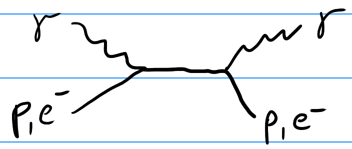
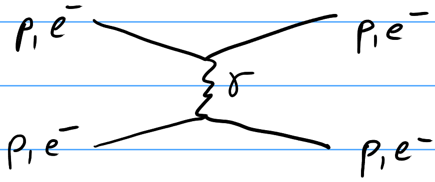
§9 Recombination

↳ Formation of neutral hydrogen
(neglect He for today)

Universe at $T = 1\,eV$:

- p, e^-, γ tightly coupled:





- Equilibrium between free particles and bound H:

$$e^- + p \longleftrightarrow H + \gamma$$

$$n_i^{eq} = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{(\mu_i - m_i)/T}$$

with $\mu_p + \mu_e = \mu_H$

$$\Rightarrow \underbrace{\left(\frac{n_H}{n_e \cdot n_p}\right)_{eq}}_{\text{charge neutrality: } \frac{n_H}{n_e^2}} = \underbrace{\frac{g_H}{g_e\, g_p}}_{= \frac{4}{2\cdot 2}} \Bigg(\underbrace{\frac{m_H}{m_e\, m_p}}_{\approx \frac{1}{m_e}} \frac{2\pi}{T}\Bigg)^{3/2} \underbrace{e^{(m_p + m_e - m_H)/T}}_{e^{B_H/T}}$$

with $B_H = 13.6\,eV$

$$\Rightarrow \left(\frac{n_H}{n_e^2}\right)_{eq} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T}$$

Convenient to define

$$X_e = \frac{n_e}{n_p + n_H} \qquad \text{(free electron fraction)}$$

Note: $n_p + n_H = 0.75 \cdot \eta_b \cdot n_\gamma$ ← $\frac{2\zeta(3)}{\pi^2} T^3$

↑ approximate fraction of H (rest is He)

$$\Rightarrow \quad 1 - X_e = \frac{\cancel{n_p} + n_H - \cancel{n_e}}{n_p + n_H}$$

$$\Rightarrow \quad \frac{1 - X_e}{X_e^2} = \frac{n_H}{n_e^2}(n_p + n_H)$$

$$\Rightarrow \quad \boxed{\left(\frac{1 - X_e}{X_e^2}\right)_{eq} = \frac{2\zeta(3)}{\pi^2}\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T} \cdot 0.75\, \eta_b}$$

(Saha equation)

For $T \gtrsim B_H$: rhs tiny (remember: $\eta_b \sim 10^{-10}$)

$$\Rightarrow X_e \approx 1$$

For $T \ll B_H$: rhs increases $\Rightarrow X_e$ decreases

Recombination happens when

$$\frac{B_H}{T} \approx -\log\left[\frac{2\zeta(3)}{\pi^2} 0.75\,\eta_b \left(\frac{2\pi T}{m_e}\right)^{3/2}\right]$$

$$= \underbrace{-\log\left[\frac{2\zeta(3)}{\pi^2} 0.75\,\eta_b \left(\frac{2\pi B_H}{m_e}\right)^{3/2}\right]}_{=\,35.9} + \underbrace{\frac{3}{2}\log\frac{B_H}{T}}_{\text{can be neglected}}$$

$\Rightarrow T_{rec} \approx 0.38\,eV$

$T_{rec} = T_0(1 + z_{rec}) \Rightarrow z_{rec} \approx 1600$

Numerical solution: $T = 0.4\,eV$: $X_e = 0.999$
$T = 0.3\,eV$: $X_e = 0.15$

$T_{rec} \ll B_H$ because $n_\gamma \gg n_e, n_p$

$\Rightarrow$ Enough high-energy photons in tail of distribution to keep H ionized

$\Rightarrow$ Recombination happened after M-R equality

$$\Rightarrow t_{rec} = \frac{2}{3} H(T_{rec})^{-1} = \frac{2}{3 H_0 \sqrt{\Omega_M (1 + z_{rec})^3}}$$

$$\approx 2.6 \cdot 10^5 \text{ years}$$

9.1 Photon decoupling

As long as $X_e \approx 1$, photons experience frequent interactions:

