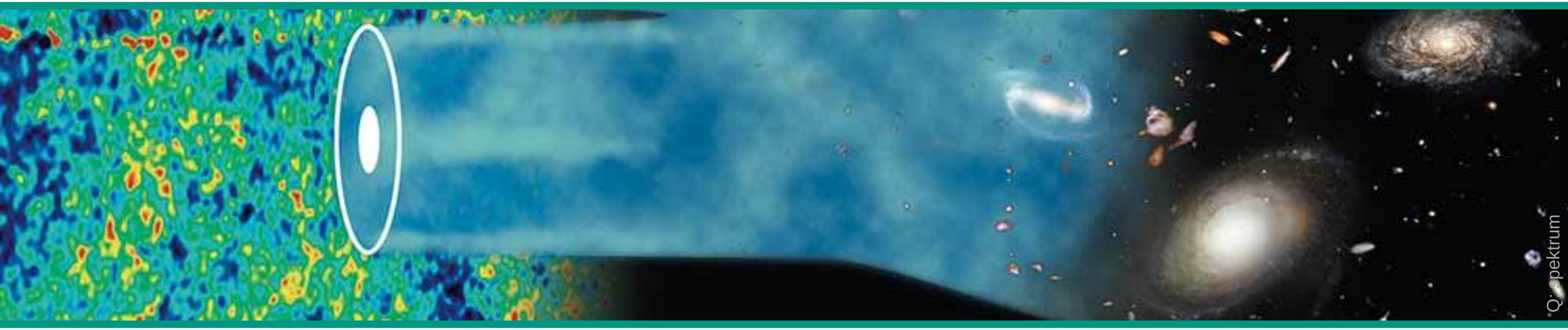
Karlsruhe Institute of Technology

Introduction to Cosmology

Winter term 23/24 Lecture 6 Nov. 28, 2023



Recap of Lecture 5

- **Thermal universe: the radiation–dominated epoch with phase transitions**

  - temperature evolution: $\mathrm{T}(t) \sim \sqrt{t}$ with ´fixpoint´ $T = 1\ MeV$ after $t = 1\ s$

  - particles (energy E_J) in thermodynamical equilibrium with heat bath (**photons**)

  - resulting **Boltzmann distribution** with $N_J(t) \sim e^{-E_J/k_B T(t)}$

  - condition for **freeze–out**: $\Gamma(t) = H(t)$ ⇔ interaction rate = Hubble expansion

  - **photons** from **matter – anti–matter** annihilation, tiny **baryon asymmetry** η

  - QCD – **phase transition**: quark–gluon plasma → nucleons (p, n)

RECAP: freeze–out of neutrinos

- during expansion of universe: freeze–out processes of ν´s are important

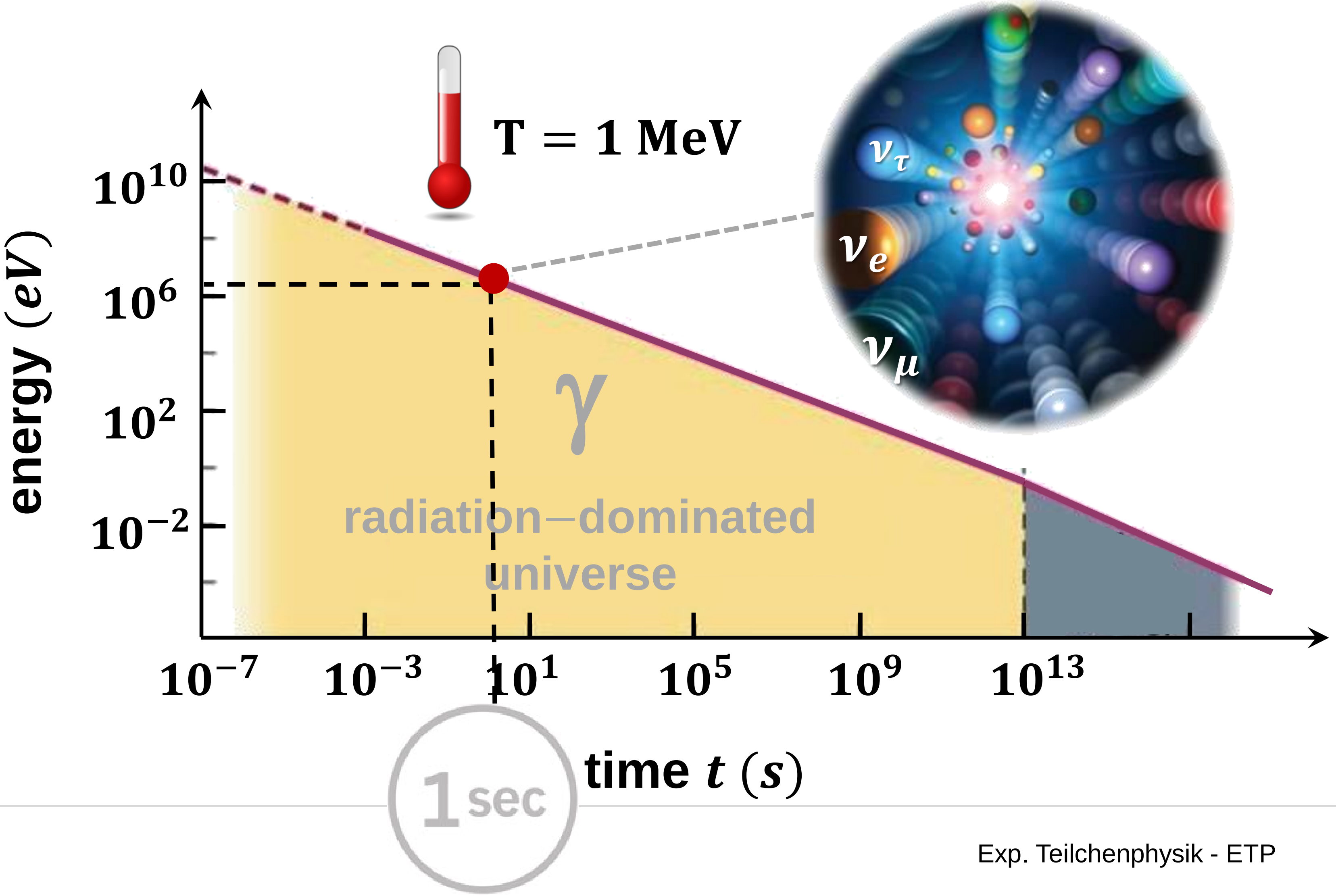
- since $t = 1\ s$ neutrinos from the Big Bang are ´**free–streaming**´

$$T(t = 1\ s) = 1\ MeV$$

$t = 1\ s$

$$\Gamma_\nu(t) = H(t)$$

weak interaction cross section



neutrino freeze−out & # of relativistic particles

- **An important time stamp: ν´s decouple at $t = 1\ s$ & $T = 1\ MeV$**

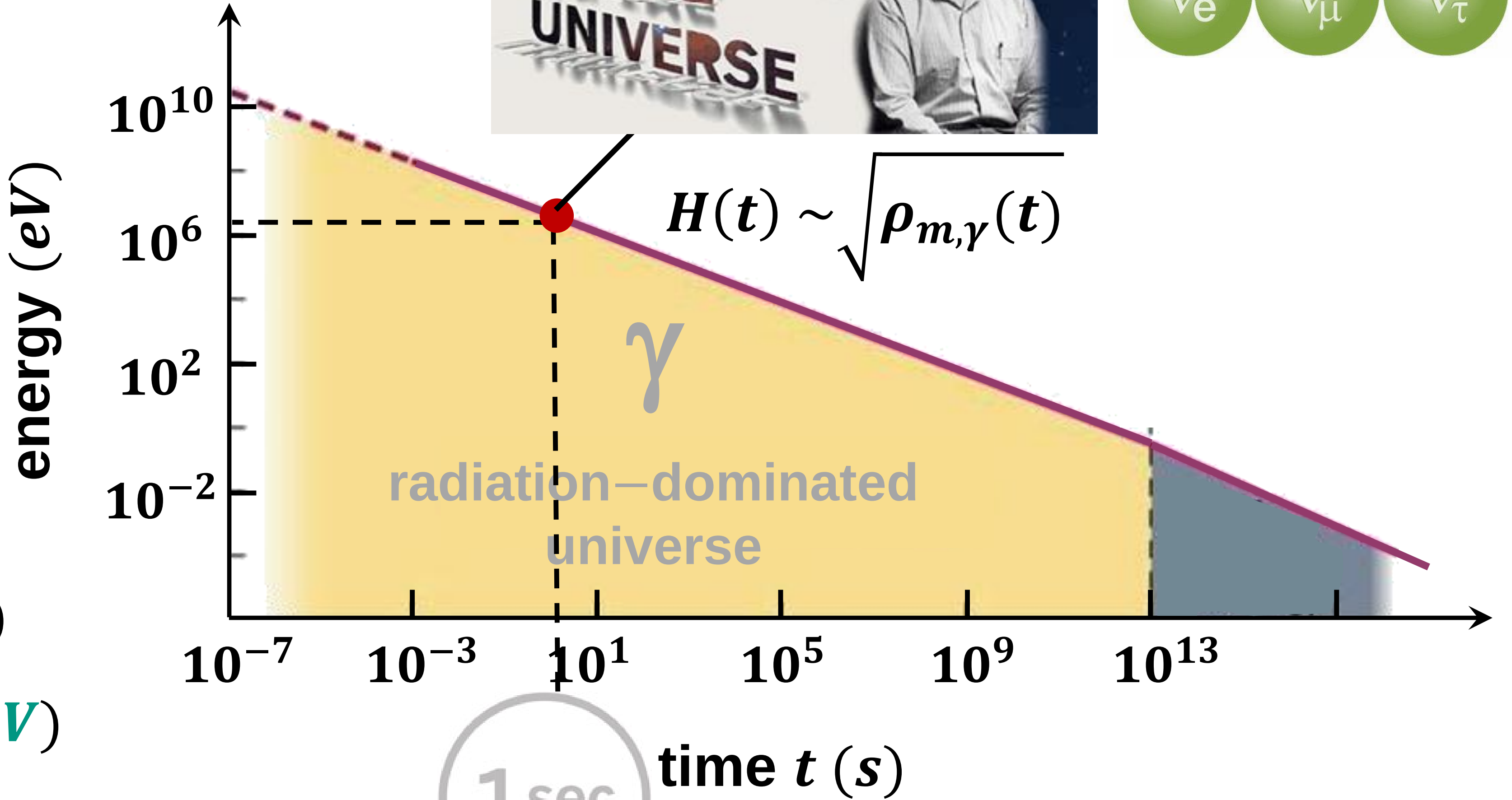
- how many **relativistic ´degrees of freedom´** are present at this time?*

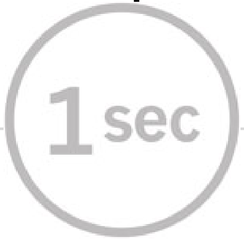
- preview: **relativistic particles** at $E = 1\ MeV$

γ: ☑ $(m_\gamma = 0)$

$\nu_e\, \nu_\mu\, \nu_\tau$: ☑ $(m_\nu < 0.8\ eV)$

$e^- e^+$: ☑ $(m_e = 0.511\ MeV)$





neutrino freeze–out & # of relativistic particles

- **An important time stamp: ν´s decouple at $t = 1\ s$ & at $T = 1\ MeV$**

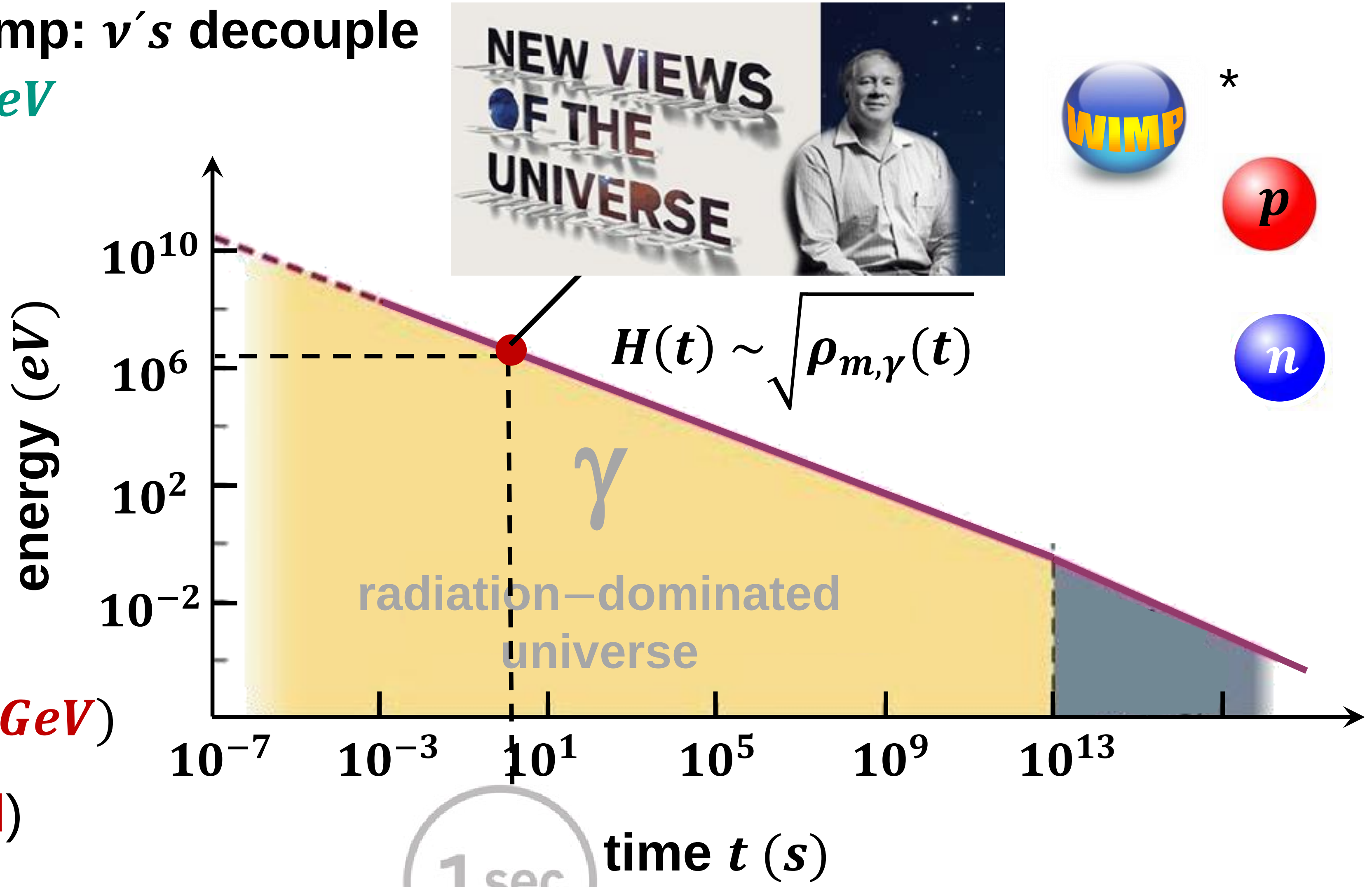
- how many **relativistic ´degrees of freedom´** are present at this time?*

- preview: **relativistic particles** at $E = 1\ MeV$

p, n: ☒ ($m_{p,n} \approx 1\ GeV$)

χ (neutralino)*: ☒ ($m_\chi \geq GeV$)

$\mu^-\mu^+, \tau^-\tau^+$: ☒ (**decayed**)



*see $ATP - I$

freeze–out of massless photons

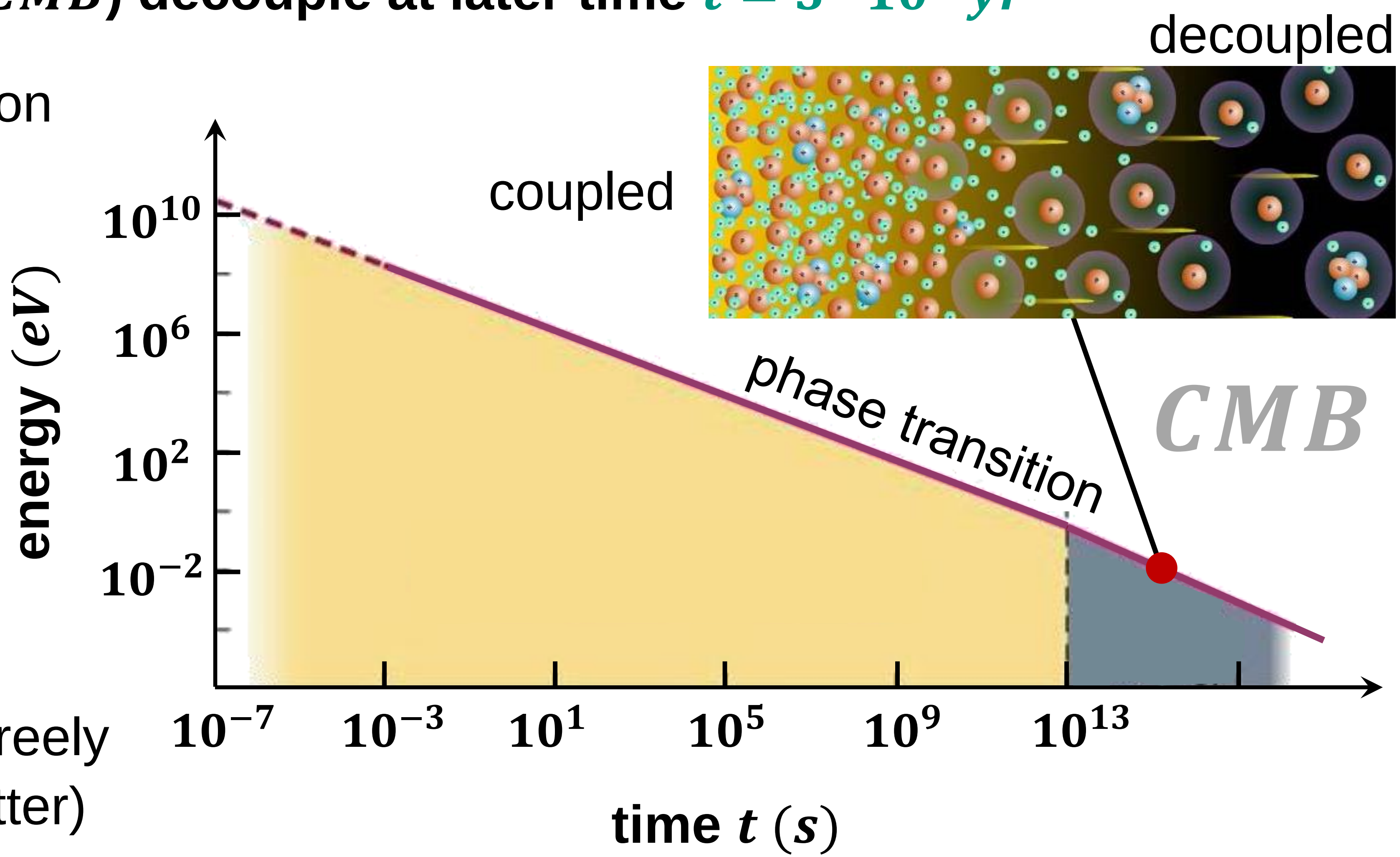
- Matter and radiation (CMB) decouple at later time $t = 3 \cdot 10^5\ yr$

