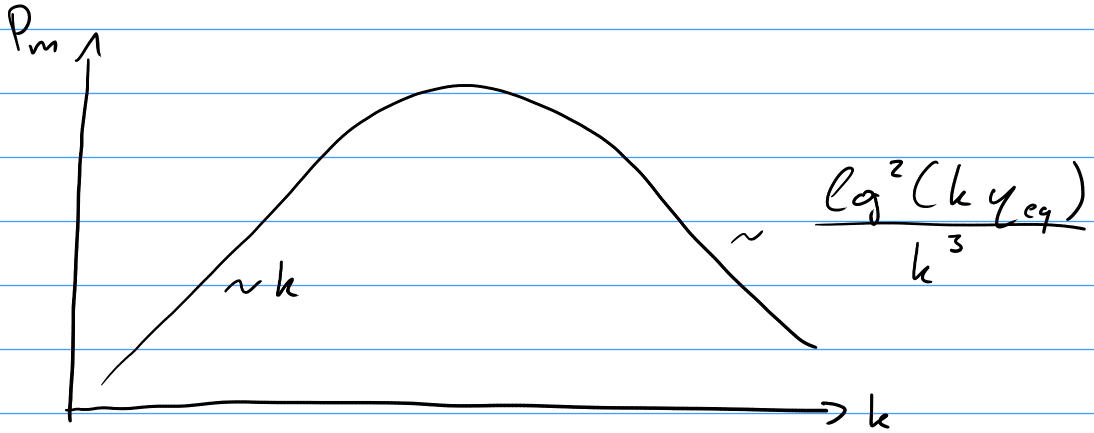
Recap:

- DM perturbations grow already during RD
- Jeans instability leads to formation of structures



Baryons:
- Strong interactions with photons
- Structure formation more complicated

11.3 Baryon Acoustic Oscillations

Consider perturbations in baryons & photons between M-R equality and photon decoupling

potential Φ dominated by DM

"tight coupling"
$v_\gamma = v_B \equiv v$

Continuity eq. (sub-horizon):

$$\delta_\gamma' + \frac{4}{3}\vec{\nabla}\cdot\vec{v} = 4\Phi' \qquad \delta_B' + \vec{\nabla}\cdot\vec{v} = 3\Phi'$$

Initial conditions: $\delta_\gamma = 4\frac{\delta T}{T}$; $\delta_B = 3\frac{\delta T}{T}$

$$\Rightarrow \delta_B = \frac{3}{4}\delta_\gamma$$

Conservation of combined momentum

$$q \equiv q_\gamma + q_b = \frac{4}{3}(1+R)\bar{\rho}_\gamma v$$

$$R = \frac{3}{4}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma}$$

$\Rightarrow$ Euler eq.:

$$[(1+R)\vec{v}]' = -\frac{1}{4}\vec{\nabla}\delta_\gamma - (1+R)\vec{\nabla}\Phi$$

increase of inertial mass — pressure term unchanged — increase of grav. mass

$$R' = \frac{3}{4}\left(\frac{\bar{\rho}_b'}{\bar{\rho}_\gamma} - \frac{\bar{\rho}_b}{\bar{\rho}_\gamma^2}\bar{\rho}_\gamma'\right) = \frac{3}{4}\left(-3\frac{a'}{a}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma} + 4\frac{a'}{a}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma}\right)$$

$$= \mathcal{H}R$$

$$\Rightarrow \vec{v}' + \frac{\mathcal{H}R}{1+R}\vec{v} = -\frac{1}{4(1+R)}\vec{\nabla}\delta_\gamma - \vec{\nabla}\Phi$$

Combining continuity & Euler eqs.:

$$\boxed{\delta_\gamma'' + \frac{\mathcal{H} R}{1+R}\delta_\gamma' + c_s^2 k^2 \delta_\gamma = -\frac{4}{3} k^2 \Phi \qquad c_s^2 = \frac{1}{3(1+R)}}$$

neglect Φ', Φ''

(sound speed of photon-baryon fluid)

Particular solution: $\delta_\gamma = -\frac{4}{3 c_s^2}\Phi$

$\Rightarrow$ grav. potential of DM induces perturbations in photons & baryons

Remember: $k^2 \Phi = -a^2 \underbrace{4\pi G \bar{\rho}_{DM}}_{\overset{MD}{=} \frac{3}{2} H_0^2 \Omega_{DM} \left(\frac{a_0}{a}\right)^3} \delta_{DM}$

$$\delta_m \overset{MD}{=} -\frac{27}{2}\Phi_i \log(k\eta_{eq}) \frac{a}{a_{eq}}$$

$$\Rightarrow \delta_\gamma = -\frac{27}{c_s^2}\Phi_i \left(\frac{H_0 a_0}{k}\right)^2 \frac{a_0}{a_{eq}} \Omega_{DM} \log(k\eta_{eq})$$

