§8 Big Bang Nucleosynthesis (BBN)

→ Formation of bound nuclei from p, n

→ Happens shortly after ν decoupling

$$50\ \text{keV} \lesssim T \lesssim 1\ \text{MeV} \overset{RD}{\Rightarrow} 1\text{s} \leq t \leq 400\text{s}$$

→ Earliest time probed by observations

8.1 Qualitative picture

- At $T > 1$ MeV reactions like $p + e \leftrightarrow n + \nu_e$ are in equilibrium
- At $T \approx 1$ MeV, neutrons freeze out, so the only relevant process is $n \to p + e + \bar{\nu}_e$
- At $T \lesssim 0.1$ MeV it becomes favourable to form bound states such as $n + p \leftrightarrow D + \gamma$

  binding energy: $B_D = 2.2$ MeV
- Only light elements (H, He, Li, Be) can be formed

$\Rightarrow$ Almost all neutrons end up in ^4He

$B_{^4He} = 28.3$ MeV

8.2 Step 1: Neutron freeze-out

Equil. at $T \sim 1$ MeV:

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} e^{(\mu_A - m_A)/T} = e^{\mu_A/T} n_A^{\mu=0}$$

$A = p, n$

$e^\pm, \nu, \bar{\nu}$ relativistic $\Rightarrow$ μ negligible

$\Rightarrow \mu_p = \mu_n$

$$\Rightarrow \frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-\Delta m/T} \approx e^{-\Delta m/T}$$

with $\Delta m = m_n - m_p = 1.3$ MeV

Neutrons freeze out when

$\Gamma_{p \leftrightarrow n} < H$

Dimensional analysis: $H \sim \frac{T^2}{M_{pl}}$ $\quad \Gamma \sim G_F^2 T^5$

$$\Rightarrow T_n \sim \left(M_p \cdot G_F^2\right)^{-1/3}$$

$$= 0.8 \text{ MeV}$$

Full calculation: $T_n = 0.75$ MeV

$$\Rightarrow \frac{n_n}{n_p}(T = T_n) = 0.18$$

Comment: $T_n \approx \Delta m$ great coincidence!

$\hookrightarrow$ Present universe would look very different for $T_n \gg \Delta m$ or $T_n \ll \Delta m$

8.3 Step 2: Neutron decay

Shortly after neutron freeze-out: e^+e^- annihilation

$\Rightarrow g_s^* = 3.94 = \text{const}$

$$\Rightarrow \eta_B \equiv \frac{n_p + n_n}{n_\gamma} \sim \frac{n_B}{s} = \text{const}$$

(baryon-photon ratio, typically $\eta_B \sim 10^{-10}$)

But $X_n = \frac{n_n}{n_n + n_p}$ changes because of neutron decays:

$$X_n(T) \overset{T<T_n}{=} e^{-t/\tau_n} X_n(T_n) \quad \text{with } \tau_n = 880\,\text{s}$$

8.4 Step 3: Deuterium bottleneck

Direct production of ^4He from $2p + 2n$ strongly suppressed

$\Rightarrow$ Need to produce D first

Consider reaction $p + n \leftrightarrow D + \gamma$

$$\Rightarrow \mu_p + \mu_n = \mu_D \qquad (\mu_\gamma = 0)$$

$$\frac{n_D}{n_p \cdot n_n} = \underbrace{\frac{e^{\mu_D}}{e^{\mu_p} e^{\mu_n}}}_{=1} \frac{n_D^{\mu=0}}{n_p^{\mu=0} \, n_n^{\mu=0}}$$

$$= \frac{g_D}{g_p g_n} \left(\frac{2\pi \, m_D}{m_n m_p T} \right)^{3/2} e^{\frac{m_n + m_p - m_D}{T}}$$