- $RECAP$: freeze–out condition

$$\Gamma_\gamma(t) = H(t)$$

photon electromagnetic interaction with ionized matter (**plasma state**)

- photons (CMB) propagate freely (without interaction with matter)



freeze–out of massless photons

- **Matter and radiation (CMB) decouple**

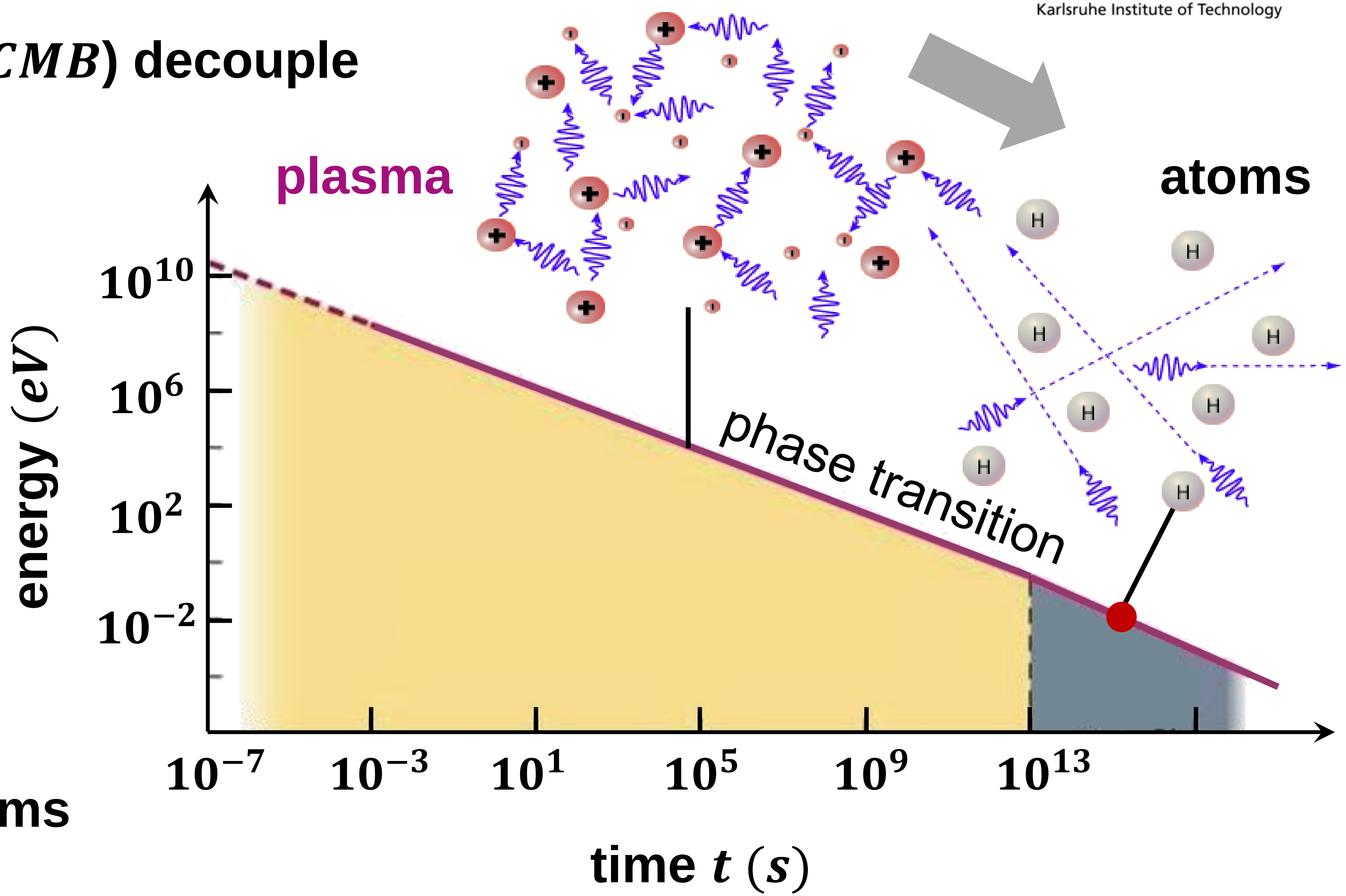
- phase transition at

$$\Gamma_\gamma(t) = H(t)$$

photon electromagnetic interaction with ionized matter (**plasma state**)

$t < 3 \cdot 10^5 yr$: **plasma**

$t > 3 \cdot 10^5 yr$: **neutral atoms**



freeze–out of massless photons

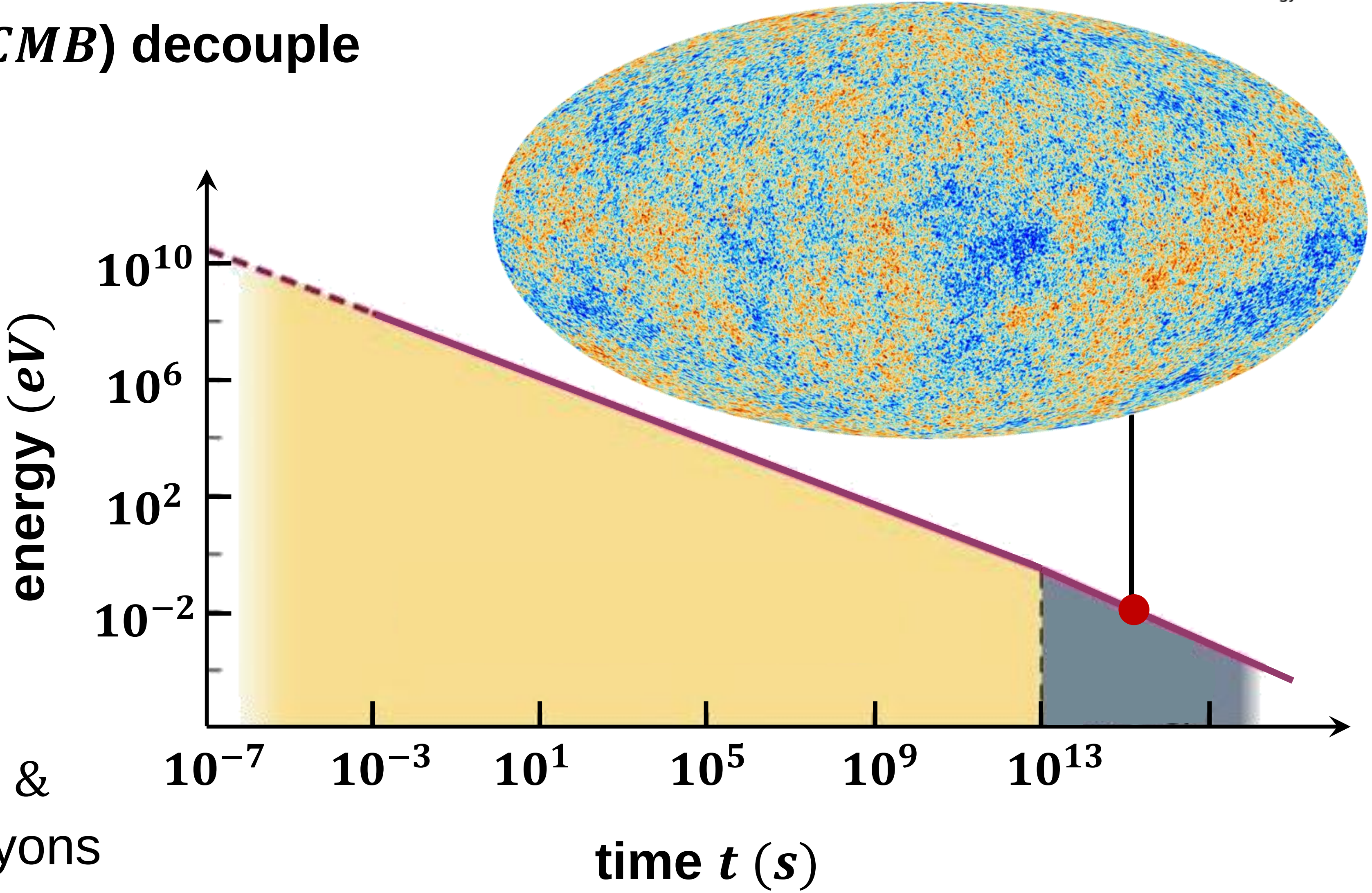
- **Matter and radiation (CMB) decouple**

- phase transition at

$$\boldsymbol{\Gamma_\gamma(t) = H(t)}$$

- universe becomes **transparaent for radiation**

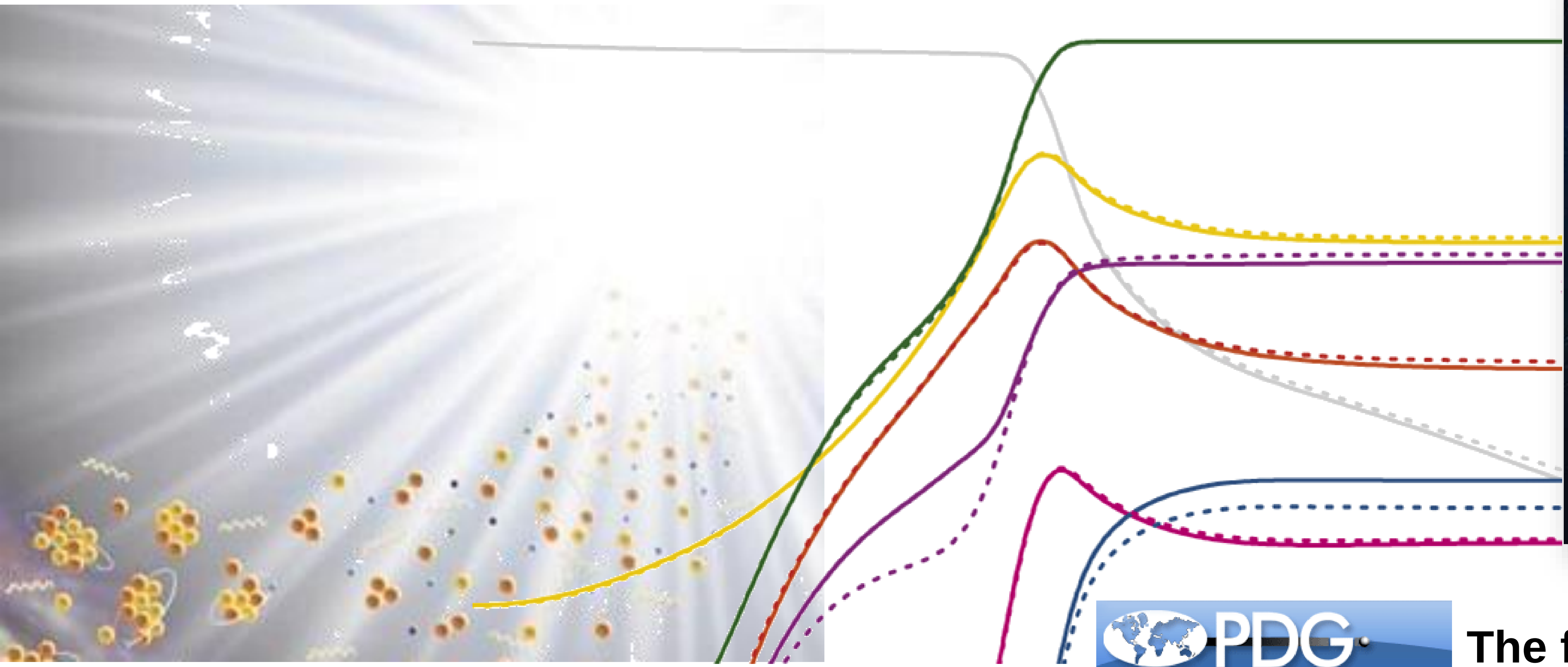
- matter no longer coupled to radiation fluid, thus **matter oscillations*** stop & **gravity** alone acts on baryons

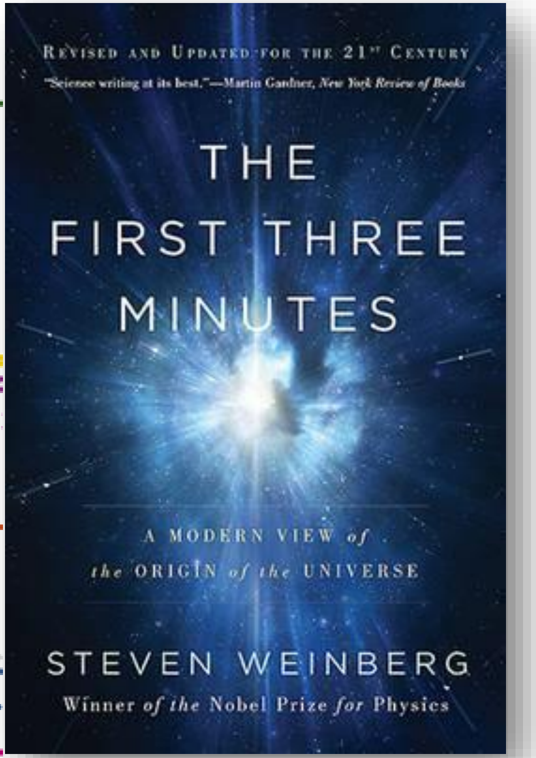


* ***B***aryon ***A***coustic ***O***scillations (***BAO***), see ch. 4.2

3.1 Primordial nucleosynthesis (*BBN**)

- **The origin of the light elements: a cornerstone for the hot Big Bang**





Steven Weinberg
The first three minutes
Basic books, **2009**

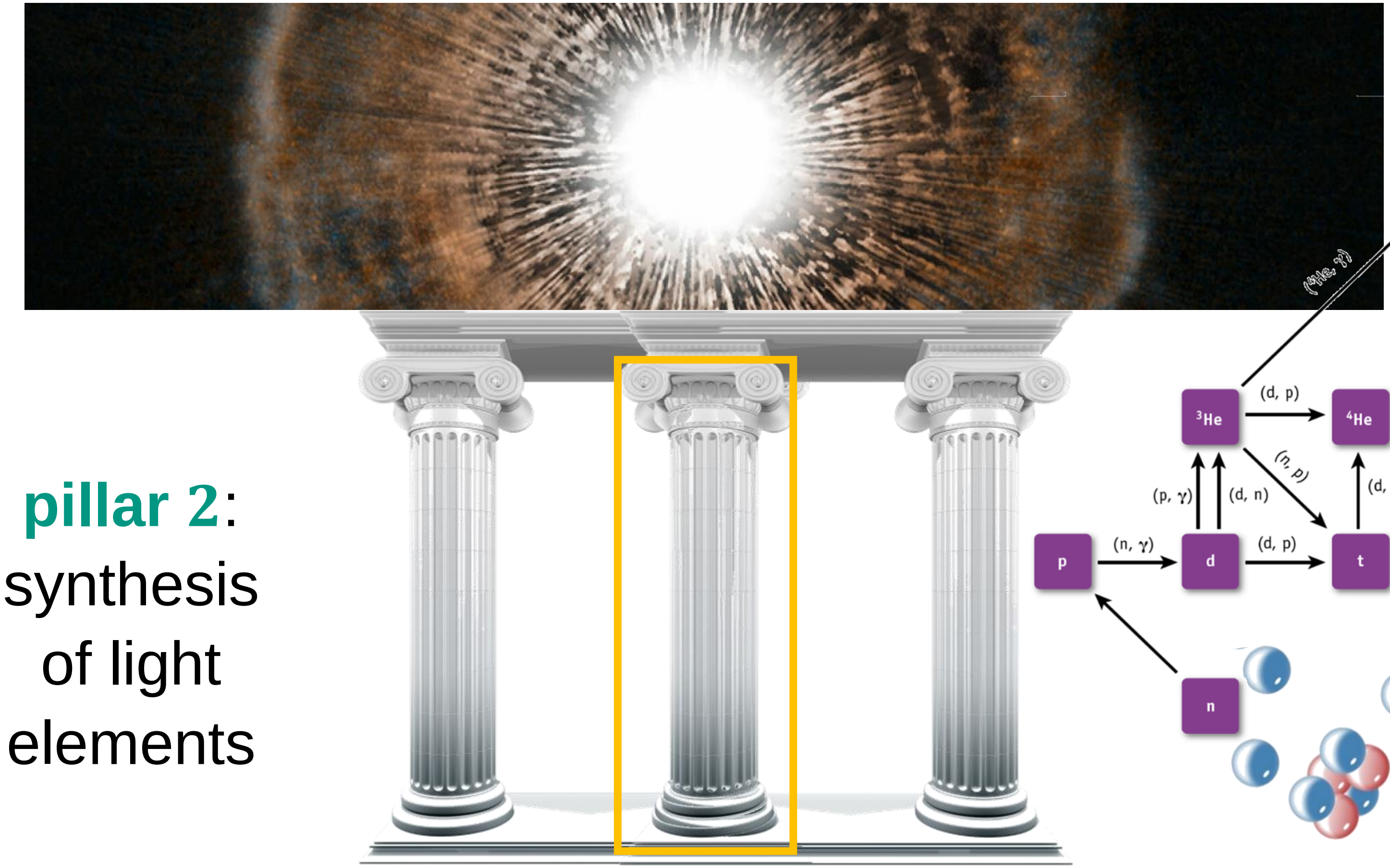
[rpp2022-rev-bbang-nucleosynthesis.pdf (lbl.gov)]

particle data group

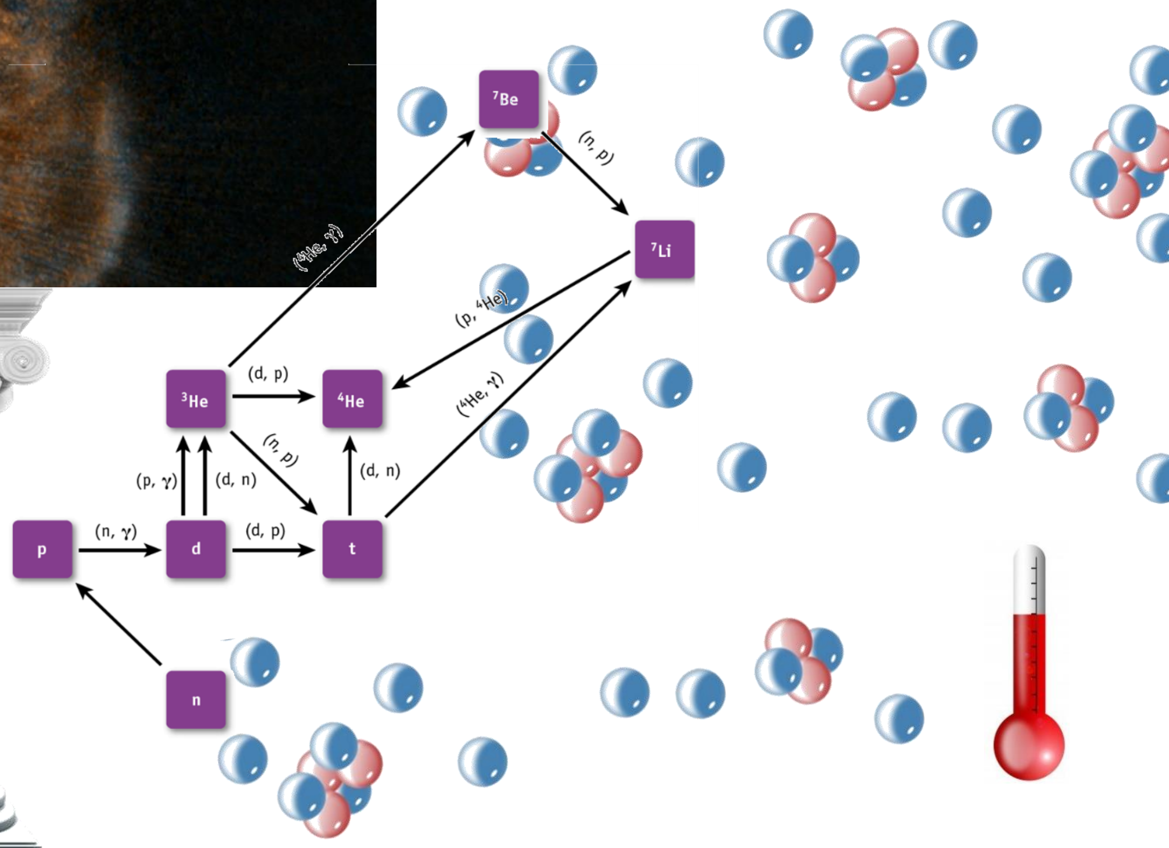
**Big Bang Nucleosynthesis*

Primordial nucleosynthesis & hot Big Bang

- **Abundance of light elements as key evidence for a hot Big Bang**



pillar 2:
synthesis
of light
elements



Primordial nucleosynthesis: breakthroughs

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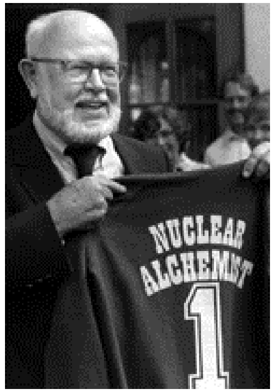
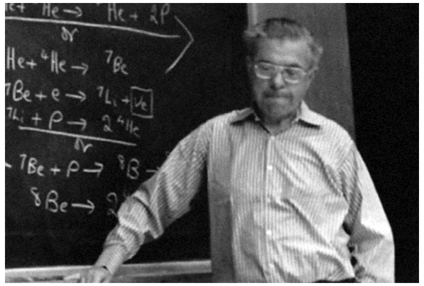
- **Phase**$-I$**: the ´**classical**´ works** & **discoveries from** $\mathbf{1948 \ldots 1977}$

$\mathbf{1948}$**:** **G. Gamov** & **R. Alpher** – **all elements** were forged in the early universe via neutron capture & $\boldsymbol{\beta}$ – decay



$\mathbf{1957}$**:** **W. Fowler** *et al.* – **heavy elements only made in stars** (Nobel prize $\mathbf{1983}$)



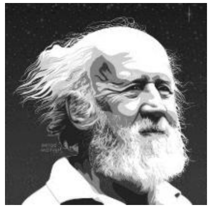
$\mathbf{1964}$**:** **F. Hoyle** *et al.* – $\boldsymbol{He} - \mathbf{4}$ **production is ´primordial´,** production of heavy elements (**no** Nobel prize)

Primordial nucleosynthesis: breakthroughs

- **Phase–I: the ´classical´ works & discoveries from $1948 \ldots 1977$**

1965: J. Peebles – first ´modern´ calculation of BBN reaction paths (Nobel prize **2019**)

1970: H. Reeves – deuterium spectroscopy as observational means to measure the **baryon density of the universe**



1977: D. Schramm *et al.* – BBN **limits the number of light ν – generations** (cosmology ⇔ particle physics)

Primordial nucleosynthesis: fundamentals

- **The Big Bang Nucleosynthesis ...**

... allows detailed investigations of the **physics of the early universe**

... correctly predicts the **abundances of light elements** (despite a huge variation of more than $\mathbf{10}\ \boldsymbol{orders\ of\ magnitude}$!)

... allows an ´in–situ´ determination of the **baryon density $\boldsymbol{\Omega_B}$** as well as of the **baryon asymmetry parameter $\boldsymbol{\eta}$ (ratio baryons / photons)**

... acts as an important test bed **for novel theories**: number of **light $\boldsymbol{\nu}$ – generations**, sterile neutrinos, novel particles (such as **gravitinos**,…)

... is a part of **nuclear astrophysics** (element synthesis in stars & elsewhere)

Primordial nucleosynthesis: the starting point

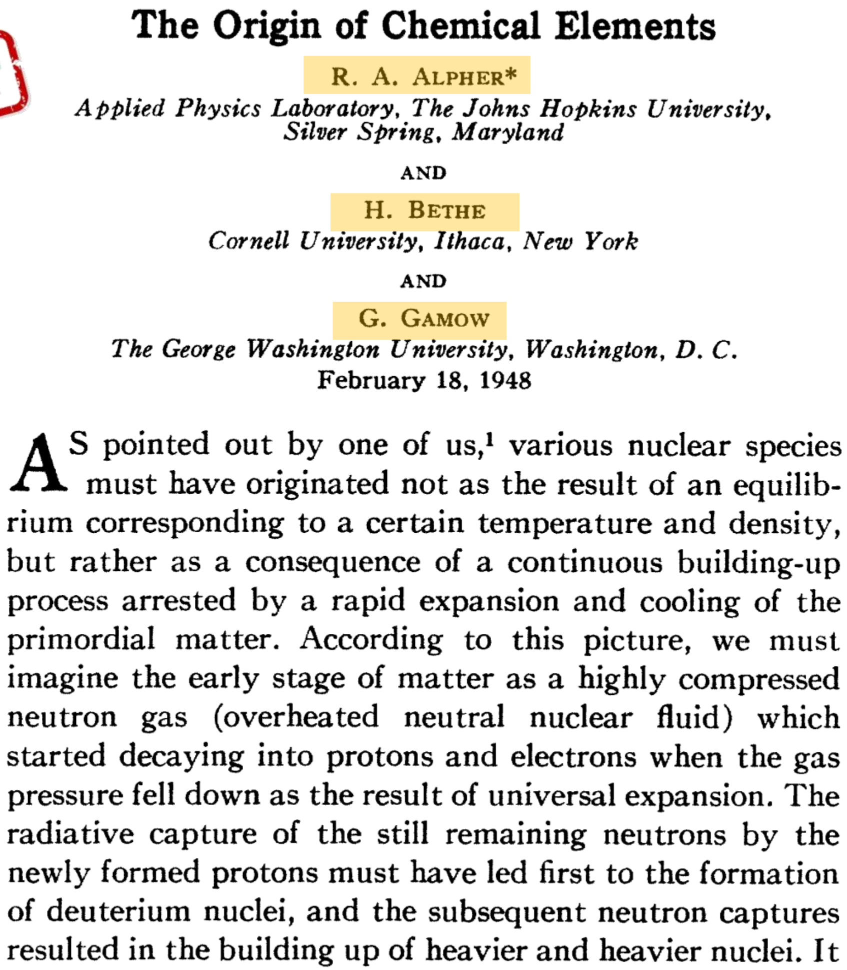
- **The world–famous ´$\alpha - \beta - \gamma$´ paper from 1948 on nucleosnthesis**

- <u>all</u> elements were forged in the very early universe via neutron capture & $\boldsymbol{\beta}$ – decay

- based on thesis $\boldsymbol{R.\,Alpher}$, supervisor: $\boldsymbol{G.\,Gamow}$

- name of ´author´ $\boldsymbol{H.\,Bethe}$ was added ´***in absentia***´

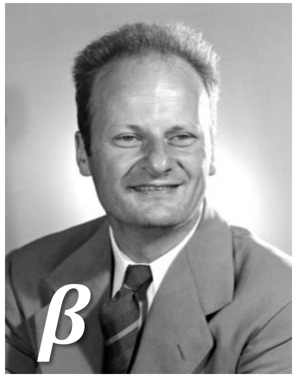
- publication date of $\boldsymbol{PRL}$* print: $\boldsymbol{April\ 1, 1948}$

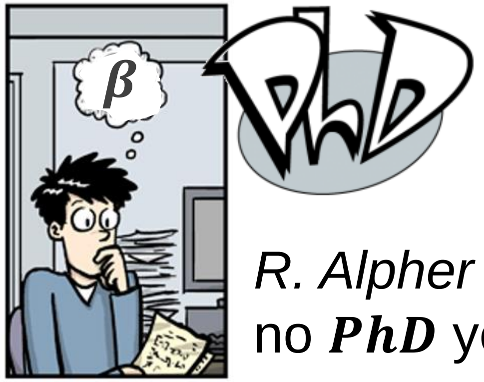


The Origin of Chemical Elements

R. A. Alpher*
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
and
H. Bethe
Cornell University, Ithaca, New York
and
G. Gamow
The George Washington University, Washington, D. C.
February 18, 1948

As pointed out by one of us,[1] various nuclear species must have originated not as the result of an equilibrium corresponding to a certain temperature and density, but rather as a consequence of a continuous building-up process arrested by a rapid expansion and cooling of the primordial matter. According to this picture, we must imagine the early stage of matter as a highly compressed neutron gas (overheated neutral nuclear fluid) which started decaying into protons and electrons when the gas pressure fell down as the result of universal expansion. The radiative capture of the still remaining neutrons by the newly formed protons must have led first to the formation of deuterium nuclei, and the subsequent neutron captures resulted in the building up of heavier and heavier nuclei. It





R. Alpher had no ***PhD*** yet!

Phys. **R**ev. **L**etters

Primordial nucleosynthesis: fundamentals

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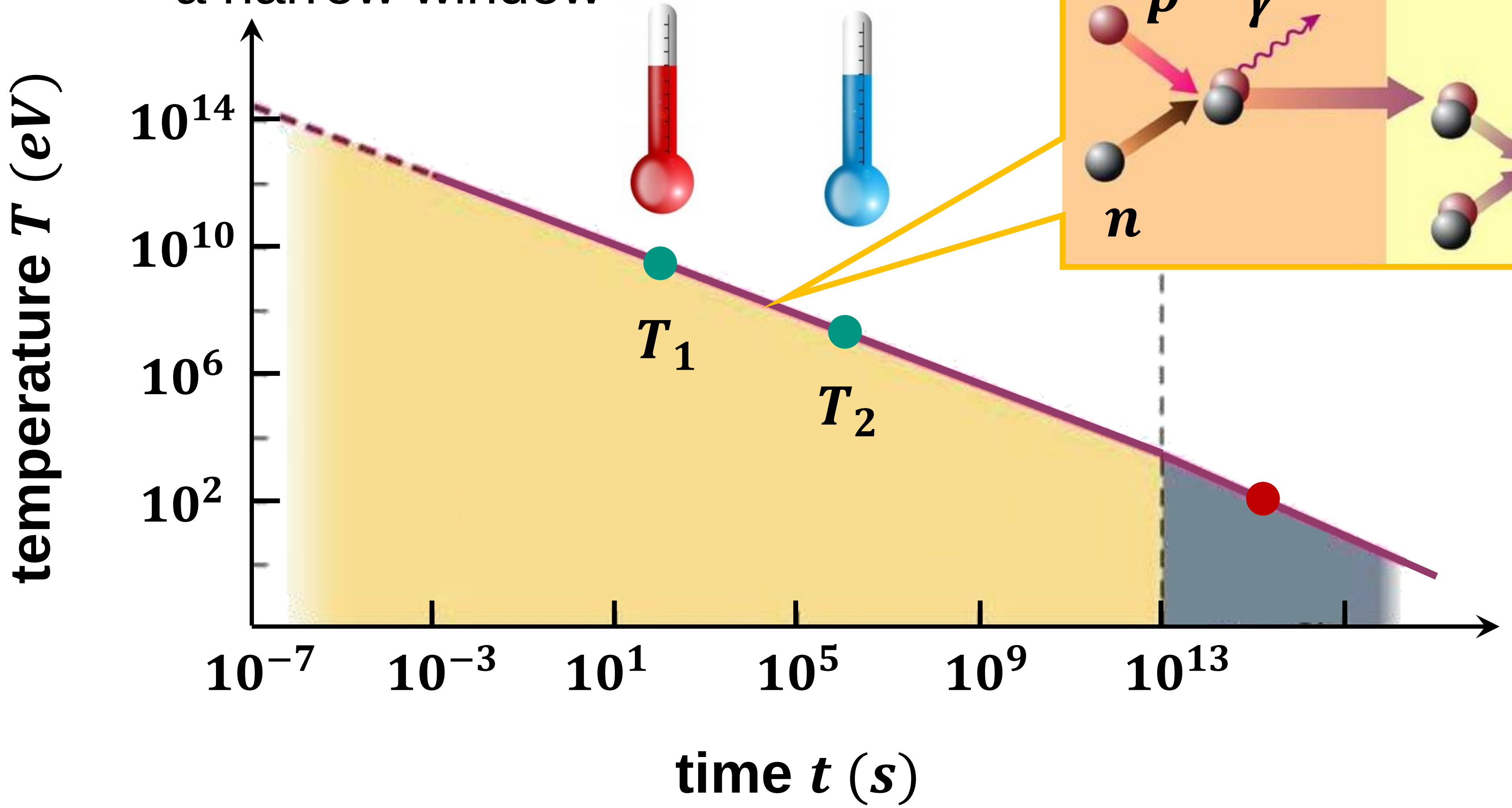
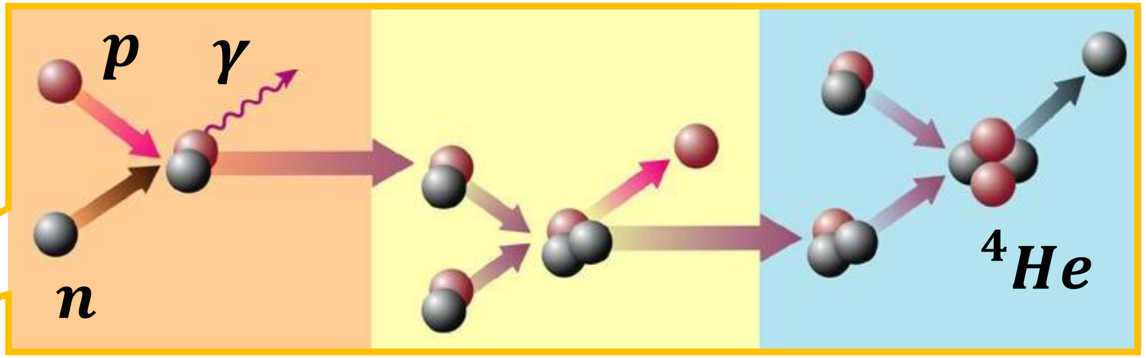
- **Modern description of processes during light element synthesis**

- a narrow window





$T_1 = 5 \cdot 10^{10} K$

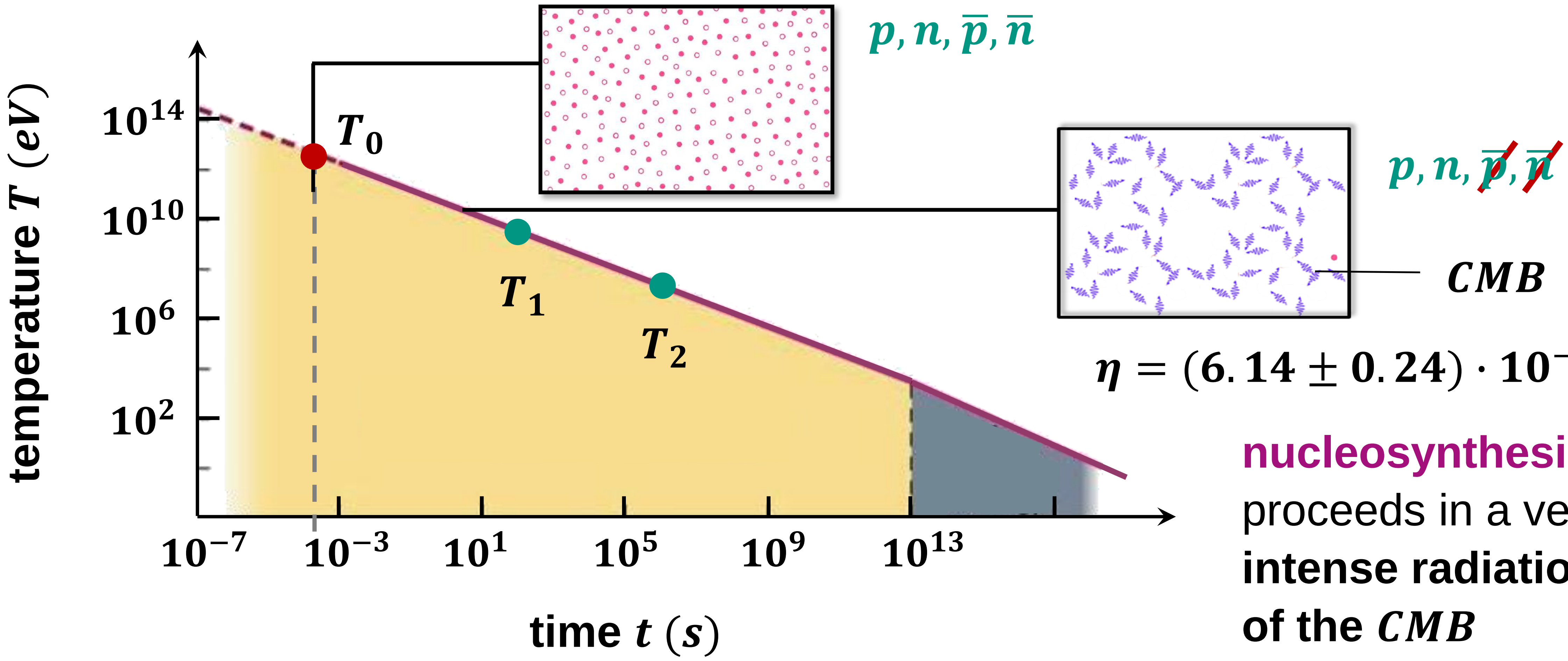
$T_2 = 5 \cdot 10^{8} K$

nucleosynthesis (BBN) ´only´ proceeds to ^4He, all heavier elements only forged later inside stars*

*see $ATP-2$ summer term 2024

Primordial nucleosynthesis: specific phases

- Our initial starting point T_0: annihilation of matter (p, n) & anti−matter $(\overline{p}, \overline{n})$



$$\eta = (6.14 \pm 0.24) \cdot 10^{-10}$$

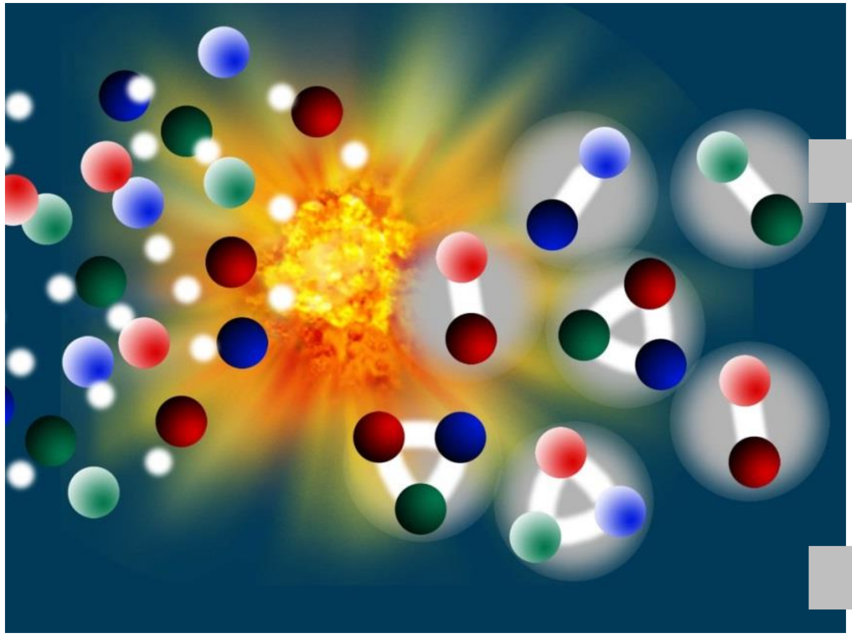
nucleosynthesis (BBN) proceeds in a very **intense radiation field of the** CMB

*see $ATP - 2$ summer term 2024

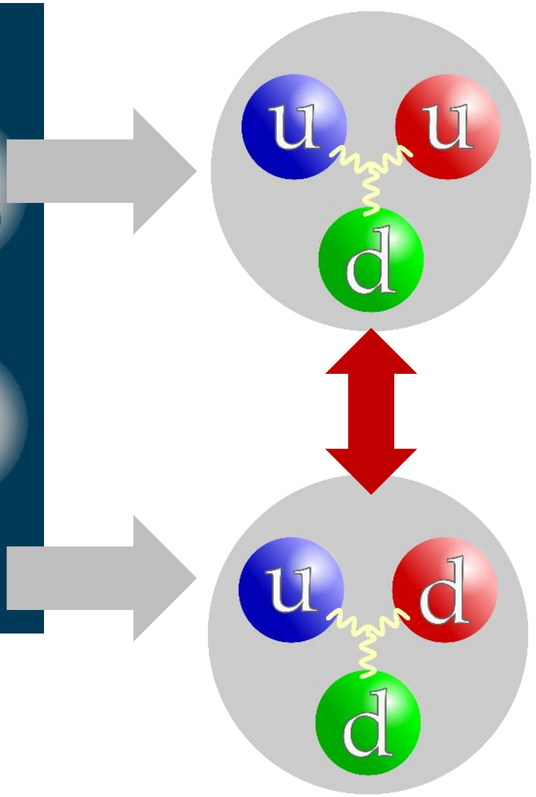
Primordial nucleosynthesis: phase 1

- **Thermodynamic equilibrium**

  - nucleons $\boldsymbol{p}, \boldsymbol{n}$ **condense** out of initial **quark−gluon plasma** (**hadronisation**)



QCD phase transition
at $T \sim 150\ MeV$



$\boldsymbol{p}$ **(proton)**
$m = 938.3\ MeV$

$W^{\pm}$ **– bosons**

$\boldsymbol{n}$ **(neutron)**
$m = 939.6\ MeV$

$T = 10^{11} K$

$E = 10\ MeV$

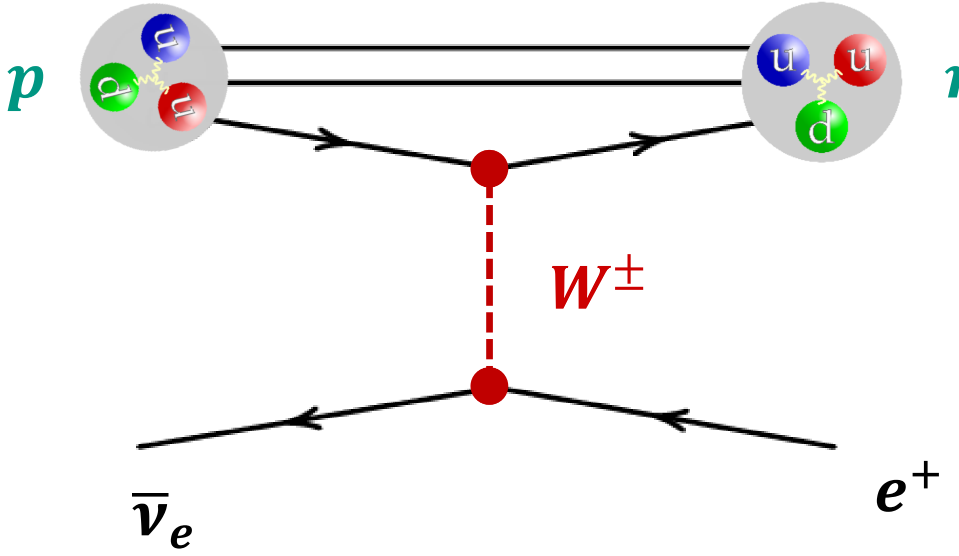
$t = 0.01\ s$

Primordial nucleosynthesis: phase 1

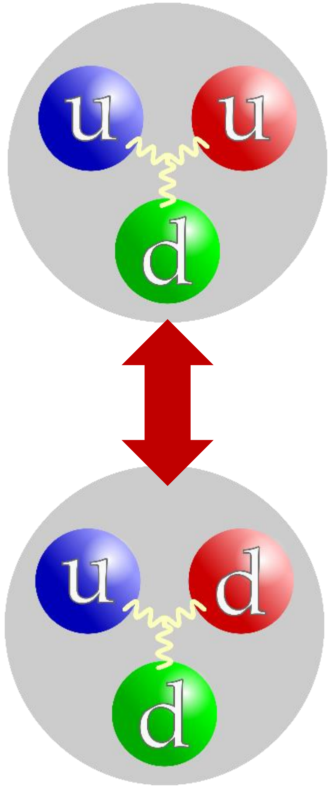
- **Thermodynamic equilibrium: example inverse $\beta-$ decay**

  - nucleons p, n transform into each other due to **weak interaction processes**

$$p + \bar{\nu}_e \Leftrightarrow n + e^+$$



inverse β – decay on p, n



p (proton)
$m = 938.3\ MeV$

$W^{\pm}$ – **bosons**

n (neutron)
$m = 939.6\ MeV$

$T = 10^{11} K$

$E = 10\ MeV$

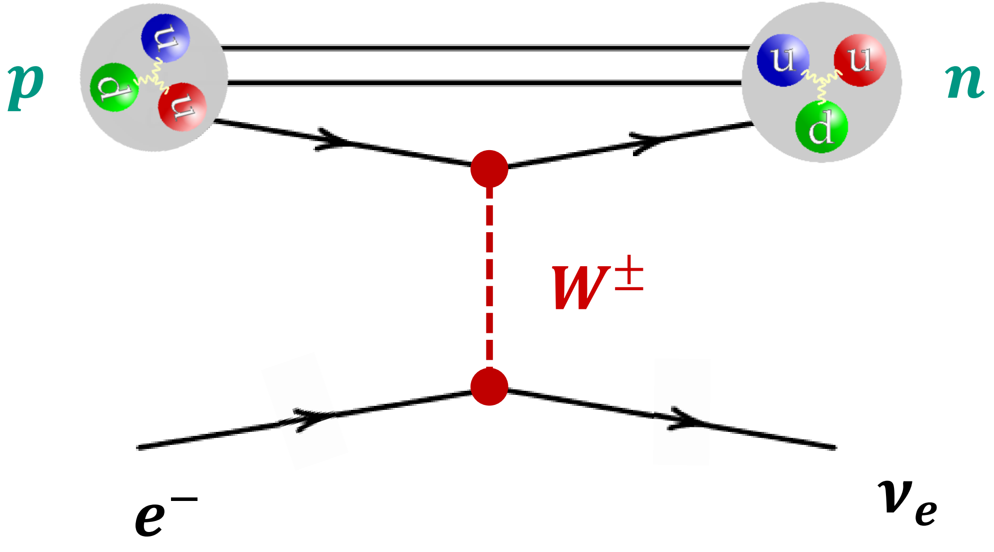
$t = 0.01\ s$

Primordial nucleosynthesis: phase 1

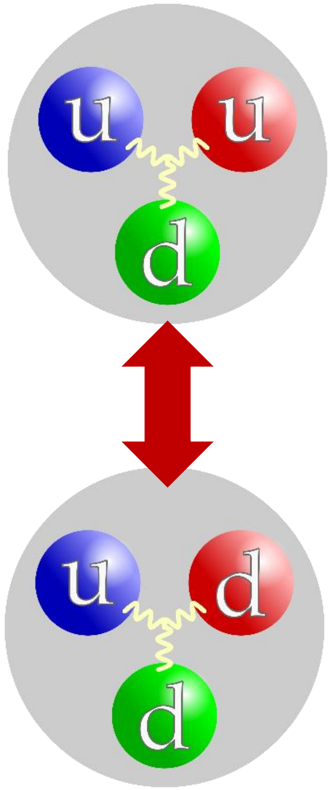
- **Thermodynamic equilibrium: example electron capture process**

  - nucleons p, n transform into each other due to **weak interaction processes**

$$p + e^- \Leftrightarrow n + \nu_e$$



electron capture on p



p **(proton)**
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$E = 10\ MeV$

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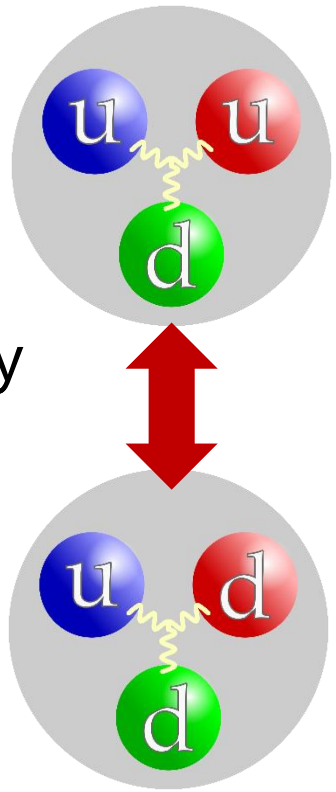
Primordial nucleosynthesis: phase 1

- **Thermodynamic equilibrium: weak interaction processes of key importance**

- nucleons $\boldsymbol{p, n}$ transform into each other due to **weak interaction processes**

- no change of $(\boldsymbol{p, n})$ **ratio** via strong/electromagnetic interactions (**strong isospin**), only via weak interactions, i.e. $\boldsymbol{\nu}$ – processes (before they are decoupling!)

- very early universe ($\boldsymbol{t < 1\ s}$): extremely high $\boldsymbol{\nu}$ – densities*: $\boldsymbol{N_\nu \approx 10^{31} \ldots 10^{32}\ cm^{-3}}$



$\boldsymbol{p}$ **(proton)**
$\boldsymbol{m = 938.3\ MeV}$

$\boldsymbol{W^\pm}$ **– bosons**

$\boldsymbol{n}$ **(neutron)**
$\boldsymbol{m = 939.6\ MeV}$

$\boldsymbol{T = 10^{11} K}$

$\boldsymbol{E = 10\ MeV}$

$\boldsymbol{t = 0.01\ s}$

*presently we have only: $\boldsymbol{N_\nu = 336\ cm^{-3}}$

Primordial nucleosynthesis: phase 1

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- **Thermodynamic equilibrium: calculating initial (p, n) – ratio at high T**

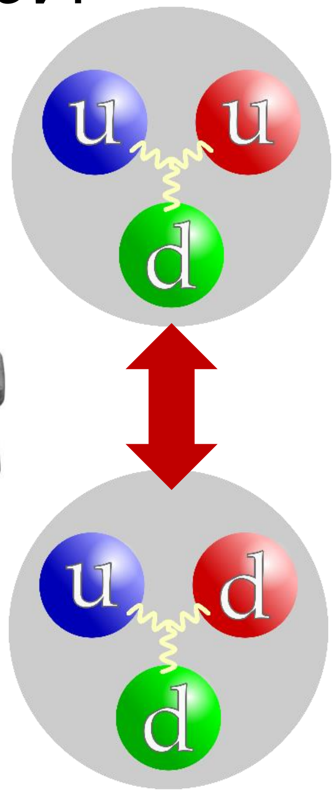
  - (p, n) – **ratio** follows a **Boltzmann distribution***
    different masses $\Delta m = 1.3\ MeV$!

  - for **high temperatures**
    $T(10\ MeV) \gg \Delta m\ (1.3\ MeV)$
    we have:

$$n/p = e^{-\Delta m \cdot c^2 / k_B \cdot T}$$

$$n/p = e^{-1.3/10} = 0.88$$

⇨ almost equal numbers



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$W^{\pm}$ – **bosons**

n (neutron)
$m = 939.6\ MeV$

$T = 10^{11} K$

$E = 10\ MeV$

$t = 0.01\ s$

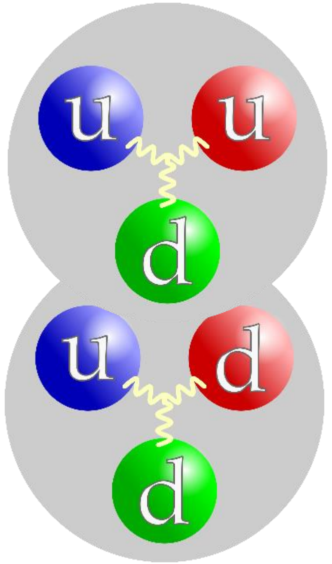
*see Lect. 5, p. 41 – 43

Primordial nucleosynthesis: phase 1

- **Thermodynamical equilibrium: fusion reactions to deuterium at high T?**

  - can nucleons (p, n) already undergo fusion processes to deuterium?

  - for **high temperatures** $T(10\ MeV)$ we have to consider whether deuterium is ***stable*** under these conditions



d **(deuterium)**
$E_B = 2.2\ MeV$

strong interaction

$T = 10^{11} K$

$E = 10\ MeV$

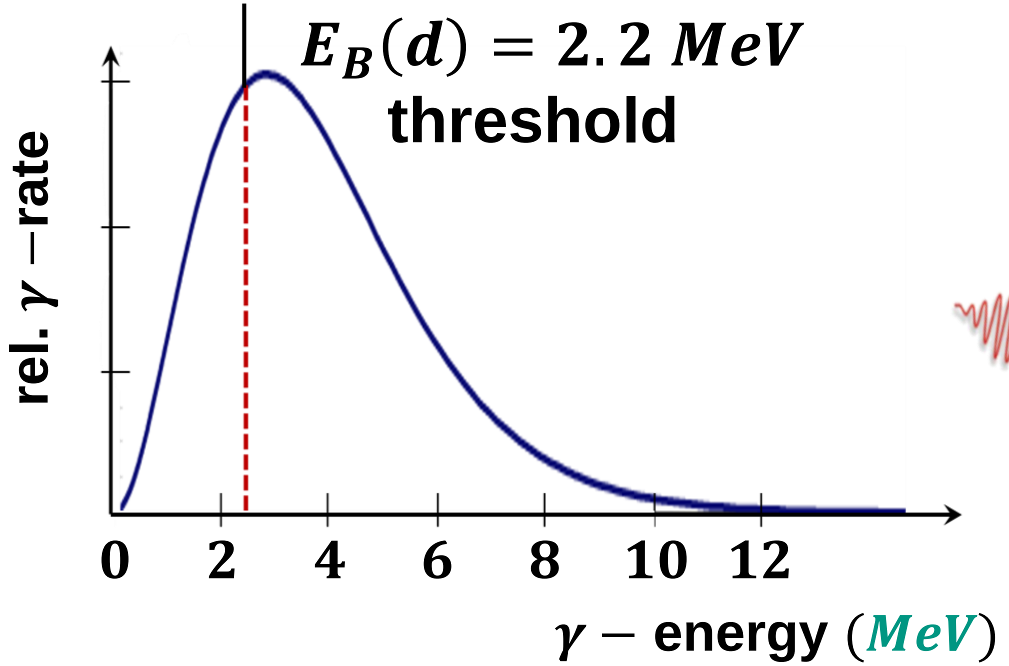
$t = 0.01\ s$

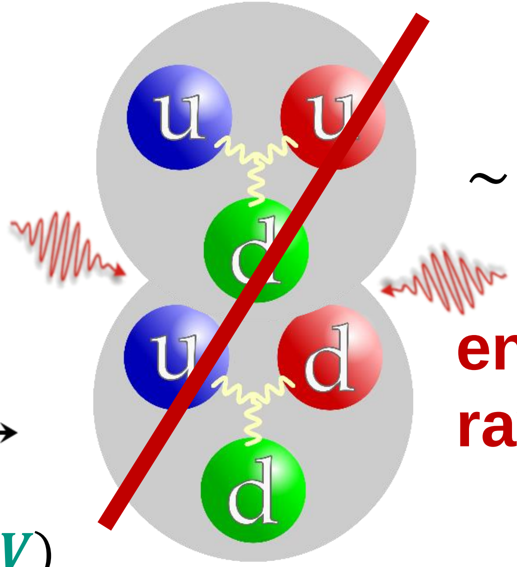
⇨ fusion reactions (?) in an **intense heat bath**

Primordial nucleosynthesis: phase 1

- **Thermodynamical equilibrium: fusion reactions to deuterium at high T?**

  - all deuterium nuclei forged at $T = MeV - scale$ are immediately **destroyed**





$\sim 10^{10}\ \gamma\text{´}s$ per d

energetic gamma rays break up d

$T = 10^{11} K$

$E = 10\ MeV$

$t = 0.01\ s$

⇨ fusion reactions cannot preceed in an **intense heat bath** & **nucleons (p, n) remain unpaired**

Primordial nucleosynthesis: phase 2

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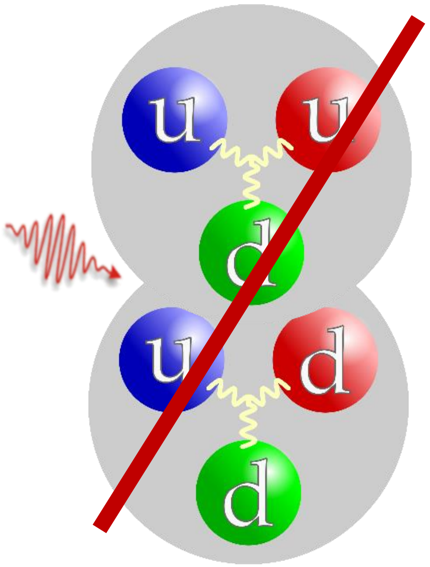
- **Universe has to expand** & **cool before fusion reactions can proceed**

  - $\boldsymbol{d}$ is **bottleneck** for fusion reactions due to **energtic $\gamma\acute{}s$** at $MeV - scale$

  - only ´solution´:



$T \ll 10^{11} K$

$E \ll 10\ MeV$

$t \gg 0.01\ s$

  - waiting for temperature T to drop: but **neutrinos are decoupling** at $t = 1\ s\ (1\ MeV)$
    ⇨ no further (n, p) transformations ⇨ (n, p) – **ratio is fixed**

Primordial nucleosynthesis: phase 2

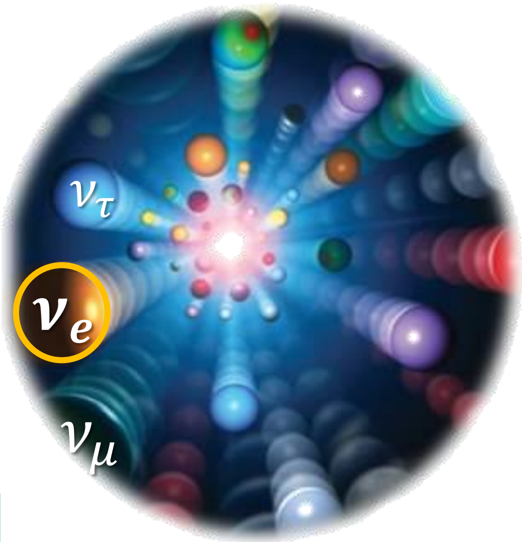
- **Freeze−out of neutrinos at $t \approx 1\ s$ of key importance for (n, p) −ratio**

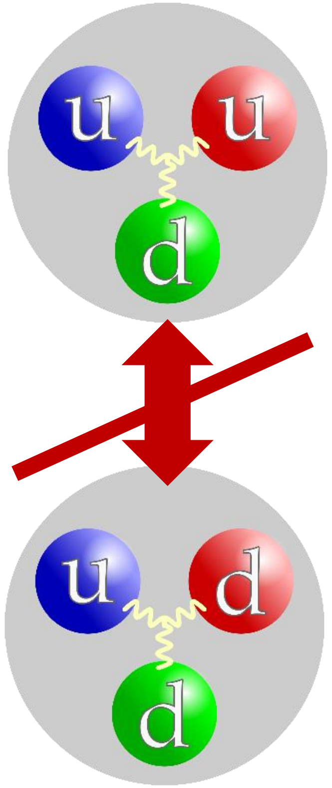
  - needed: more detailed calculations for **neutrino freeze−out** energy for ν_e

$$\Gamma_\nu(t) = H(t)$$

$\sim G_F \cdot T^5$ $\sim T^2$

$$T_{fr} = 0.7\ MeV$$





p (proton)
$m = 938.3\ MeV$

~~$W^\pm$ − bosons~~

n (neutron)
$m = 939.6\ MeV$

$T \approx 10^{10} K$

$E = 0.7\ MeV$

$t \approx 1\ s$

Primordial nucleosynthesis: phase 2

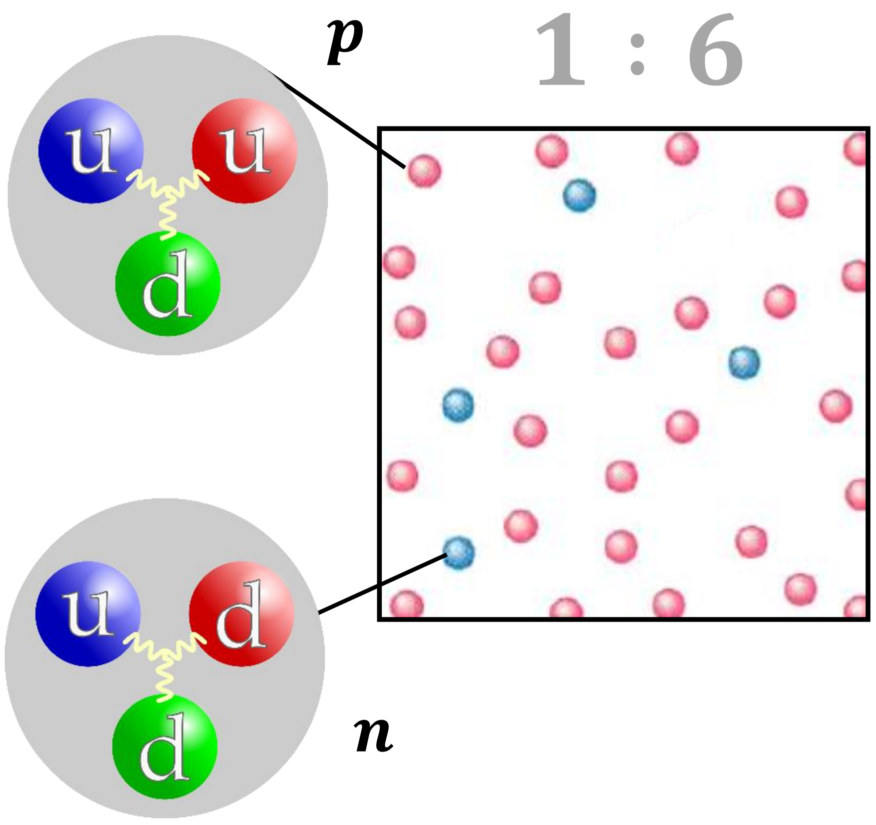
- **Freeze–out of neutrinos at $t \approx 1\ s$ of key importance for (n, p) – ratio**

  - calculated (n, p) – **ratio** at $T = 0.7\ MeV$

$$n/p = e^{-\Delta m \cdot c^2 / k_B \cdot T}$$

$$n/p = e^{-1.3/0.7} = 0.15 \approx \frac{1}{6}$$

  - (n, p) – **ratio** not further modified by thermodynamics



$T \approx 10^{10} K$

$E = 0.7\ MeV$

$t \approx 1\ s$

SIDE REMARK: Primordial nucleosynthesis´

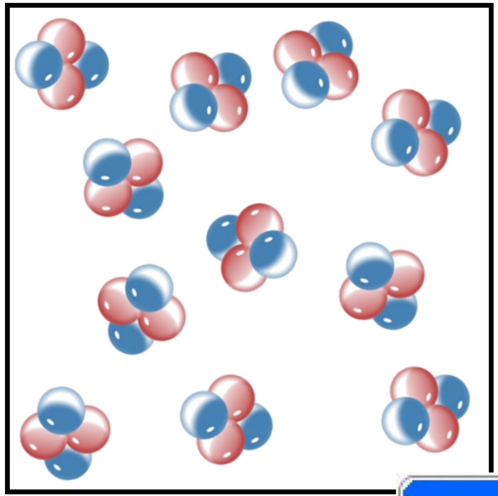
- **Consider a different universe** (#42*) **with larger freeze−out temperature**

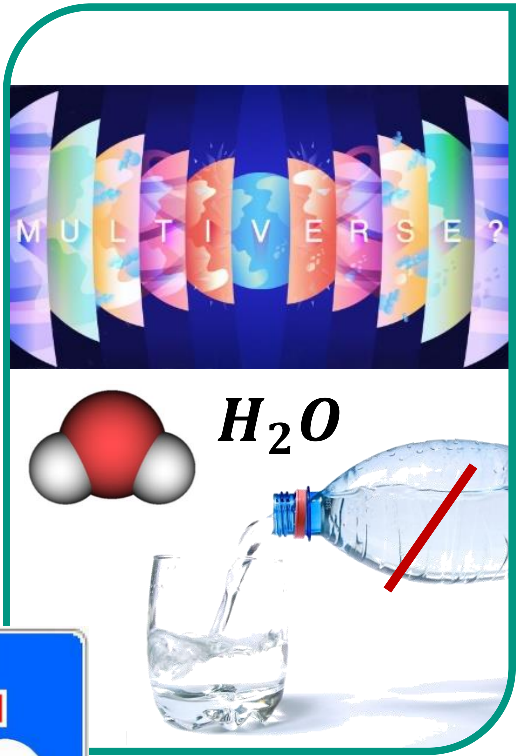
  - in case of significantly changed physical constants we have for example:

$$T_{fr} = 70\ MeV$$

$$n/p = e^{-1.3/70} = 0.981 \ \approx 1$$

formation of ${}^{4}He$





  - no free protons p remain to form

    molecules: H_2O and CH_2

    molecules: H_2 (thus: only fast−burning He − stars...)

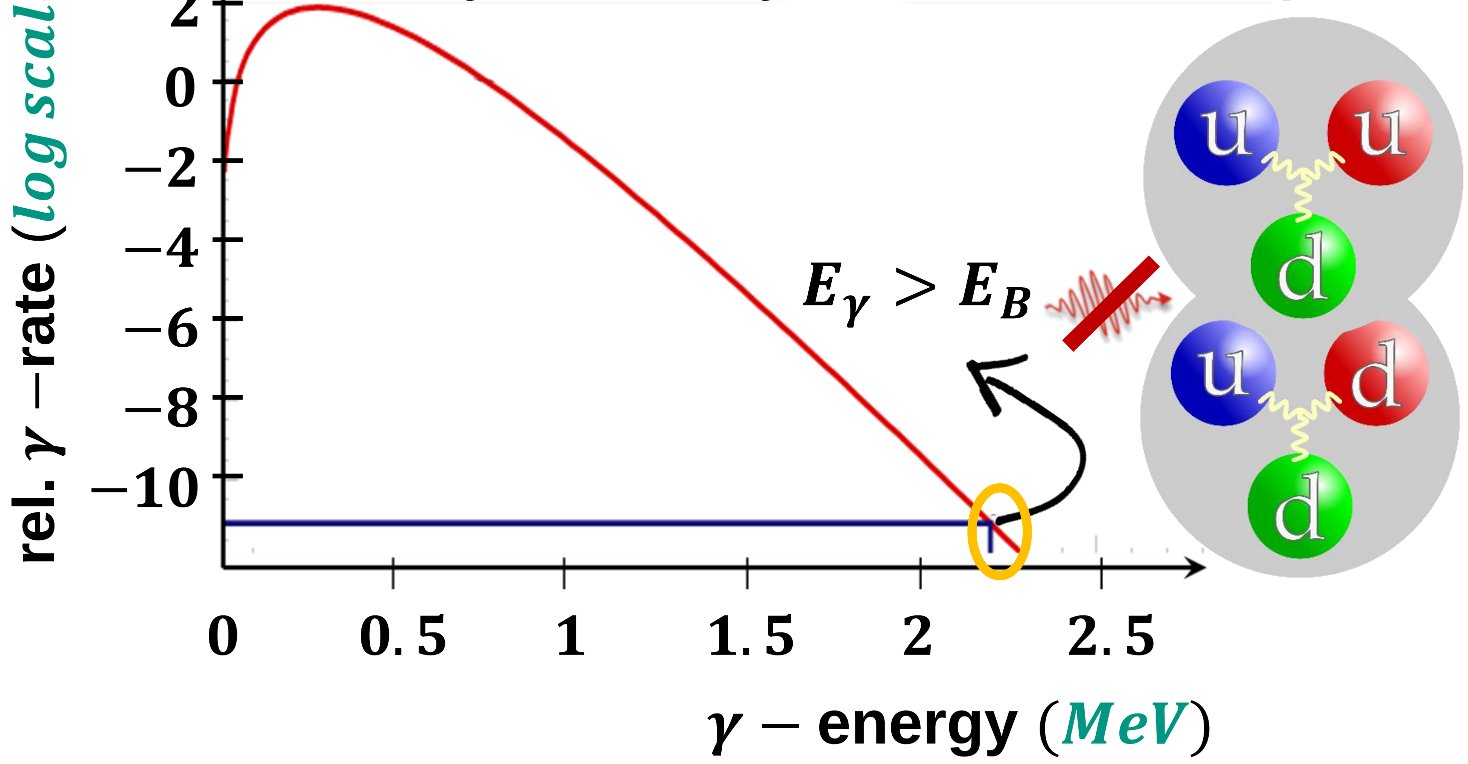
* The answer to life, the universe, and everything | MIT News | Massachusetts Institute of Technology

Primordial nucleosynthesis: phase 2 ends

- **We need to wait for the universe to cool and expand, until d can be formed**

- only after $t = 60\ s$ the universe is cool enough ($T < 0.1\ MeV$) so that d is stable

- only extremely small fraction of γ´s with $E > 2.2\ MeV$!



d (deuterium)
$E_B = 2.2\ MeV$
is finally stable!

