§7 Relic neutrinos

Neutrinos decouple from thermal bath at $T_{dec} \sim 2-3$ MeV. What happens then?

Without interactions, the coordinate momentum $k = a \cdot p$ of each neutrino is time-independent

$$\Rightarrow \quad f(p,t) = f(k) = f_{dec}\left(\frac{a(t)}{a_{dec}} p\right)$$

$\uparrow$ scale factor at decoupling

$$\text{with} \quad f_{dec}(p) = \frac{1}{(2\pi)^3} \frac{1}{e^{p/T_{dec}} + 1}$$

$$\Rightarrow \quad f(p,t) = \frac{1}{(2\pi)^3} \frac{1}{e^{p/T_{eff}(t)} + 1} \qquad \text{with} \quad T_{eff}(t) = \frac{a_{dec}}{a(t)} T_{dec}$$

↳ Neutrinos maintain thermal distr. even without interactions

↳ Effective temperature decreases as $T_{eff} \sim a^{-1}$

$\Rightarrow n_\nu \sim a^{-3}$

↳ True even for $T_{eff} < m_\nu$

$\Rightarrow$ Neutrinos have rel. distribution even in present universe ("hot relic")

$\Rightarrow$ Very different from equil. dist. (i.e. Maxwell-Boltzmann)

Since $T_{dec} > m_e$, there are still many e^+, e^- in thermal bath when neutrinos decouple

↳ Annihilate away for $T \ll m_e$: $e^+ e^- \to \gamma\gamma$

↳ Energy and entropy transferred to photons, but not to neutrinos

$\Rightarrow$ T_γ does not decrease as $a(t)^{-1}$

$\Rightarrow$ For $T_\gamma \ll m_e$: $T_\gamma \neq T_{\nu,eff}$

Make use of entropy conservation in electron-photon system:

$$g_*^{e\gamma}(T_\gamma)\, a^3 T_\gamma^3 = \text{const}$$

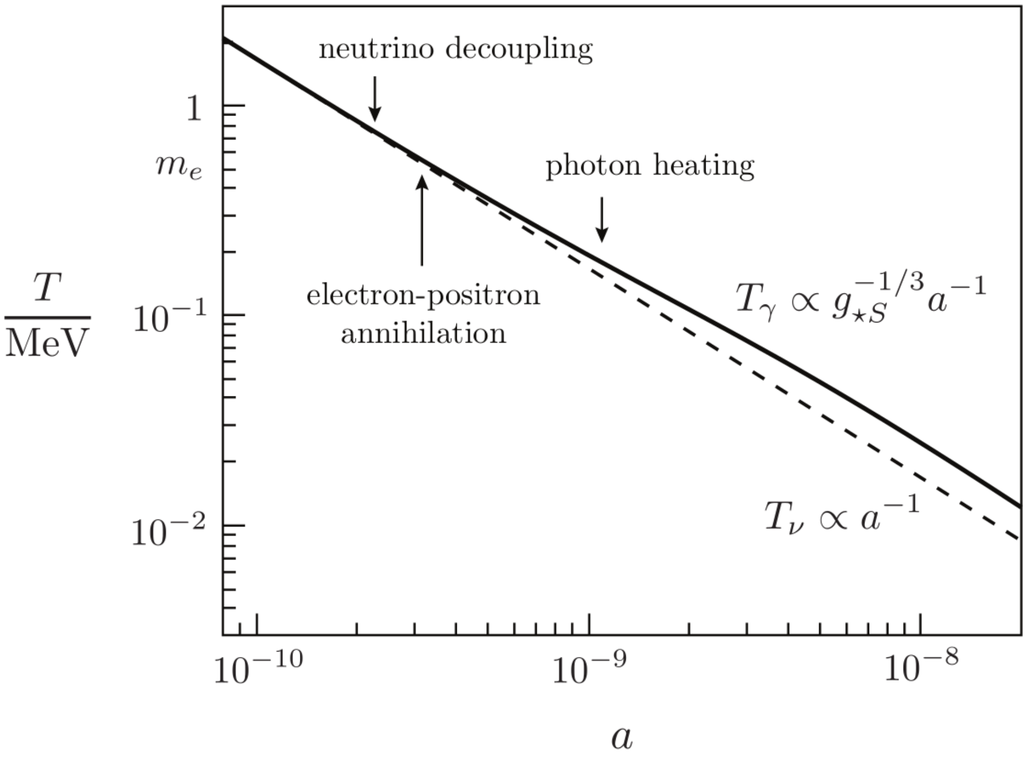
For $T_{dec} > T_\gamma > m_e$: $g_*^{e\gamma} = \underset{\gamma}{\underset{\uparrow}{2}} + \frac{7}{8}(\underset{e^-}{\underset{\uparrow}{2}} + \underset{e^+}{\underset{\uparrow}{2}}) = \frac{11}{2}$