To solve homogeneous eq. consider

$$\delta_\gamma = A(k,\eta) \cos\left(k \int^\eta d\tilde{\eta}\, c_s\right)$$

varies slowly with η

Approximate solution for

$$2A' + \frac{\mathcal{H} R}{1+R} A = 0$$

$$\Rightarrow A \sim \exp\left(-\int^{\eta} d\tilde{\eta} \frac{\mathcal{H} R}{2(1+R)}\right)$$

$$= \exp\left(-\int_0^R \frac{d\tilde{R}}{2(1+\tilde{R})}\right) = \frac{1}{\sqrt{1+R}}$$

Initial condition:

Recover formulas for pure radiation in limit $R \to 0$, $c_s^2 \to \frac{1}{3}$

$$\delta_r = -2\Phi = 6\Phi_i \cos\left(\frac{1}{\sqrt{3}} k\eta\right)$$

$$\Rightarrow A = 6\Phi_i \cdot \frac{1}{\sqrt{1+R}}$$

Putting everything together:

$$\boxed{\begin{aligned} \delta_r &= \frac{4}{3}\delta_b = \Phi_i\left[-\frac{27}{c_s^2}\left(\frac{H_0 a_0}{k}\right)^2 \frac{a_0}{a_{eq}} \Omega_{0m} \log(k\eta_{eq}) + \frac{6}{\sqrt{1+R}} \cos\left(k\int^{\eta} d\tilde{\eta}\, c_s\right)\right] \\ v &= \frac{3}{4k^2}\delta_r' \end{aligned}}$$

Physical interpretation:

(1) Baryons fall into potential wells created by DM

=> density + temperature increase

(2) Photon pressure stops infall

=> turn-around

(3) Baryons get pushed away

=> density + temperature decrease

(4) Gravity wins over photon pressure

Cycle interrupted at photon decoupling

$$\Rightarrow \delta_b(k, \eta_{dec}) = \frac{3}{4}\delta_\gamma(k, \eta_{dec})$$

$$= \Phi_i(k)\left[F_1(k, \eta_{dec}) + F_2(k, \eta_{dec})\cos(k r_s)\right]$$

$\sim \frac{1}{k^2}$

only mild k dependence

$r_s = \int_0^{\eta_{dec}} c_s \, d\eta$ (sound horizon)

After decoupling δ_b and δ_γ evolve independently

$\Rightarrow$ no more photon pressure on baryons

$$\left.\begin{aligned} \delta_b' - k^2 v_b &= 3\phi' \\ v_b' + \frac{2}{\eta} v_b &= -\phi \end{aligned}\right\} \text{ same as for DM}$$

Define $\Delta \equiv \delta_{DM} - \delta_b$

$$\Rightarrow \Delta'' + \frac{2}{\eta}\Delta' = 0$$

$$\Rightarrow \Delta(\eta) = C_1 + \frac{C_2}{\eta}$$

Initial conditions at decoupling:

$$C_1 = \delta_{DM}^{dec} - \delta_b^{dec} + k^2 \eta_{dec} \left(v_{DM}^{dec} - v_b^{dec}\right)$$

$$C_2 = -k^2 \eta_{dec}^2 \left(v_{DM}^{dec} - v_b^{dec}\right)$$

For $\eta \to \infty$: $\Delta \to C_1 = \text{const}$

$$\delta_{DM} \sim a \sim \eta^2$$

$$\Rightarrow \Delta \ll \delta_{DM} \Rightarrow \delta_{DM} \approx \delta_b$$

$\hookrightarrow$ baryons "catch up" with DM perturbations

In the process, acoustic oscillations in baryons get transferred to DM

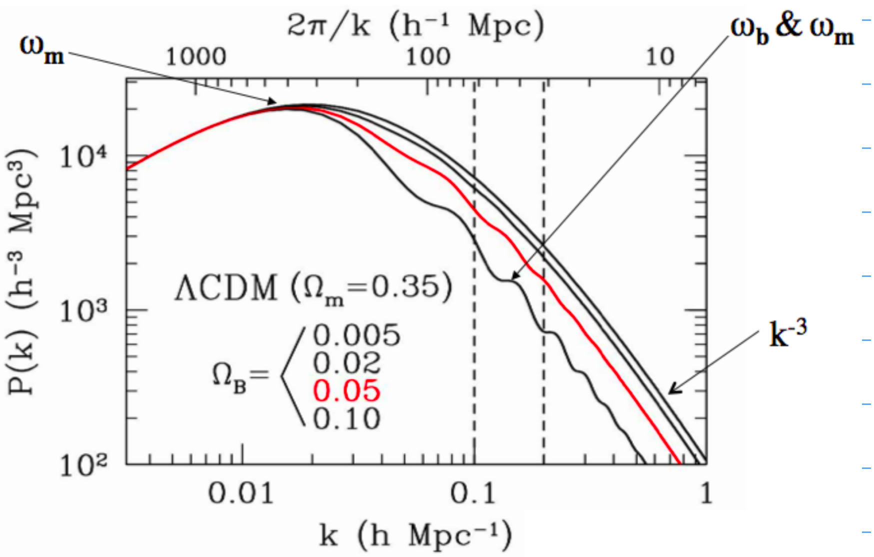
Lengthy calculation:

$$\delta_b \approx \delta_{DM} \approx \left[\frac{\Omega_{DM}}{\Omega_m} \delta_{DM}^{dec} + \frac{\Omega_b}{5\,\Omega_m} k^2 \eta_{dec} \underbrace{v_b^{dec}}_{\sim \sin(k r_s)} \right] \frac{a}{a_{dec}}$$