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

$$\sigma_T = \frac{8\pi}{3} \frac{\alpha^2}{m_e^2} \approx 0.67 \cdot 10^{-24}\ cm^2$$

(Thomson cross section)

$$\Gamma_\gamma = n_e \sigma_T = X_e \sigma_T (n_p + n_H)$$

With decreasing X_e, Γ_γ decreases as well

=> Photons decouple for $\Gamma_\gamma(T_{dec}) = H(T_{dec})$

MD: $$H(T_{dec}) = H_0 \sqrt{\Omega_m} \left(\frac{T_{dec}}{T_0}\right)^{3/2}$$

$$\Rightarrow X_e(T_{dec})\, T_{dec}^{3/2} = \frac{\pi^2}{2\zeta(3)} \frac{H_0 \sqrt{\Omega_m}}{0.75\, \eta_b\, \sigma_T\, T_0^{3/2}}$$

Can be estimated from Saha eq.

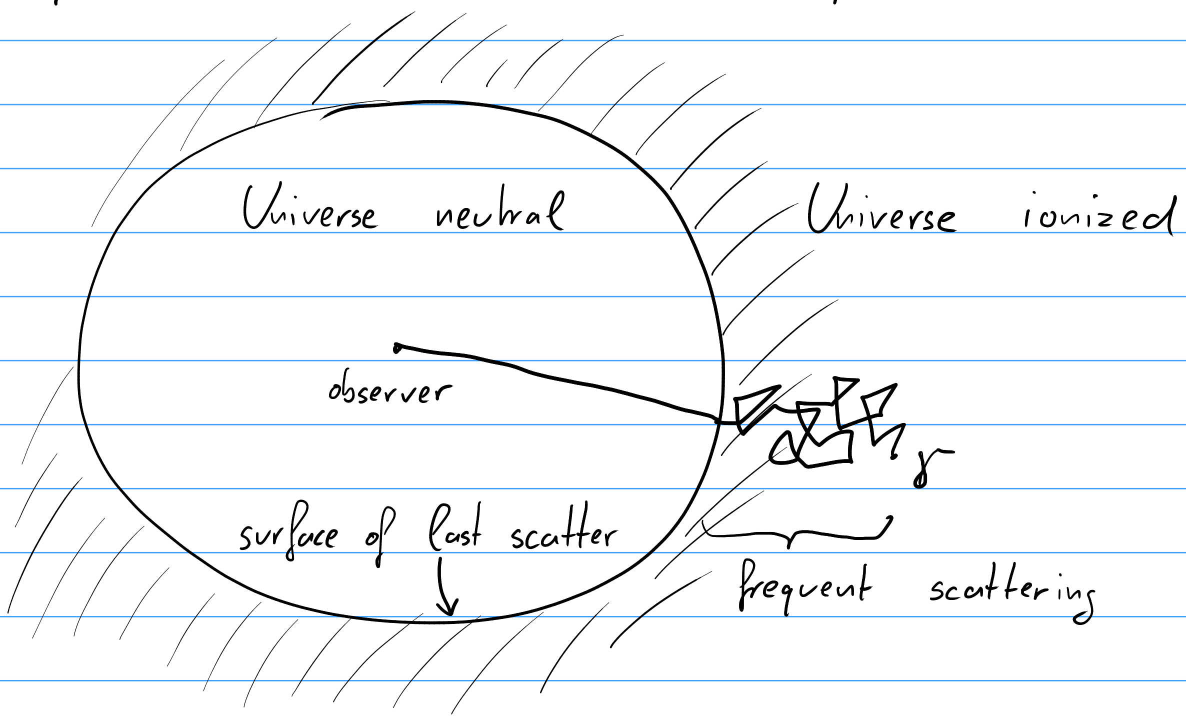
$$\Rightarrow T_{dec} \approx 0.27\ eV \Rightarrow z_{dec} \approx 1100$$

$$t_{dec} \approx 380\,000\ yrs$$

Many refinements needed:

- Electrons not in perfect eq. at T_{dec}
  $\Rightarrow$ Need to solve Boltzmann eq. to get X_e
- Need to include excited hydrogen (2s, 2p) in calculation