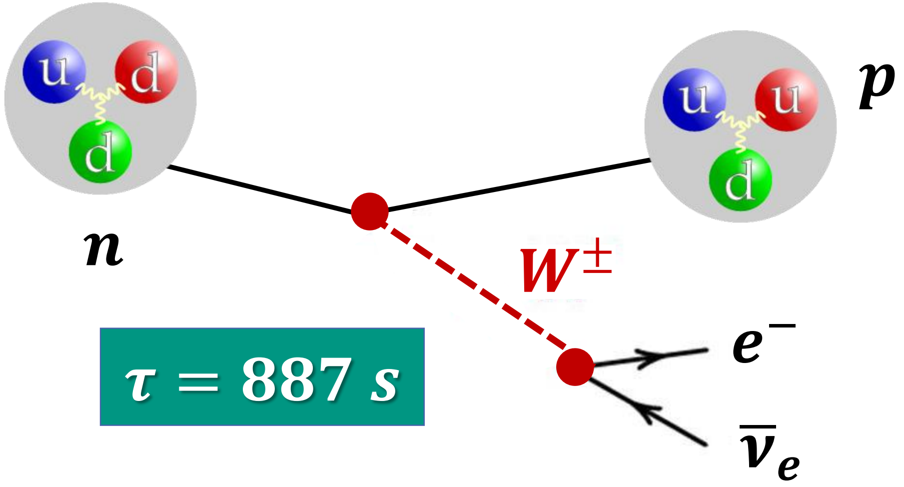
$T < 10^9 K$

$E \approx 0.1\ MeV$

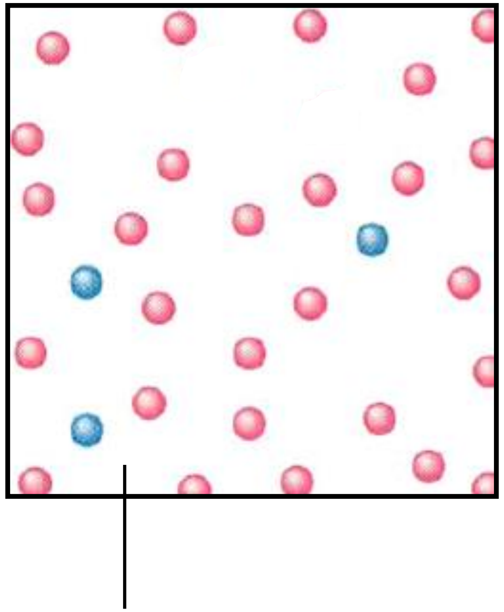
$t \approx 60\ s$

Primordial nucleosynthesis: phase 2 ends

- **We now need to take into account the finite life–time $\tau_n = 887\ s$ of neutrons**
  - after $t = 60\ s$ a fraction of neutrons will have decayed ($\beta - decay$)



1 : 7



$T < 10^9 K$

$E \approx 0.1\ MeV$

$t \approx 60\ s$

  - (n, p) – **ratio** after $t = 60\ s$ decreases to

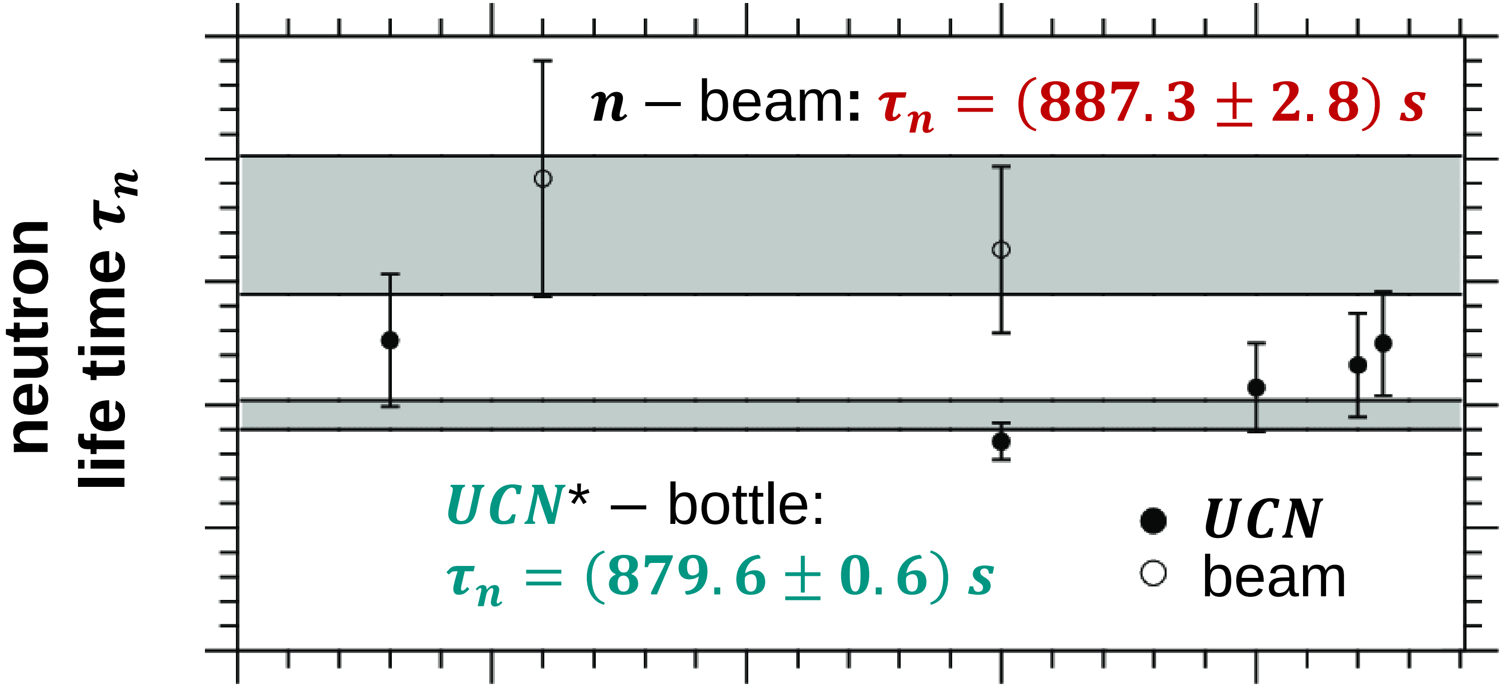
$$\frac{n}{p} = \frac{1}{6} \cdot e^{-(60/887)} \approx \frac{1}{7}$$

(n, p) – ratio now at $1/7$

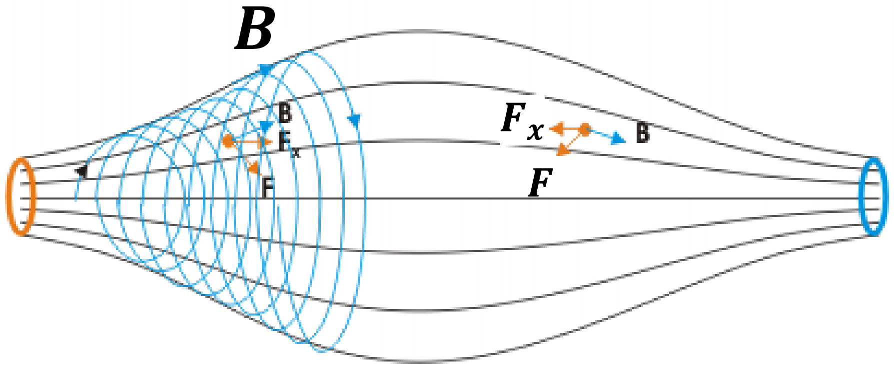
Particle physics & primordial nucleosynthesis

- **finite life–time τ_n of neutrons: overview of experimental results**

  - **systematic effects** in measurements of n – life times τ_n over past decades



magnetic bottle technique



*Ultra–Cold Neutrons

Primordial nucleosynthesis: phase 3

- **12 main pathways** of BBN

1 $n \rightarrow p + e^- + \bar{\nu}_e$

2 $p + n \rightarrow d + \gamma$

3 $d + p \rightarrow {}^3He + \gamma$

4 $d + d \rightarrow {}^3He + n$

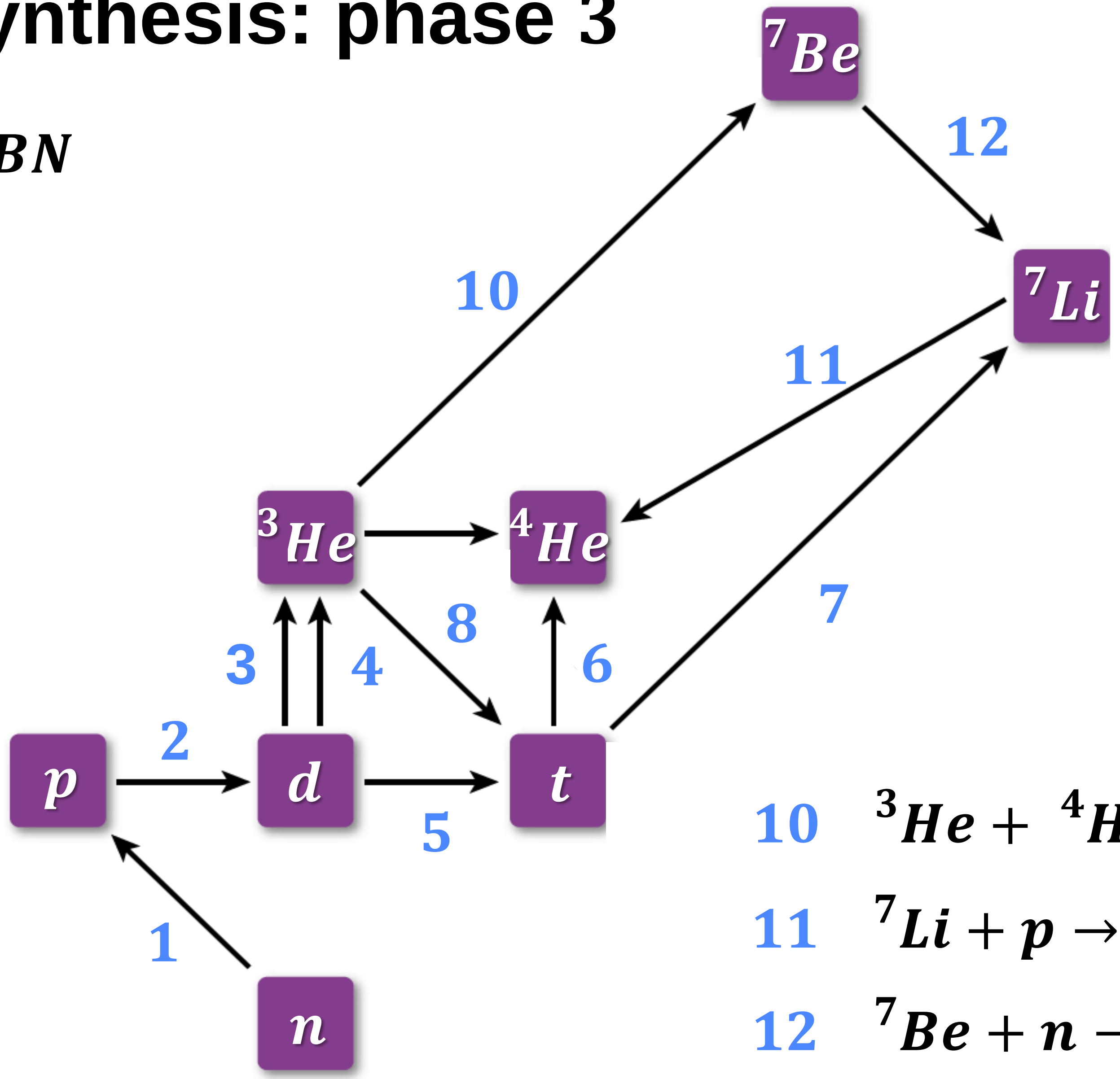
5 $d + d \rightarrow {}^3H + p$

6 $d + {}^3H \rightarrow {}^4He + n$

7 ${}^3H + {}^4He \rightarrow {}^7Li + \gamma$

8 ${}^3He + n \rightarrow {}^3H + p$

9 ${}^3He + d \rightarrow {}^4He + p$



10 ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

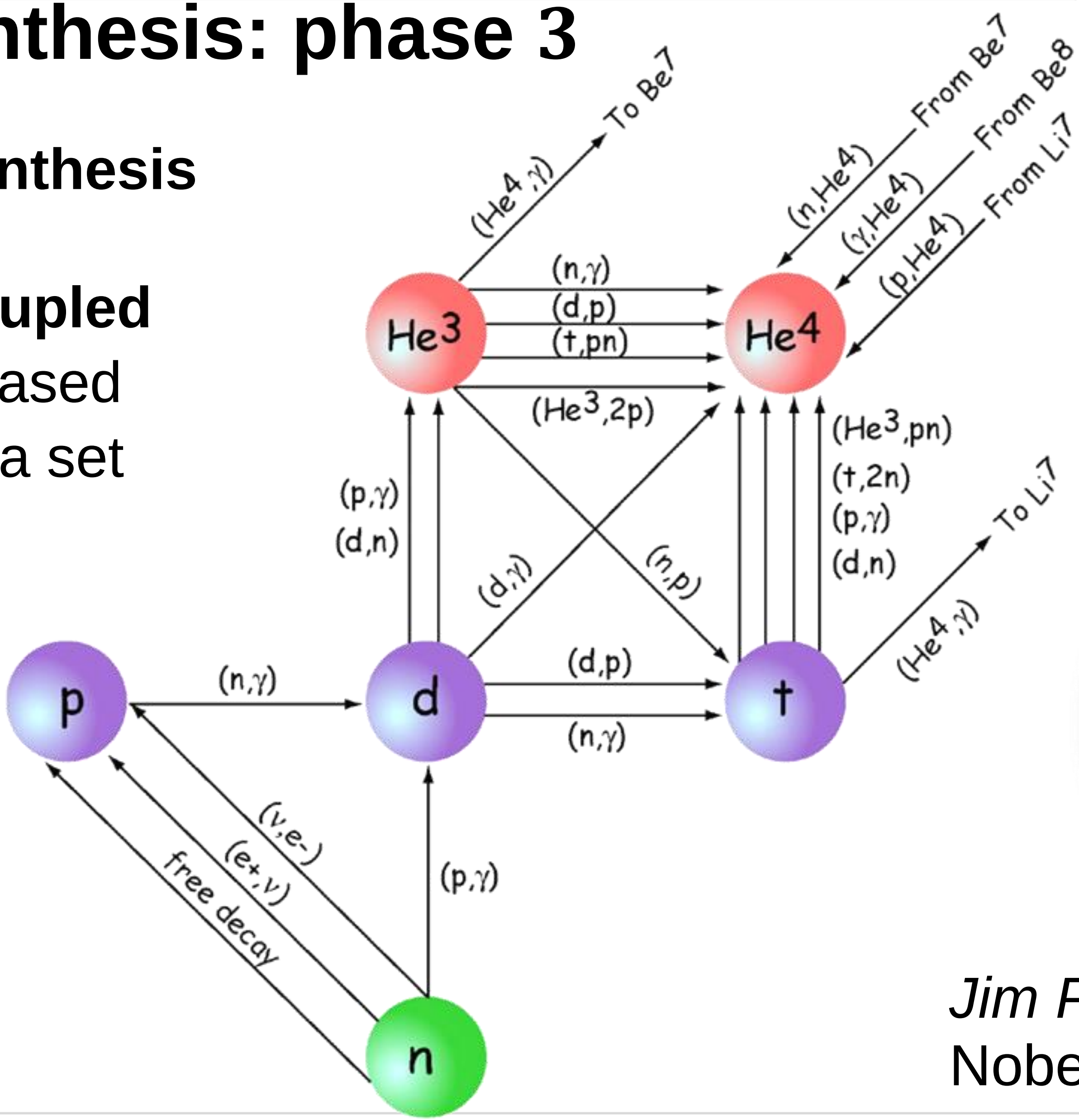
11 ${}^7Li + p \rightarrow {}^4He + {}^4He$

12 ${}^7Be + n \rightarrow {}^7Li + n$

Primordial nucleosynthesis: phase 3

- **all pathways of nucleosynthesis**

  - an extended system of **coupled differentiual equations** based on entire experimental data set



Jim Peebles
Nobel prize 2019

Primordial nucleosynthesis: phase 3 channels

- **Grouping the different reaction pathways into channels (reaction ´ladder´)**

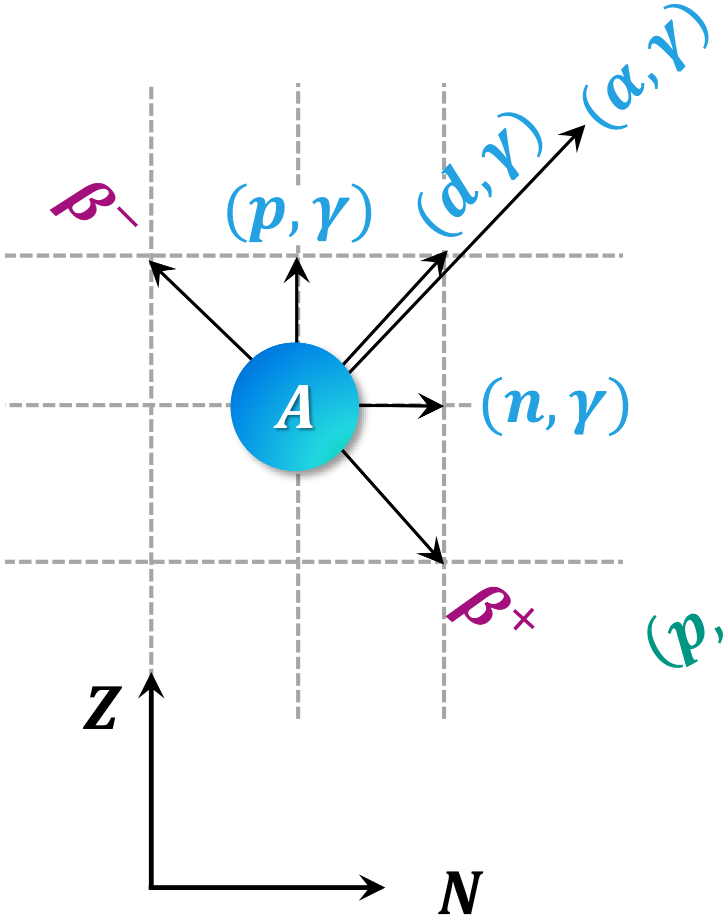
$\beta^{\pm}$ – **decays**

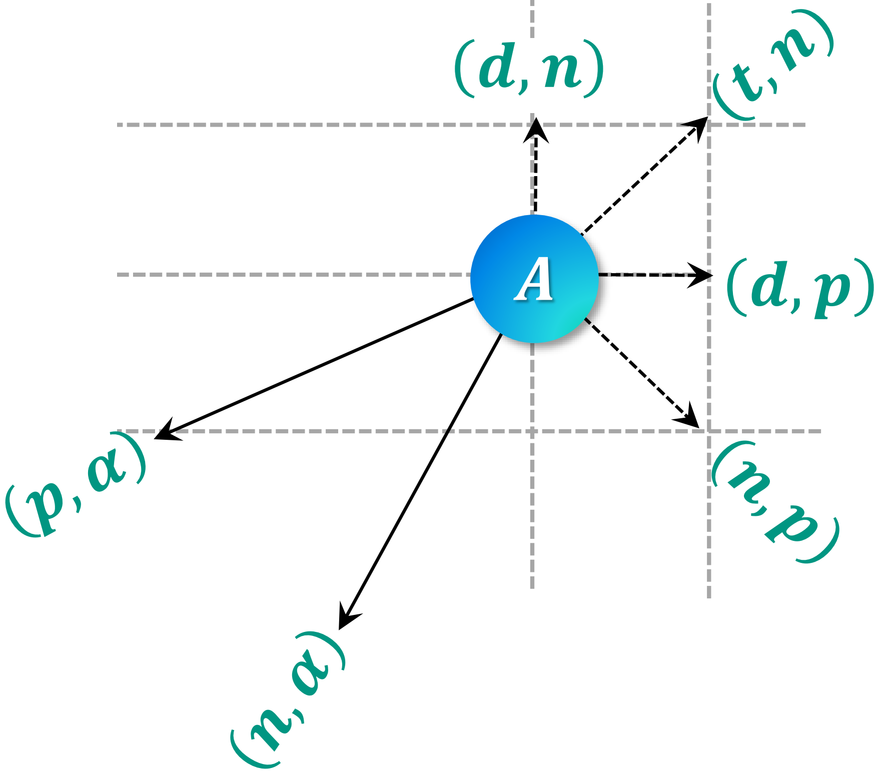
capture reactions:

$(p,\gamma),(n,\gamma),$
$(d,\gamma),(\alpha,\gamma)$

transfer processes:

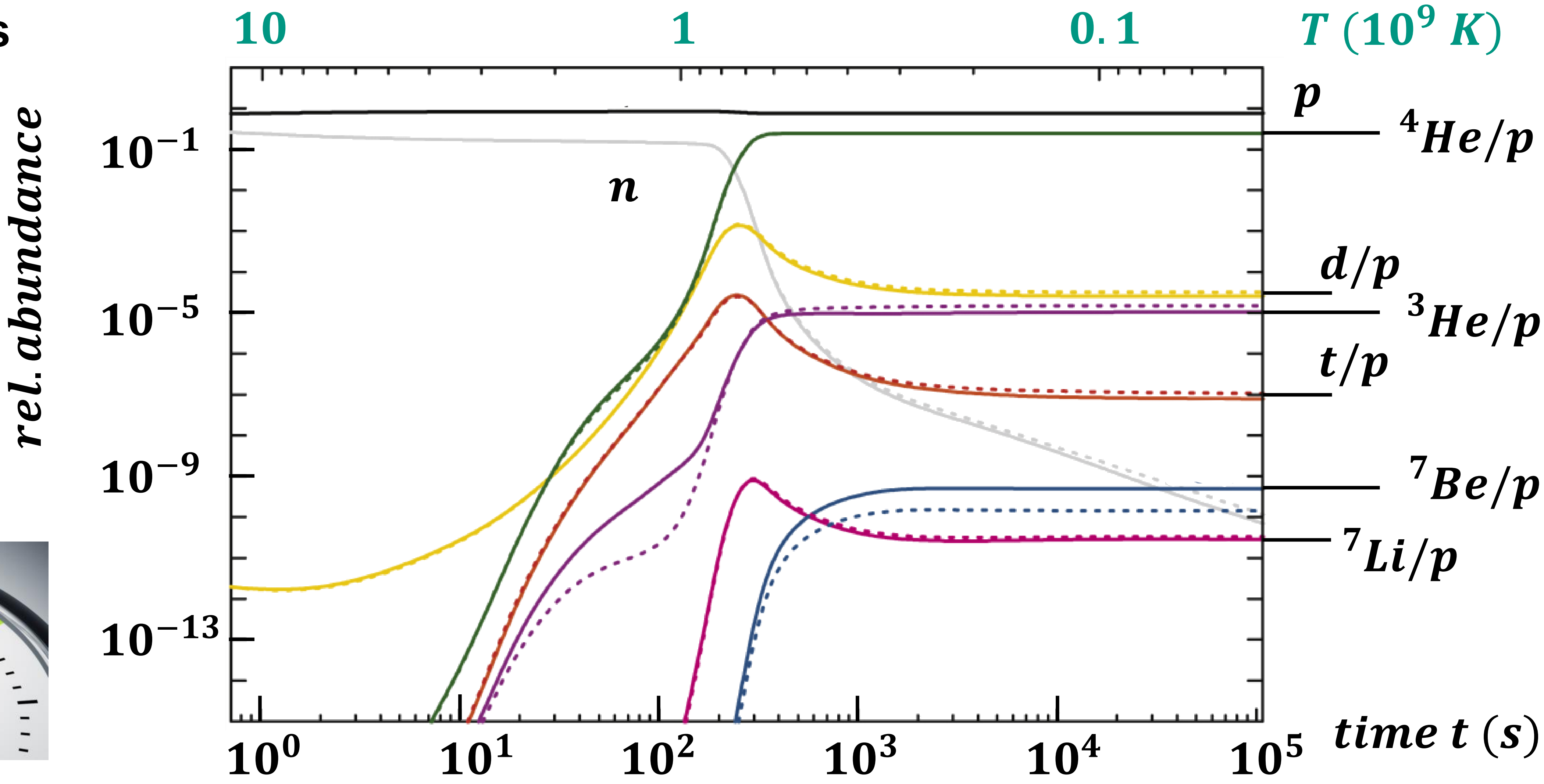
$(n,p),(d,p),$
$(d,n),(t,n),\ldots$





Primordial nucleosynthesis: fusion yields

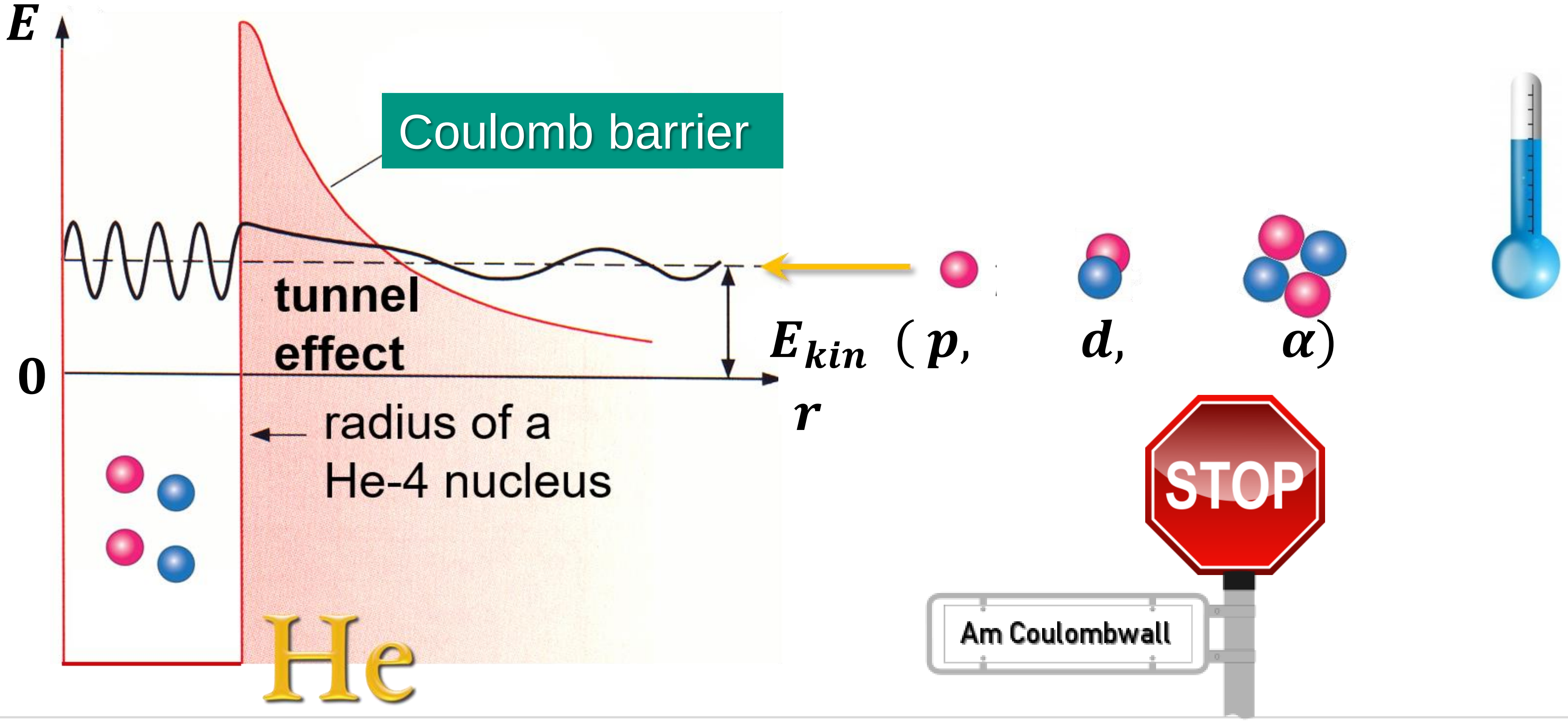
- Results



Primordial nucleosynthesis: fusion stops

- **Synthesis of light elements at low temperature ends due to Coulomb barrier**

  - energy $E_{p,d,\alpha}$ of **charged particles** no longer sufficient to sustain fusion reactions



$T < 5 \cdot 10^8 K$

$E \ll 0.1\, MeV$

$t \approx 300\, s$

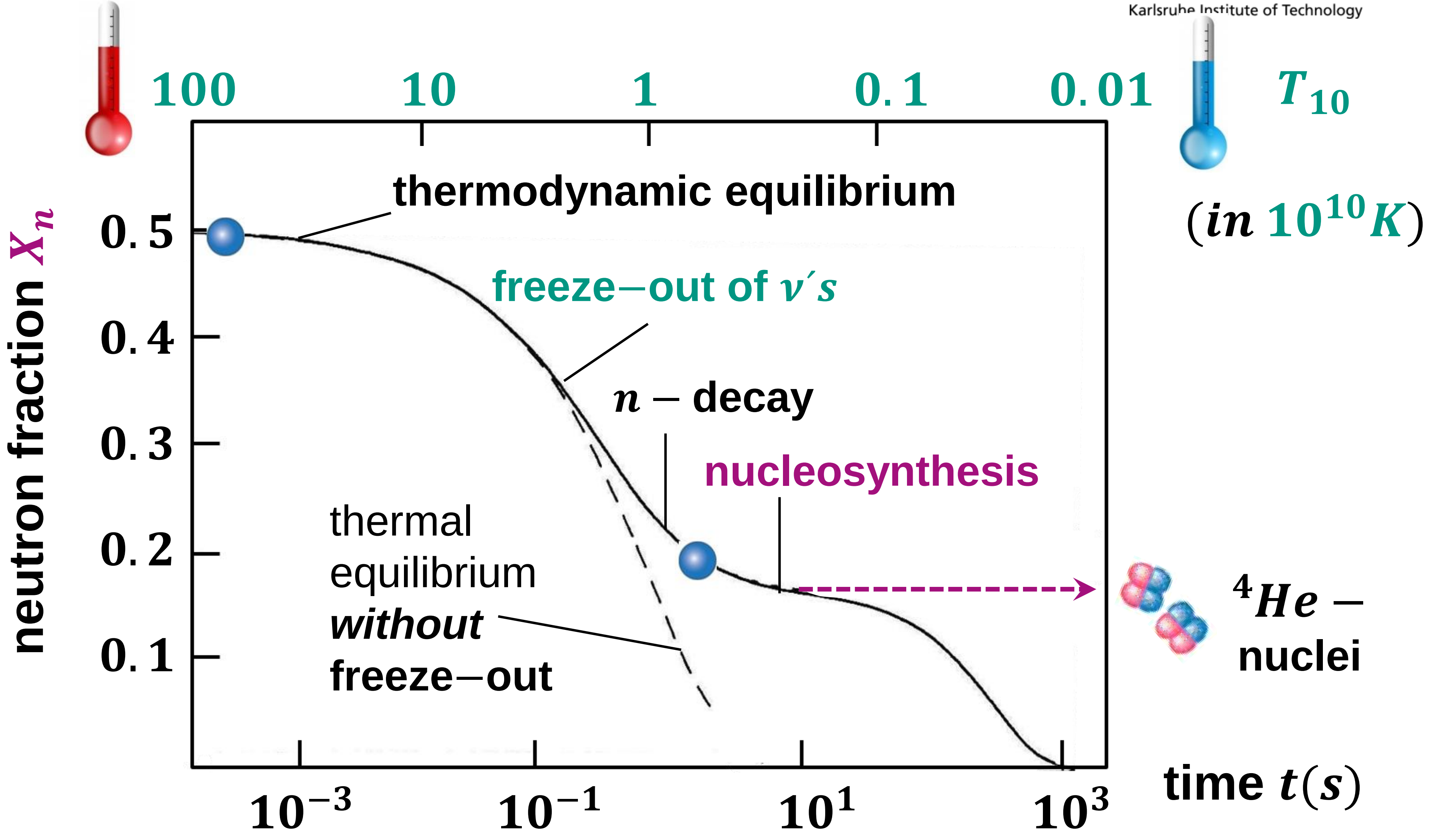
Primordial nucleosynthesis: all steps

- **Overview BBN**

- fraction X_n of free n

$$X_n = \frac{N(n)}{N(p) + N(n)}$$

- **after $t > 60\ s$: all neutrons end up in ^4He**



Primordial nucleosynthesis & parameter η

- **Fusion yield of light elements depends on the size of baryon asymmetry η**

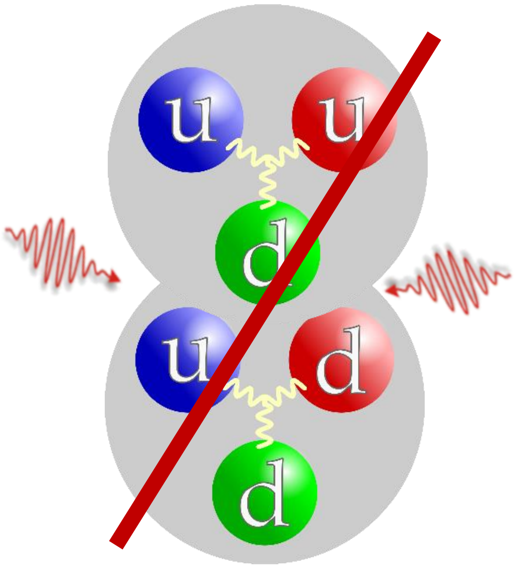
  - theoretical estimates of the **light element fusion yields** have been performed for **different values of η**:

$$\eta = \frac{n_B - n_{\overline{B}}}{n_\gamma}$$

$$\eta = 10^{-6} \dots 10^{-12}$$

η

different fusion yields



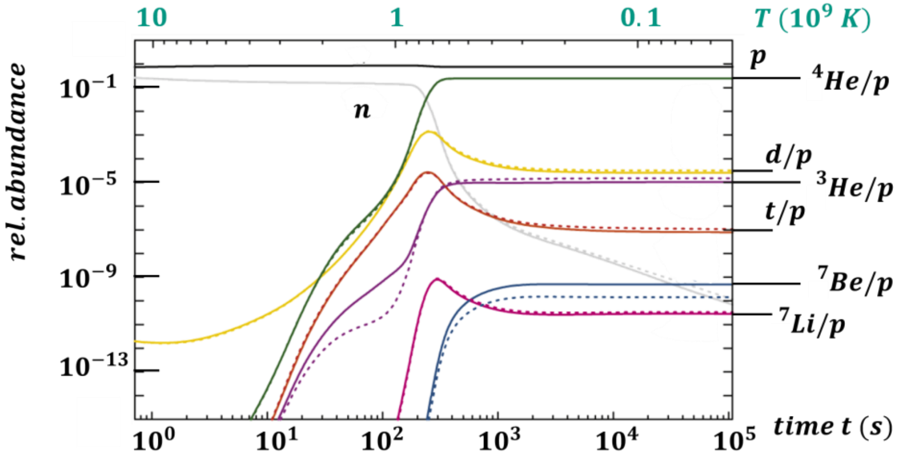
observed precision **fusion yields** of light elements in BBN

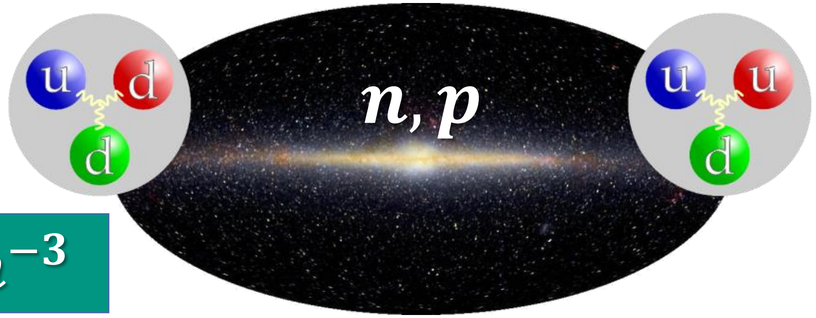
estimates of **baryon asymmetry paramter η**

Primordial nucleosynthesis & baryon density ρ_B

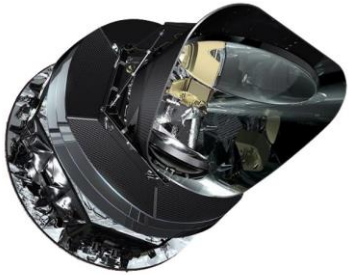
- Fusion yield of light elements depends on the value of the **baryon density ρ_B**

BBN calculations

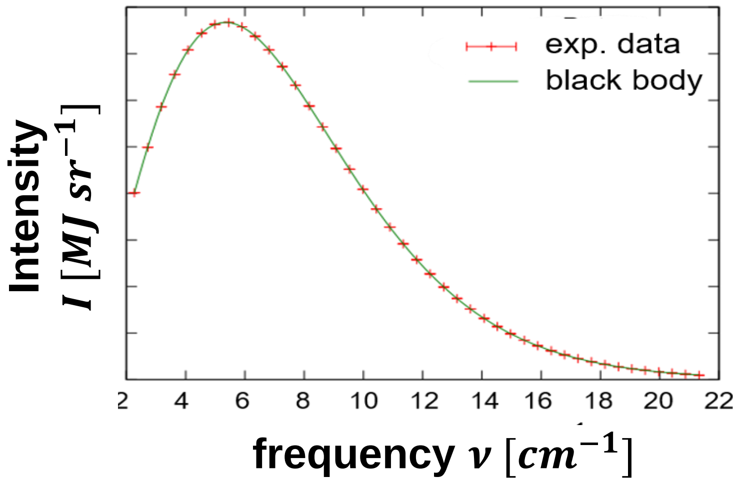


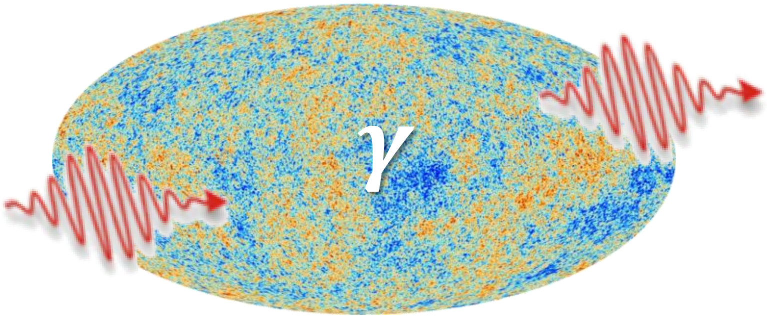


$\rho_B = ?\ cm^{-3}$



Planck distribution*





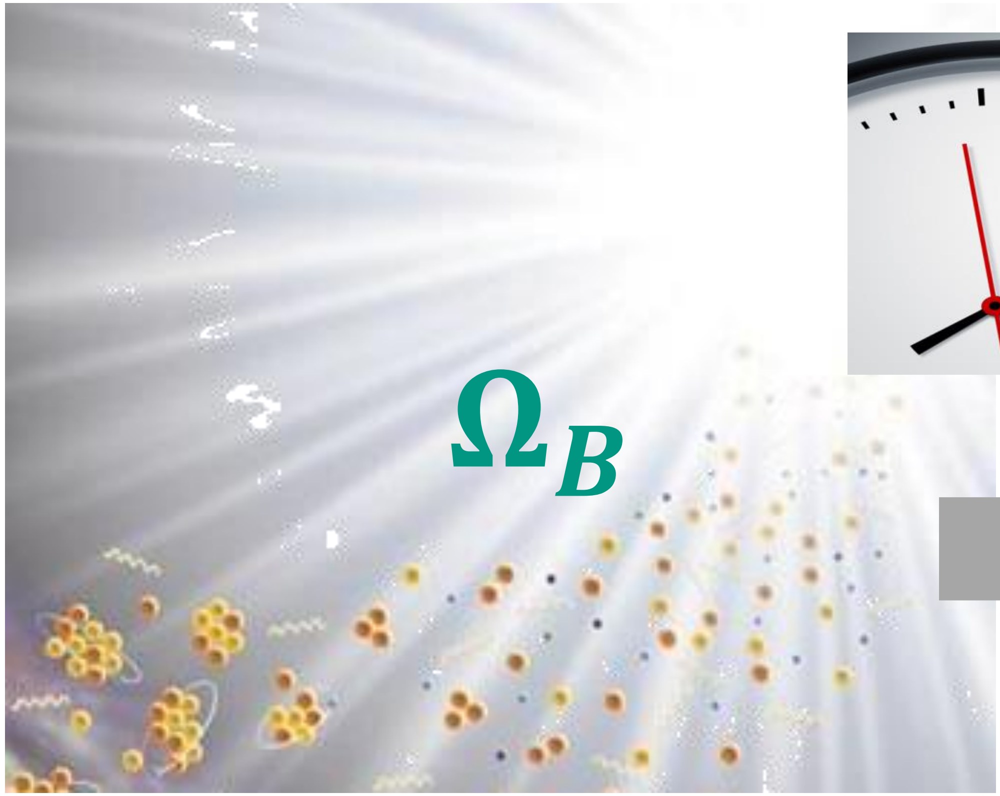
$n_{photons} = (412 \pm 2)\ cm^{-3}$

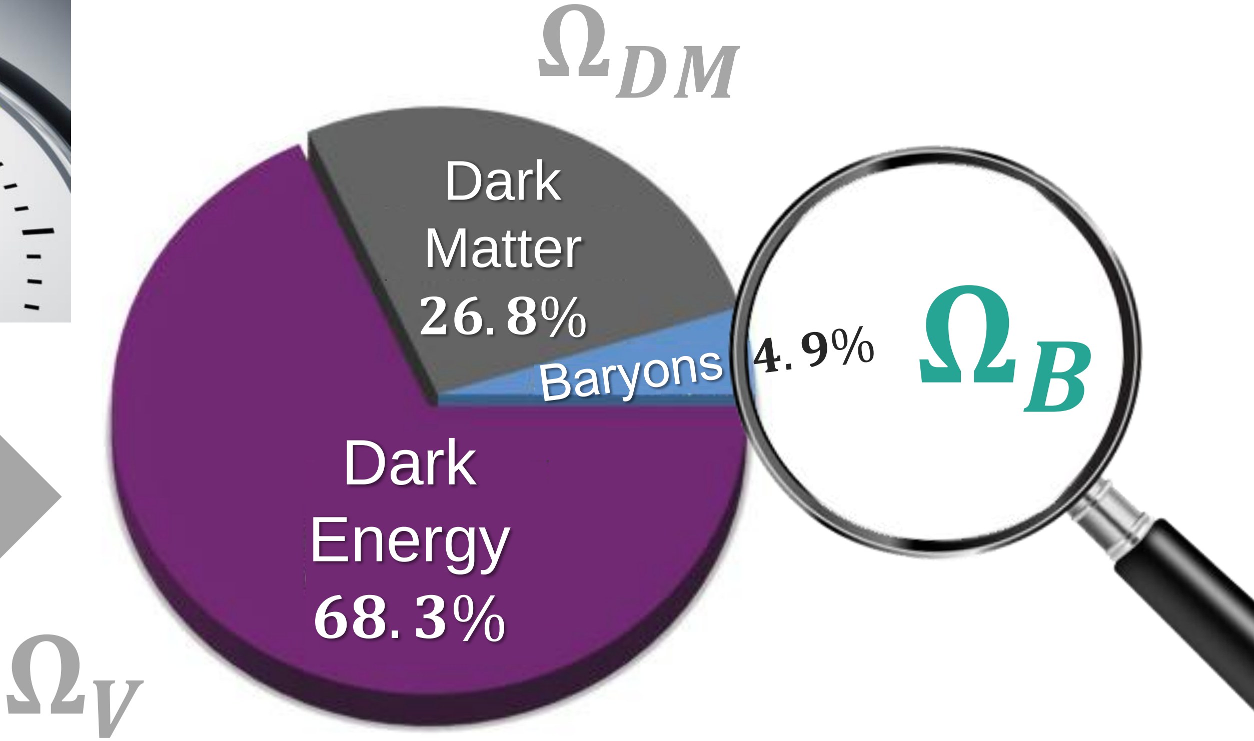
*see Chap. 3.2

Primordial nucleosynthesis & baryon density Ω_B

- **Extracting the baryon density parameter Ω_B from BBN synthesis of elements**

  - a spectroscopic determination of **light elements d, ^3He, $^{6,7}Li$** determines Ω_B





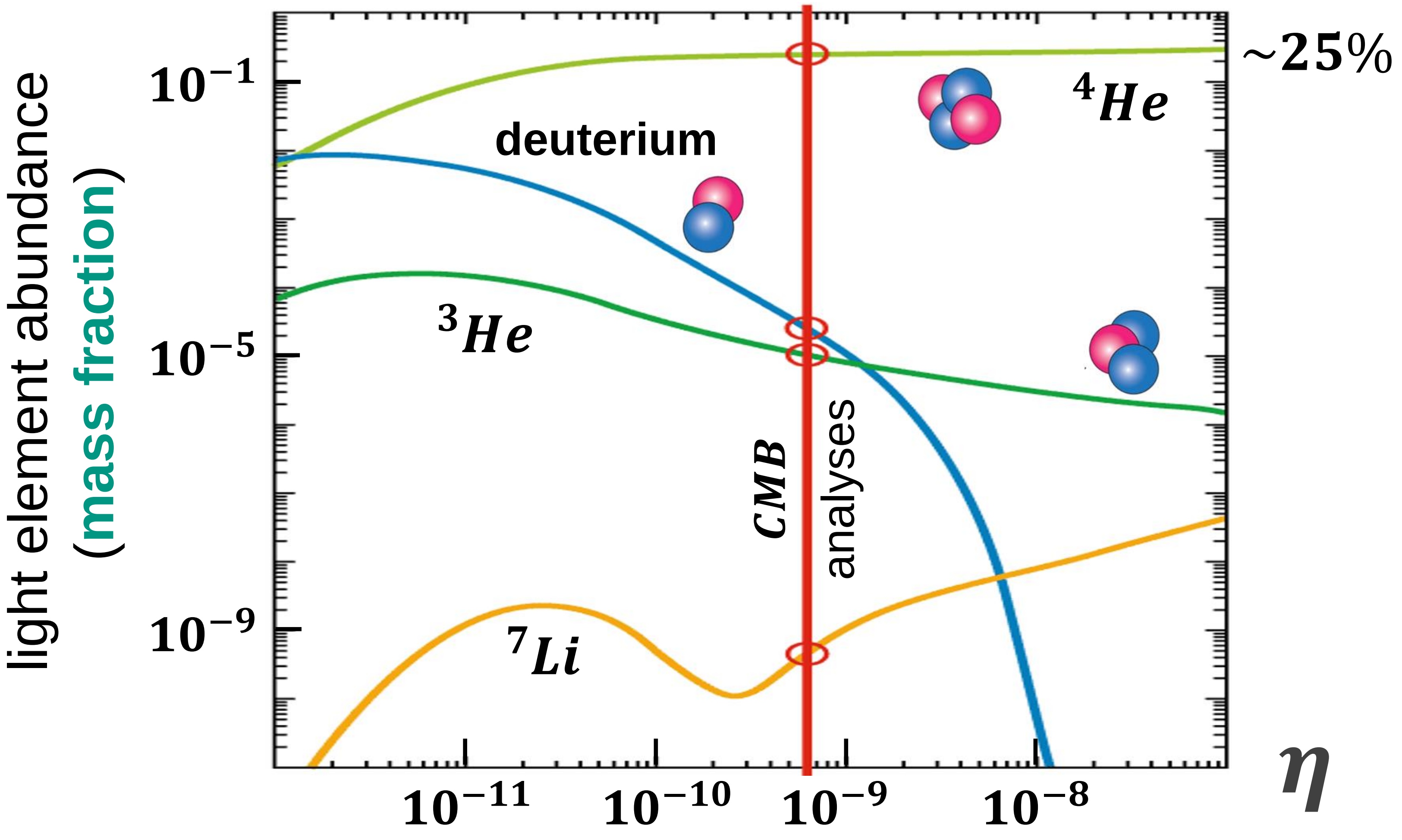
Primordial nucleosynthesis: calculations

- **Results – fusion yields for light elements as function of parameter η**

- fusion yield of ^4He **is dominating** & increases for a **larger baryon asymmetry η**

- yield of **other isotopes** is much smaller & covers many orders of magnitude

- yield of **each isotope** as $f(\eta)$ **is unique** & thus important for cross–checks

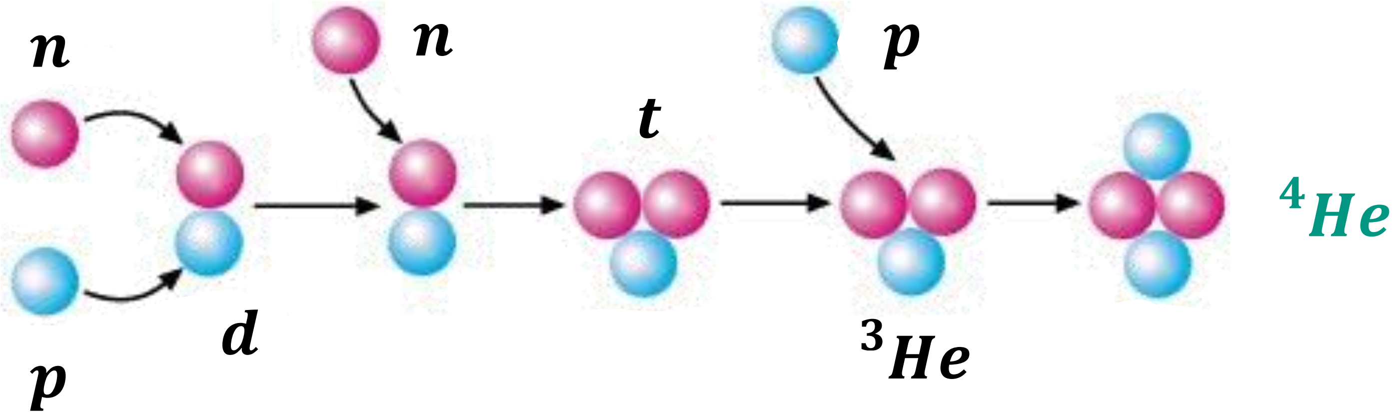


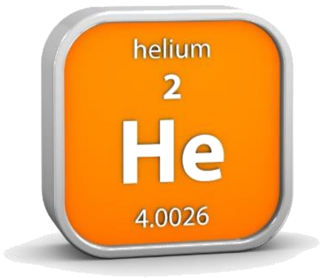
Primordial nucleosynthesis & yield of ^4He

- **Synthesis of light elements essentially stops at ^4He**

  - fusion beyond ^4He is difficult: this is a **very stable** (doube magic) **nucleus**
  - ^4He is **difficult to destroy**: also tiny cross–sections for capture of p, n, d
  - almost all neutrons are incorporated into ^4He (⇨ available n limits its formation)
  - usual parameter in literature: primordial ^4He – **mass fraction Y_p**

one of many BBN **reaction pathways** to ^4He – formation





Primordial nucleosynthesis & yield of ^4He

- **A quick estimate on the ^4He – mass fraction Y_p as function of (n,p) ratio**

  - we start with the calculated (n,p) – ratio of $\mathbf{1:7}$ at the onset of BBN

  - we then form ^4He via the initial ratio out of $(2n + \underbrace{2p) + 12\,p}_{}= {}^4He + 12\,p$

$$2\,n \quad : \quad 14\,p$$

  - we now calculate the **^4He mass fraction Y_p** $= {}^4He/(2\,n + 14\,p)$

    with $m_n = m_p = m_{nucl}$ we have $= 4\,m_{nucl}\,/\,16\,m_{nucl} = 0.25$

  - for the (n,p) – ratio of $\mathbf{1:7}$ we expect a **^4He mass fraction $Y_p = 0.25$**