For $T_\gamma \ll m_e$: $g_*^{e\gamma} = 2$

$$\Rightarrow \frac{11}{2} a_{dec}^3 T_{dec}^3 = 2 a^3 T_\gamma^3$$

$$\Rightarrow T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_{dec} \frac{a_{dec}}{a}$$

$$\Rightarrow T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_{\nu,eff}$$



7.1 Neutrinos in the present Universe

Use CMB temperature $T_{\gamma,0} = 2.73\,\mathrm{K}$ to predict present-day temperature of cosmic neutrino background ($C\nu B$):

$$T_{C\nu B} = T_{\gamma,\mathrm{eff},0} \approx 1.95\,\mathrm{K}$$

$$\Rightarrow n_{\nu,0} = \frac{3}{4} \cdot 2 \cdot \frac{\zeta(3)}{\pi^2} T_{C\nu B}^3 \approx 112\,\mathrm{cm}^{-3} \text{ per species}$$

So far not detected. Promising idea: PTOLEMY

$$\nu_e + {}^3H \to {}^3He + e^-$$

↳ Tiny energy release, very challenging!

What about indirect effects?

↳ Contribution of ρ_ν modifies expansion rate

$$\rho_{\nu,0} = \sum m_\nu \cdot n_{\nu,0}$$

↳ Require $\Omega_\nu = \frac{\rho_{\nu,0}}{\rho_c} < 1 \Rightarrow \sum m_\nu < 50\,\text{eV}$

Require $\Omega_\nu < \Omega_M \Rightarrow \sum m_\nu < 15\,\text{eV}$

KATRIN experiment: $m_{\nu_e} < 0.8\,\text{eV} \Rightarrow \sum m_\nu < 2.4\,\text{eV}$

$$\Rightarrow \Omega_\nu < 0.05$$

Neutrinos cannot be all of dark matter!

Neutrino oscillation experiments: $\sum m_\nu > 0.06\,\text{eV}$

$$\Rightarrow \Omega_\nu > 10^{-3} \gg \Omega_{rad}$$

7.2 Neutrinos during radiation domination

General treatment: Consider non-interacting rel. species with $T_{ni} \neq T_\gamma$

$$\Rightarrow \rho = \rho_{eq} + \rho_{ni}$$

$$= \left(\sum_{\substack{\text{bosons} \\ \text{in eq.}}} g_i + \frac{7}{8} \sum_{\substack{\text{fermions} \\ \text{in eq.}}} g_i \right) \frac{\pi^2}{30} T_\gamma^4 + \left(\frac{7}{8}\right) g_{ni} \frac{\pi^2}{30} T_{ni}^4$$

$$= g_* \frac{\pi^2}{30} T_\gamma^4$$

with $$g_* = \sum_{\substack{\text{bosons} \\ \text{in eq.}}} g_i + \frac{7}{8} \sum_{\substack{\text{fermions} \\ \text{in eq.}}} g_i + \left(\frac{7}{8}\right) g_{ni} \left(\frac{T_{ni}}{T_\gamma}\right)^4$$

Analogous: $$s = g_{*,s} \frac{2\pi^2}{45} T_\gamma^3$$

with $$g_{*,s} = \sum_{\substack{\text{bosons} \\ \text{in eq.}}} g_i + \frac{7}{8} \sum_{\substack{\text{fermions} \\ \text{in eq.}}} g_i + \left(\frac{7}{8}\right) g_{ni} \left(\frac{T_{ni}}{T_\gamma}\right)^3$$

$\Rightarrow$ $g_* \neq g_{*,s}$ in general

In our case: $T_{ni} = T_{\gamma,\text{eff}} = \left(\frac{4}{11}\right)^{1/3} T_\gamma$

$\Rightarrow$ For $T_\gamma < m_e$: $$g_* = 2 + 6 \cdot \frac{7}{8} \cdot \left(\frac{4}{11}\right)^{4/3}$$
$$= 3.36$$

$$g_{*,s} = 2 + 6 \cdot \frac{7}{8} \cdot \frac{4}{11}$$
$$= 3.91$$

Convenient to define

$$\boxed{N_{eff} = \frac{\rho_\nu}{\frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_\gamma}}$$

"effective number of neutrino species"

Calculation so far: $N_{eff} = 3$

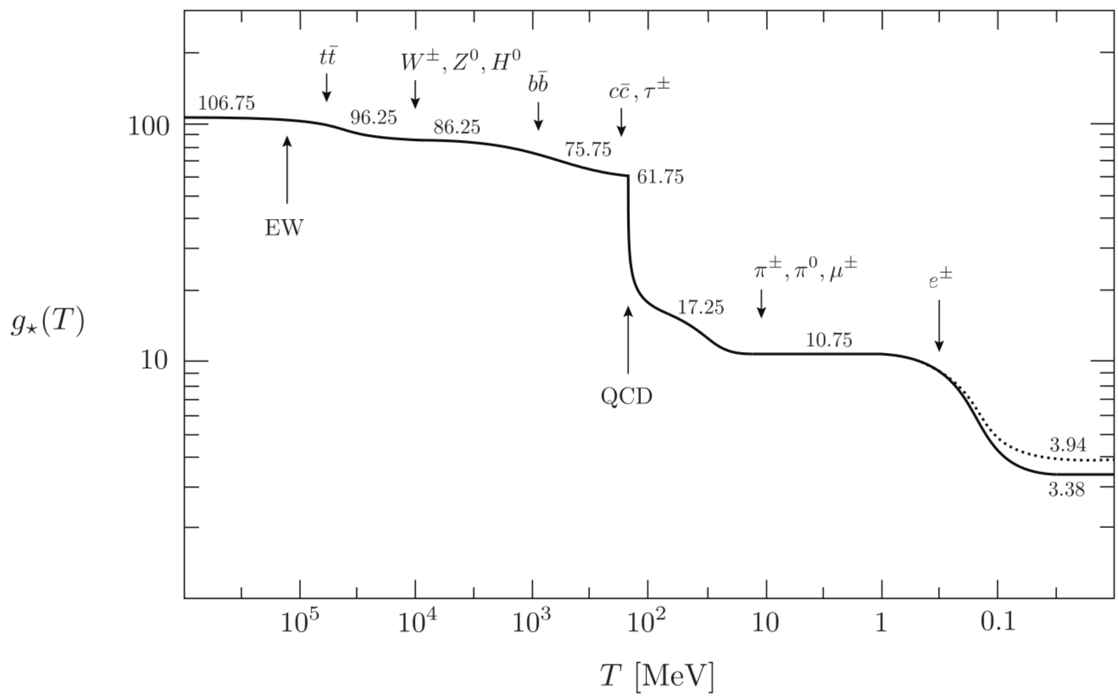
↳ Assumes instant decoupling

↳ More realistic: Neutrinos benefit slightly from e^+e^- annihilation

↳ Detailed calculation: $N_{eff} = 3.0440$

$\Rightarrow$ $g_* = 3.38$

$g_{*,s} = 3.94$



Hubble rate during RD: $H = 1.66 \sqrt{g_*} \frac{T^2}{M_{pl}}$

↳ highly sensitive to contribution from νs

7.3 Dark radiation

Consider particle N that decouples from thermal bath at $T_{N,dec}$

$\Rightarrow$ Hot relic for $m_N \ll T_{N,dec}$

$$T_{N,eff} = \left(\frac{g_*}{g_{*,dec}}\right)^{1/3} T_\gamma$$

$$\rho_N = \xi \frac{g_N}{2} \left(\frac{g_*}{g_{*,dec}}\right)^{4/3} \rho_\gamma \quad \text{with } \xi = \begin{cases} 1 & \text{boson} \\ \frac{7}{8} & \text{fermion} \end{cases}$$

Contribution to $N_{eff} = \underbrace{N_{eff,SM}}_{3.0440} + \Delta N_{eff}$

$$\Delta N_{eff} = \frac{\rho_N}{\frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_\gamma} = \frac{4}{7} \xi g_N \left(\frac{11}{4} \frac{g_{*,s}}{g_{*,dec}}\right)^{4/3}$$

↑ before neutrino decoupling

Consider a Majorana fermion (right-handed neutrino) with $\xi = 7/8$, $g_N = 2$, $T_{dec} > 100$ GeV ($g_{*,dec} \approx 100$)

$\Rightarrow \Delta N_{eff} \approx 0.05$

↳ Important target for cosmology