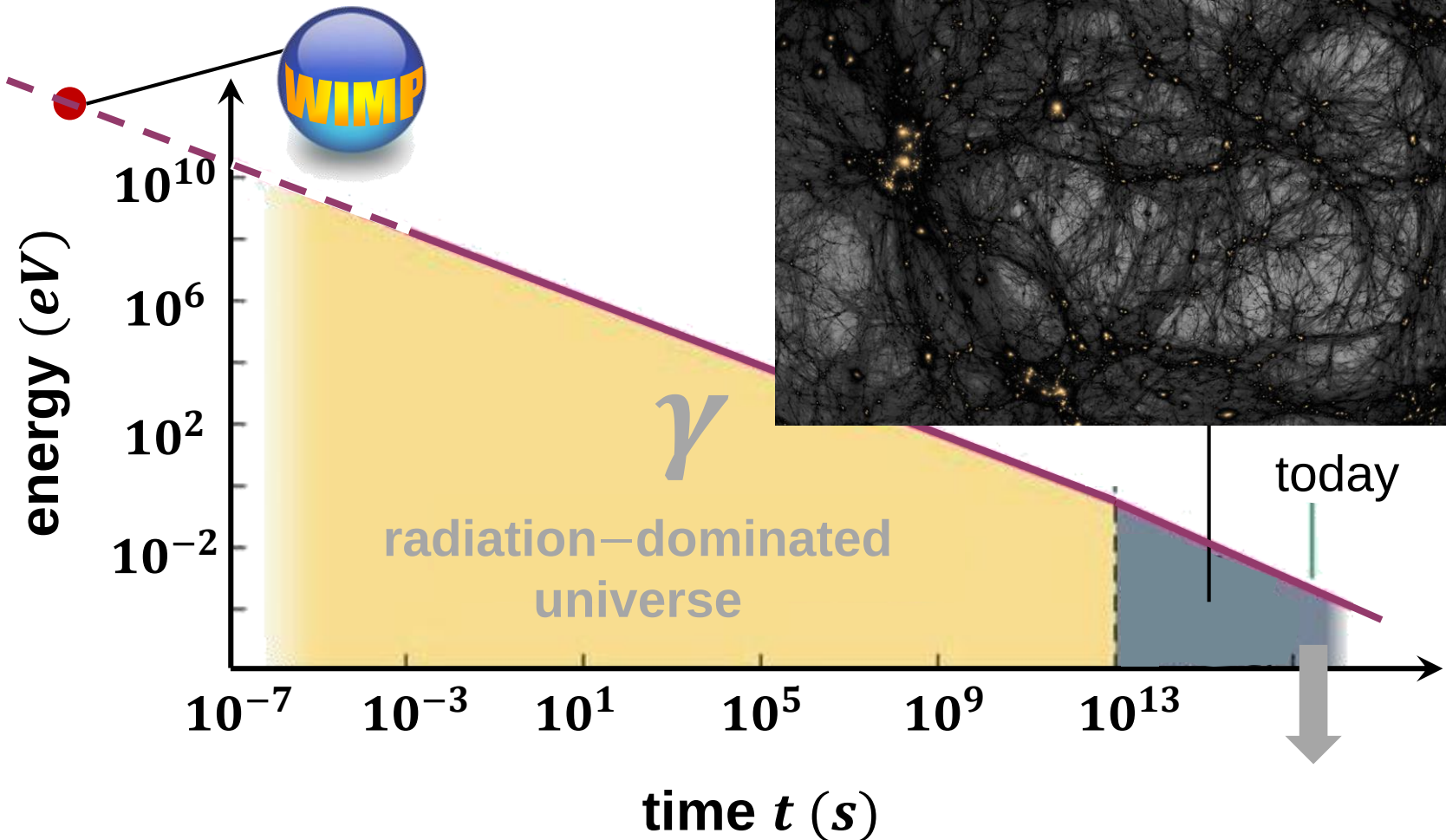
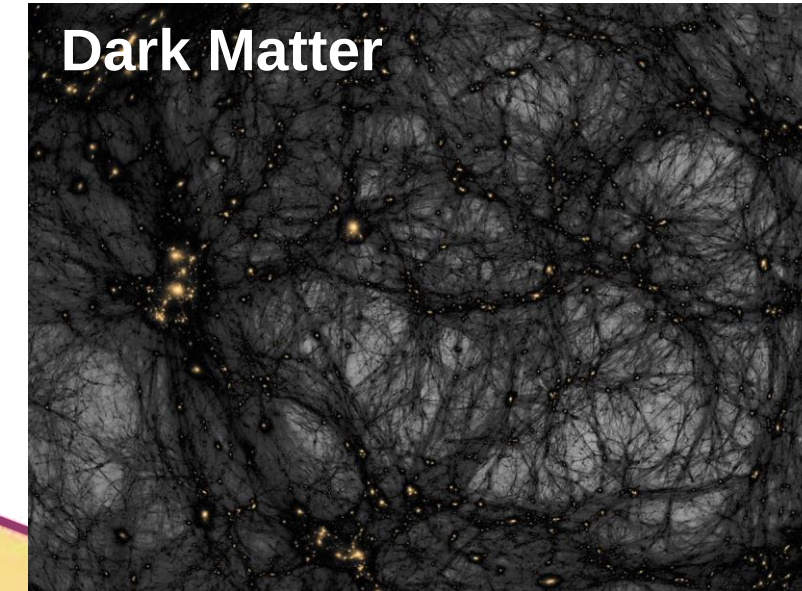
- particles **without** coupling to heat bath*: production via oscillations in case of **sterile (*RH*) neutrinos**