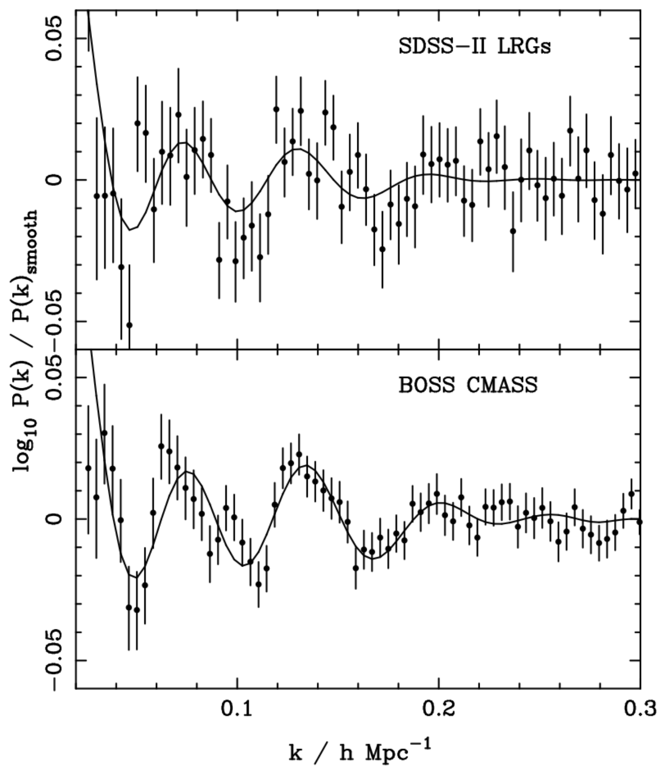
Note: $\dfrac{\Omega_b}{5\,\Omega_m} \approx 0.05 \ll 1$

$$\Rightarrow P_m(k,\eta) \sim \delta_{DM}^2 \sim P(k) \left(A + B \sin(k r_s) + \mathcal{O}\left(\frac{\Omega_b^2}{\Omega_m^2}\right)\right)$$

Matter power spectrum for different Ω_b



Deviation from smooth matter power spectrum



Calculate correlation function

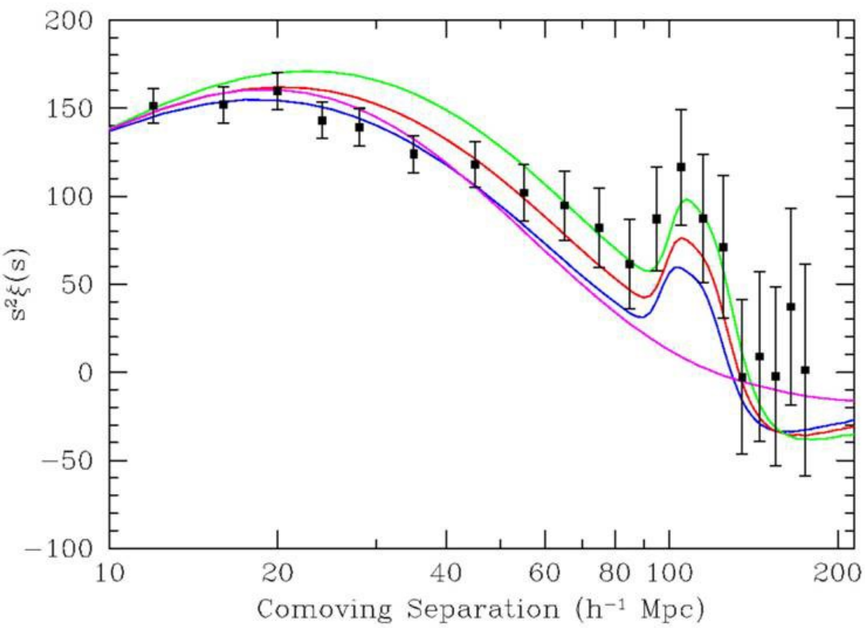
$$\xi_m(y,\eta) = \frac{1}{(2\pi)^3} \int d^3k \; e^{-i\vec{k}\cdot\vec{y}} P_m(k,\eta)$$

$$= \frac{1}{2\pi^2} \int_0^\infty dk \, \frac{k}{y} \sin(ky) \, P_m(k,\eta)$$

$$= \xi_0(y,\eta) + \xi_{osc}(y,\eta)$$

$$\xi_{osc}(y,\eta) = \frac{1}{2\pi^2} \int_0^\infty dk \, \frac{k}{y} \, B \, \sin(ky) \sin(k r_s) \, P(k)$$

$\hookrightarrow$ Peak for $y \approx r_s$



Observations give $a_0 r_s \approx 150$ Mpc

$\hookrightarrow$ physical size of sound horizon at photon decoupling