Using $g_D = 3$, $g_p = g_n = 2$, $m_n \approx m_p \approx \frac{m_D}{2}$, $m_n + m_p - m_D = B_D$

$$\frac{n_D}{n_p \cdot n_n} = \frac{3}{4} \left(\frac{4\pi}{m_p T} \right)^{3/2} e^{B_D/T}$$

Using $X_n (1 - X_n) = \frac{n_p \cdot n_n}{n_B^2}$ and $n_B = \eta_B n_\gamma$

$$\Rightarrow \frac{n_D}{n_B} = \frac{3}{4} X_n (1 - X_n) \eta_B n_\gamma \left(\frac{4\pi}{m_p T} \right)^{3/2} e^{B_D/T}$$

$$\sim 10^{-10} \left(\frac{T}{m_p} \right)^{3/2} e^{B_D/T}$$

For $T \gtrsim 100$ keV : $\frac{n_D}{n_B} \ll 1$

D production efficient $\Leftrightarrow$ $\frac{n_D}{n_B} = 1$

$$\Rightarrow \frac{B_D}{T} = - \ln \left[10^{-10} \left(\frac{T}{m_p} \right)^{3/2} \right] \sim 30$$

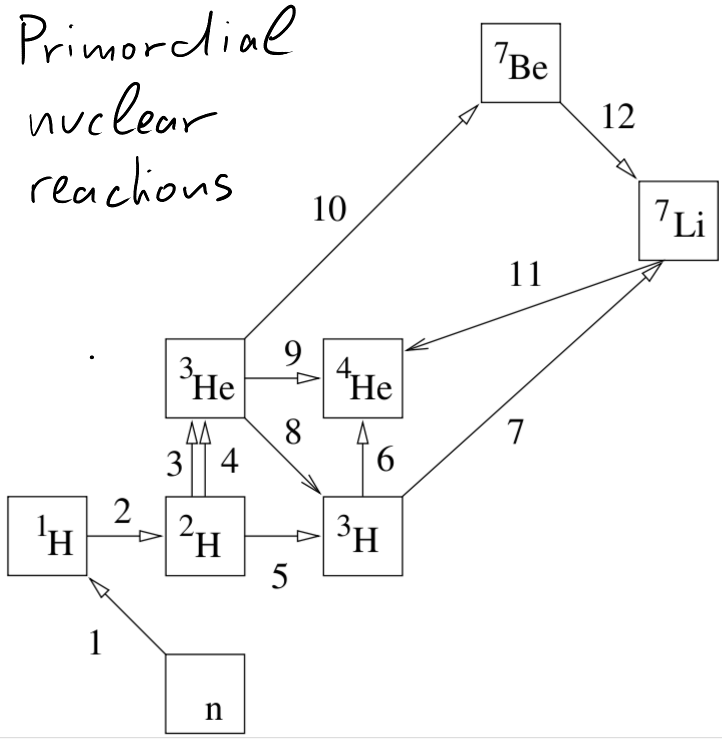
$$\Rightarrow T_D \approx 80 \text{ keV}$$

$$\Rightarrow t_D \approx 150 \text{ s}$$

8.5 Step 4: Deuterium burning

For $T < T_D$: Chain of nuclear reactions

Primordial nuclear reactions



1. $p \longleftrightarrow n$
2. $p(n,\gamma)d$
3. $d(p,\gamma)^3\mathrm{He}$
4. $d(d,n)^3\mathrm{He}$
5. $d(d,p)t$
6. $t(d,n)^4\mathrm{He}$
7. $t(\alpha,\gamma)^7\mathrm{Li}$
8. $^3\mathrm{He}(n,p)t$
9. $^3\mathrm{He}(d,p)^4\mathrm{He}$
10. $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$
11. $^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$
12. $^7\mathrm{Be}(n,p)^7\mathrm{Li}$

$$\left.\begin{array}{l} D + D \rightarrow {}^3H + p \\ D + D \rightarrow {}^3He + n \\ D + n \rightarrow {}^3H + \gamma \\ D + p \rightarrow {}^3He + \gamma \end{array}\right\} \Rightarrow \begin{array}{l} D + {}^3He \rightarrow {}^4He + p \\ D + {}^3H \rightarrow {}^4He + n \end{array}$$

$\Rightarrow$ Almost all neutrons end up in ^4He

$$\Rightarrow \frac{n_{^4He}}{n_B} = \frac{1}{2} X_n (T = T_D) = \frac{1}{2} e^{-t_D/\tau_n} X_n (T = T_n)$$

$$= 0.063 \approx \frac{1}{16}$$

$$Y_p = \frac{\rho_{^4He}}{\rho_B} \approx 4 \frac{n_{^4He}}{n_B} = 0.25$$

Fraction of ^4He remains constant until star formation starts, which produces heavier elements (e.g. $^4He + ^4He + ^4He \to ^{12}C$)

In some regions with low star formation rates $Y_p(\text{today}) \approx Y_p(\text{BBN})$

$\Rightarrow$ Possible to directly measure Y_p

Result: $Y_p = 0.245 \pm 0.003$

$\to$ Spectacular success!

8.6 Determining η_B

Y_p depends on η_B only logarithmically through T_D

↳ insufficient for measuring η_B

Instead: Consider end of D burning

$$\Gamma_D = n_D \cdot \langle \sigma v \rangle_D < H$$

Complicated nuclear physics

$\to$ can be measured

Freeze-out happens for $T_{fo} \approx 65$ keV

$$\Rightarrow \quad n_D(T_{fo}) = \frac{H(T_{fo})}{\langle\sigma v\rangle_D (T_{fo})} \approx 10^{14}\ cm^{-3}$$

Using $\quad n_p = n_B - 4\, n_{^4He} \approx \frac{3}{4} n_B = \frac{3}{4} \eta_B\, n_\gamma$

$$\frac{D}{H} = \frac{n_D}{n_p} = \frac{1}{\eta_B} \frac{4}{3} \frac{n_D(T_{fo})}{n_\gamma(T_{fo})} = \frac{1}{\eta_B} \cdot 1.6 \cdot 10^{-14}$$

Observations: $\quad \frac{D}{H} = (2.55 \pm 0.03) \cdot 10^{-5}$

$$\Rightarrow \quad \eta_B \approx (6.2 \pm 0.2) \cdot 10^{-10}$$

η_B remains constant until today

$$\Rightarrow \Omega_B h^2 = \frac{m_p \cdot \eta_B \cdot n_{\gamma,0}}{\rho_c / h^2} = 0.022 \pm 0.001$$

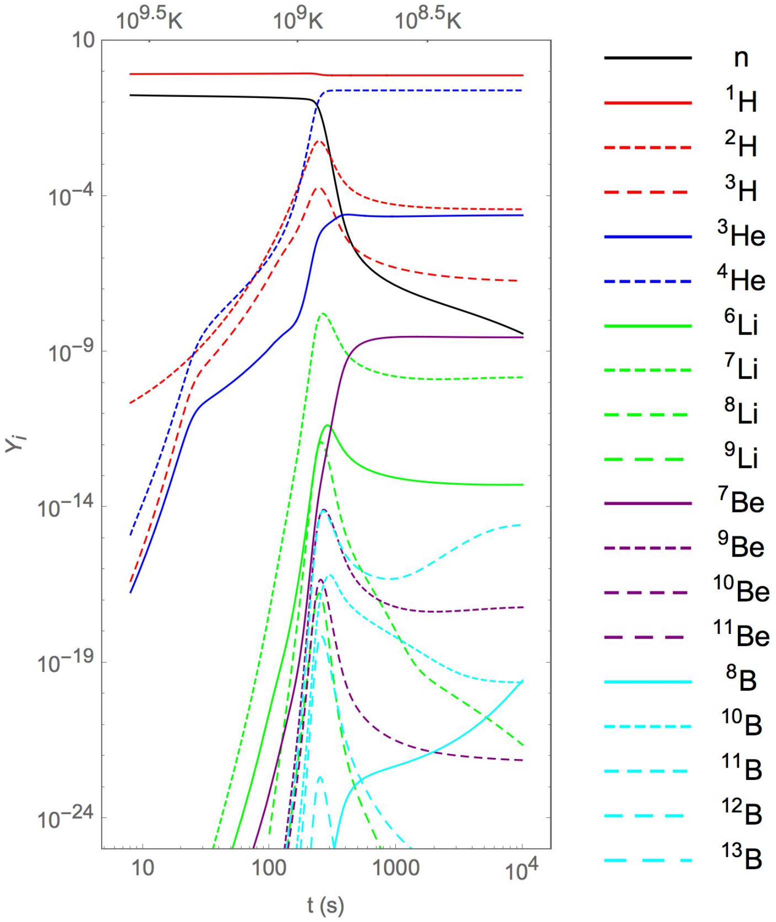
↳ Baryons only constitute ~4% of the energy density of the present universe!

Comment: Also possible to predict $\frac{^7Li}{H}$

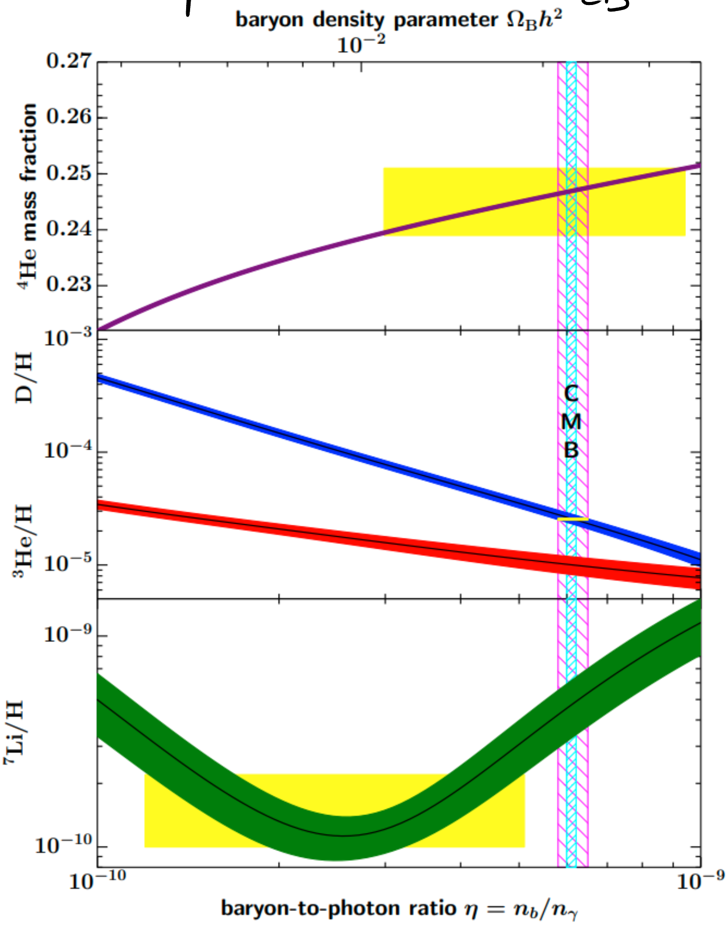
↳ Inferred value of η_B too small

↳ Theory uncertainty? Measurement error? New physics?

Dependence on t



Dependence on η_B



Note: BBN powerful probe of physics beyond Standard Model

Example: Y_p very sensitive to Hubble rate at $T \sim 1$ MeV

$\Rightarrow$ Confirms SM prediction $N_{eff} = 3.0440$

$\Rightarrow$ Upper bound: $\Delta N_{eff} < 0.4$ (95% C.L.)

$\Rightarrow$ Excludes additional light particles in thermal equilib.