Properties of the matter–dominated universe

■ Thermal processes: *WIMPs*

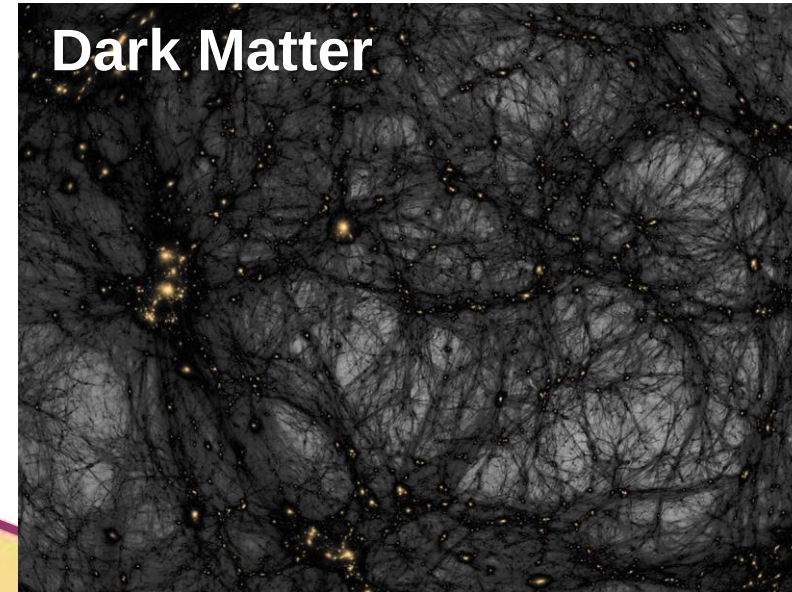
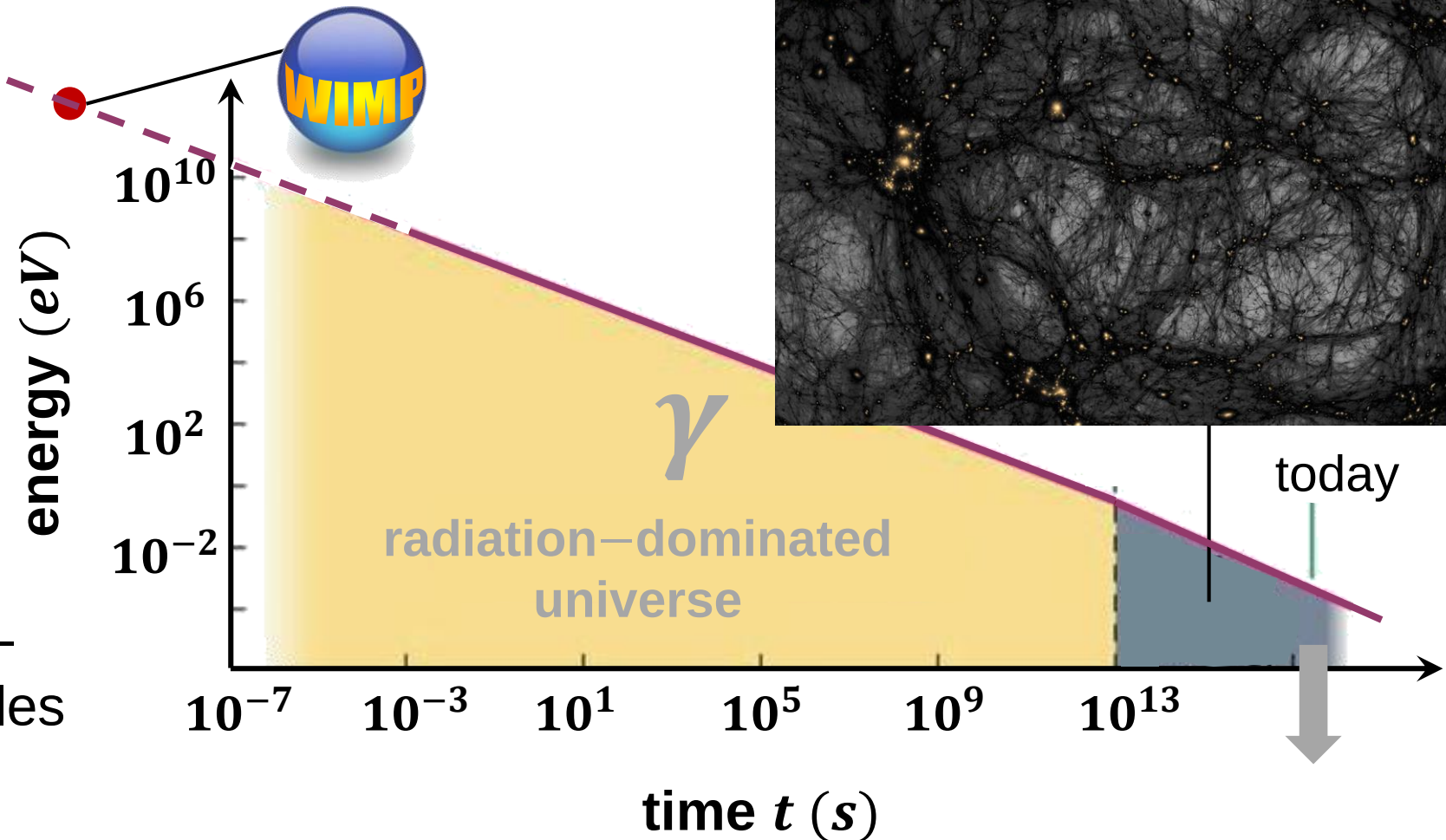
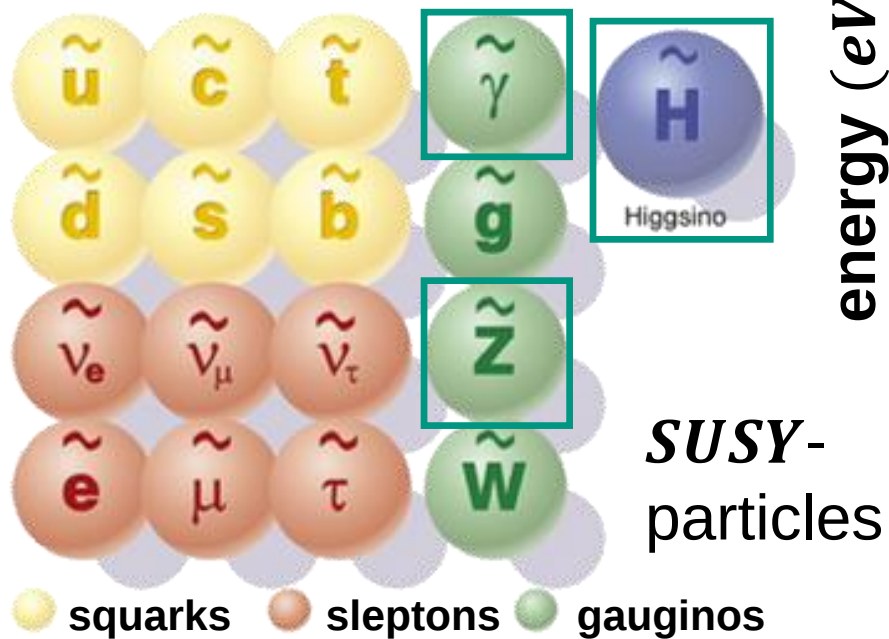
- coupling to heat bath:
particle interactions
generate dark matter



Thermal particle production: case of *WIMPs*

■ Particle production in the primordial heat bath

- **particle interactions** generate dark matter*

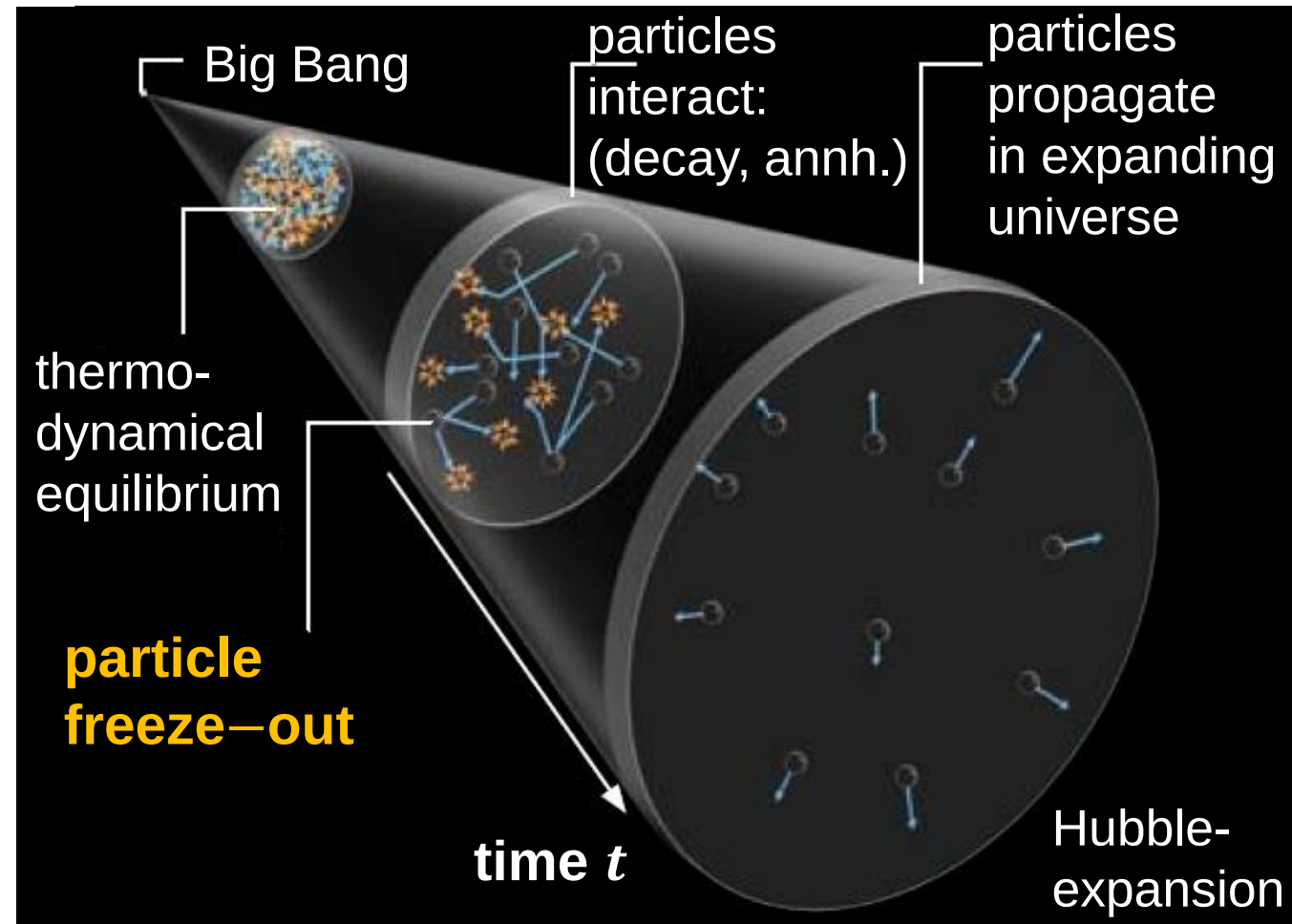


Thermal particle production: key processes

■ During expansion of universe: three key processes are of relevance

characteristic thermal phases:

- **1** : thermodynamical equilibrium: Boltzmann-distribution
- interactions with neighbours
- **freeze-out** & de-coupling
- free propagation

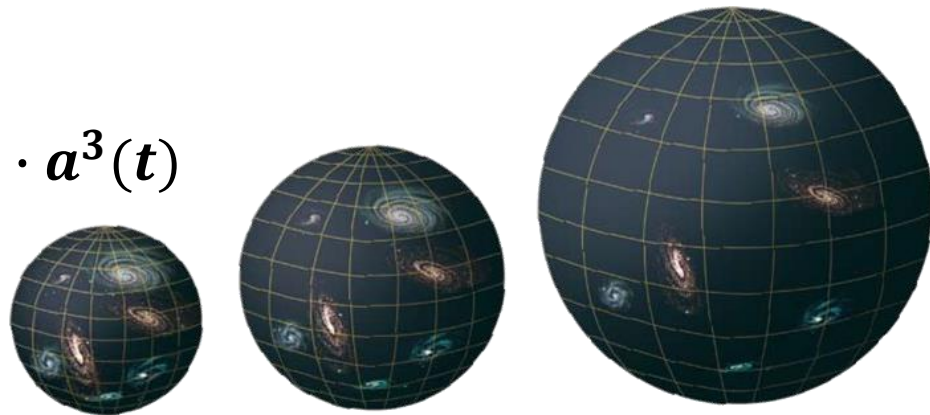


Thermal particle production: key processes #1

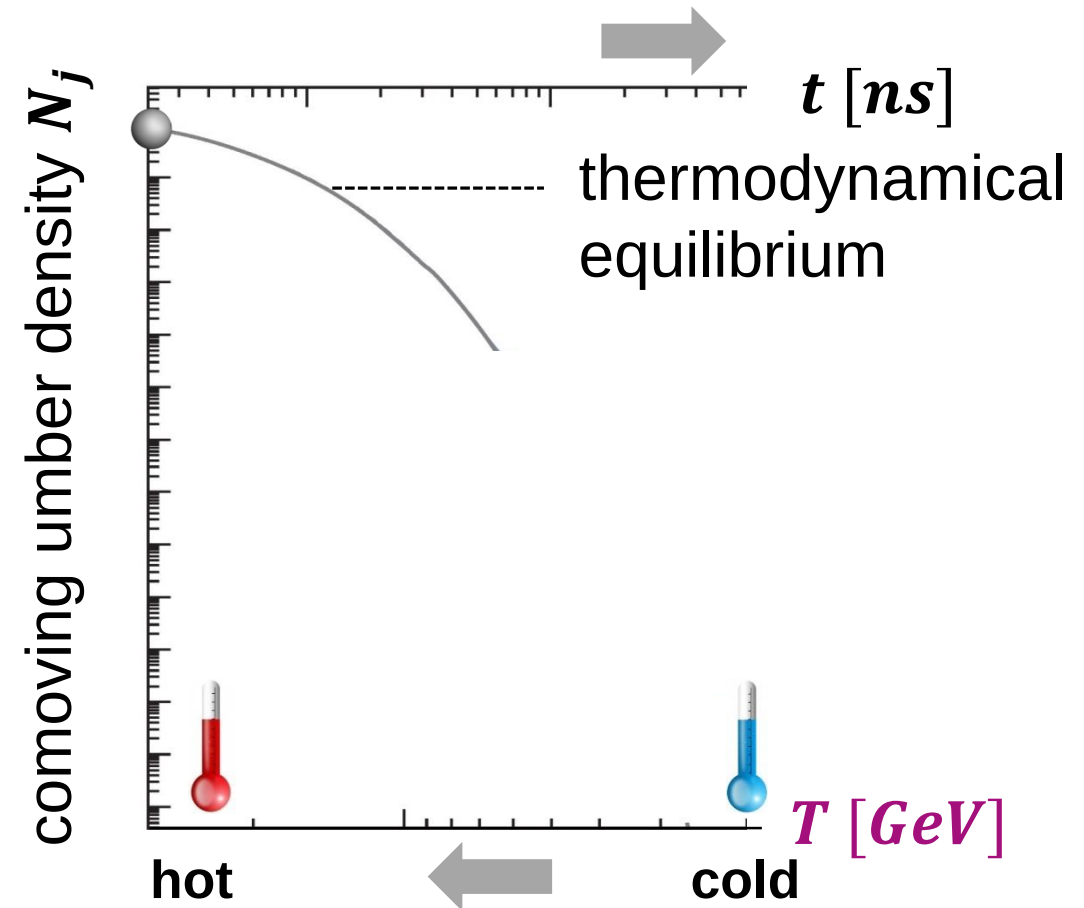
■ During expansion of universe: **phase 1 – thermal equilibrium**

- all particle number densities in **comoving coordinates $\vec{x}$**
⇒ density decrease due to the expansion of the universe thus taken into account

$$\rho_0 = \rho(t) \cdot a^3(t)$$



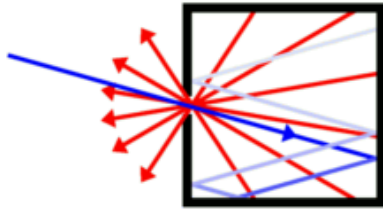
$$\vec{r}(t) = a(t) \cdot \vec{x}$$



Thermal particle production: key processes #1

■ During expansion of universe: **phase 1 – Boltzmann distribution**

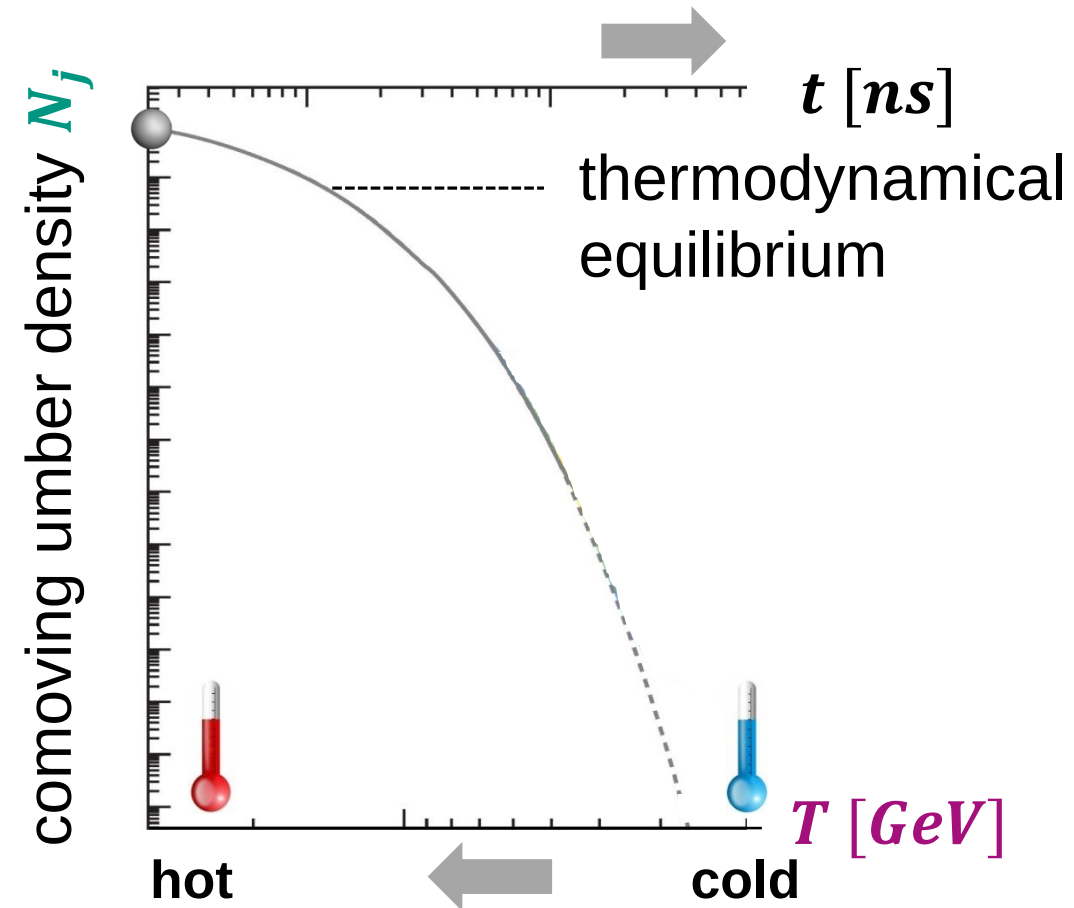
- particle species: **production** & **annihilation** (thermal heat bath)



- particle number density N_j defined by 2 free parameters:

particle energy (mass) $E_j = M_j \cdot c^2$

temperature heat bath $E_\gamma = k_B \cdot T$



Thermal particle production: key processes #1

■ During expansion of universe: phase 1 – Boltzmann distribution

- evolution of number density N_j

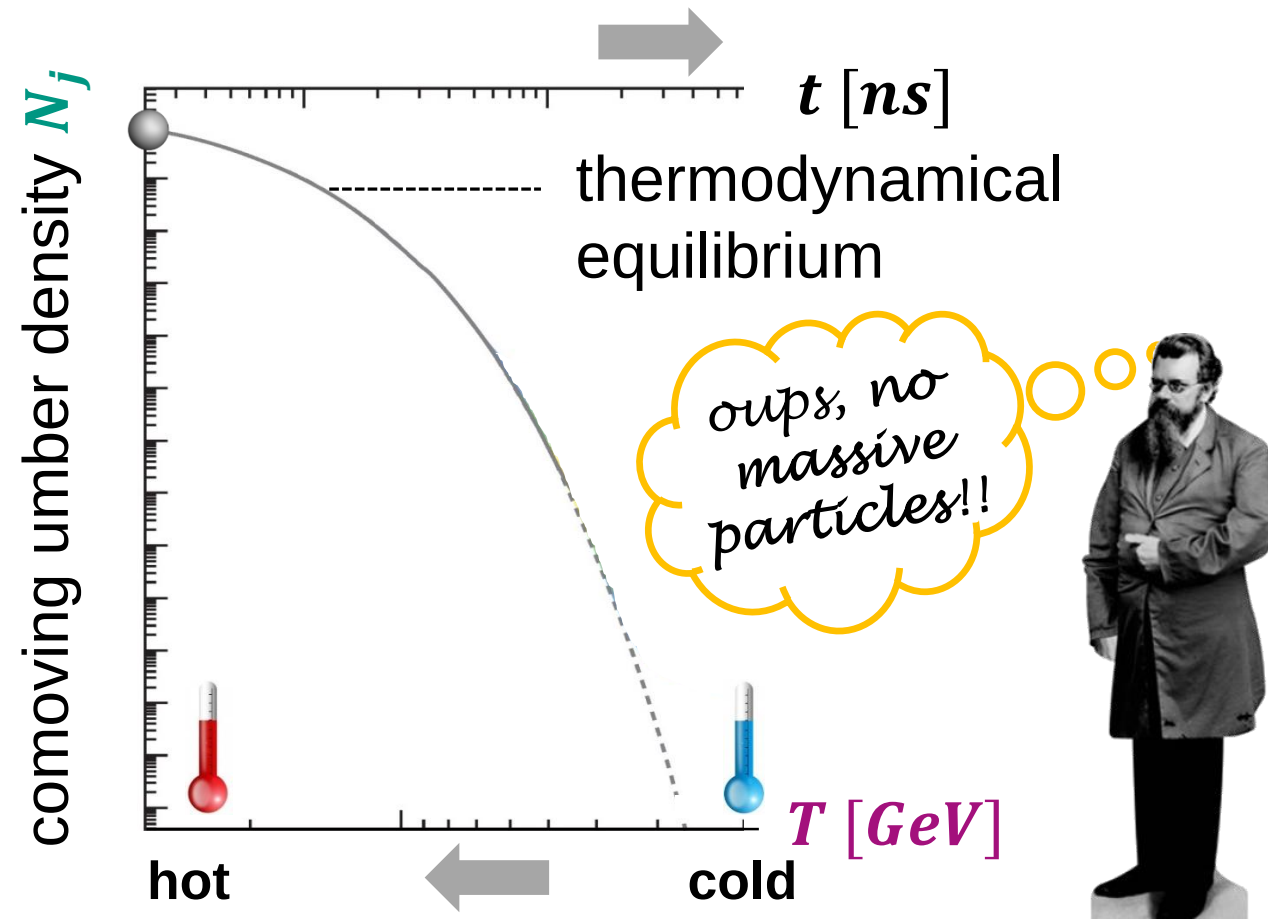
$$N_j = N_0 \cdot g_j \cdot e^{-E_j/k_B T}$$

N_0 : primary particle number density

N_j : actual particle number density
at temperature T (energy E_j)

g_j : intrinsic degree of degeneracy
(fermions, bosons)

⇒ for $T \rightarrow 0$ we have $N_j \rightarrow 0$



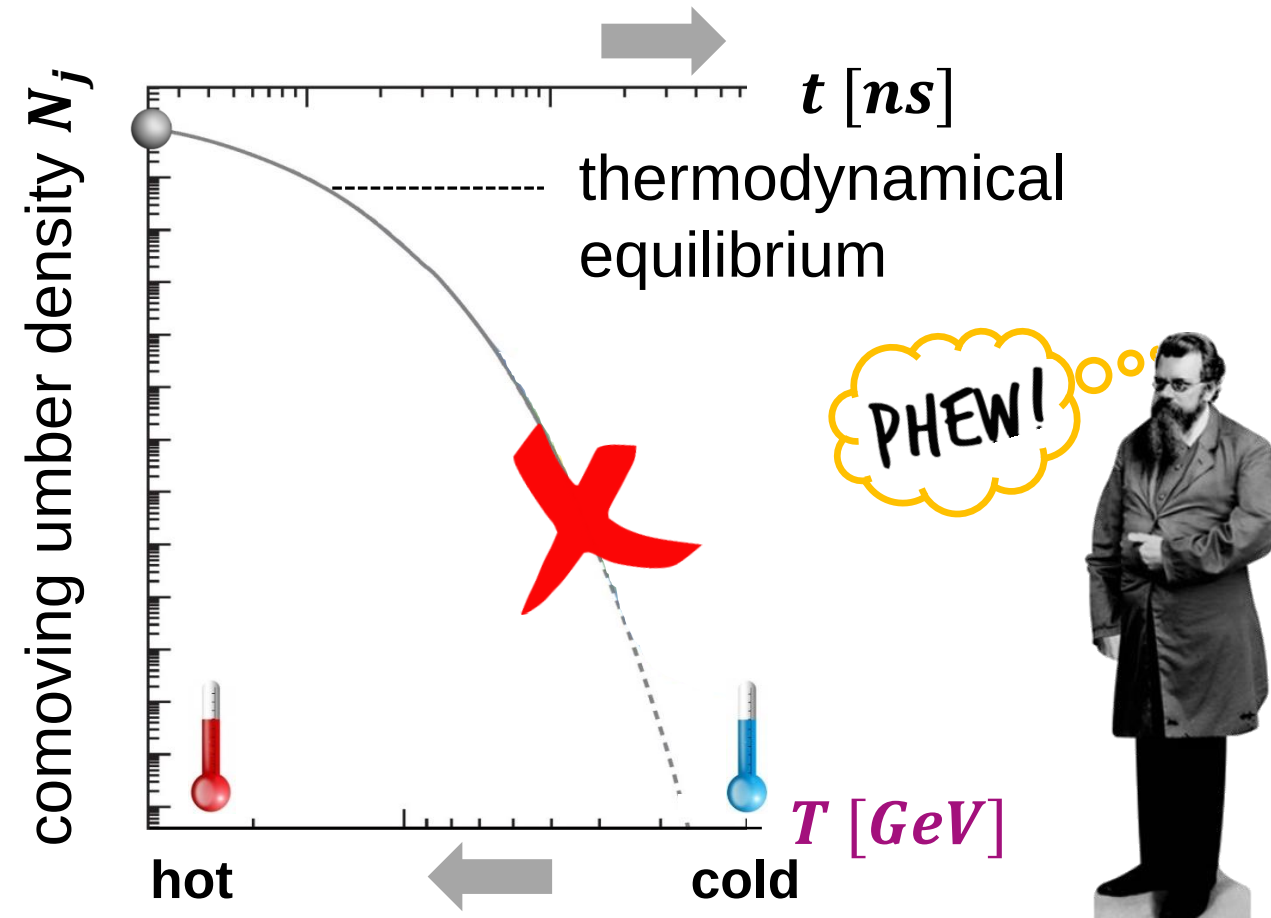
Thermal particle production: key processes #2

■ During expansion of universe: phase 2 – breaking of thermal equilibrium

- evolution of number density

$$N_j = N_0 \cdot g_j \cdot e^{-E_j/k_B T}$$

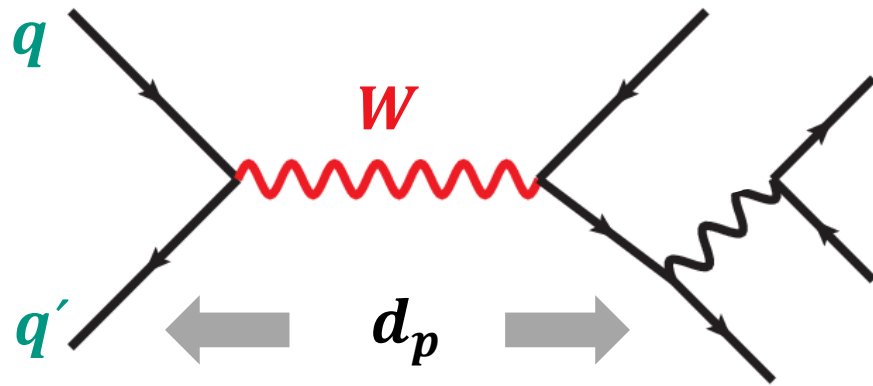
- finite number density $N_j \neq 0$
requires **breaking of thermal equilibrium**
(**decoupling from heat bath**)