But final result robust, because X_e drops exponentially $\Rightarrow$ photon decoupling very sudden



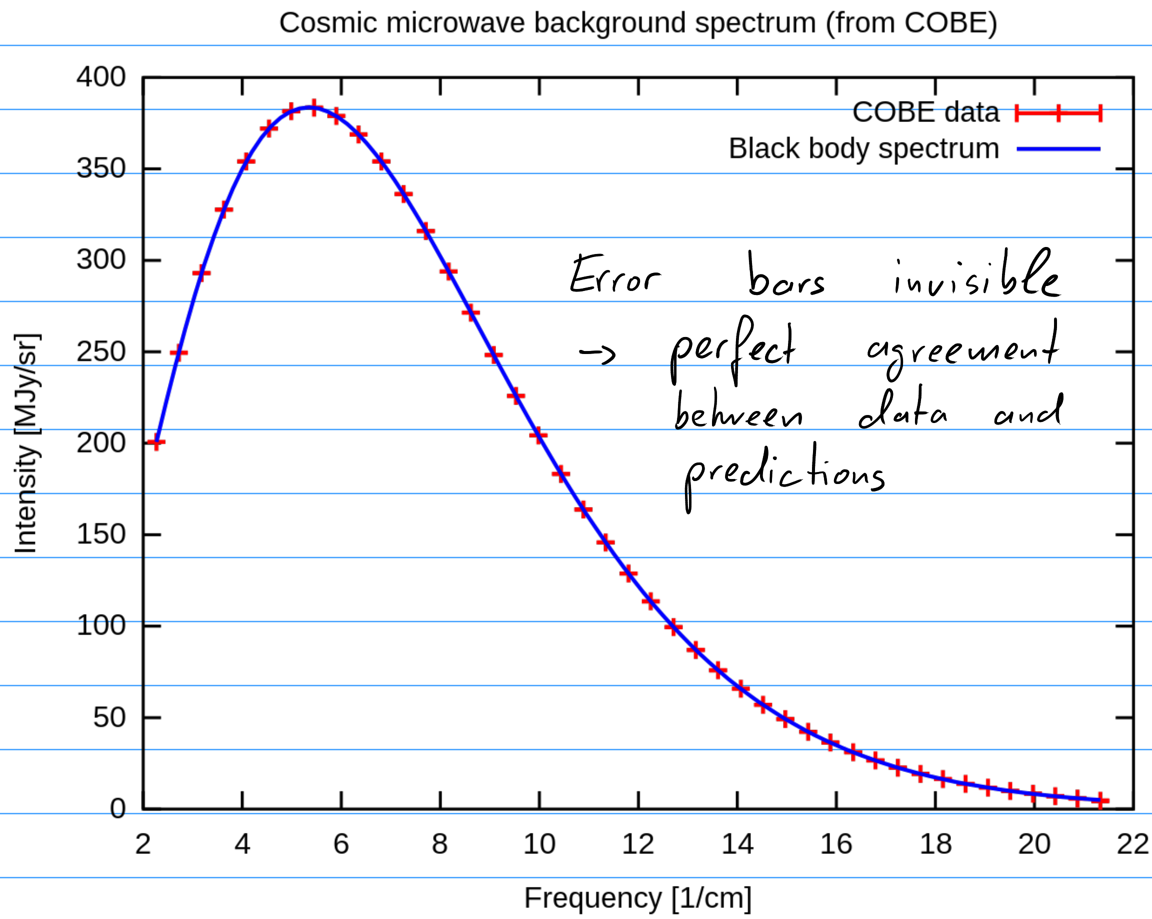
We see the surface of last scatter in every direction at a distance of ~ 13.5 Glyr

