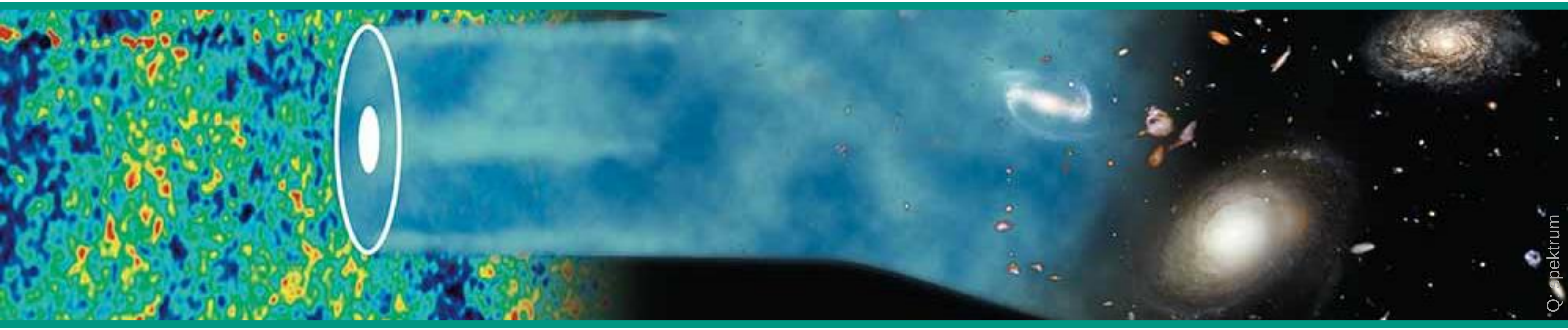
Karlsruhe Institute of Technology

Introduction to Cosmology

Winter term 23/24 Lecture 9 Dec. 19, 2023



Recap of Lecture 8

- Cosmic Microwave Background radiation CMB: a unique tool in cosmology

  - detected by ***Penzias*** & ***Wilson*** $(\mathbf{1964/65})$, using the Holmdel horn antenna

  - **perfect black–body** spectrum: $\boldsymbol{\nu = 100 \ldots 200\ GHz, \lambda = 0.5 \ldots few\ mm}$

  - origin: matter–antimatter annihilation (tiny **baryon asymmetry parameter** $\boldsymbol{\eta}$)

  - $\mathbf{3}$ **Sakharov** conditions: $\mathbf{1.}$ violation of $\boldsymbol{CP, C}$ $\mathbf{2.}$ $\boldsymbol{B}$ – violation $\mathbf{3.}$ ~~therm. equilibr.~~

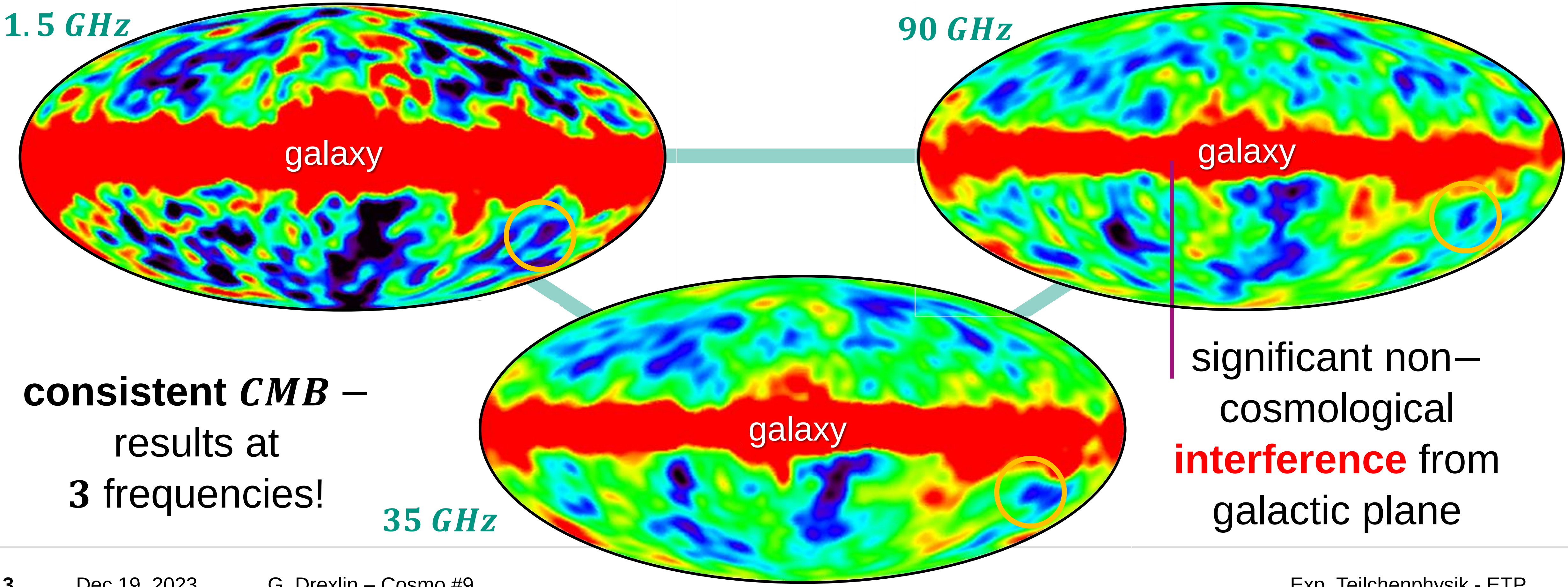
  - $\boldsymbol{COBE}$: $\boldsymbol{FIRAS}$ (*J. Mather*) & $\boldsymbol{DMR}$ (*G. Smoot*): $\boldsymbol{T = 2.7\ K}$ & $\boldsymbol{\Delta T/T}$ fluctuations

  - separate the primordial $\boldsymbol{CMB}$ – **signal** from galactic ´**foreground**´ noise

DMR: CMB – anisotropies at different frequencies

- **Noise (´foreground´) is superimposed on primordial CMB signal**

  - key method to model the foregound noise: observe at different **frequency** bands

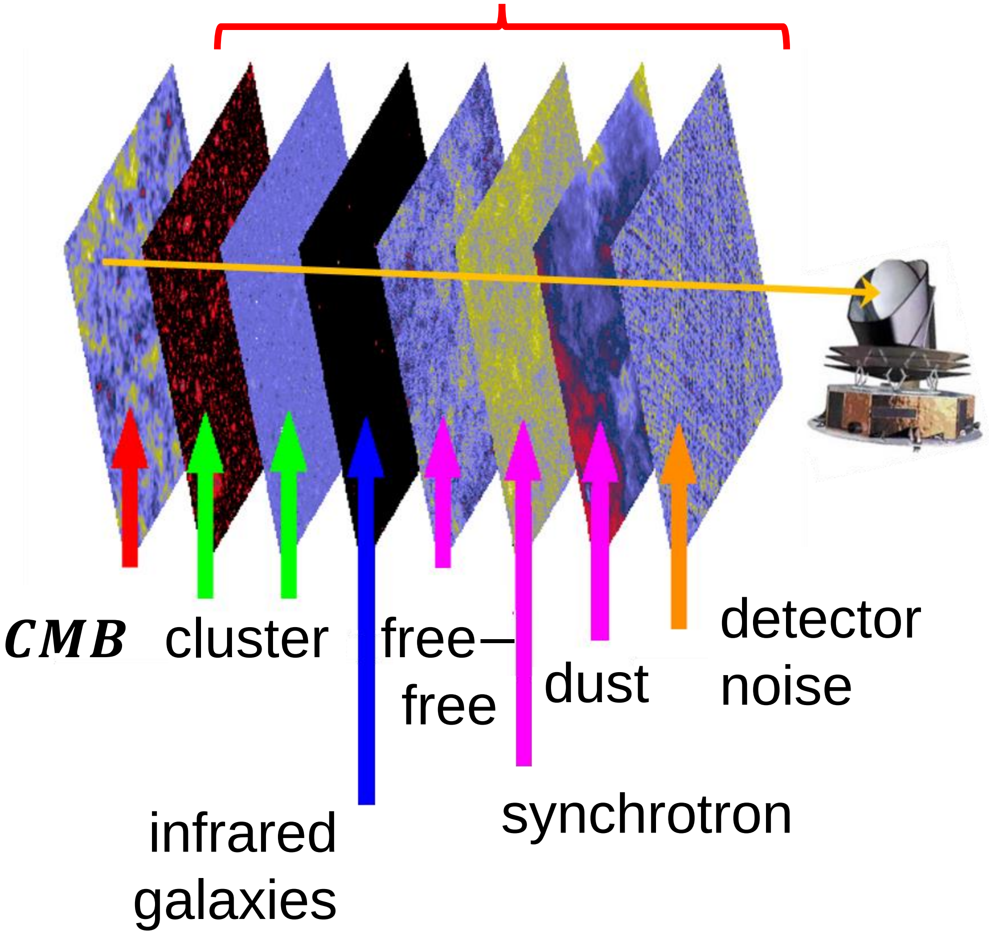


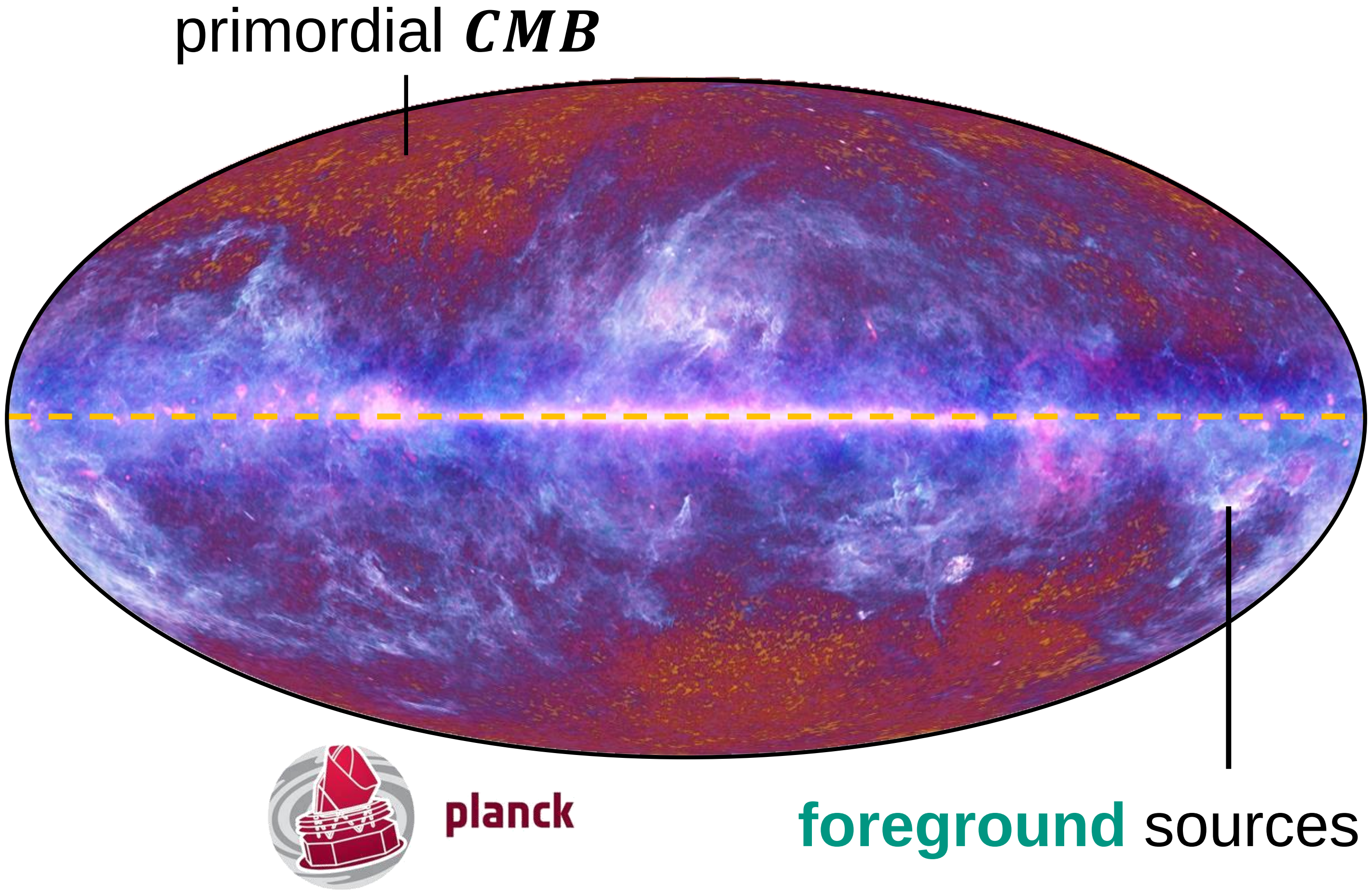
consistent CMB –
results at
3 frequencies!

significant non–
cosmological
interference from
galactic plane

CMB and galactic foreground

- **Noise−layers (´foreground´) are superimposed on primordial CMB signal***

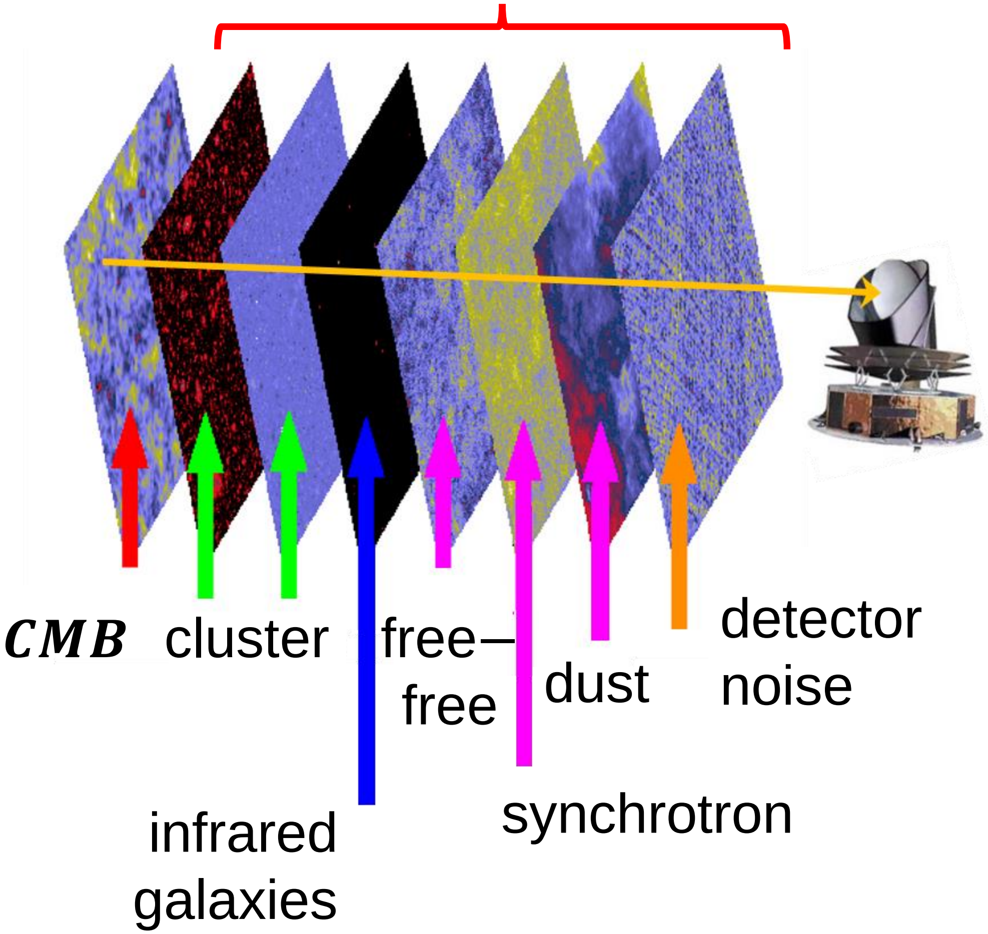


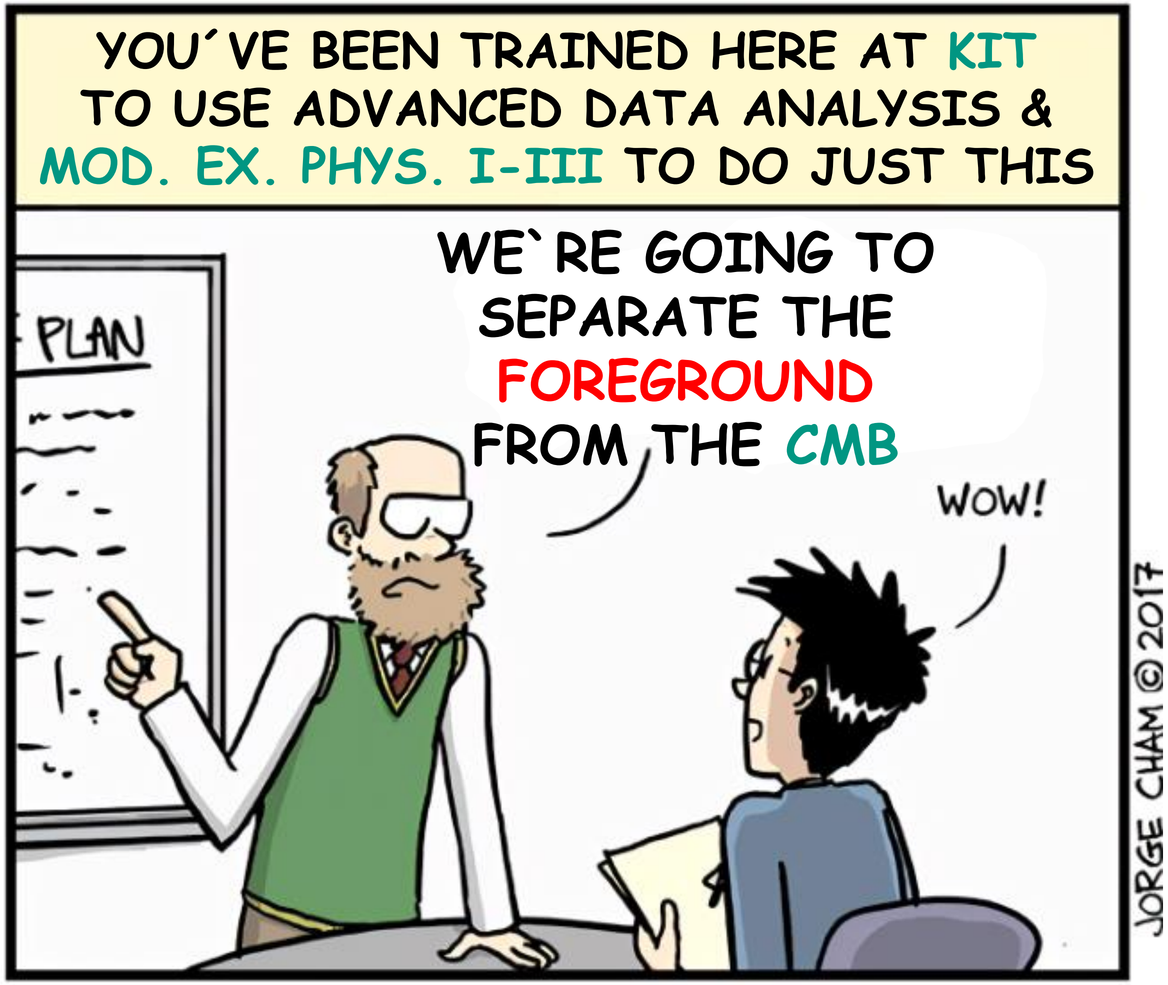


***Mollweide projection**: aligned to galactic plane

CMB and galactic foreground

- Noise−layers (´foreground´) are superimposed on primordial CMB signal*





CMB and galactic foreground: cold dust clouds

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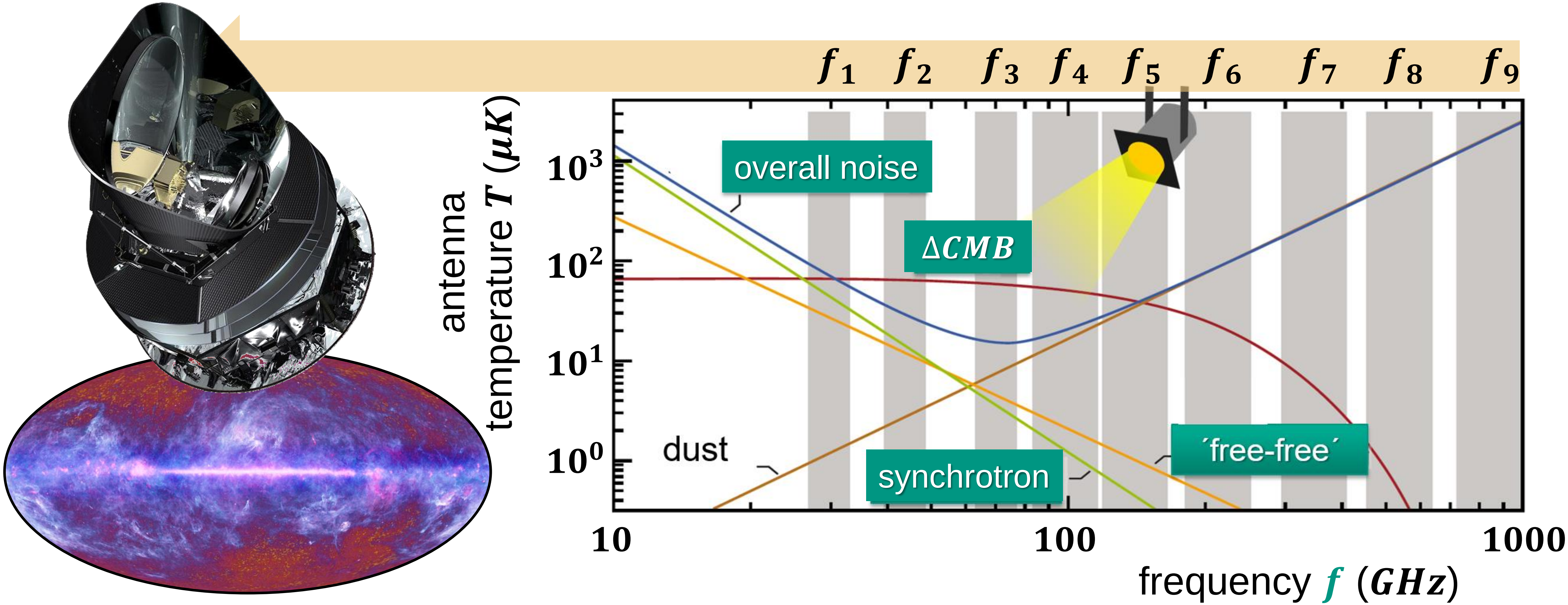
- **Noise (´foreground´) has different f – dependence as primordial CMB signal**



CMB and galactic foreground: cold dust clouds

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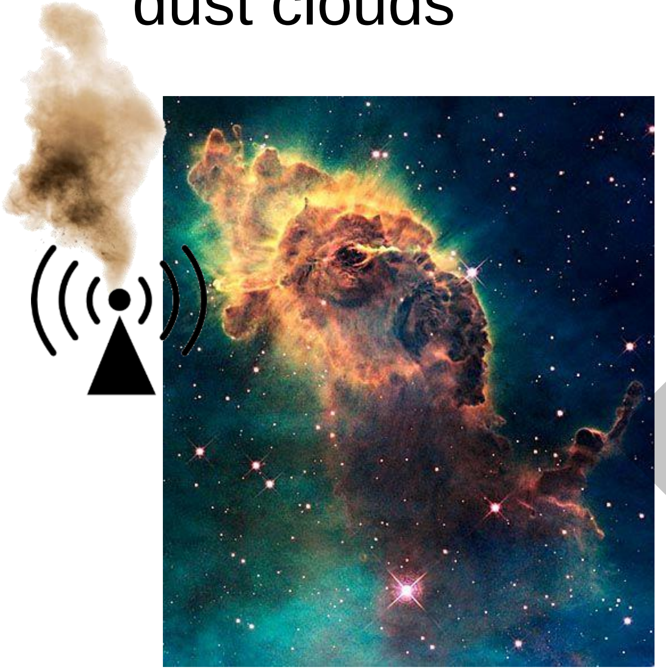
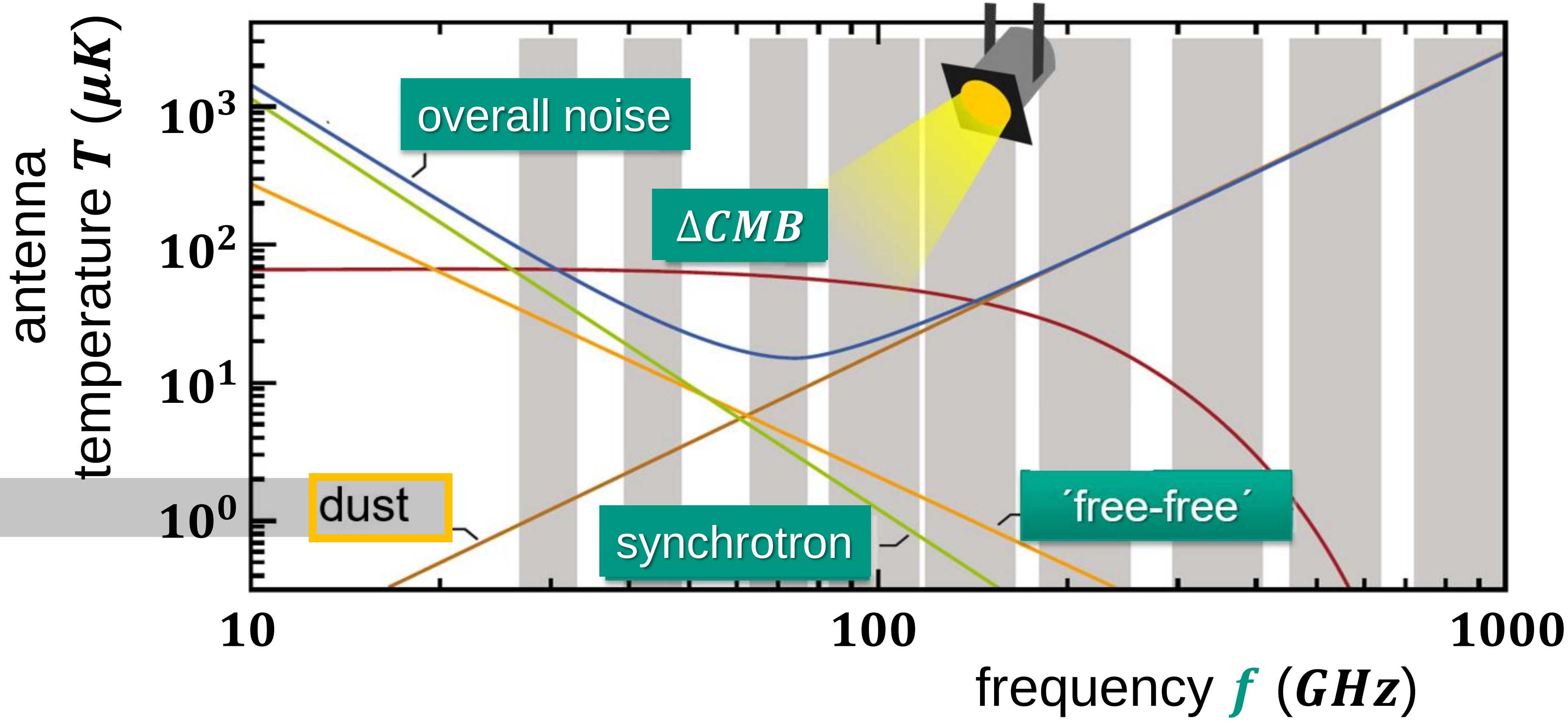
- **Noise (´foreground´): cold dust clouds emit thermal radiation at high f**

- **thermal** & **non−thermal** foreground sources in our galaxy

- cold, interstellar dust clouds



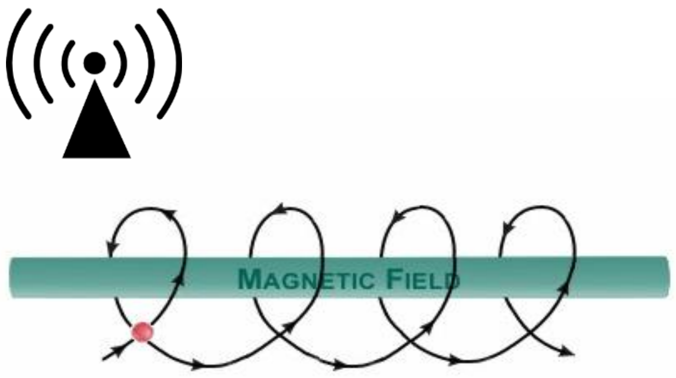


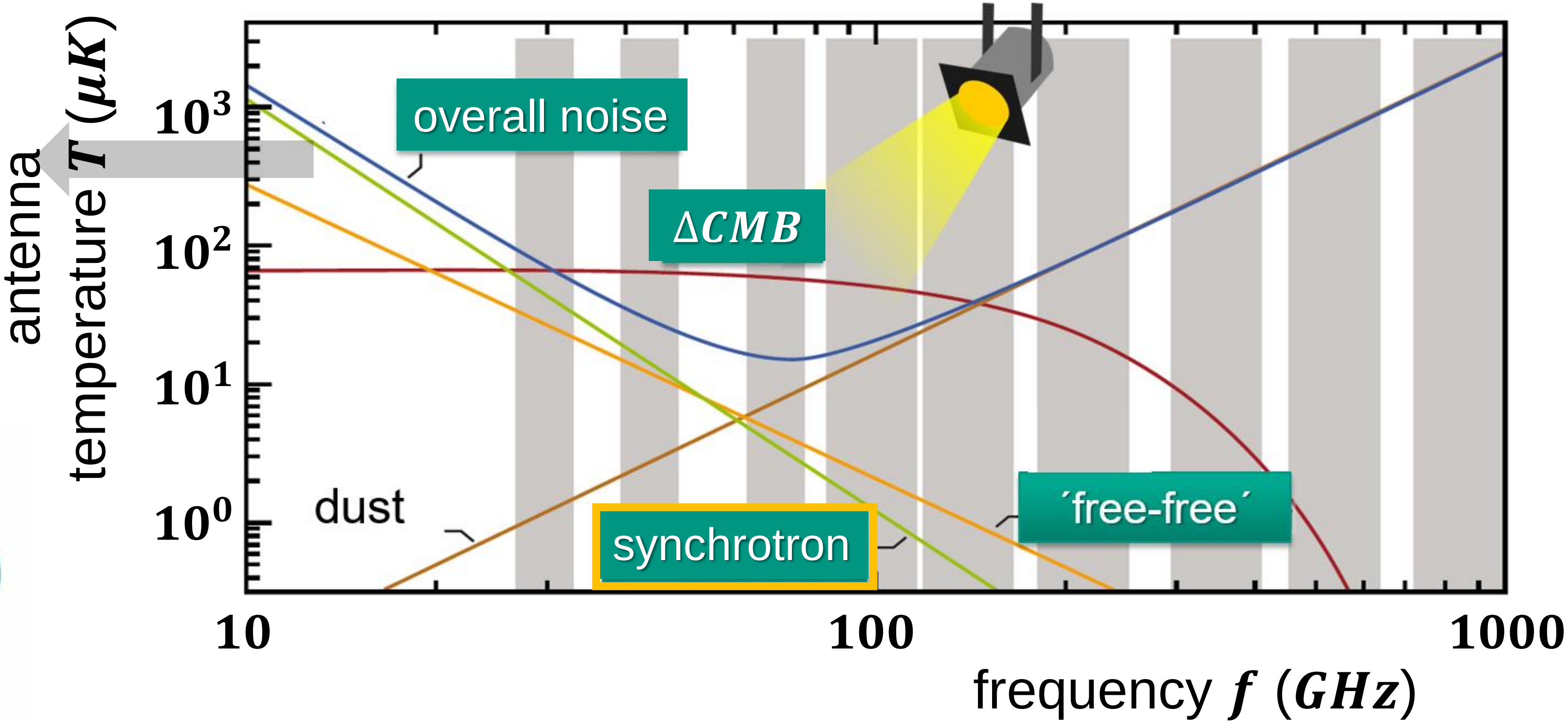
CMB and galactic foreground: relativistic e^-

- **Noise (´foreground´): GeV – electrons emit synchrotron radiation at low f**

- thermal & **non–thermal** foreground sources in our galaxy:

- **relativistic e^-**
  $E = 10 \ldots 100\ GeV$
  spiral in B – fields
  (few nT)



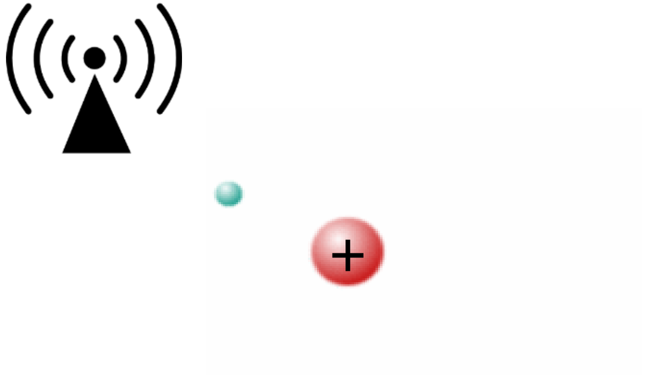


CMB and galactic foreground: free−free emission

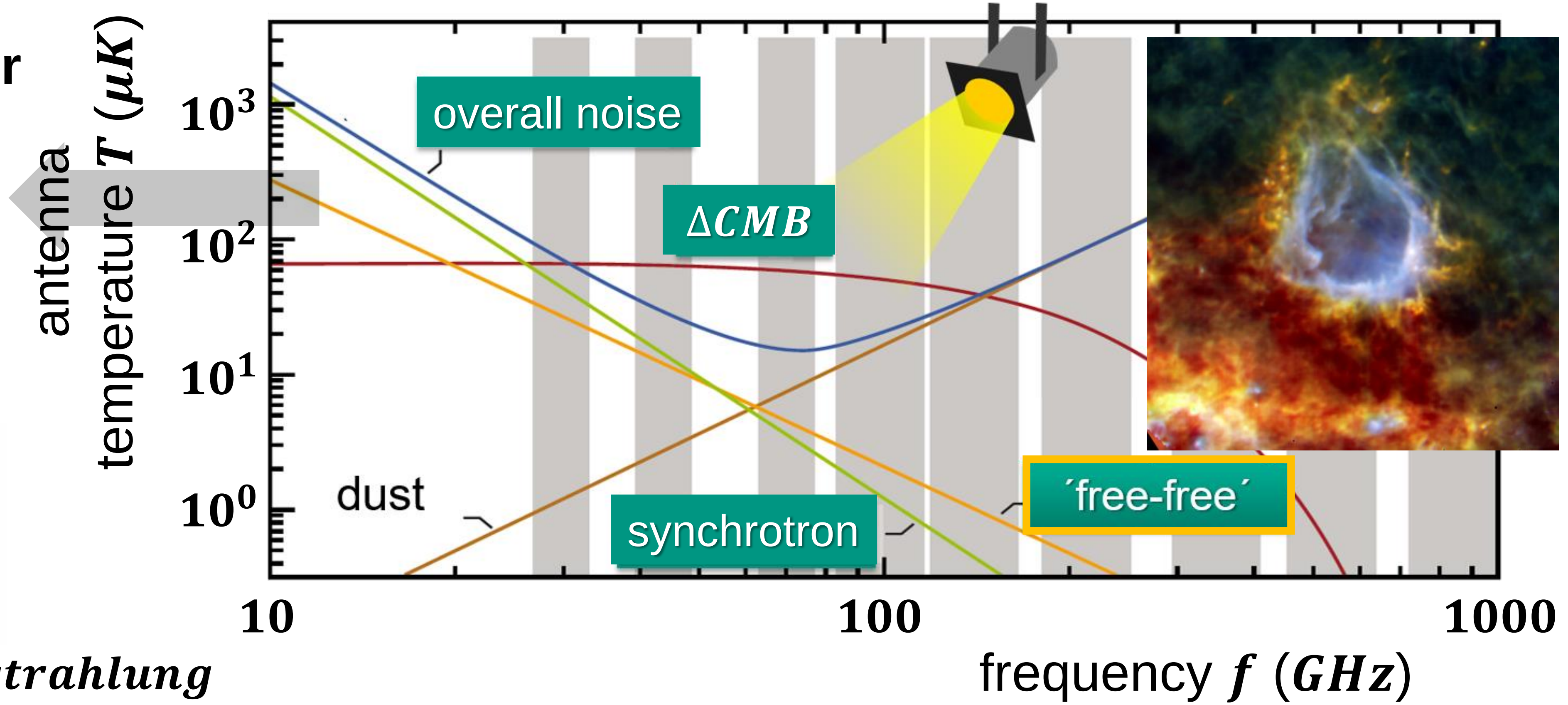
- Noise (´foreground´): thermal electrons emit bremsstrahlung at low f

- **thermal** & **non−thermal** foreground sources in our galaxy: **free−free scattering**

- **thermal e^- scatter** off of **ions** in gal. **$H - II$ regions***



bremsstrahlung



*ionised atomic hydrogen H

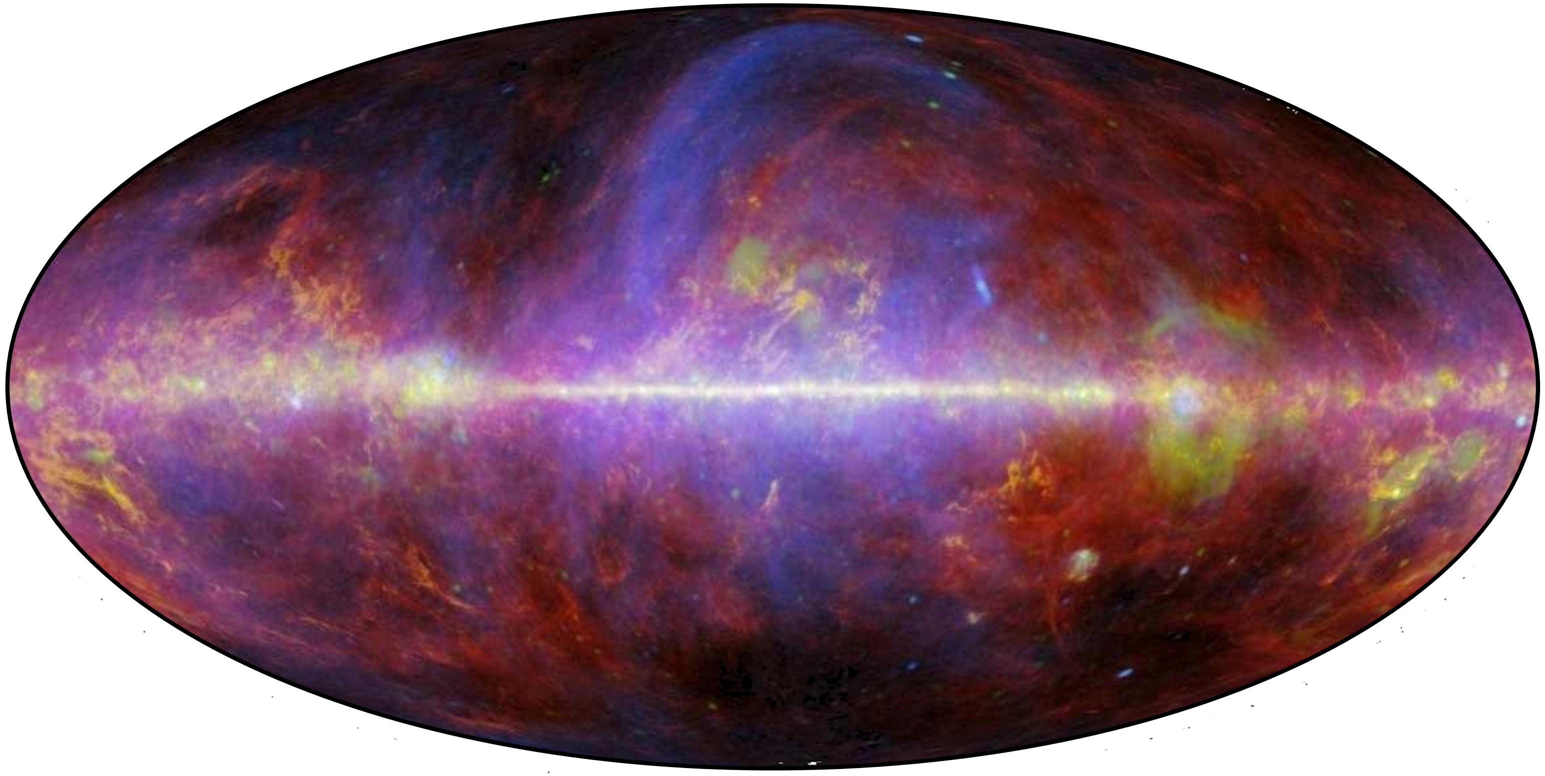
CMB and galactic foreground: separation ansatz

- **Noise (´foreground´)**

  - turn your noise signal to an **interesting measurement**

  - detailed **Planck maps on**

    - **galactic dust**
    - **galactic B – fields**
    - **galactic HII – regions**

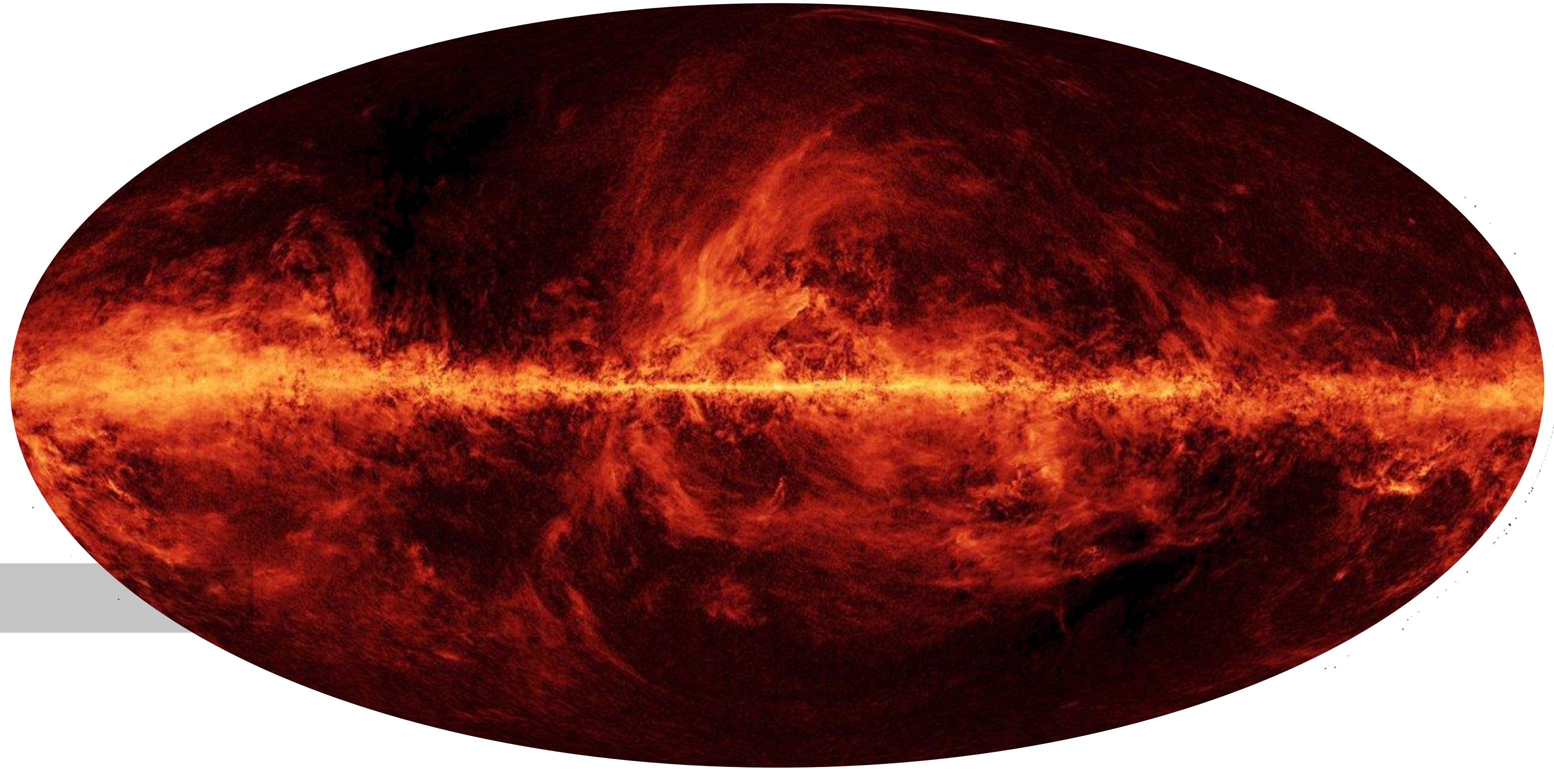


CMB and galactic foreground: cold dust clouds

- **Noise (´foreground´) has different $f-$ dependence: component separation**

- **thermal** & **non−thermal** foreground sources in our local galaxy: **cold dust clouds**

- cold, interstellar
  dust clouds



Planck map of the galactic distribution of **dust***

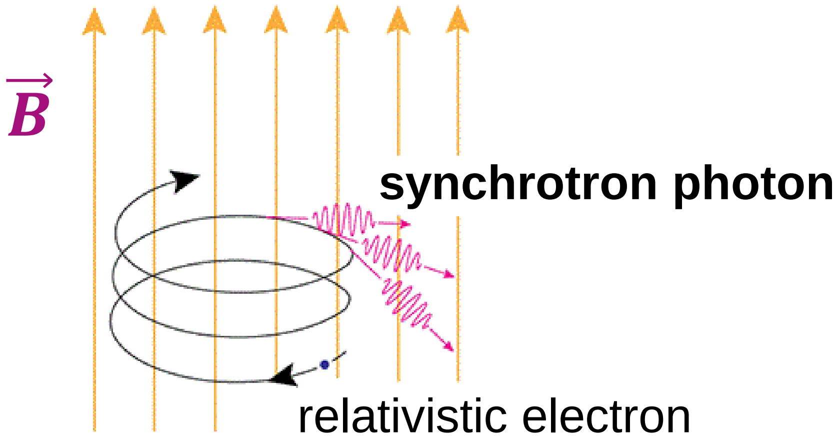
*will be of key impact for ***BICEP2***, see lect. #11

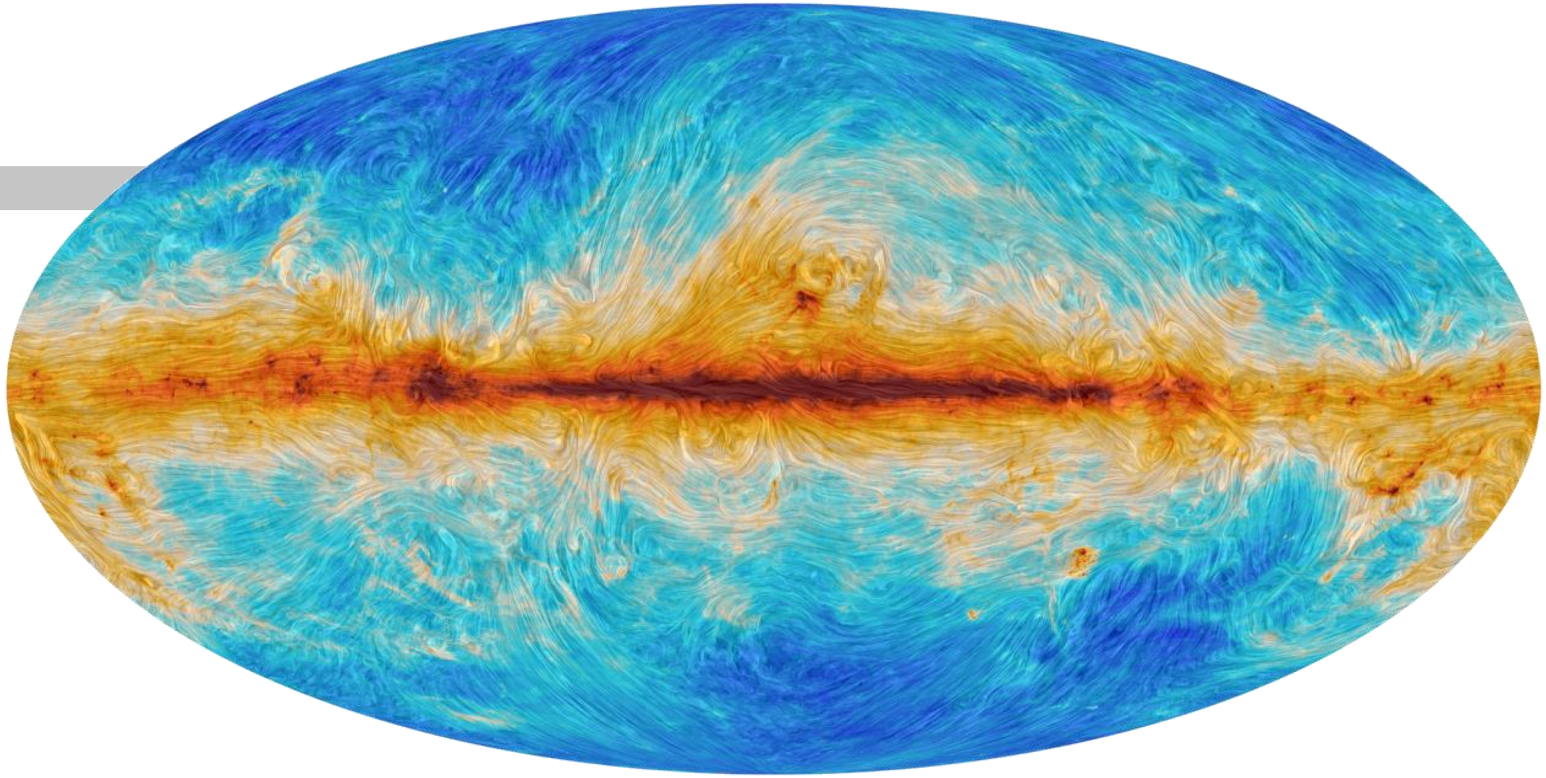
CMB and galactic foreground: synchrotron emiss.

- **Noise (´foreground´) has different f – dependence: component separation**

- unprecedented **mapping the galactic B – field** via **synchrotron radiation**

- **relativistic e^-**
$E = 10 \dots 100\ GeV$
spiral in B – fields
(few nT)





$Planck$ map of the galactic magnetic field*

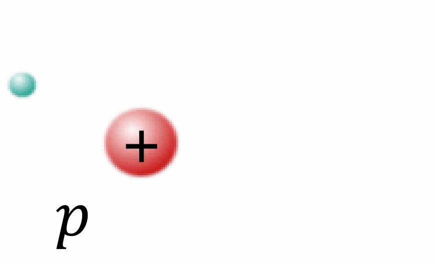
*B – field is coupled to matter, see previous page

CMB and galactic foreground: free–free process

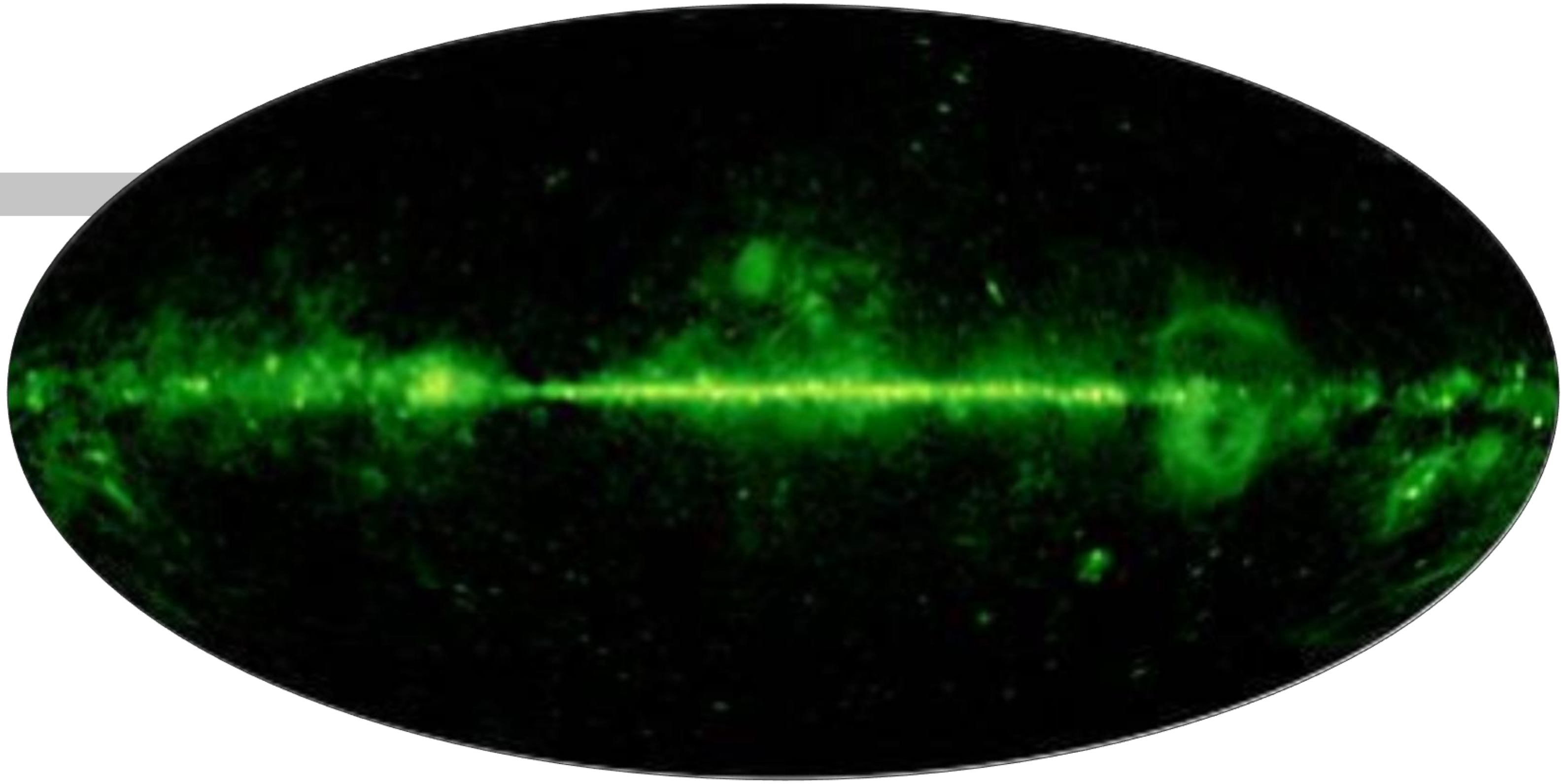
- Noise (´foreground´) has different f – dependence: component separation

- unprecedented **mapping the galactic $H-II$ – regions** via **free–free scattering**

- **thermal e^- scatter** off of **ions** in galactic **$H-II$ regions**



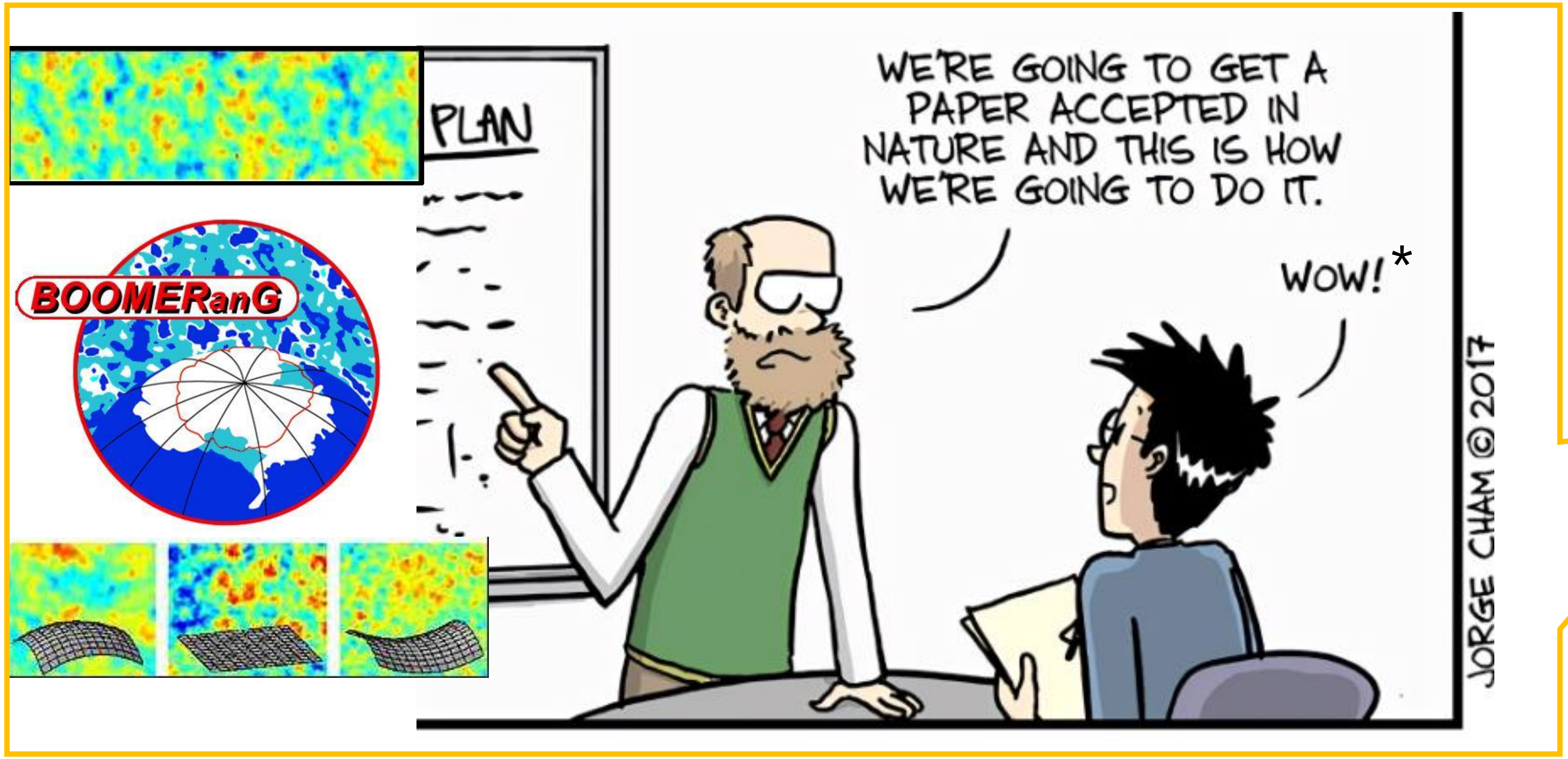
bremsstrahlung

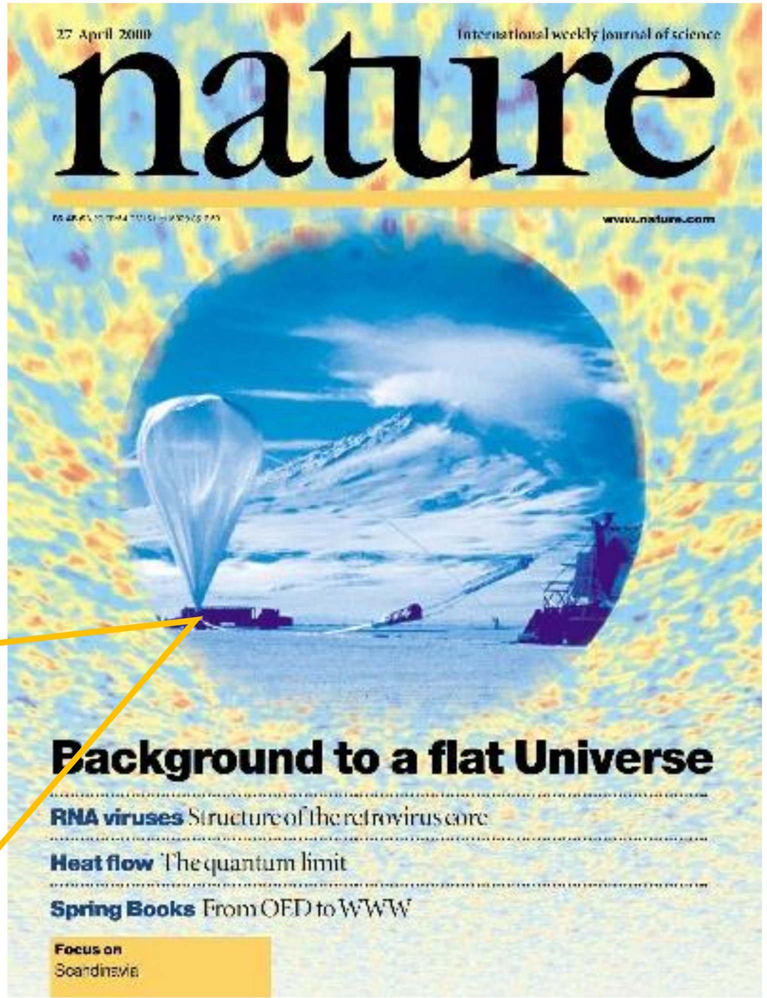


Planck map of the galactic HII – regions

The next step after $COBE$

- 2000: on the track of small CMB angular scales





* [Nature Impact Factor 2022 - Journal Impact Factor (impactfactorforjournal.com)]

The next step after $COBE$: $BOOMERanG$*

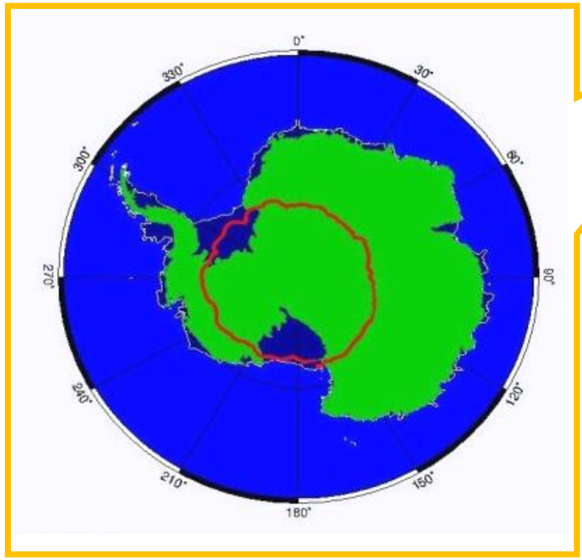
- **2000: on the track of small CMB angular scales**

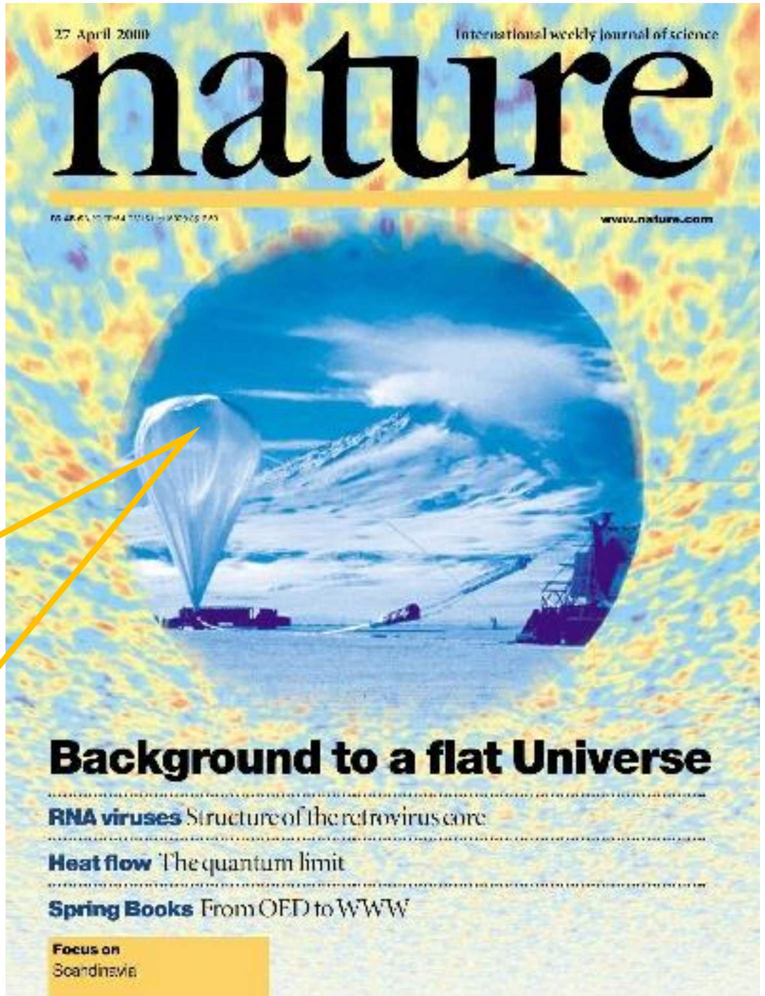
  - **ballon−based** mission from McMurdo in **Antarctica**: $\mathbf{2-week}$ mission possible in circum−polar flight

  - scientific goal: investigate the temperature fluctuations $\Delta T/T$ on **small angular scales $\delta\theta < 1°$**

  - advantage: first CMB map at high resolution

  - disadvantage: $\mathbf{2-week}$ mission limits observed fraction of the sky to a **small patch**





*Ballon Observations Of Millimetric Extragalactic Radiation and Geophysics

The next step after $COBE$: $BOOMERanG$

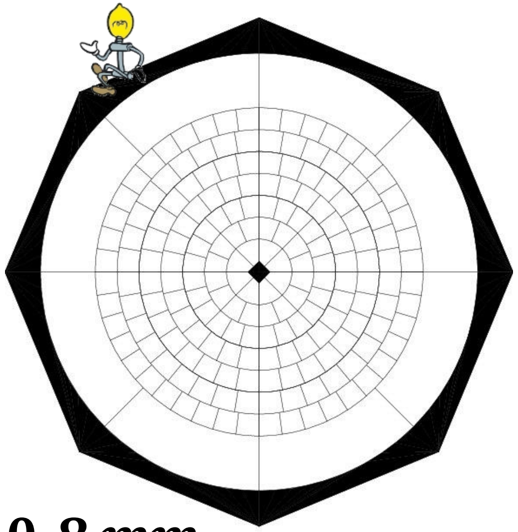
■ **2000: on the track of small CMB angular scales**

- mission in the outer stratosphere ($h = 42\ km$) to avoid the absorption of microwaves of the CMB by atmosphere

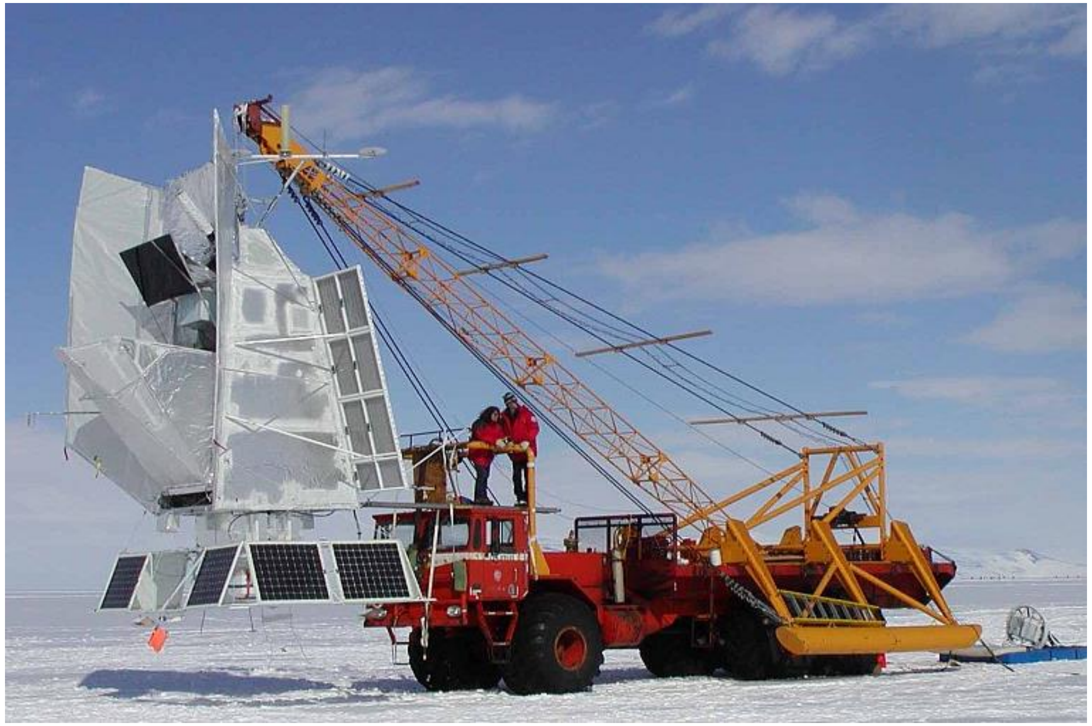
- **mirror with** $\emptyset = 1.3\ m$ focuses CMB to 16 horns, 3 frequencies: 145 / 245 / $345\ GHz$

- radiation detected by **spider−bolometers** ($T = 0.27\ K$)

- wires absorb CMB: small mass → **small heat capacity**



$0.8\ mm$



$BOOMERanG$: evidence for a flat universe

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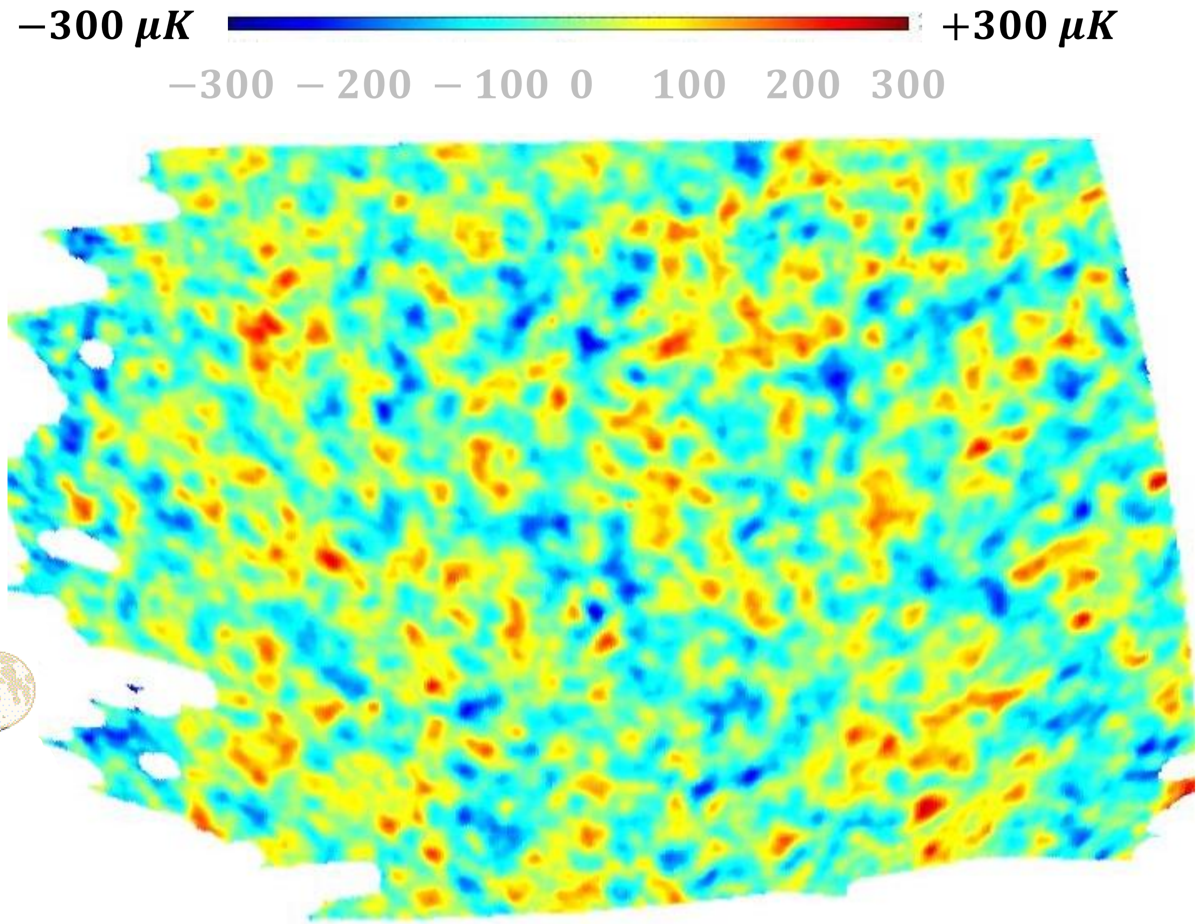
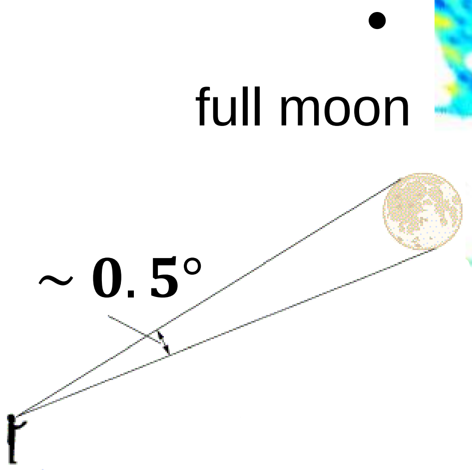
- A small CMB mission provides the first high$-$resolution map of the CMB

  - $BOOMERanG$ observes $\Delta T/T$ fluctuations down to $\delta\theta = 10'$

  - multipole analysis reveals: **maximum amplitude of $\Delta T/T \sim 1°$**

topology of the universe is flat
(Euclidian space)
$k = 0$





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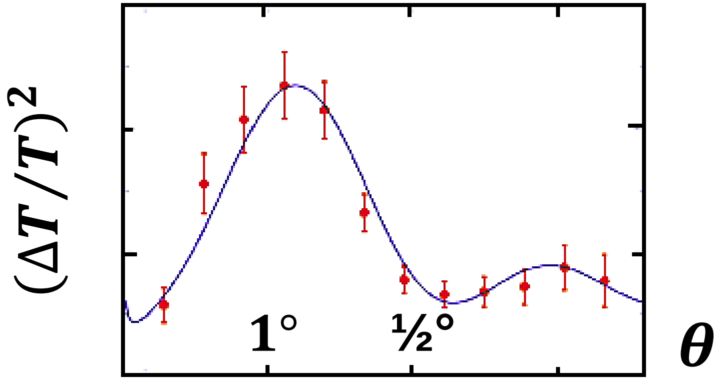
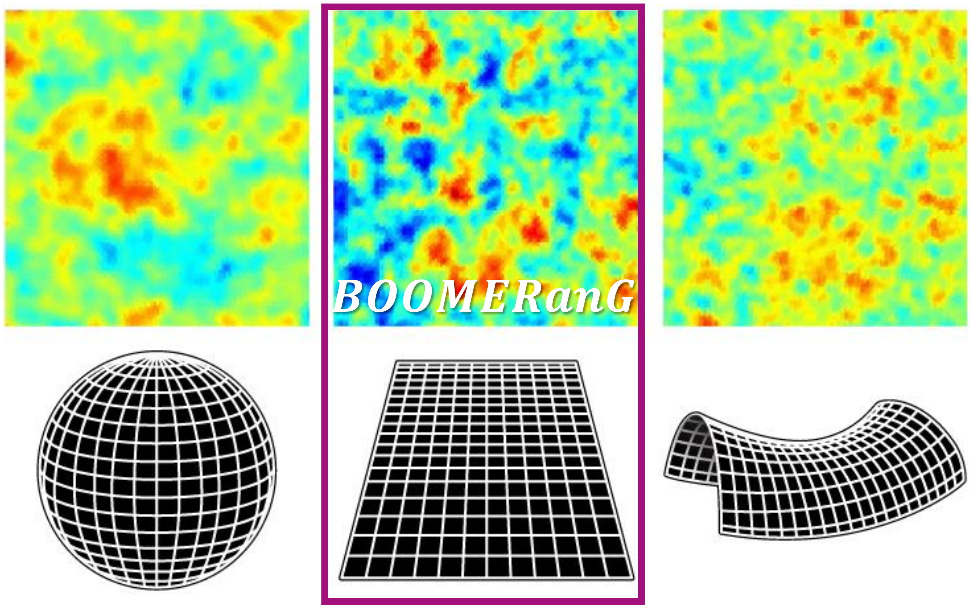
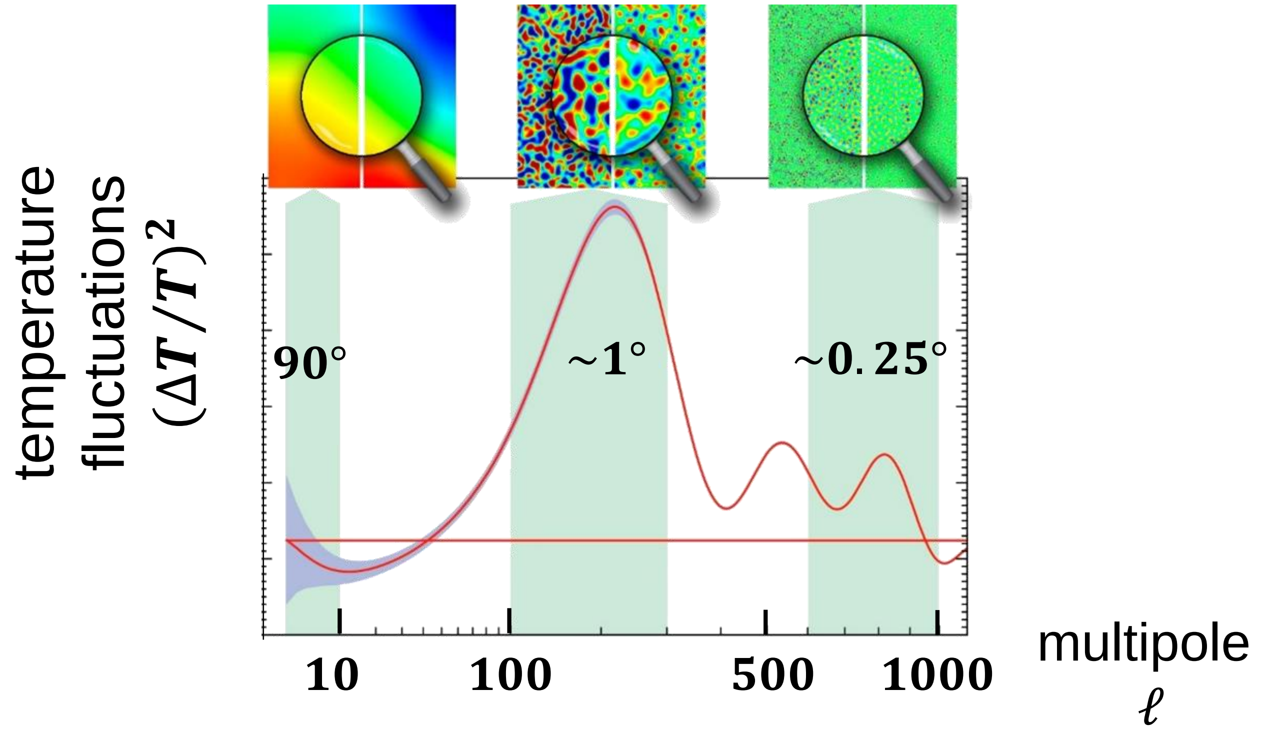
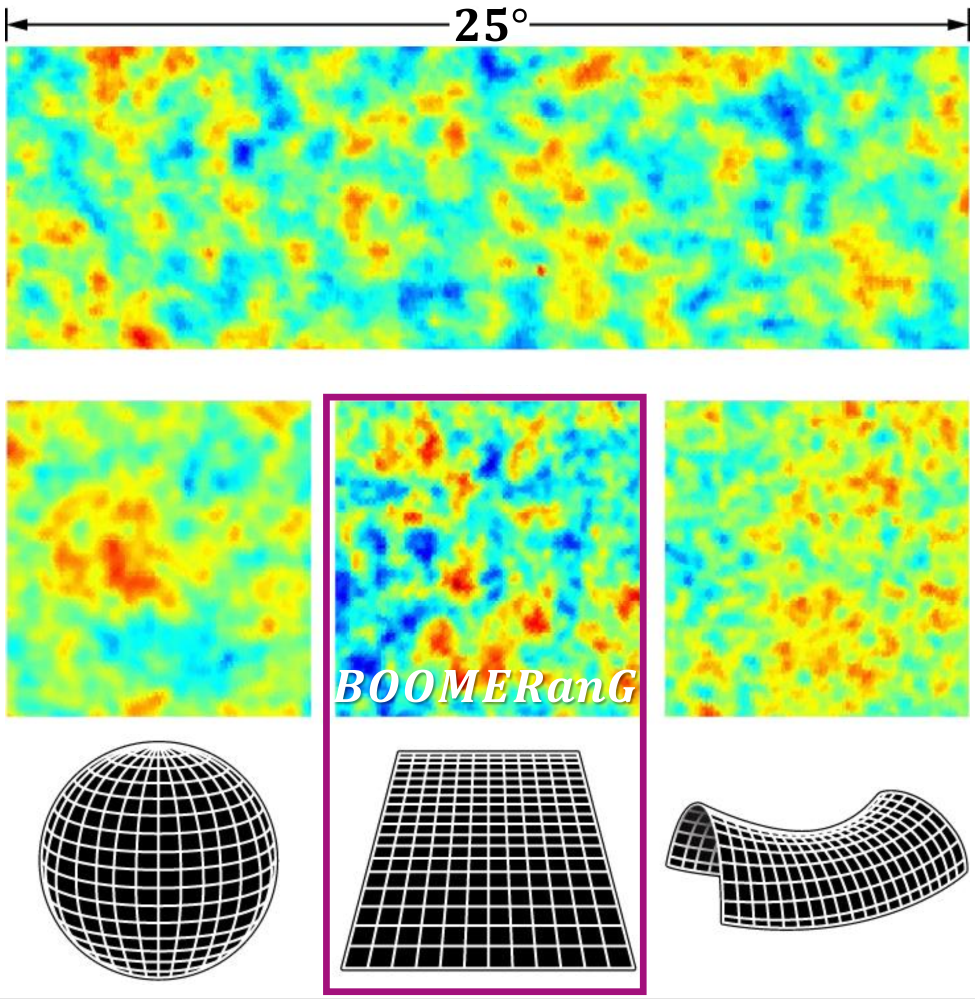




$BOOMERanG$: evidence for a flat universe

- **A small CMB mission provides the first high–resolution map of the CMB**
  - $BOOMERanG$ multipole analysis for $\Delta T/T$ peaks at $\ell = 200$ $\delta\theta = 1°$

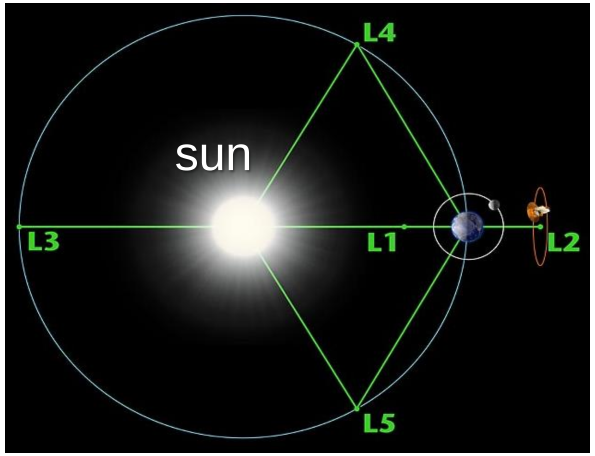


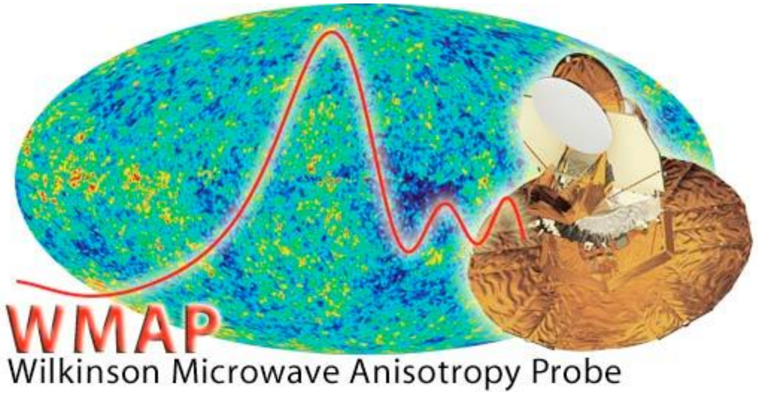


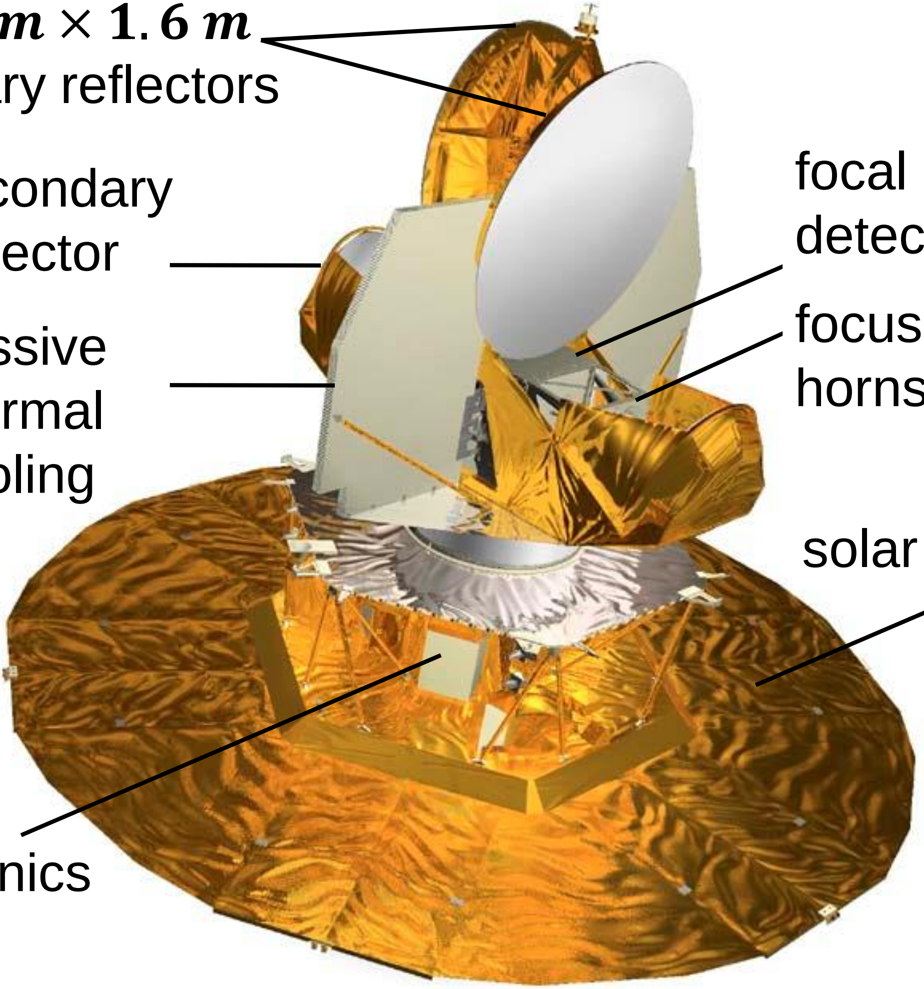
The next big step: $WMAP$ mission at $L2$

- Wilkinson Microwave Anisotropy Probe: $WMAP$

- $NASA$ space probe at the $L2$ Lagrange point for **long–term mission**

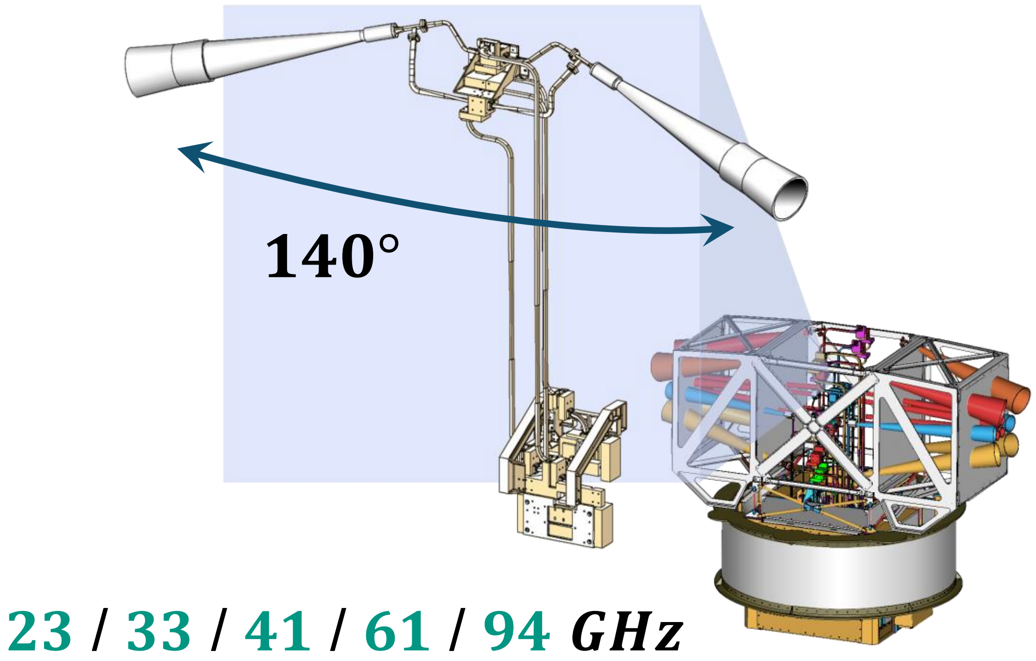




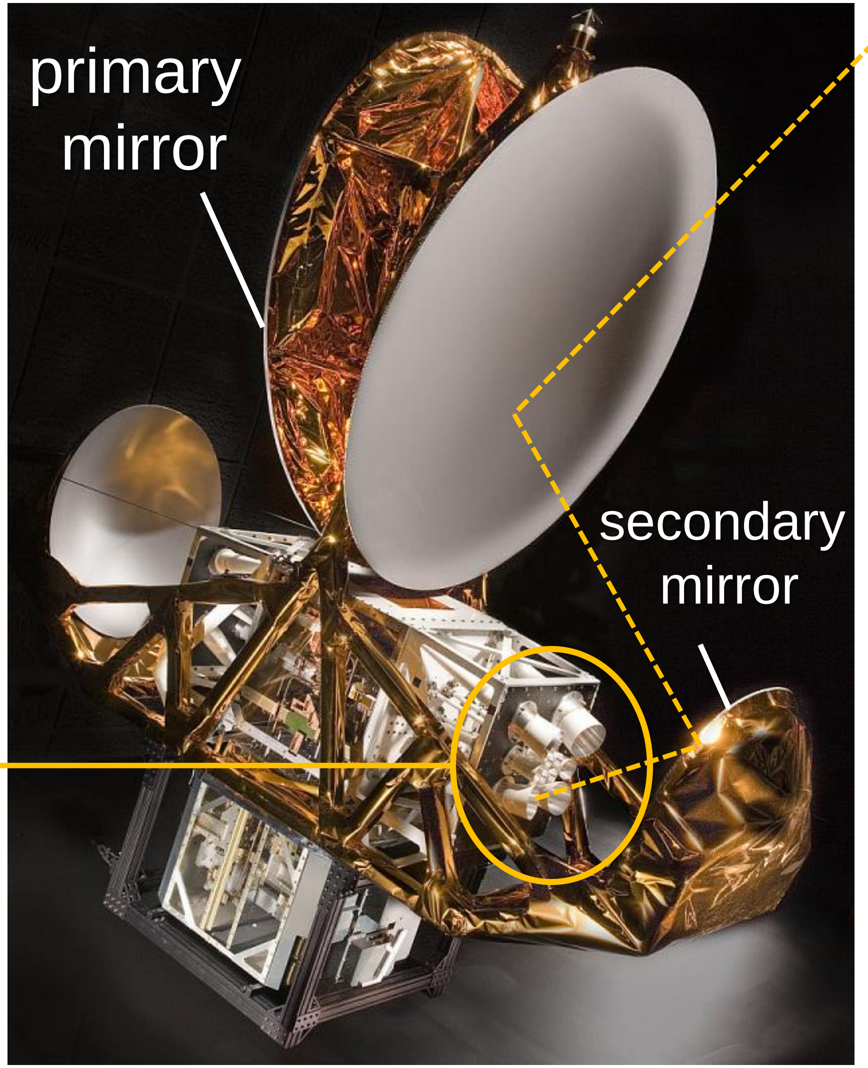


$WMAP$ mission: measurement principle

- $WMAP$: **observing CMB at 5 $frequencies$**
  - **2** large primary mirrors (back–to–back) & **2** secondary mirrors focus microwaves from CMB onto the **focal plane**



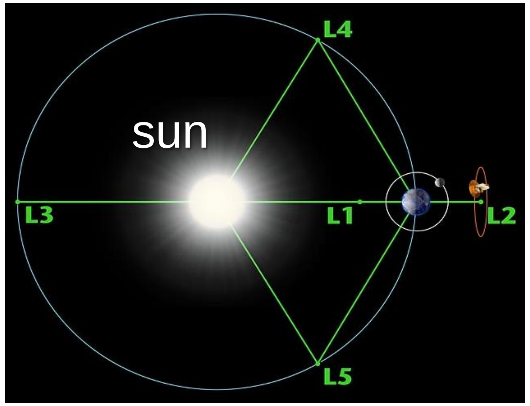
horns of the focal plane array



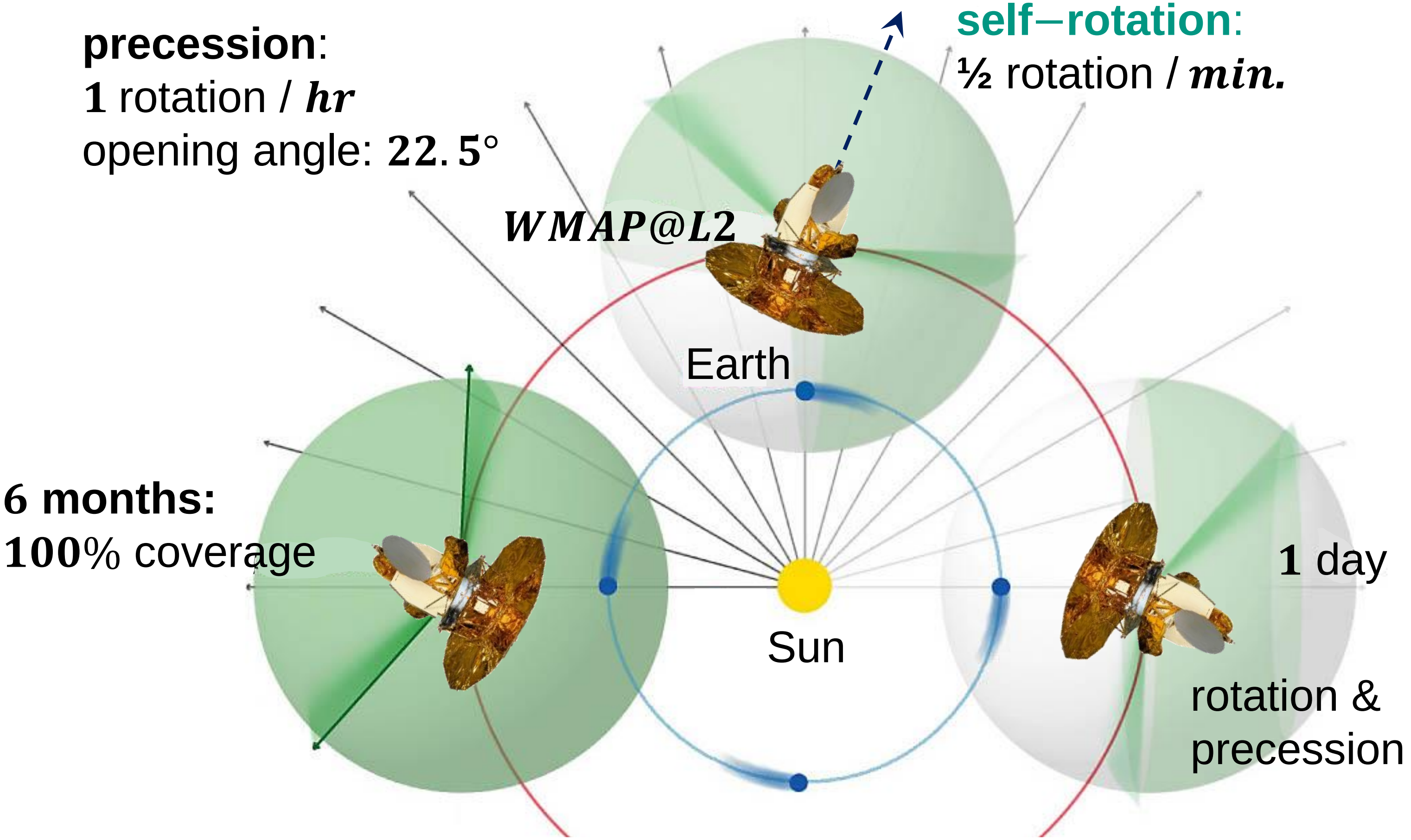
$WMAP$: scanning the sky over many years

- **Observations at the $L2$ point: effects of rotation and precession of probe**

  - coverage of the full sky sphere each 6 months

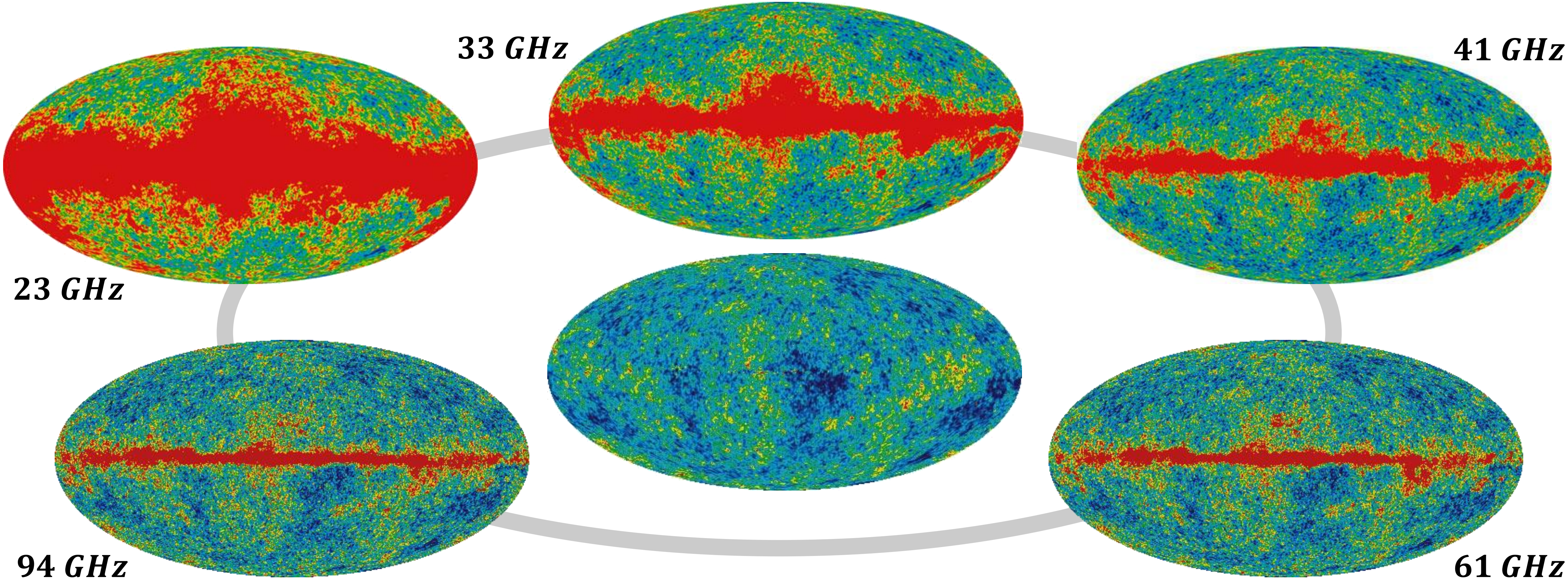


  - $WMAP$ **data analysis**:
    2003: 1 measuring year
    2006: 3 measuring years …
    2012: 9 measuring years



$WMAP$ – measurements at 5 frequencies

- Combining $WMAP$ all−sky maps at different f and superimposed CMB map



$\boldsymbol{WMAP}$: the first all–sky high–resolution map

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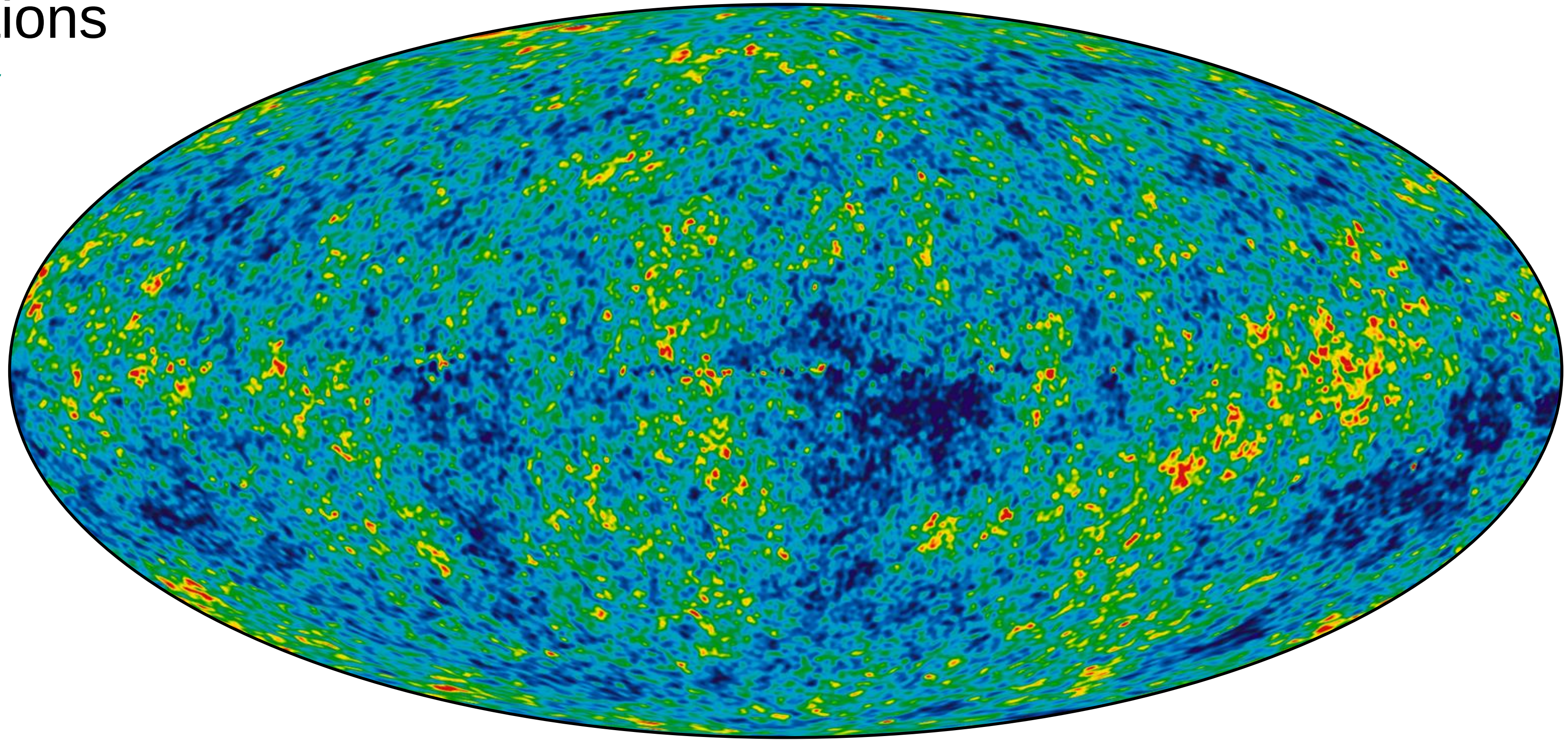
- **Resulting $\boldsymbol{CMB}$ map from $\boldsymbol{WMAP}$ after subtracting galactic foreground noise**

- temperature fluctuations with $\Delta T = \pm\, \mathbf{200}\ \boldsymbol{\mu K}$

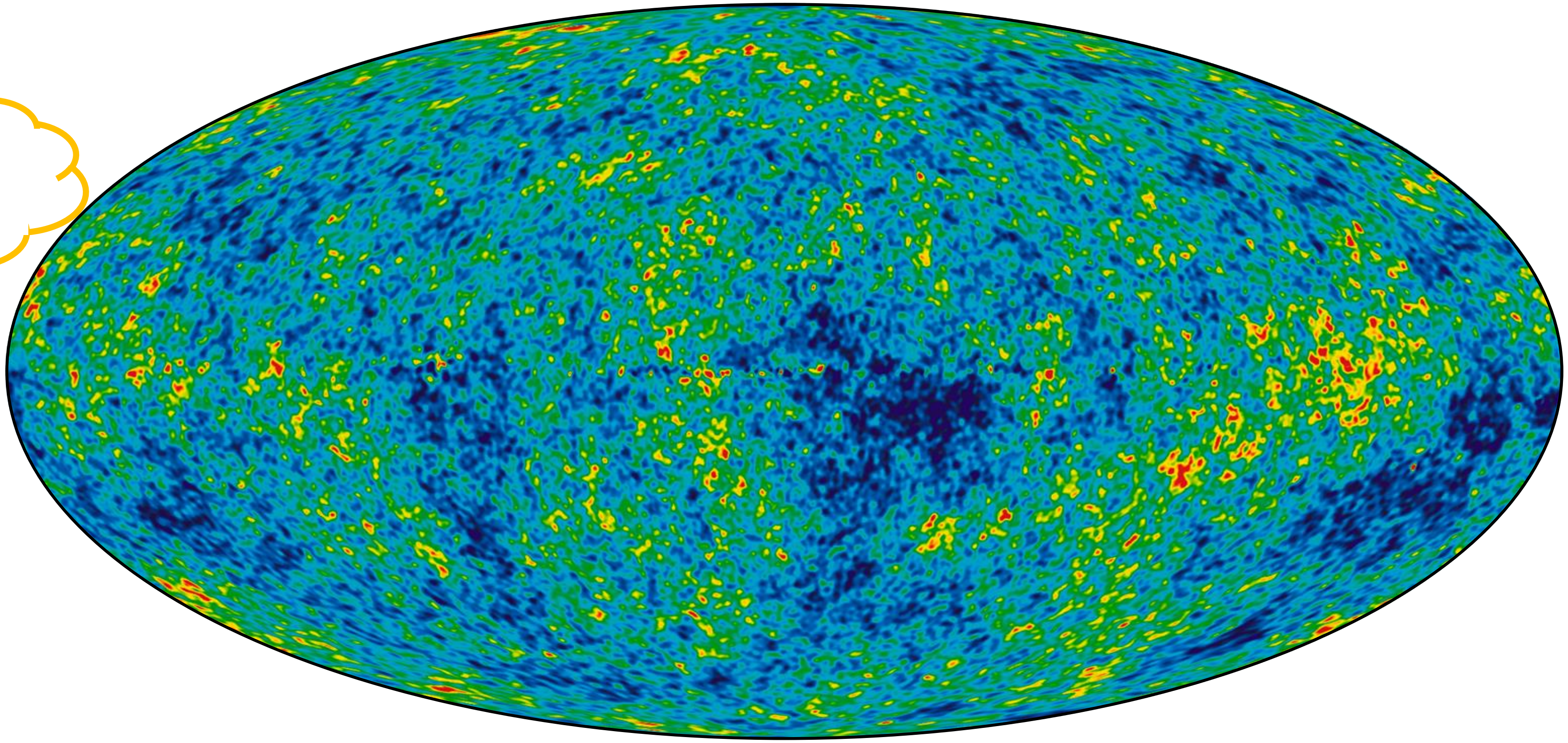
- $\boldsymbol{WMAP}$ sky map based on overall $\mathbf{9 - \boldsymbol{year} - \boldsymbol{long}}$ measuring time $(\mathbf{2003} \ldots \mathbf{2012})$



$WMAP$ & $COBE$: a comparison of maps

- **$WMAP$ has a much higher angular resolution than $COBE$ ...**
  - … but both results on temperature **fluctuations $\Delta T/T$ are consistent**

consistent results!!



$WMAP$ & $COBE$: a comparison of maps

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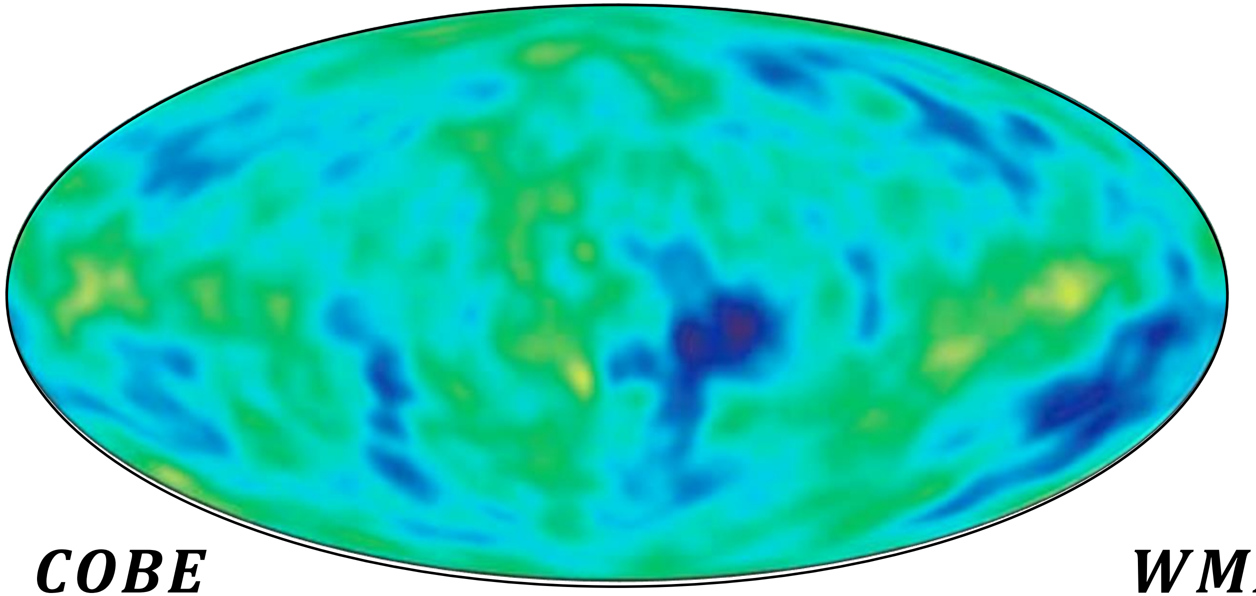
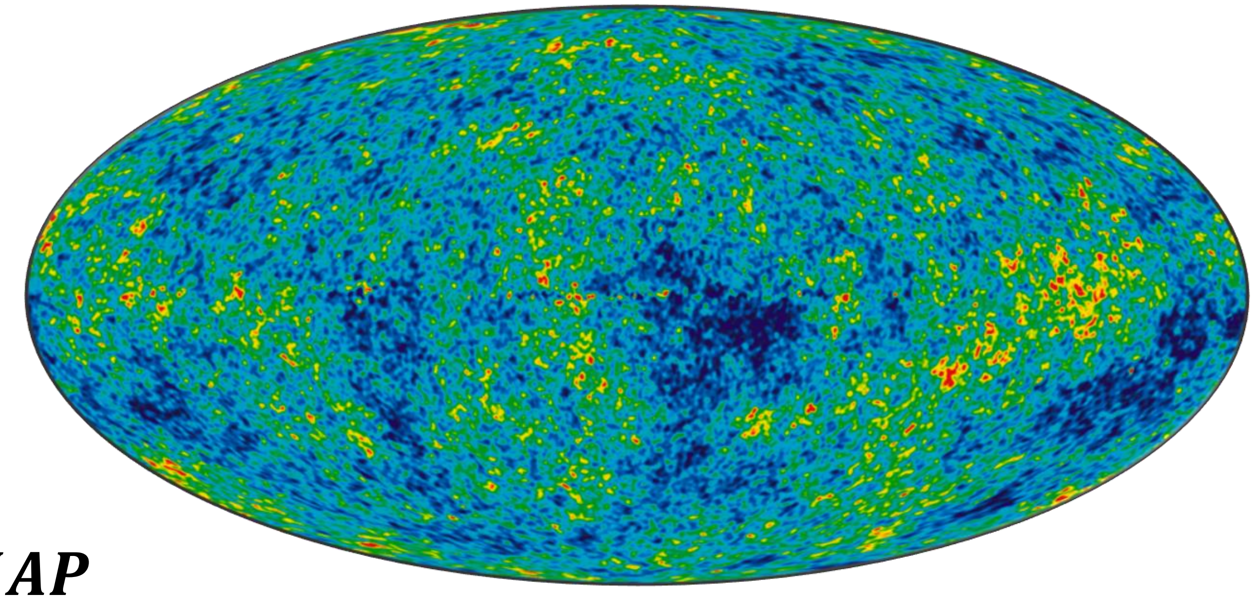
- $WMAP$ has a much higher angular resolution than $COBE$… due to mirror−∅





$COBE$

horn antenna: $\delta\theta$ independent of f

$\nu\ (GHz)$	31.5	53	90
$FWHM$	$\sim 7°$		

$WMAP$

mirror: (Rayleigh−criterion $\delta\theta \sim \lambda$)

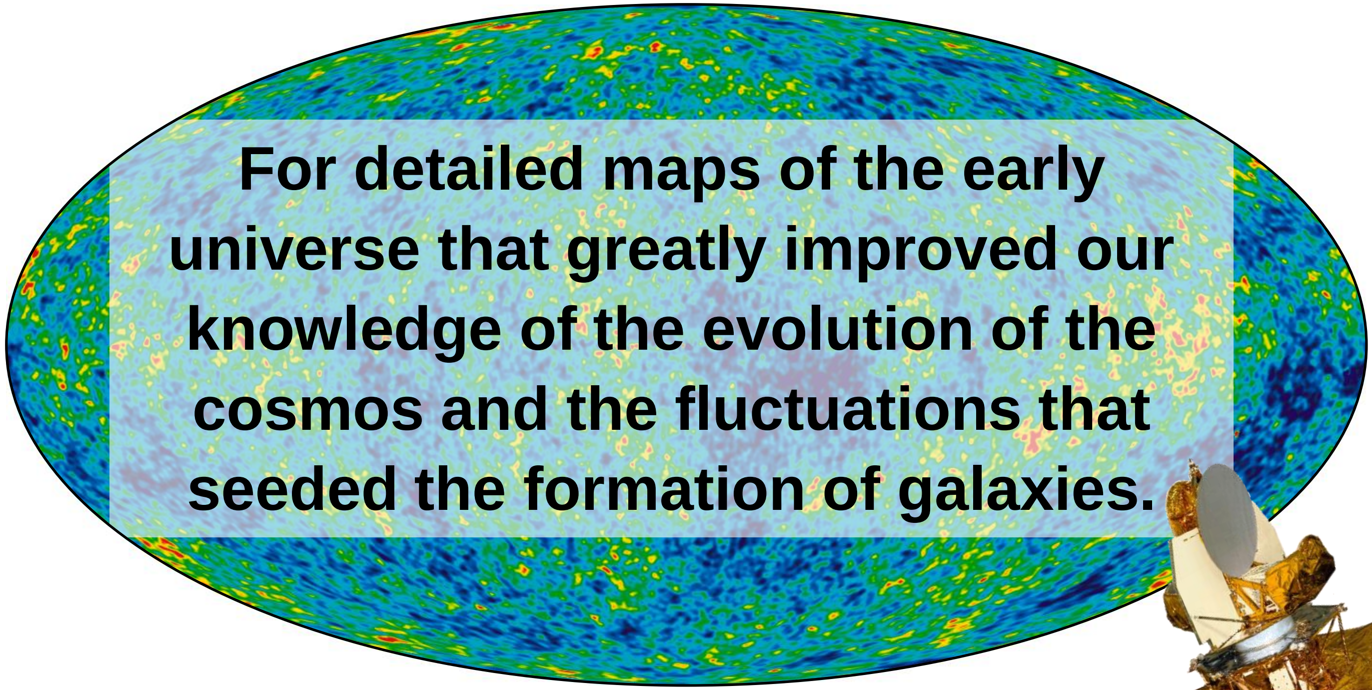
$\nu\ (GHz)$	22	30	40	60	90
$FWHM$	$0.93°$	$0.68°$	$0.53°$	$0.35°$	$< 0.23°$

WMAP – Breakthrough Prize 2018

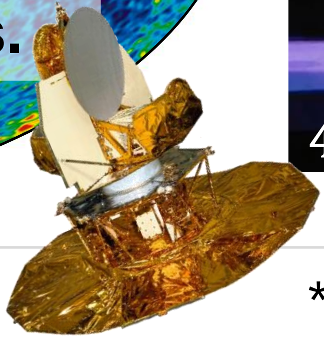
- **Fundamental Physics Breakthough Prize**

  - **award sum***: **3 *M*$**
  - laureates **2018**: ***WMAP*** science team (**all 27** members)

FUNDAMENTAL PHYSICS BREAKTHROUGH PRIZE



For detailed maps of the early universe that greatly improved our knowledge of the evolution of the cosmos and the fluctuations that seeded the formation of galaxies.



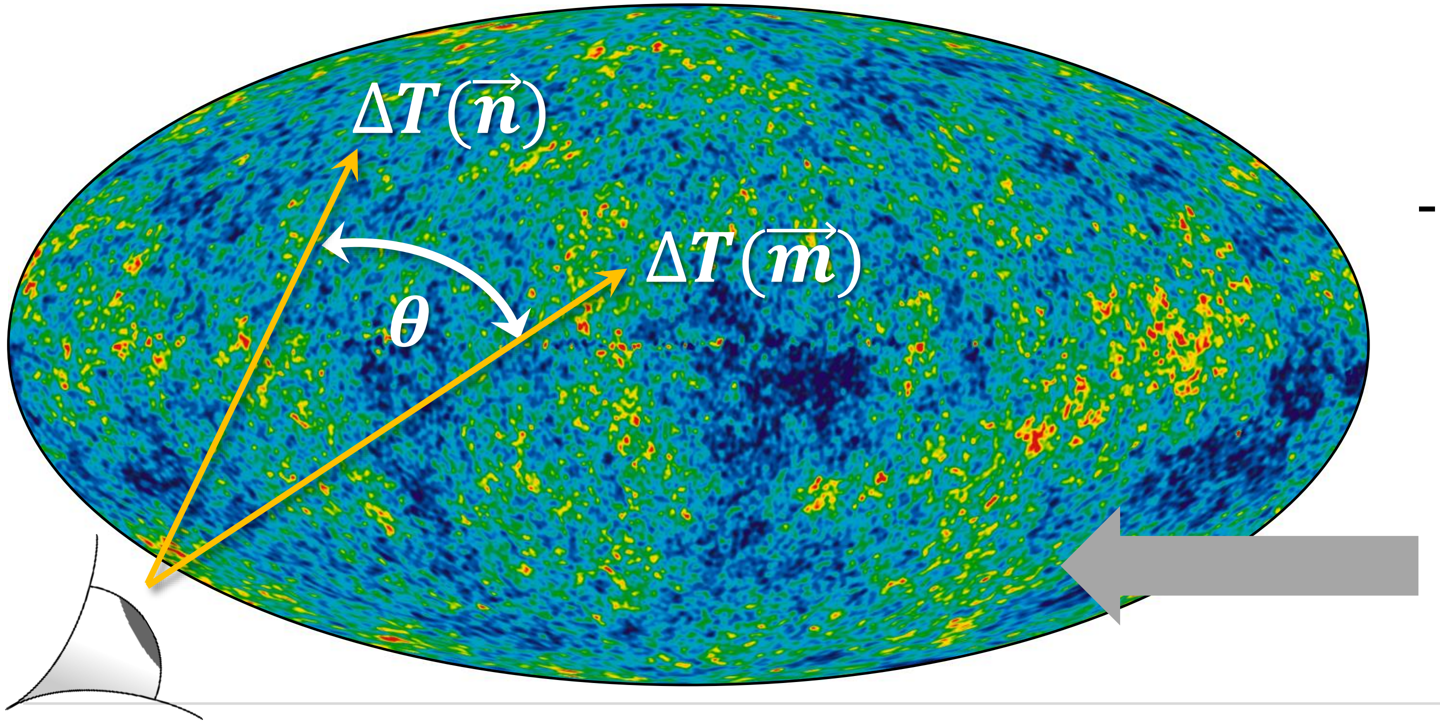
$* = 2 \times$ Nobel prize sum

CMB – analysis of angular correlations

- **statistical analysis of fluctuations ΔT around the CMB mean value T_0**

  - we are interested in the **correlation function $C(\theta)$** between two directions $\vec{n}$ and $\vec{m}$ observed under a varying **angle θ** (from small to large)

$$C(\theta) = \left\langle \left(\frac{\Delta T(\vec{n})}{T_0}\right) \cdot \left(\frac{\Delta T(\vec{m})}{T_0}\right) \right\rangle$$



  - we compare a large data set of measured **temperature fluctuations** as function of **angular distance θ**

*„on which angular scale do we observe the **largest temperature fluctuations**?“*

CMB – multipole analysis of angular correlations

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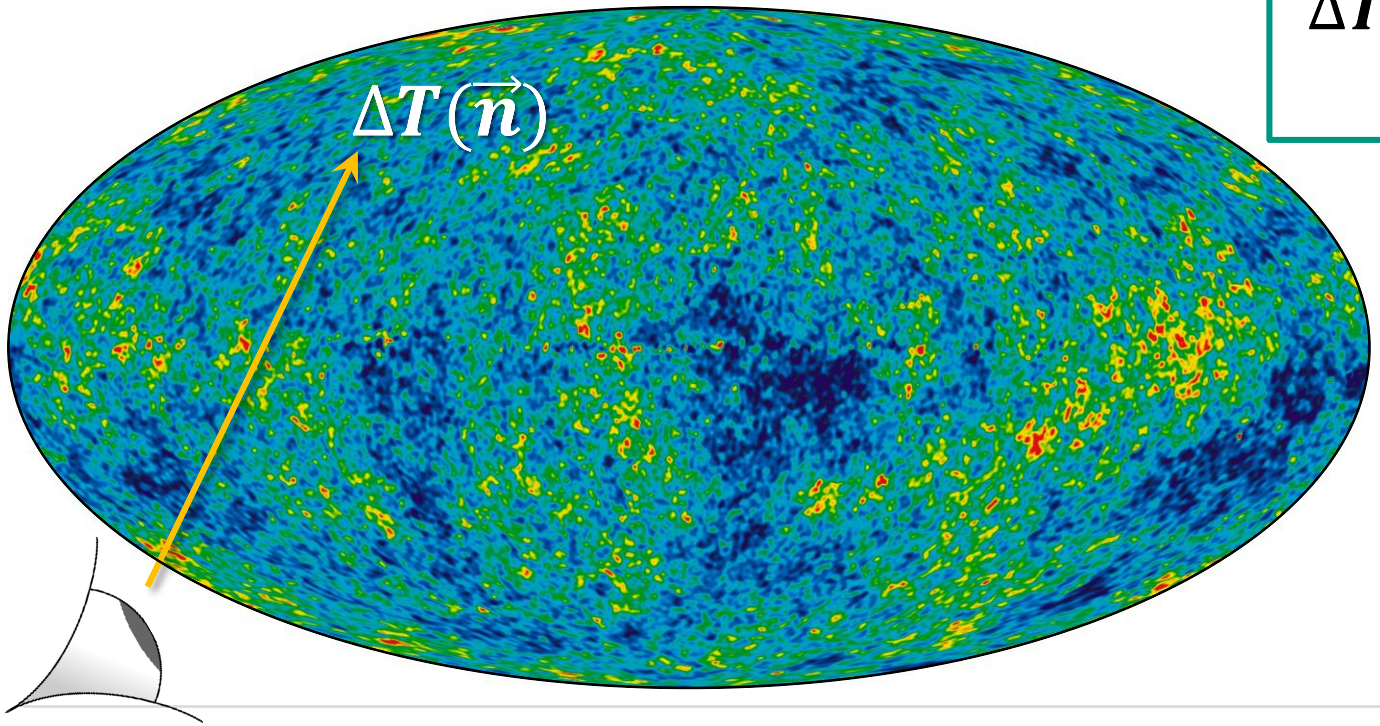
- **multipole analysis* of fluctuations ΔT around the CMB mean value T_0**
  - we express the observed temperature fluctuation, say in **specific direction $\vec{n}$**, via **spherical coordinates, using** **spherical harmonics $Y_{\ell m}$**



$$\Delta T(\vec{n}) = \frac{T(\vec{n}) - T_0}{T_0} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{m=+\ell} a_{\ell m} Y_{\ell m}$$

ℓ: **multipole**

$(2\ell + 1)$ orthogonal, independent **coefficiants $a_{\ell m}$**

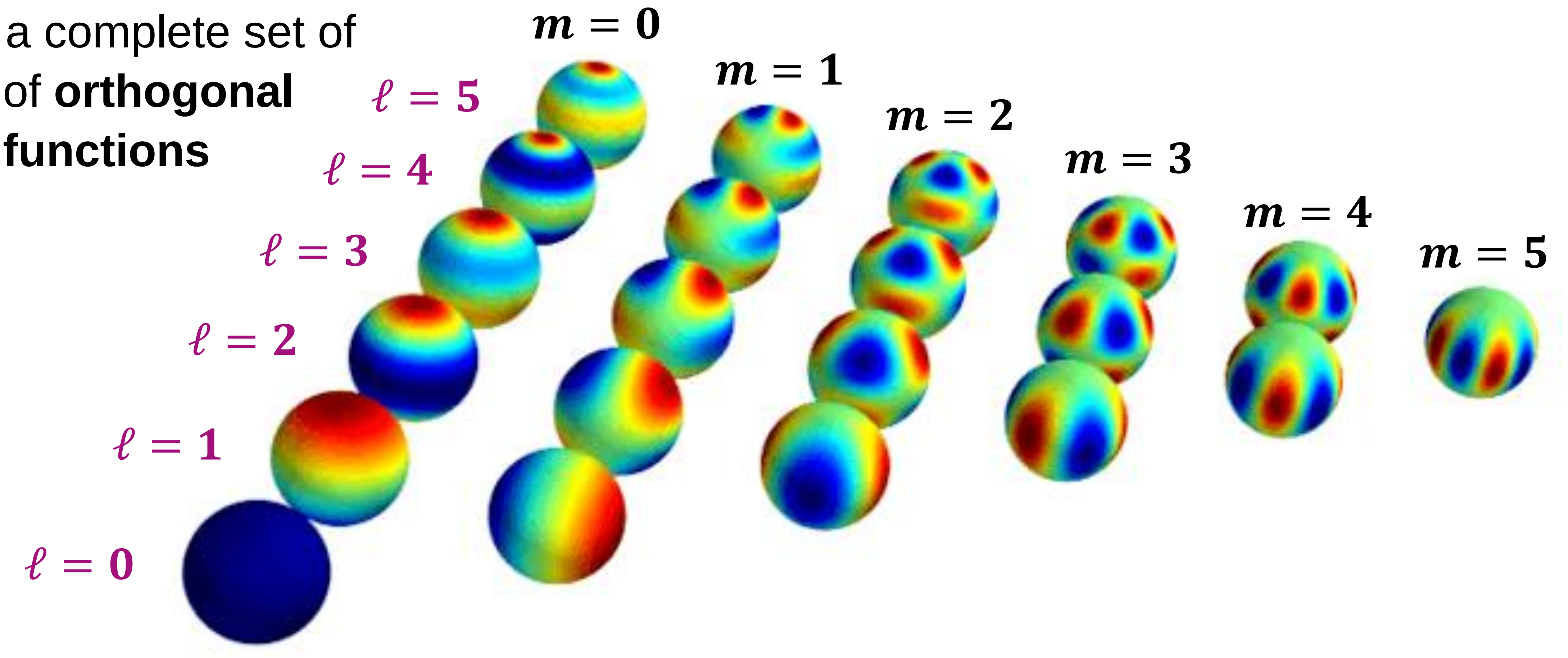
- **ortho−normal function set $Y_{\ell m}$**

*see your electromagnetics & atomic physics

CMB – multipole analysis of angular correlations

- **Sphercial harmonics: lowest orders visualized***

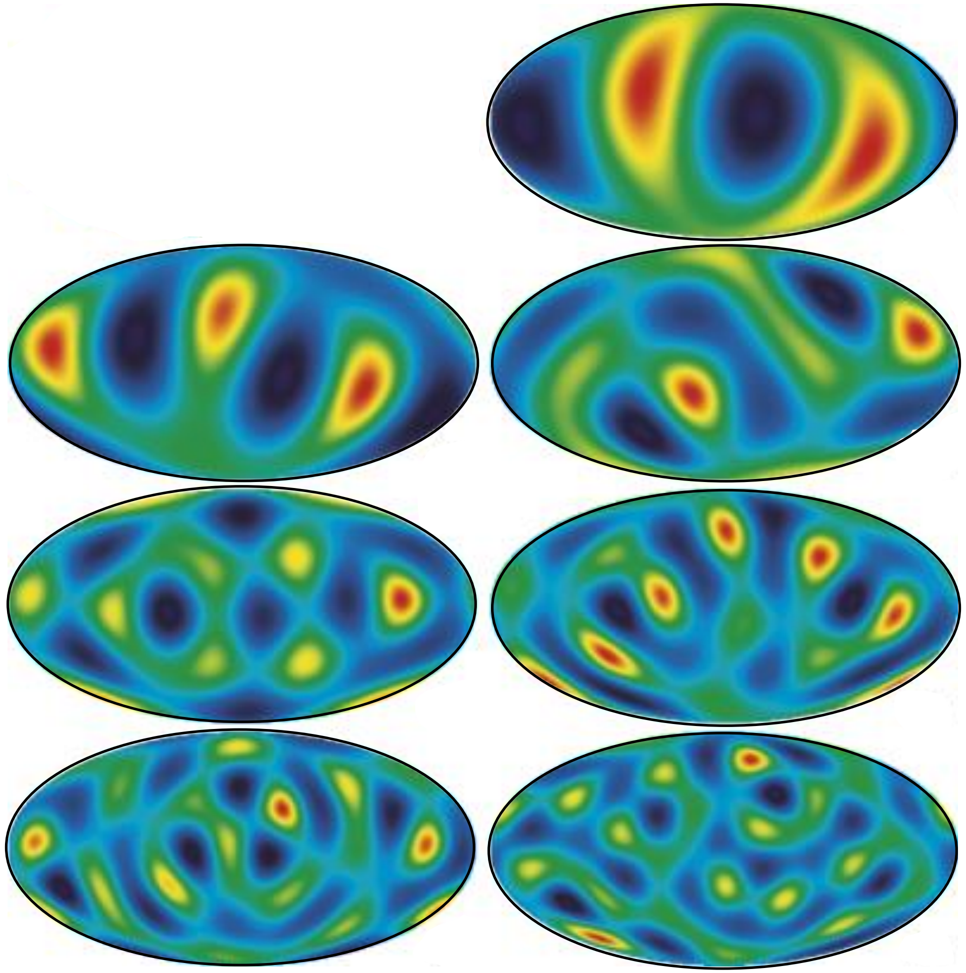
  - a complete set of of **orthogonal functions**



*see [A Poor Man's CMB Primer: Bumps on a Blackbody | Physics Forums]

CMB – multipole analysis of angular correlations

- multipole analysis: lowest orders visualized in Mollweide projection



$\ell = 2$: quadrupole

$\ell = 3, 4$: octupole, hexadecapole

$\ell = 5, 6$:

$\ell = 7, 8$

CMB – multipole analysis of angular correlations

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- correlation function $C(\theta)$ of temperature fluctuations ΔT only depends on θ

correlation function $C(\theta)$

$$C(\theta) = \left\langle \left(\frac{\Delta T(\vec{n})}{T_0}\right) \cdot \left(\frac{\Delta T(\vec{m})}{T_0}\right) \right\rangle$$

decompose to spherical harmonics $Y_{\ell m}$

$$\Delta T(\vec{n}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{m=+\ell} a_{\ell m} Y_{\ell m}$$

we develop $C(\theta)$ into (associated)
Legendre polynominals $P_\ell(\cos\theta)$
⇨ CMB data provide **coefficiants C_ℓ**

for **Gaussian fluctuations** we have:

coefficiants C_ℓ

$$C_\ell = \frac{1}{2\ell + 1} \cdot \sum_{m=-\ell}^{m=+\ell} |a_{\ell m}|^2$$

$$C(\theta) = \frac{1}{4\pi} \cdot \sum_{\ell} (2\ell + 1) \cdot C_\ell \cdot P_\ell(\cos\theta)$$

CMB – multipole analysis of angular correlations

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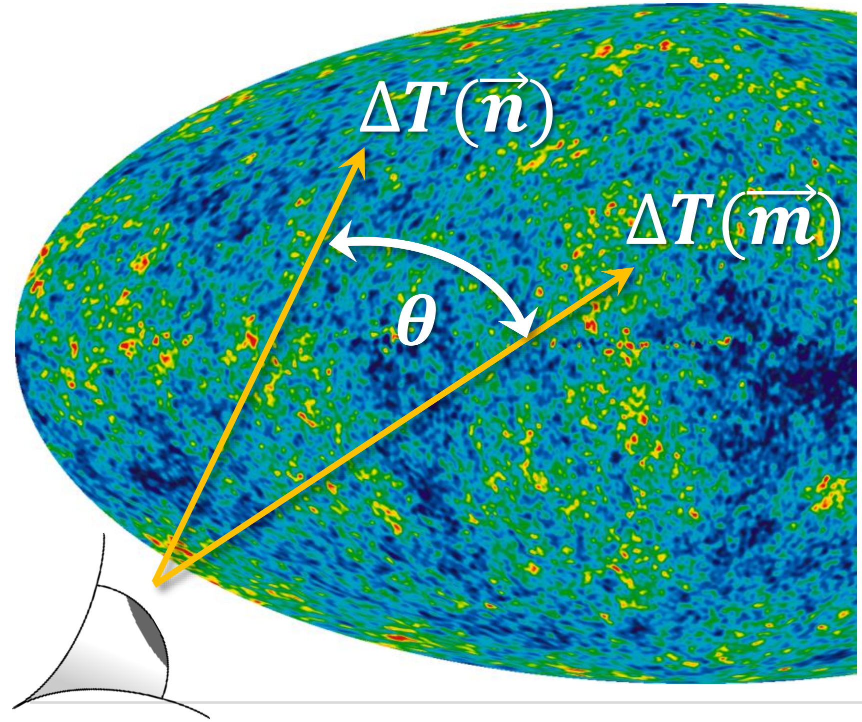
- **For each multipole ℓ we can perform $(2\ell + 1)$ orthogonal measurements**

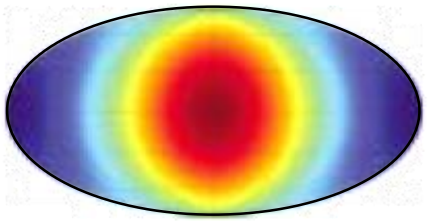
  - T – **data** provide us with the **correlation coefficiants C_ℓ**

$$C_\ell = \frac{1}{2\ell + 1} \cdot \sum_{m=-\ell}^{m=+\ell} |a_{\ell m}|^2$$



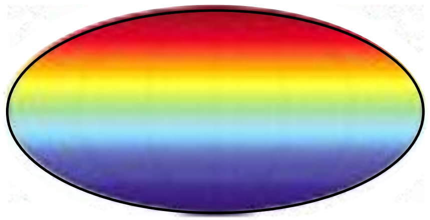
- example #1: dipole with $\ell = 1$

  ⇨ perform $(2\ell + 1) = 3$ orthogonal measurements



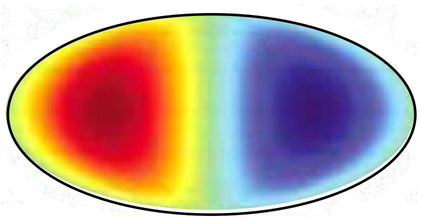
$m = +1$

´front–back´



$m = 0$

´top–bottom´



$m = -1$

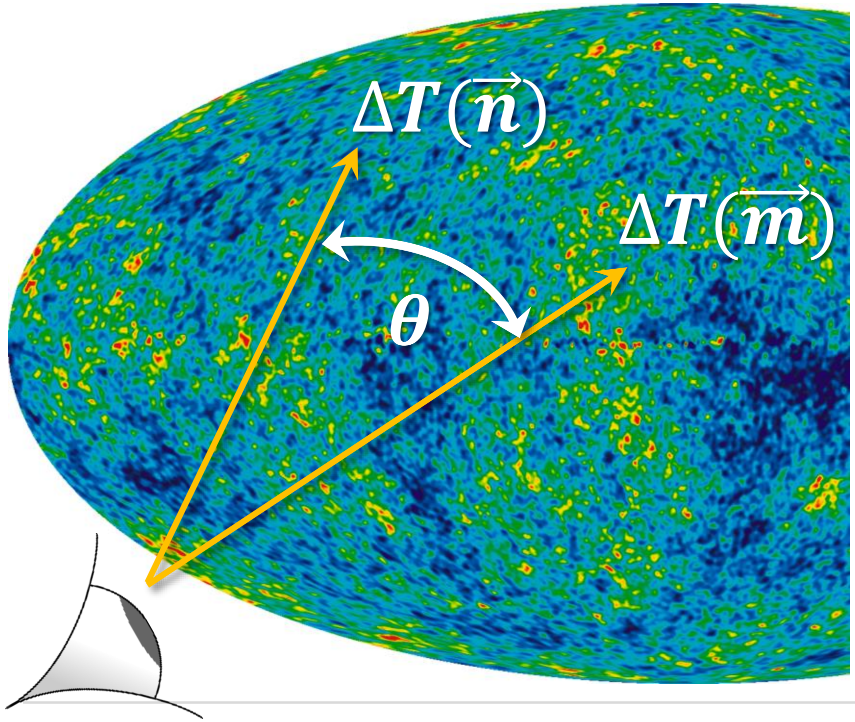
´left–right´

CMB – multipole analysis of angular correlations

- **For each multipole ℓ we can perform $(2\ell + 1)$ orthogonal measurements**

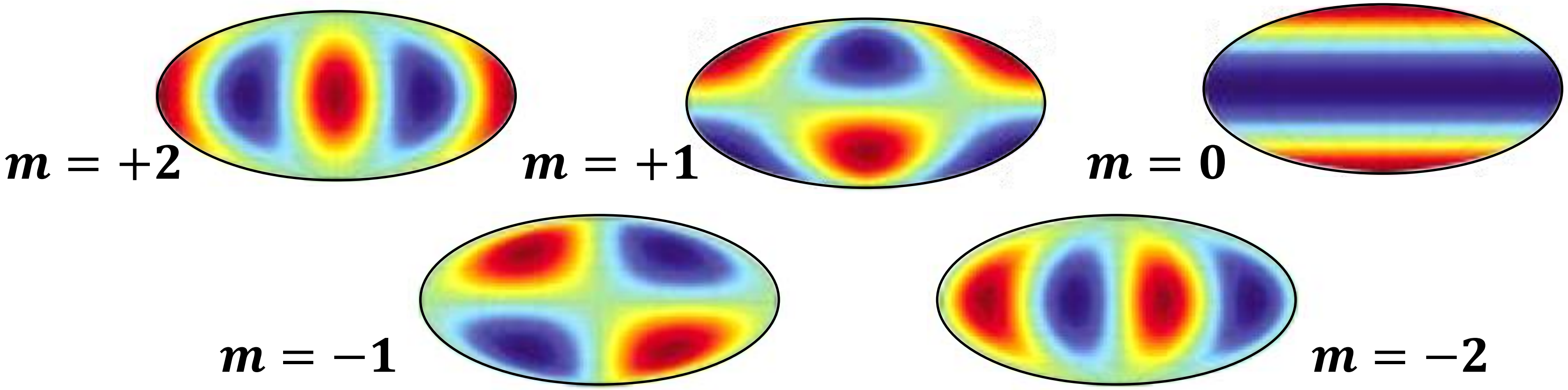
  - T – **data** provide us with the **correlation coefficiants C_ℓ**

$$C_\ell = \frac{1}{2\ell + 1} \cdot \sum_{m=-\ell}^{m=+\ell} |a_{\ell m}|^2$$



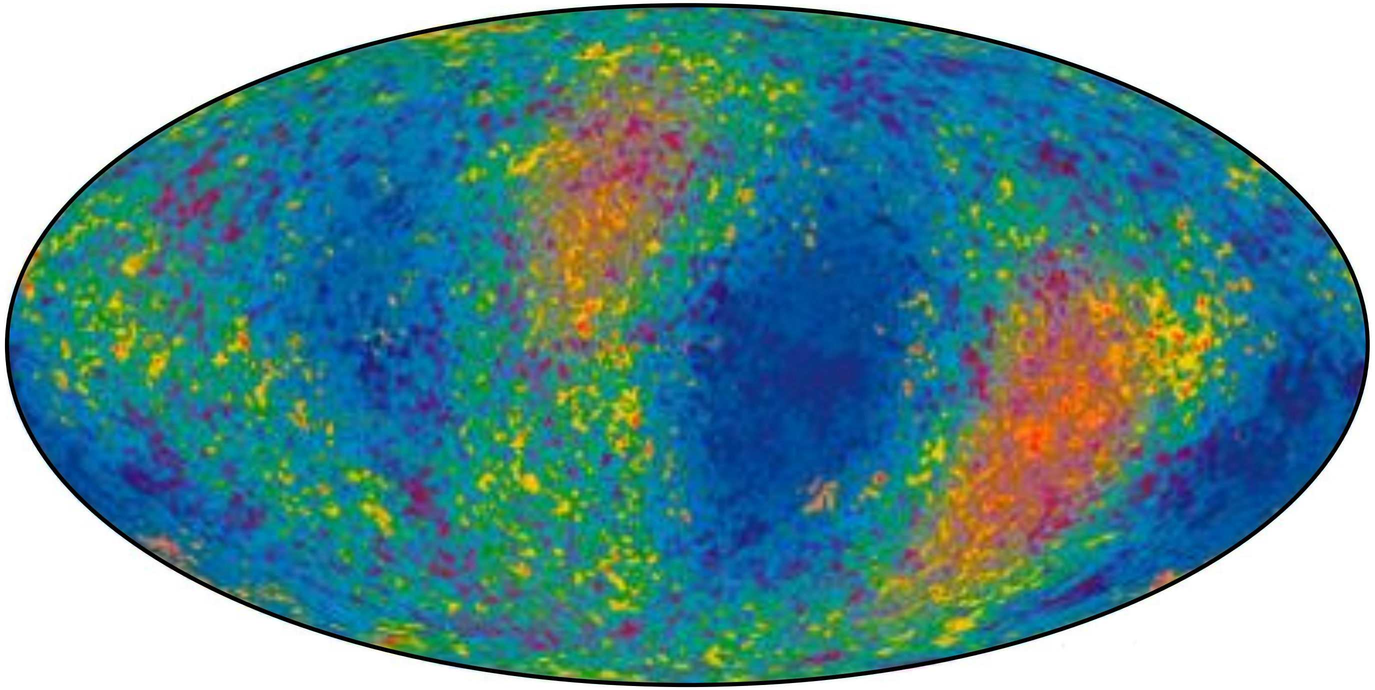
- example #2: dipole with $\ell = 2$

  ⇨ perform $(2\ell + 1) = 5$ orthogonal measurements



CMB – multipole analysis: fit of results

- **For each multipole ℓ we fit the data: resulting octupole $\ell = 3$ contribution**



$$(\Delta T)^2 = \ell \cdot (\ell + 1) \cdot C_\ell \,/\, 2\pi$$

CMB – multipole analysis: display of results

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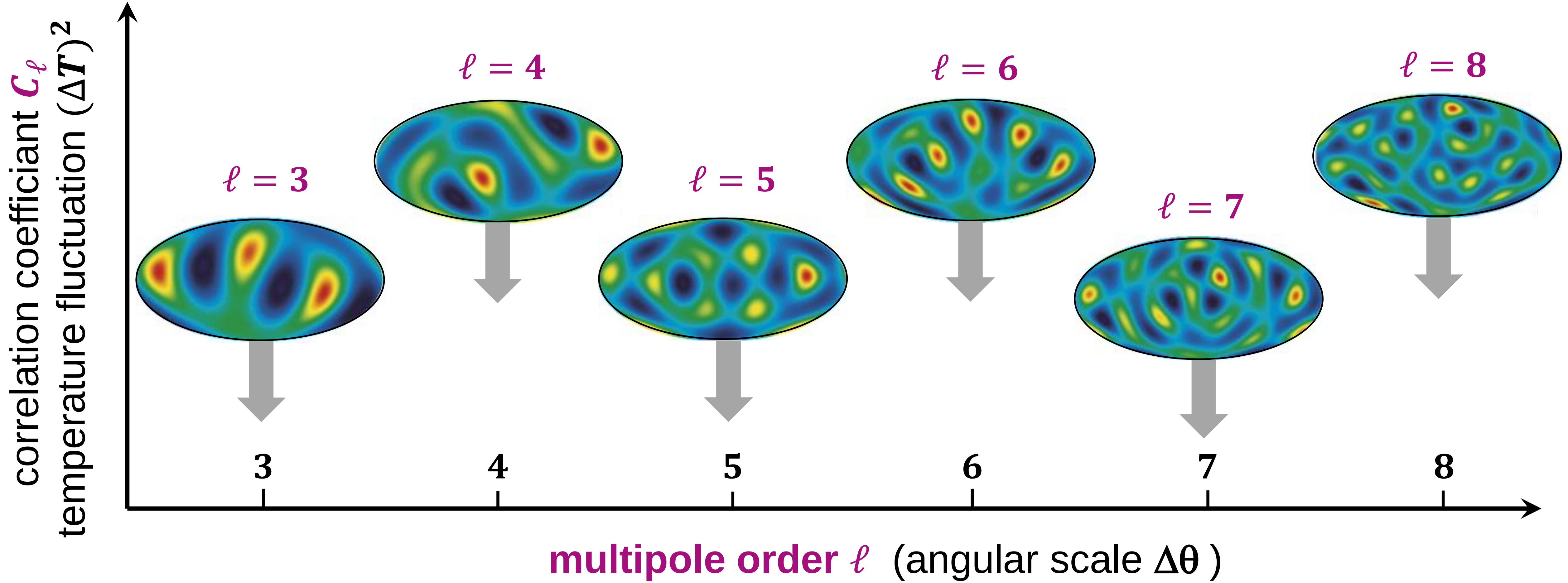
- For each **multipole** ℓ we display the fitted data: coefficiants C_ℓ *vs.* ℓ



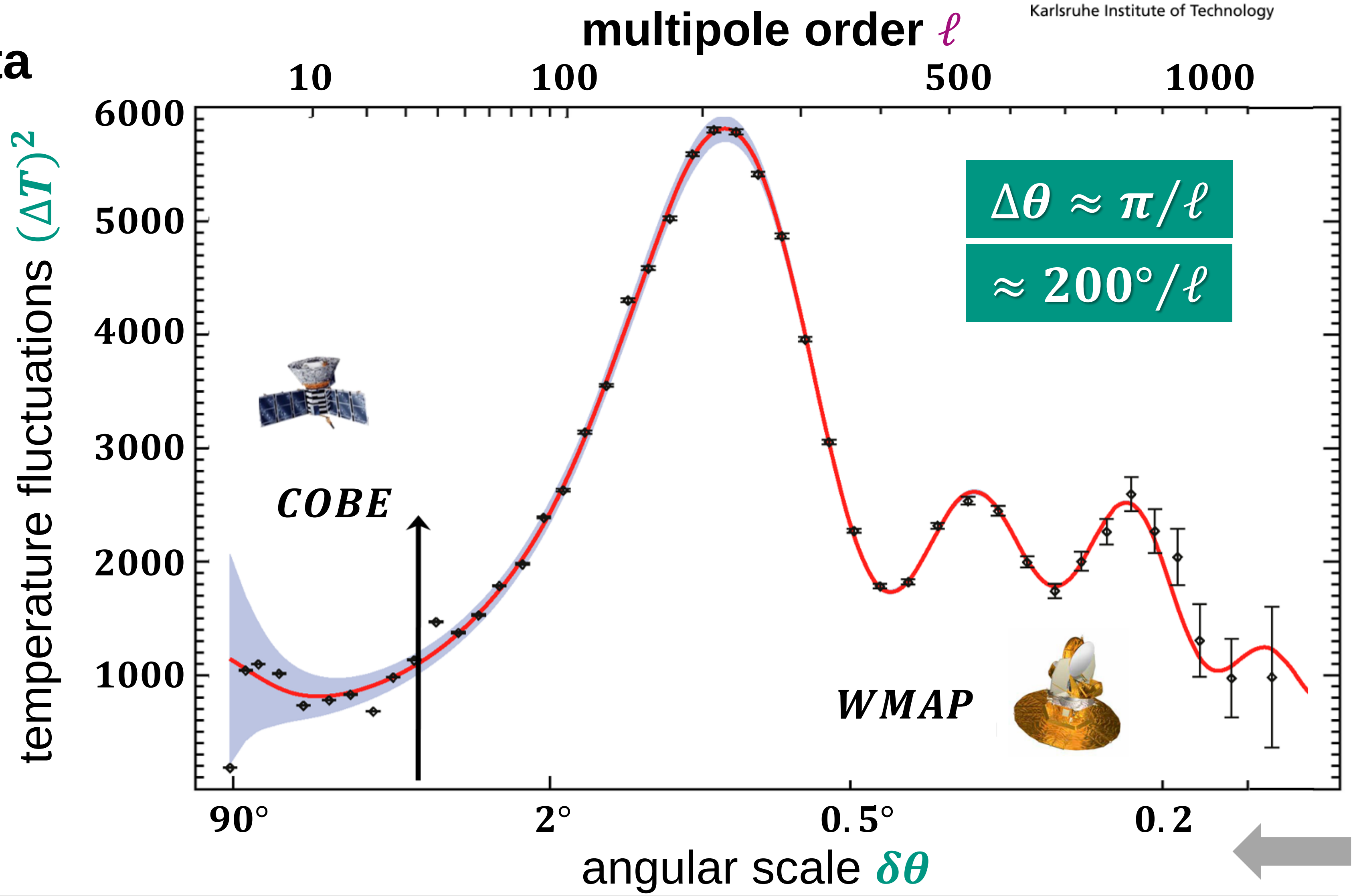
CMB – multipole analysis: $WMAP$ results

- **7 – *year* multipole data**

- first all–sky and high–resolution CMB multipole analysis up to $\ell = 1000$ (*aka* **CMB power spectrum**)

$$(\Delta T)^2 = \ell \cdot (\ell + 1) \cdot C_\ell / 2\pi$$

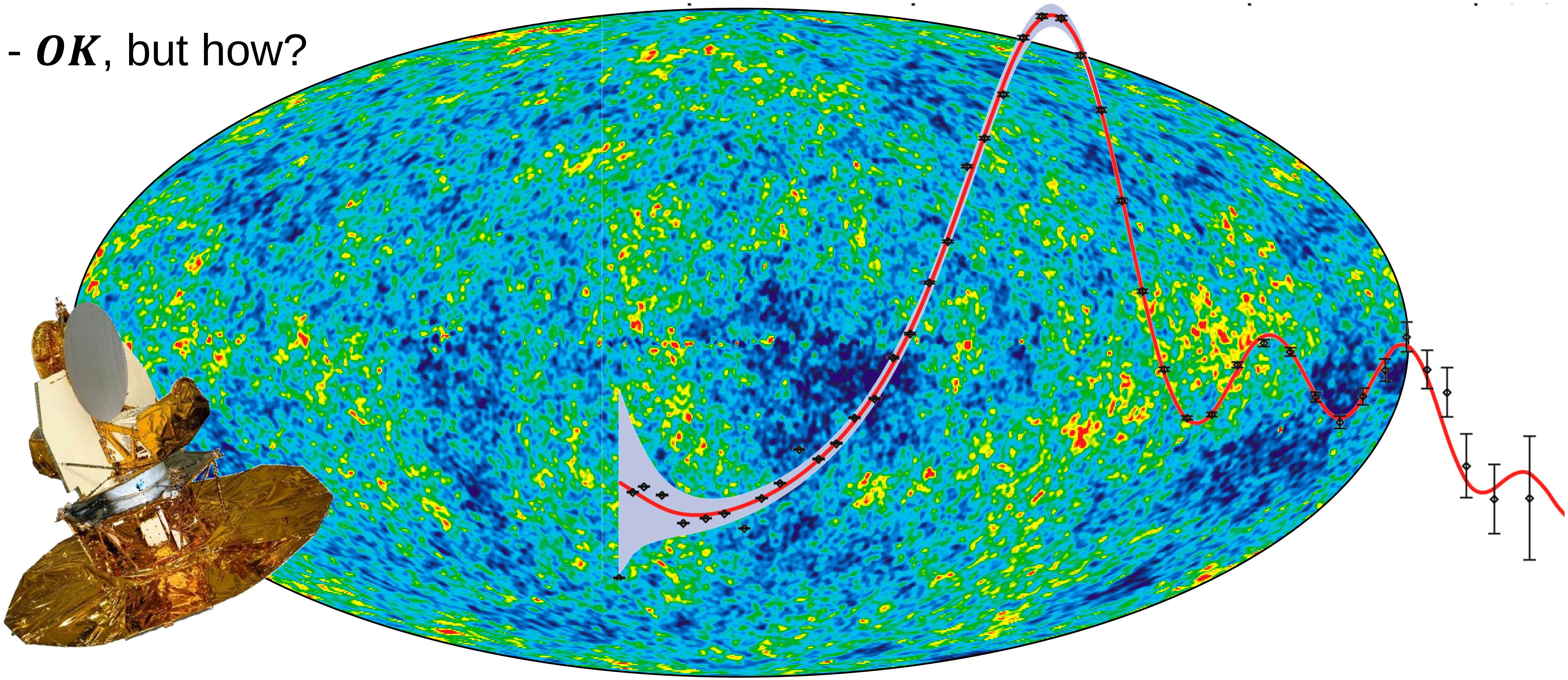
precision age of cosmology



CMB – multipole analysis: what can we learn?

- CMB – a key to unlock the most fundamental information on the universe

  - OK, but how?



CMB – multipole analysis: different regions

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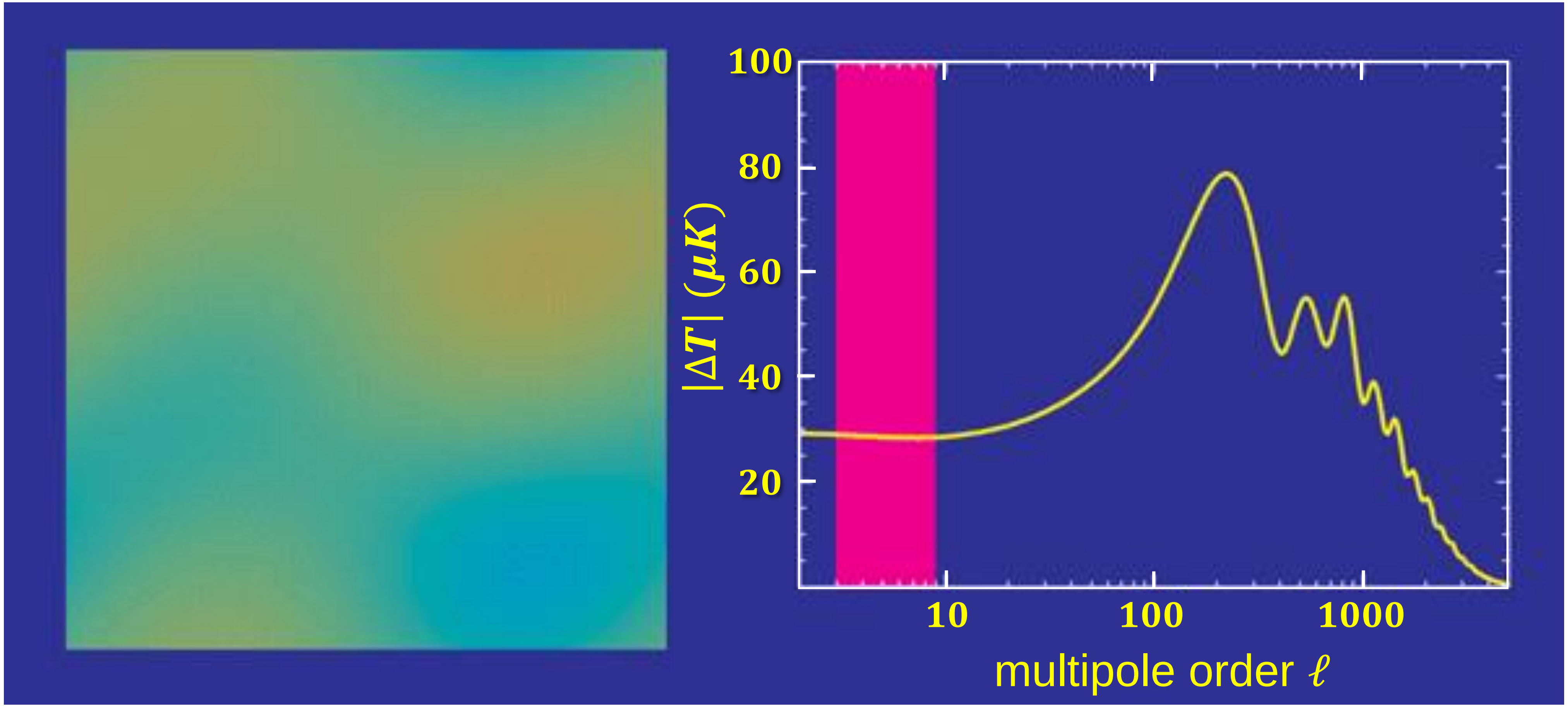
- CMB multipoles visualized: from large patches of the sky to tiny regions

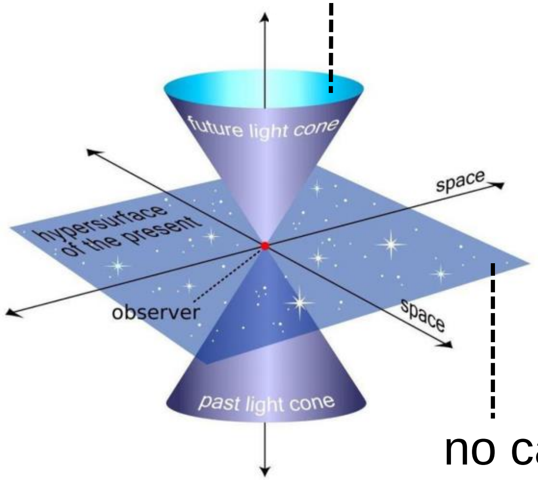
size does matter



CMB – multipole analysis: 3 different regions

- CMB **theory**

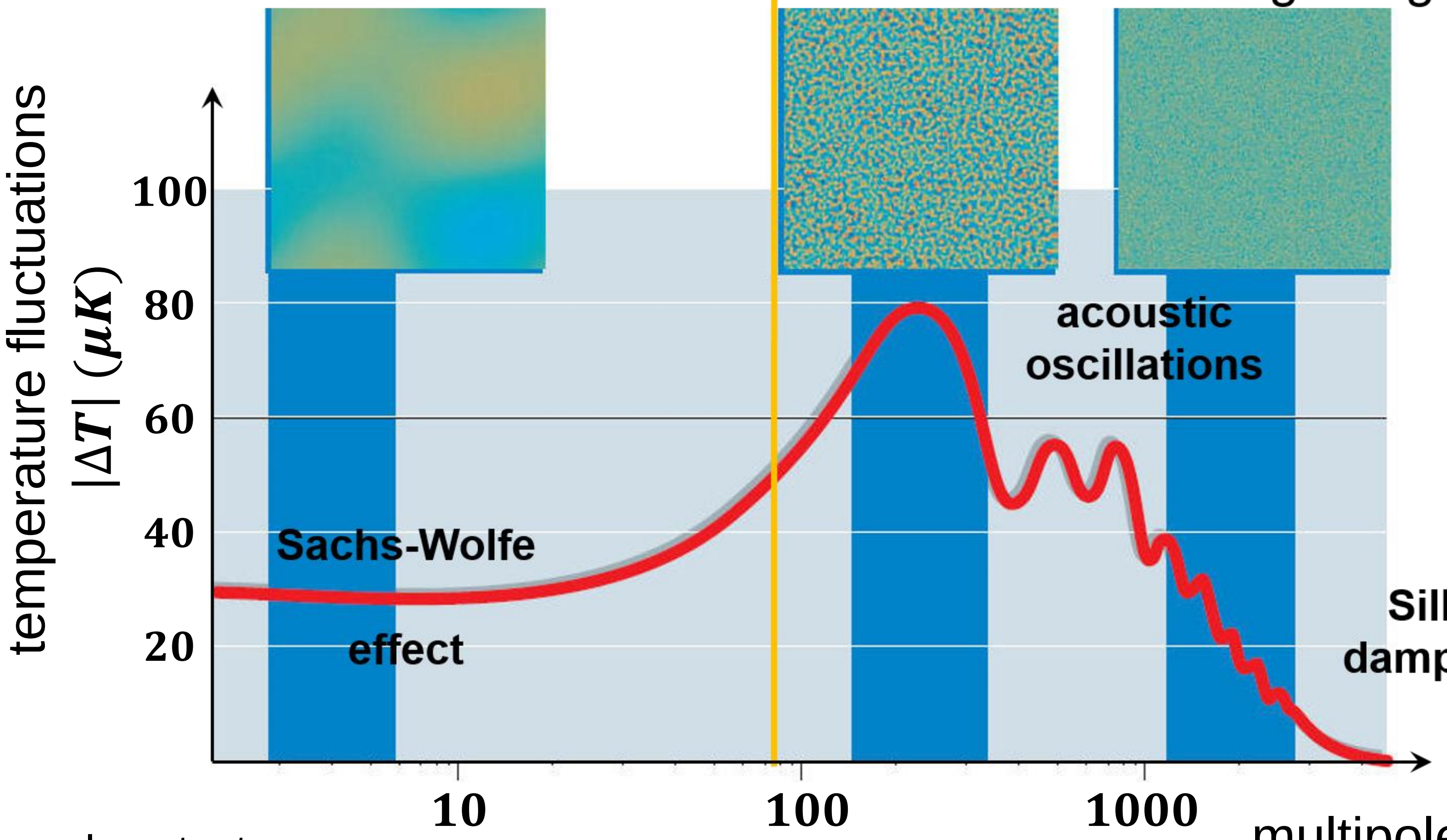
- **small ℓ: no** causal contact!
- **large ℓ: yes,** causal contact!



no causal contact

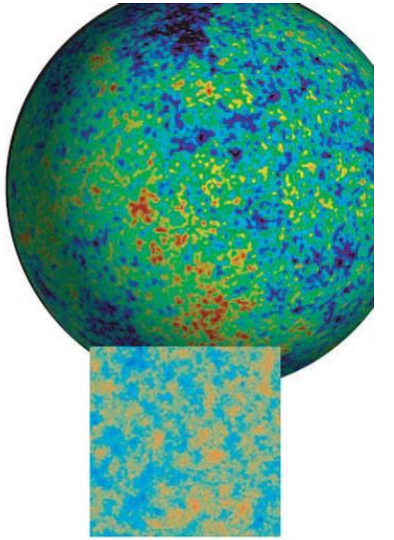
regions *without* causal contact

regions which are causally connected since Big Bang



Silk-
damping

multipole ℓ



J. Silk

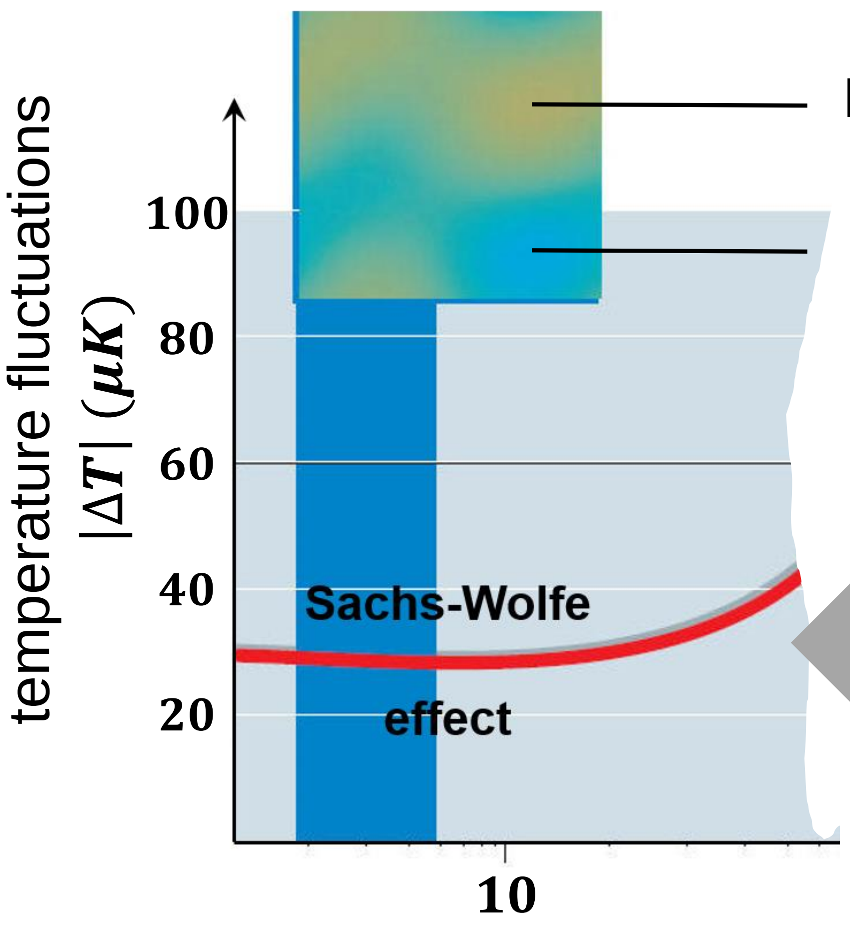
CMB – multipole analysis: largest sizes

- CMB theory

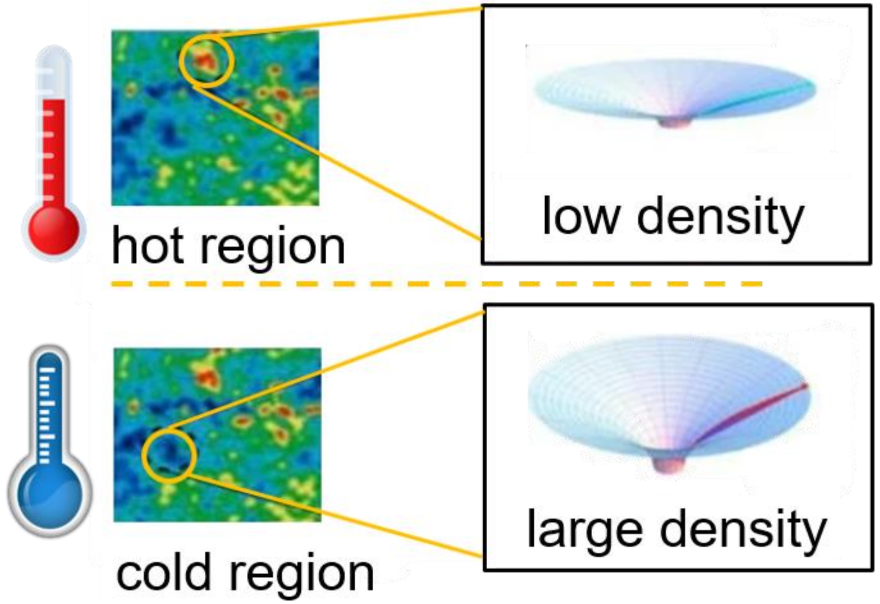
- small ℓ: no causal contact!

amplitude of fluctuations are (quasi) scale–invariant

why?



Sachs–Wolfe effect: CMB leaves gravity wells of **Dark Matter**



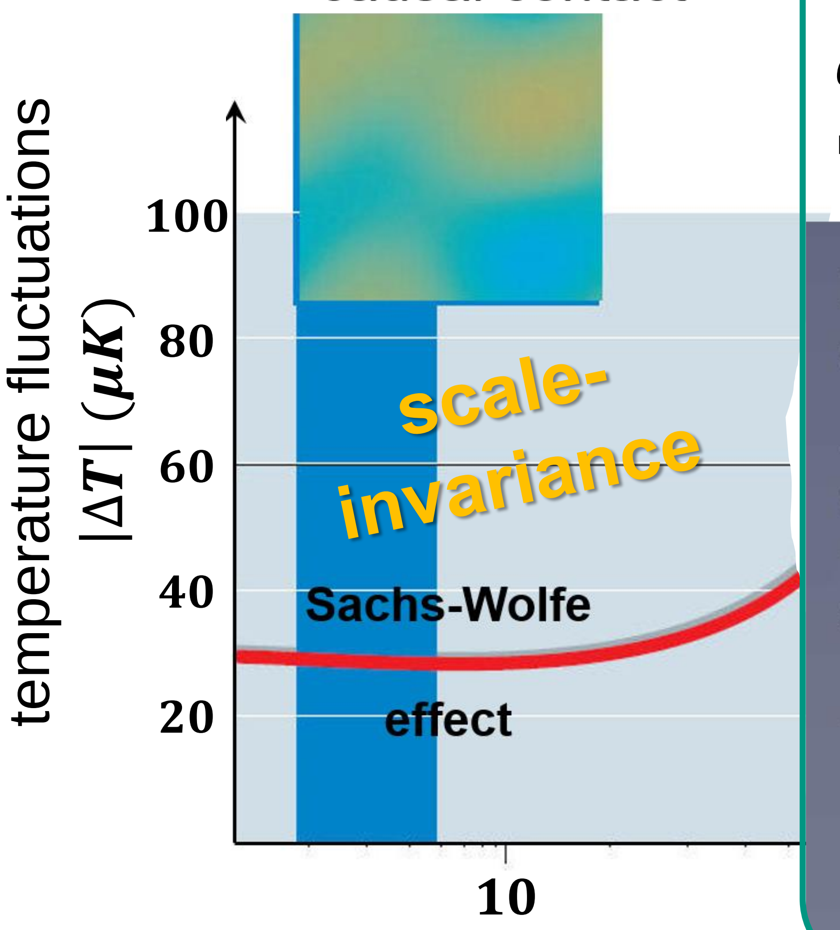
CMB – multipole analysis: a sign of inflation?

- CMB & inflation

- small ℓ: no causal contact!

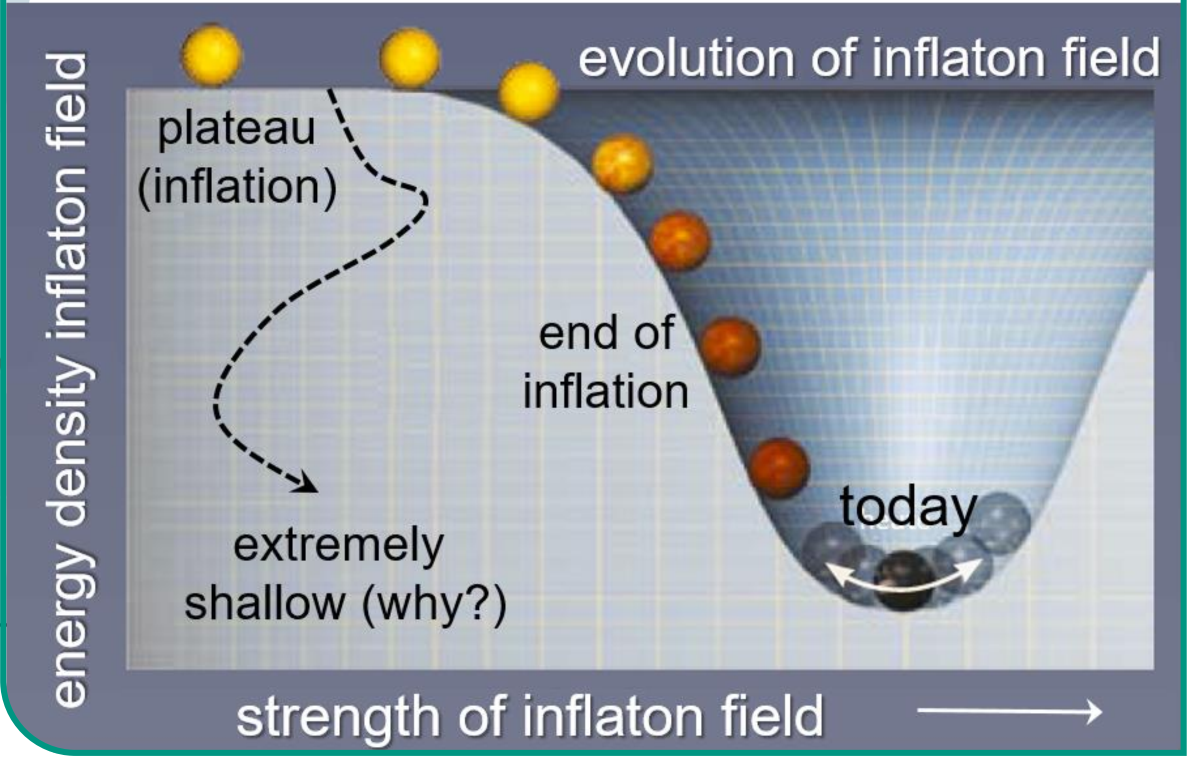
amplitude of fluctuations are (quasi) scale–invariant

a possible solution



exponential increase of scale factor $a(t)$ **from** $t = 10^{-36} \ldots 10^{-32}$ s by e^{100}

⇨ new scalar field: **the ´inflaton´**



CMB – multipole analysis: Harrison–Zel´dovich

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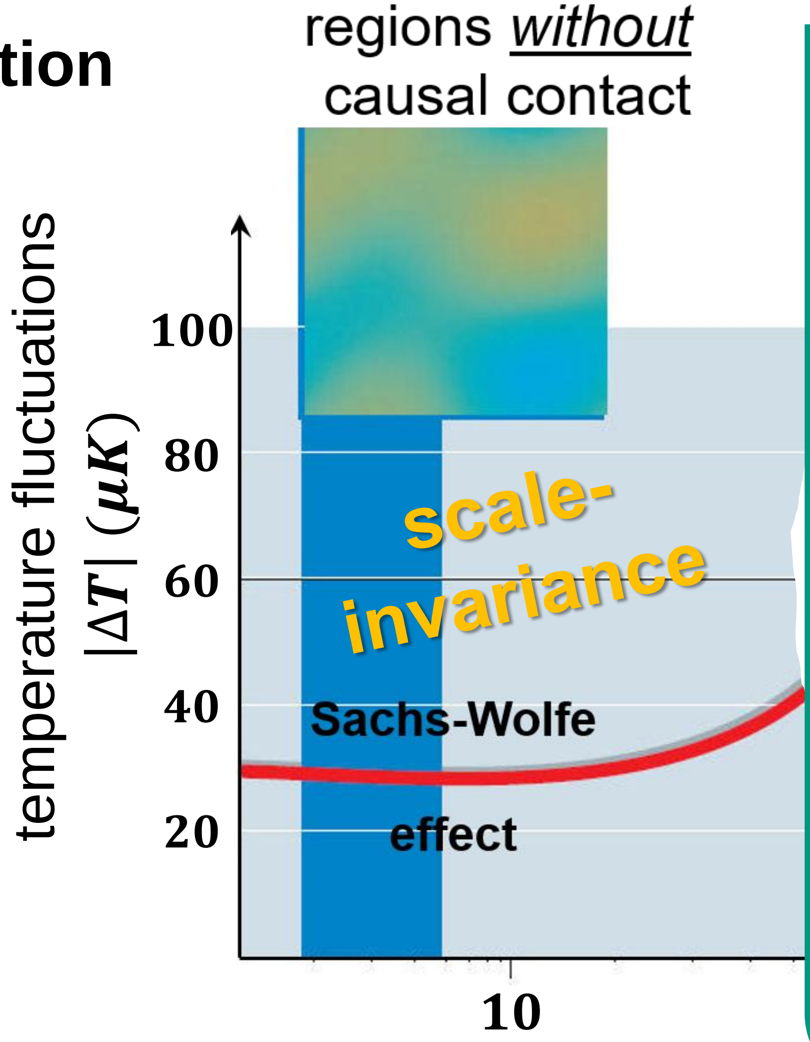
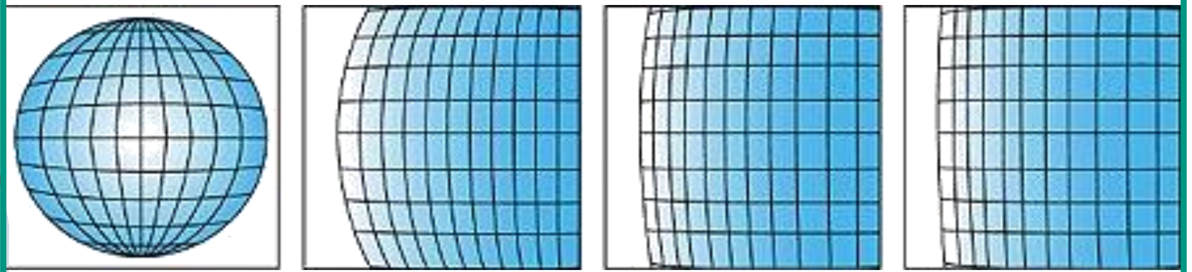
- CMB & inflation

- **small ℓ: flat Harrison– Zel´dovich (HZ)** spectrum

Y. Zel´dovich*



exponential increase of scale factor $a(t)$: flat, Euclidean space ($k = 0$)
also: no GUT – magnetic monopoles,
$\Delta\rho/\rho$ via quantum–mechanical fluctuations of the inflaton field



HZ: all primordial perturbations have the **same amplitude**, independent of their size (⇨ **flat power spectrum**)

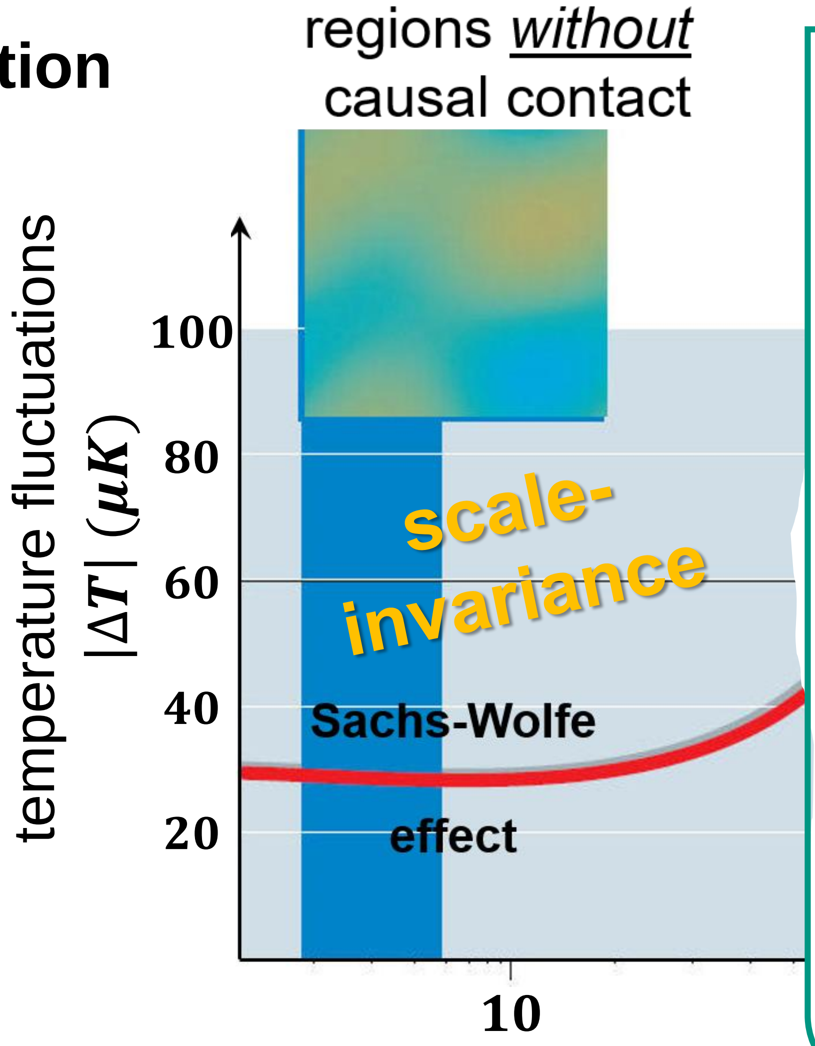
*see also next lecture (SZ effect)

CMB – multipole analysis: spectral index n

- CMB & inflation

- small ℓ: flat **Harrison–Zel´dovich (HZ) spectrum**

Y. Zel´dovich*



exponential increase of scale factor $a(t)$ causes $\dot{a}(t)/a(t)$ to be scale-invariant ⇨ use **spectral index n**

$$\frac{\ell \cdot (\ell + 1)}{2\pi} \cdot C_\ell \equiv \left(\frac{\Delta T}{T}\right)^2 \propto k^{n-1}$$

k: wave–number ($= \lambda^{-1}$)

inflatation: $n = 0.92 \ldots 0.98$

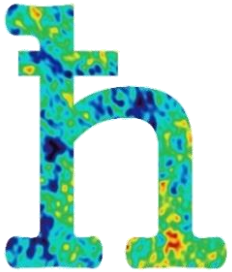
observation: $n = 0.967 \pm 0.004$

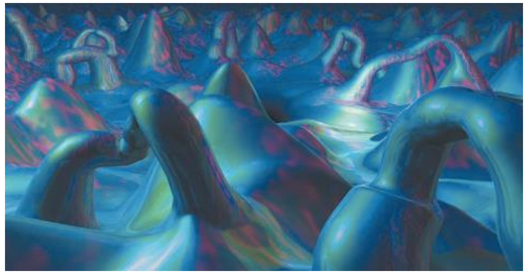
⇨ good agreement, but still no proof...

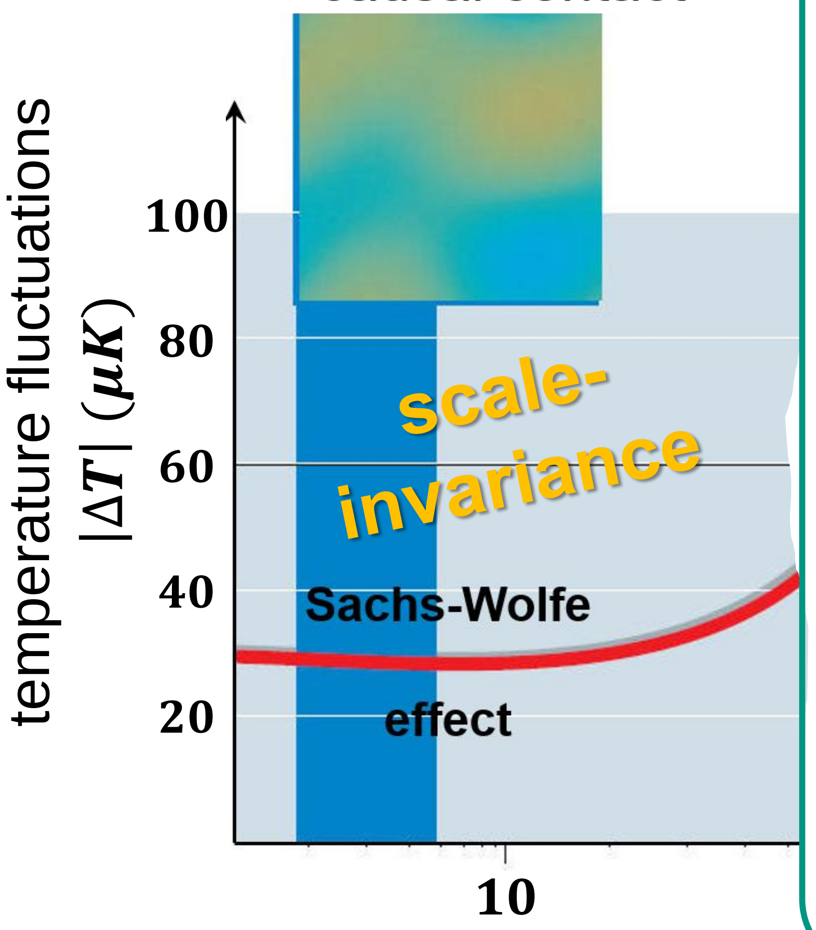
CMB – multipole analysis: QM* on largest scales

- CMB & inflation

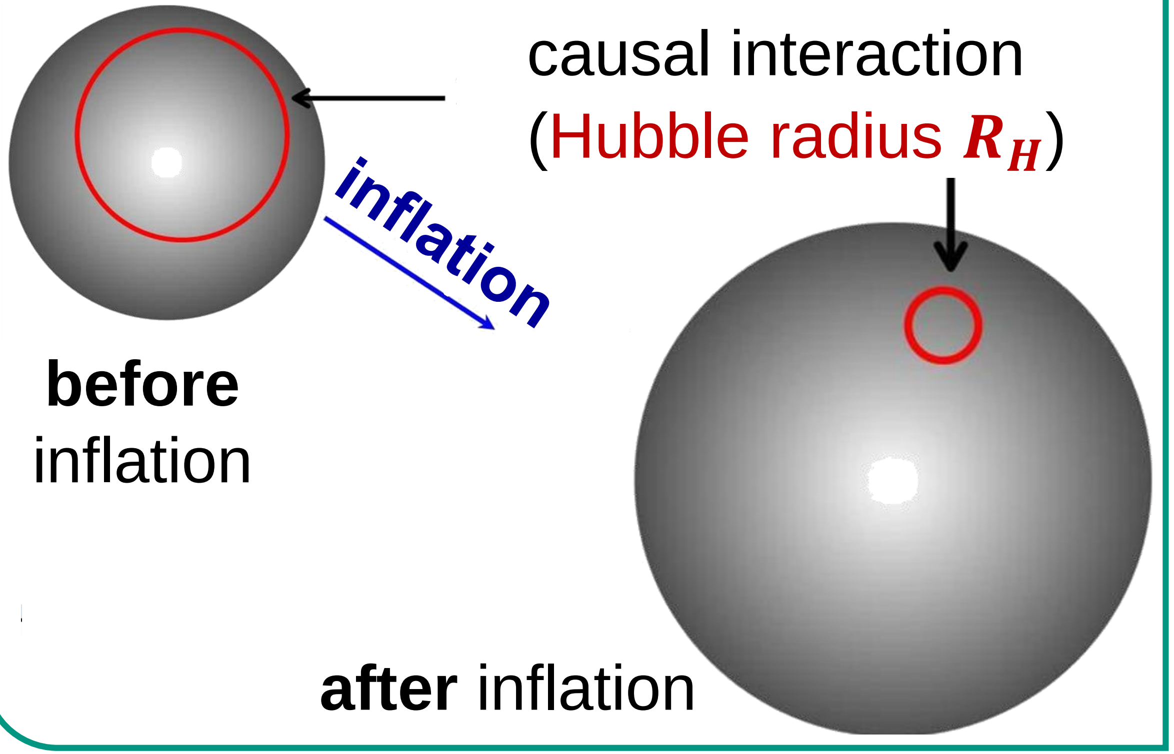
- small ℓ: QM is visible in the CMB sky







CMB temperature fluctuations at the largest scales: ⇨ we ´see´ **zero–point fluctuations** of the early inflaton field



*Quantum Mechanics

Sachs–Wolfe effect due to inflation, part 1

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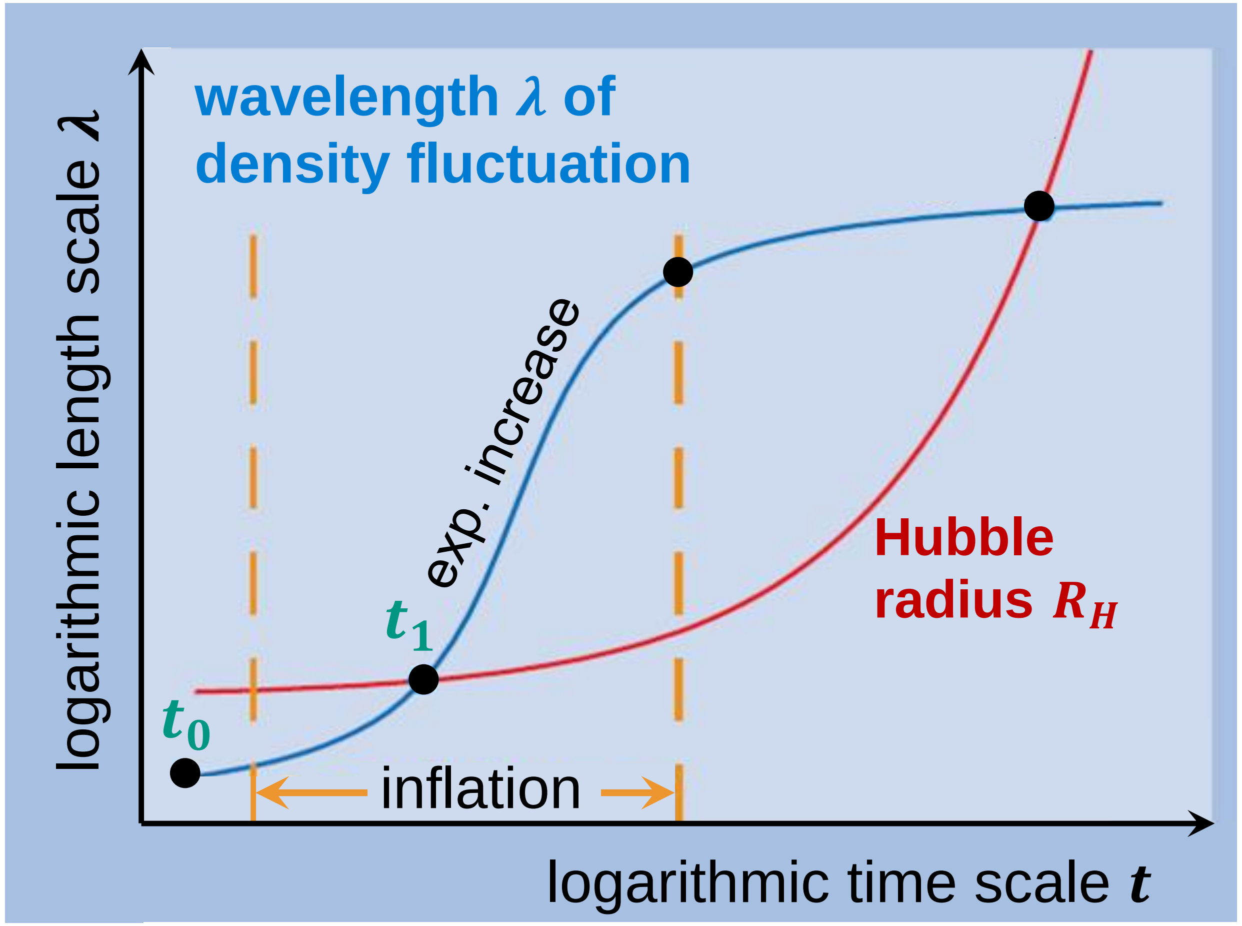
- **from the zero–point fluctuation before inflation to inflation…**

- density fluctuations appear on **all length (λ –) scales** (QM: zero–point)

- before onset of inflation $t = t_0$:
  regions are in full causal contact,
  as **Hubble radius $R_H > \lambda$**

- **inflationary phase** $t = t_1\ (< t_2)$ **:**
  exponential increase of the **length λ**
  of a density fluctuation $\Delta\rho$, we
  now have **Hubble radius $R_H \ll \lambda$**

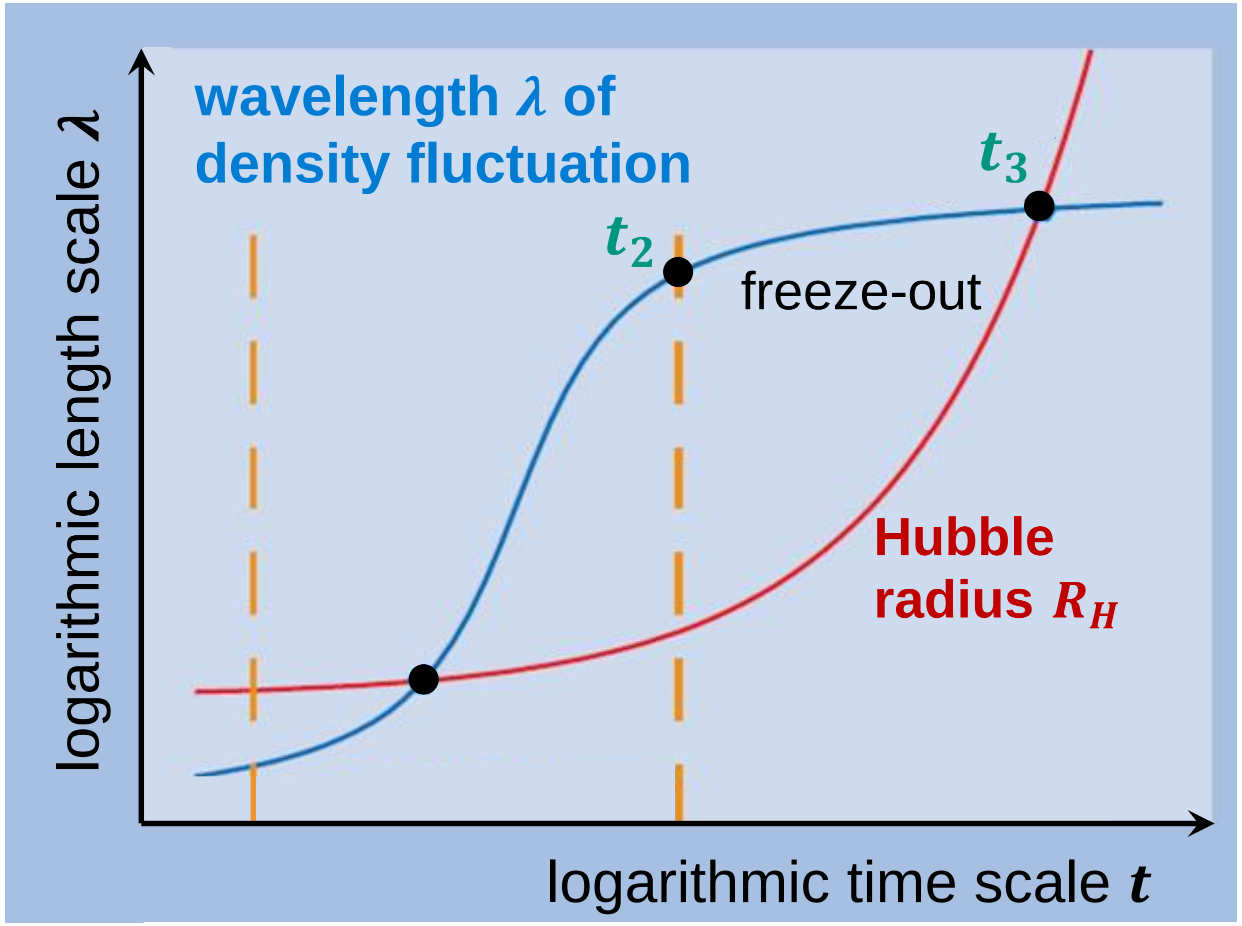


Sachs–Wolfe effect due to inflation, part 2

■ **From the freeze–out of the density mode to the re–entry into Hubble radius**

- after the end of inflation $t_2 < t < t_3$:
  the perturbation with the large size
  $\lambda \gg R_H$ cannot further grow in density
  contrast (no causal contact!)
  thus it remains ´**frozen**´

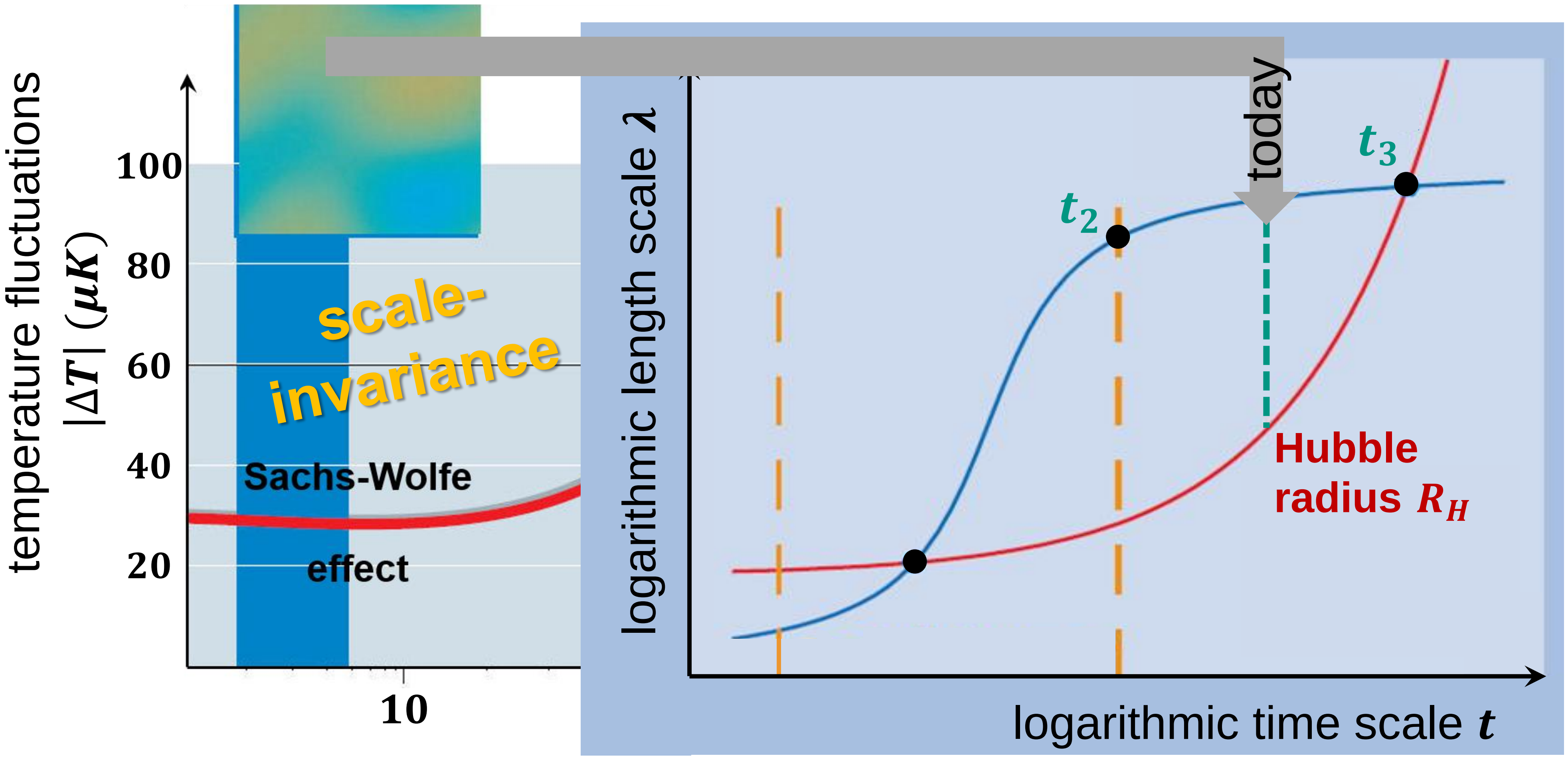
- after the Hubble radius has increased
  at later times $t > t_3$:
  the perturbation now has $\lambda < R_H$
  i.e. it can finally grow in density
  contrast (causal contact!),
  thus it is **no longer ´frozen´**



Sachs–Wolfe effect due to inflation: wrap–up

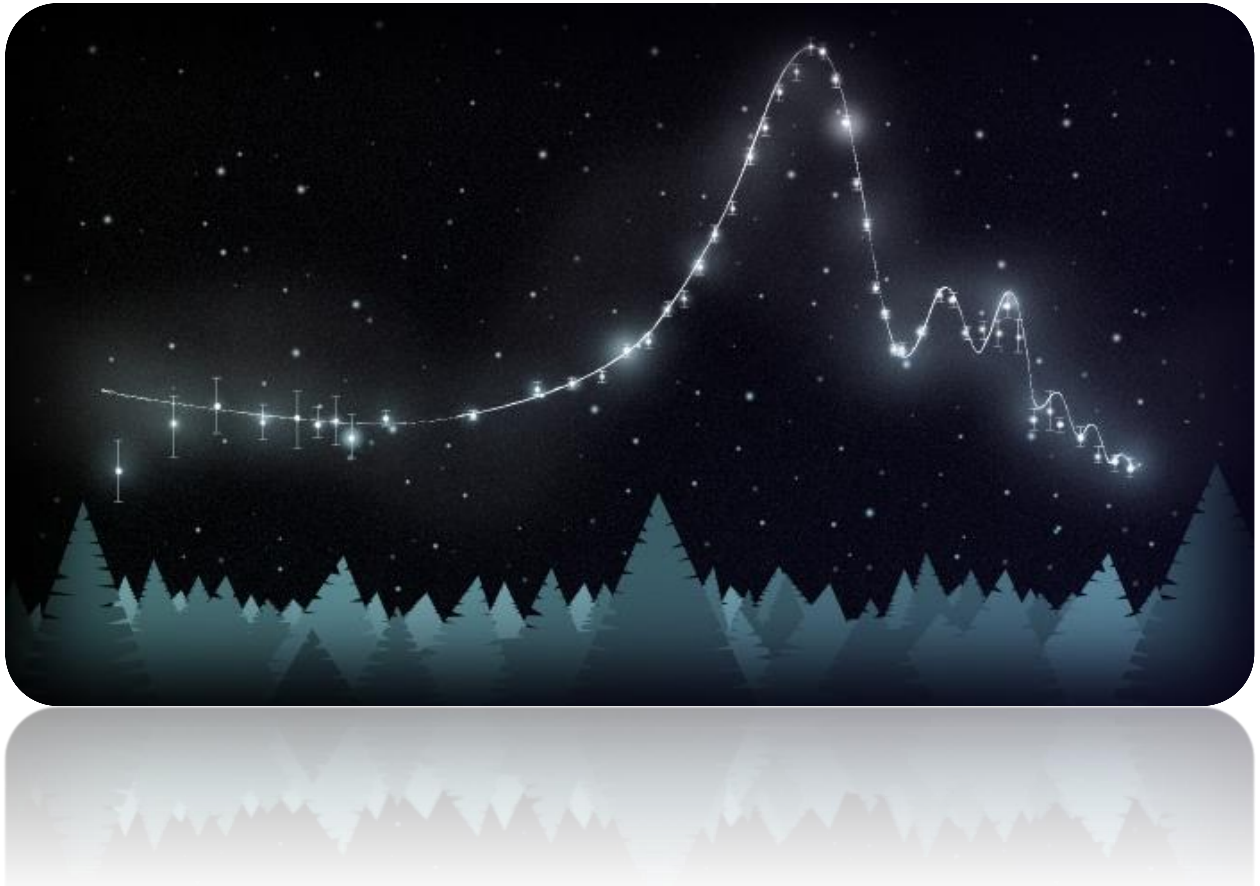
- **In the CMB we see ´frozen´ density perturbations at the largest scales**

- we see a $\boldsymbol{CMB}$ – **mode** in **frozen state** after increase of $\boldsymbol{\lambda}$ due to the **very early inflationary phase**



Legacy of cosmology: written in CMB multipoles

- **Happy Holidays** & **a Happy New Year 2024**



FROHE FESTTAGE UND EIN GUTES NEUES JAHR