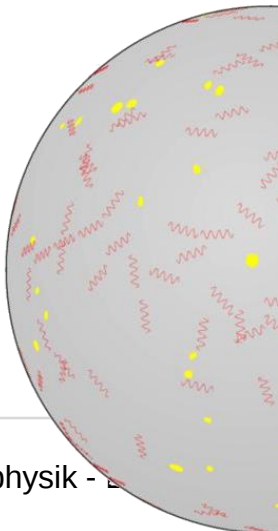
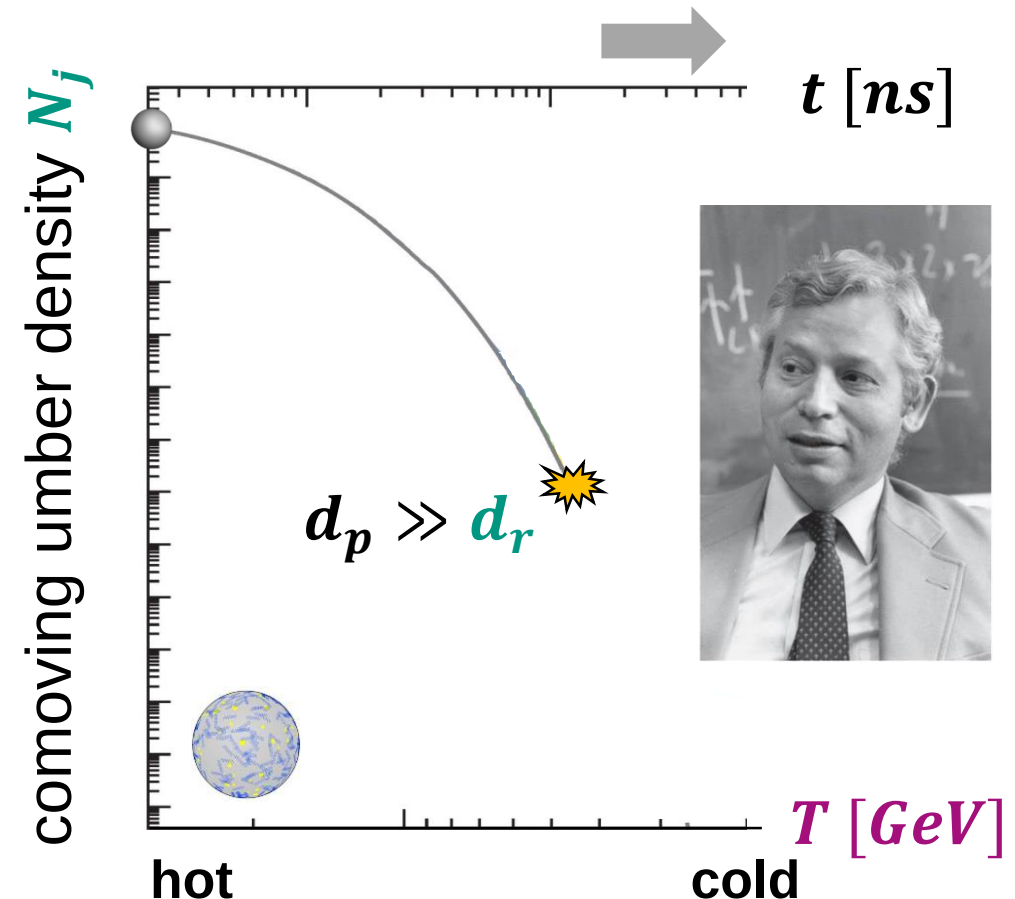
Thermal particle production: key processes #2

■ During expansion of universe: phase 2 – breaking of thermal equilibrium

- expansion of the universe takes place in 'real' coordinates $\vec{r}$



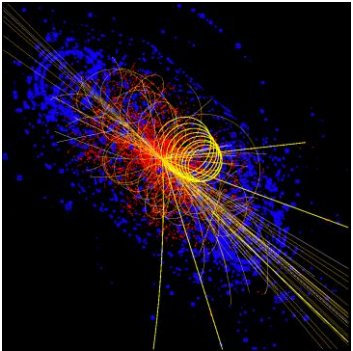
- exchange bosons W, Z, g with finite range d_r (or lifetime τ)
- yet: increase of particle distances d_p



Thermal particle production: key processes #3

■ During expansion of universe: phase 3 – freeze-out of particles

- decoupling from heat bath at a specific time t / temperature T :

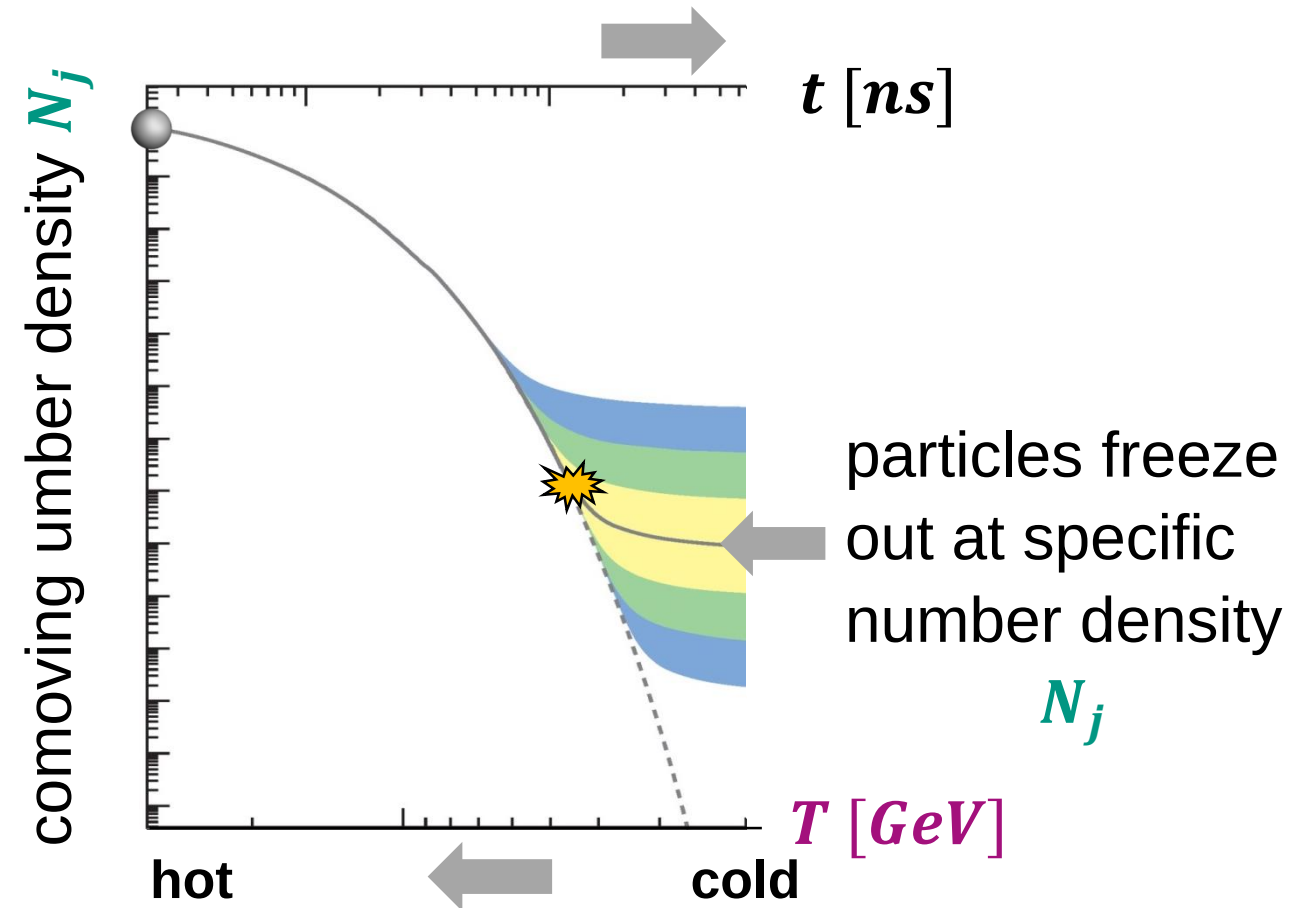


$$\Gamma(t) = H(t)$$



Hubble expansion rate $H(t)$

particle interaction rate $\Gamma(t)$



Thermal particle production: key processes #3

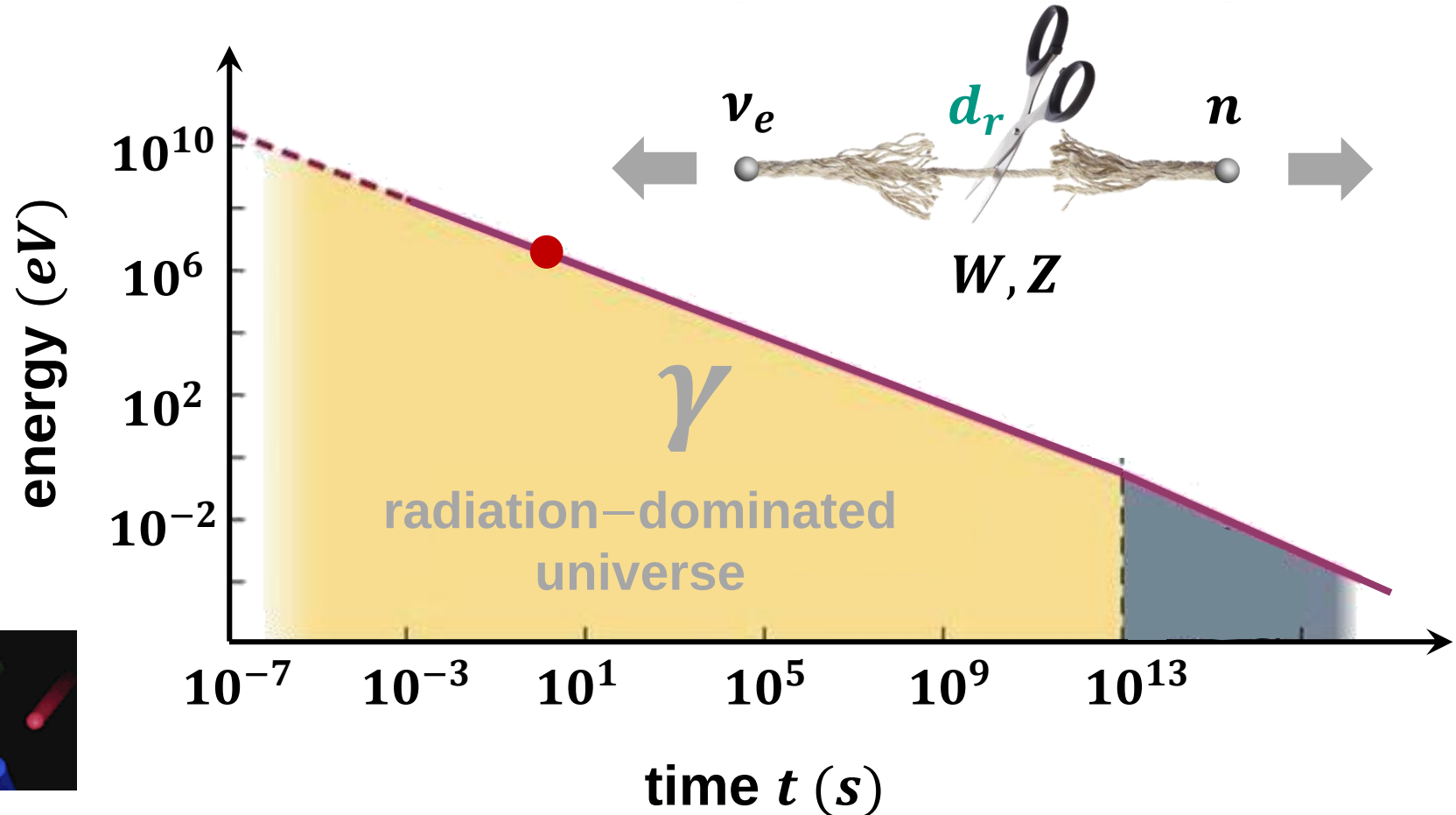
■ During expansion of universe: **freeze-out** processes of ν 's are important

- example:

neutrinos from Big Bang
decouple once we have

$$\Gamma_\nu(t) = H(t)$$

weak interaction
cross section



BIG FREEZE OUT

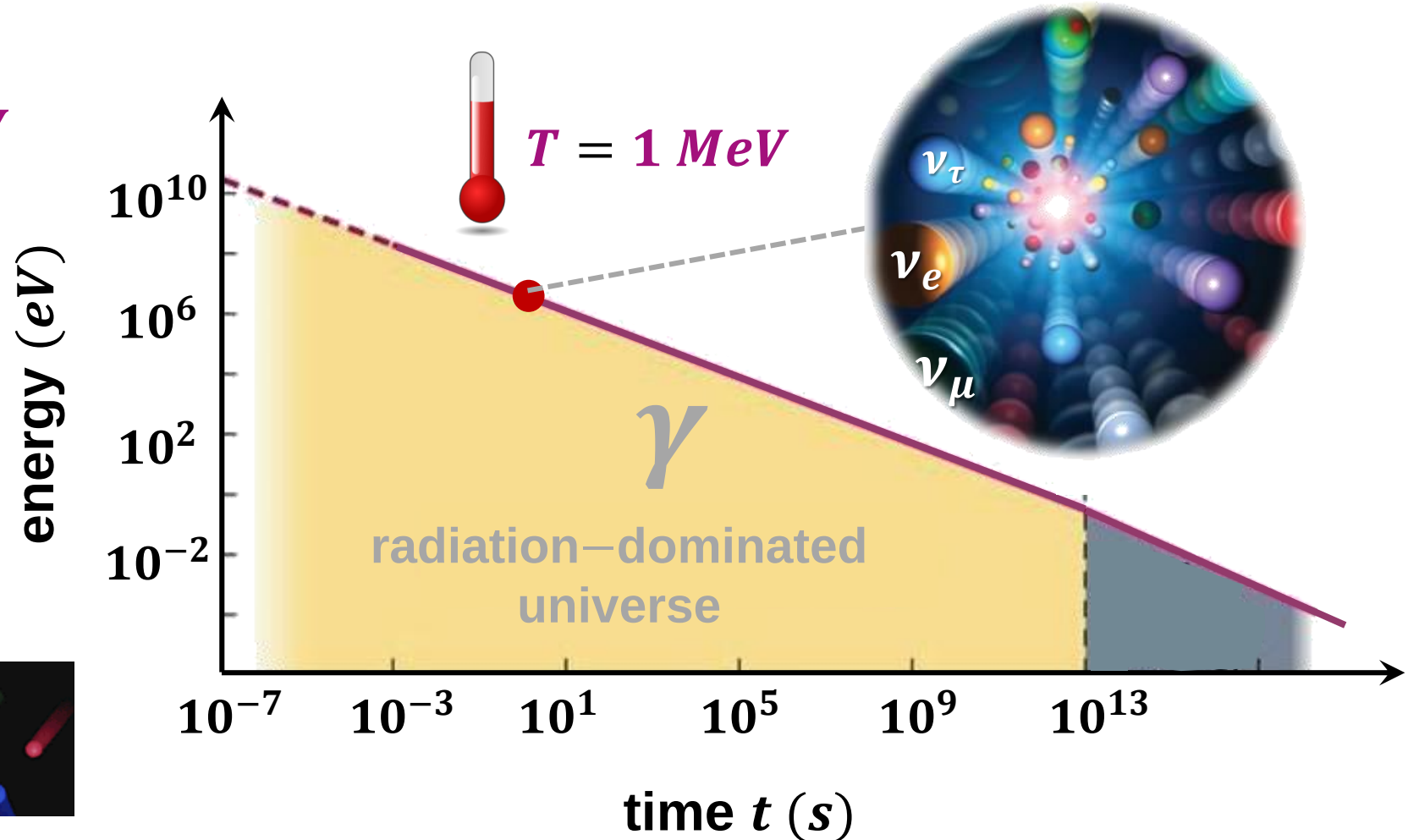
Thermal particle production: key processes #3

- During expansion of universe: **freeze-out** processes of ν 's are important

$$T(t = 1\text{ s}) = 1\text{ MeV}$$

$$\Gamma_\nu(t) = H(t)$$

weak interaction
cross section

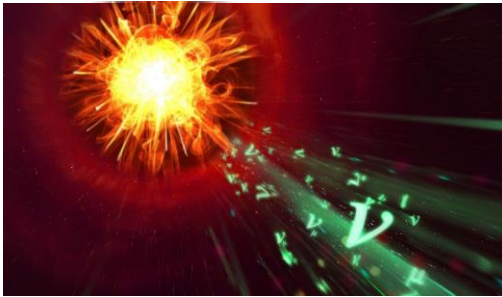


BIG FREEZE OUT

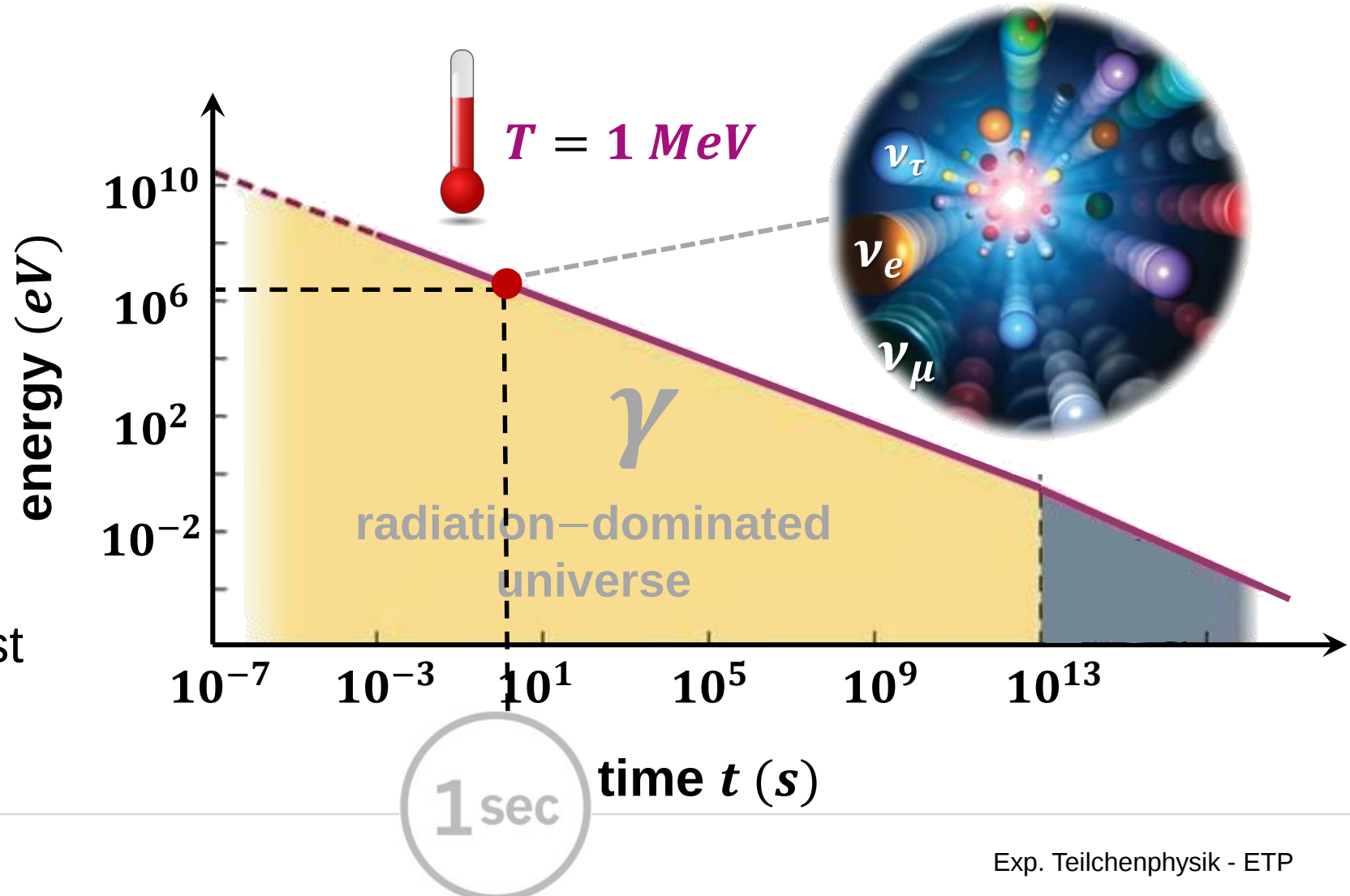
Thermal particle production: key processes #3

■ During expansion of universe: **freeze-out** processes of ν 's are important

- since $t = 1\text{ s}$ neutrinos from the Big Bang are 'free-streaming'



- detection of 'relic ν 's' would allow to take a first **picture of the universe** already at $t = 1\text{ s}$



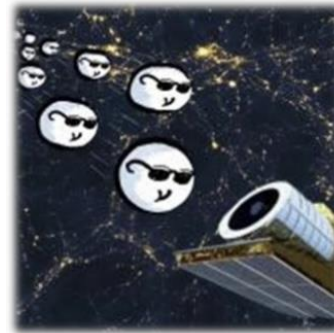
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Friday, 24 November 2023, 15:45-17:15

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Speaker

Julien Lesgourgues

RWTH Aachen



Department of Physics

Calendar of Events