Beyond it, the Universe is intransparent

The photons emitted from this surface form the CMB

9.2 Cosmic Microwave Background

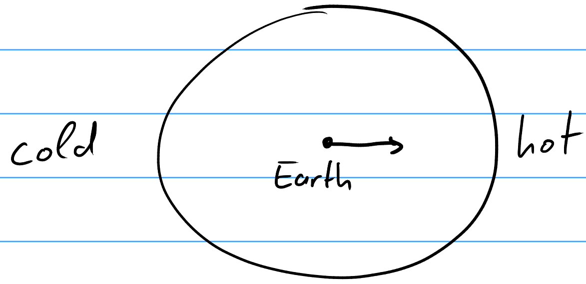
↳ First discovered in 1965 by accident



$$T_{CMB} = 2.7255 \pm 0.0006\,K$$

When seen from the Earth, T_{CMB} is not the same in every direction

↳ Doppler effect due to relative motion between Earth and CMB



$$P_{obs}(\vec{n}) = \frac{P}{\gamma(1-\vec{n}\cdot\vec{v})} \overset{v\ll 1}{\approx} P(1+\vec{n}\cdot\vec{v})$$

unit vector

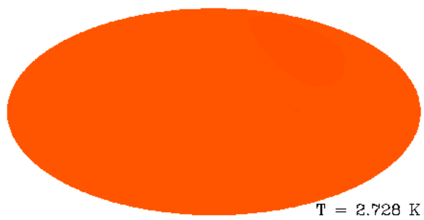
$$\Rightarrow \frac{\delta T(\vec{n})}{T} = \frac{P_{obs}(\vec{n}) - P}{P} = \vec{n}\cdot\vec{v}$$

$\Rightarrow$ dipole anisotropy

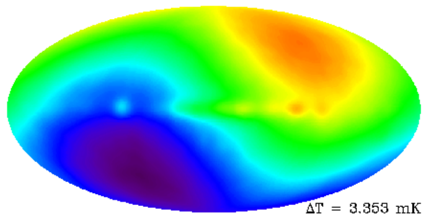
Fit to data gives $v = 368$ km/s.

→ Subtract dipole to reveal primordial anisotropy

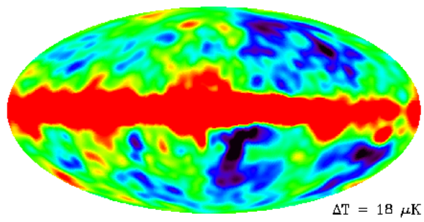
CMB monopole:



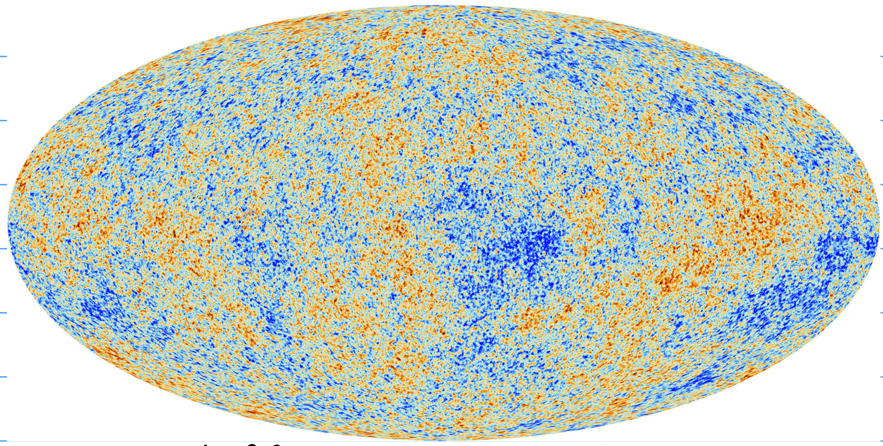
CMB dipole:



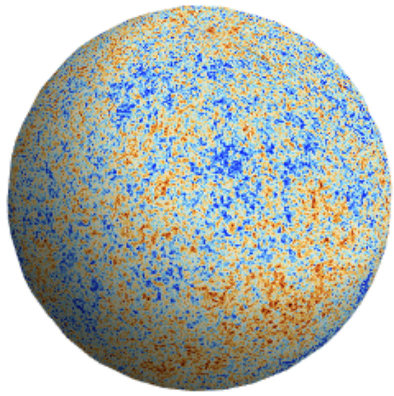
Primordial anisotropy:



Removing galactic foregrounds:



Mollweide projection



without projection

Conclusion: Universe is not perfectly homogeneous at $T = T_{dec}$

$\Rightarrow$ need to study perturbations

Summary: