11.4 CMB anisotropies

Want to compare statistical properties of perturbations with CMB

For direction $\vec{n}$ define

$$\delta T_0(\vec{n}) = T_0(\vec{n}) - T_0$$

CMB average

Expand in spherical harmonics

$$\frac{\delta T_0(\vec{n})}{T_0} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\vec{n})$$

$$= \int d\vec{n} \, \frac{\delta T_0(\vec{n})}{T_0} Y^*_{\ell m}(\vec{n})$$

For Gaussian perturbations, expect

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \delta_{\ell \ell'} \delta_{m m'}$$

with $C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$

(angular power spectrum)

$$\Rightarrow \langle \delta T_0(\vec{n})\, \delta T_0(\vec{n}') \rangle$$

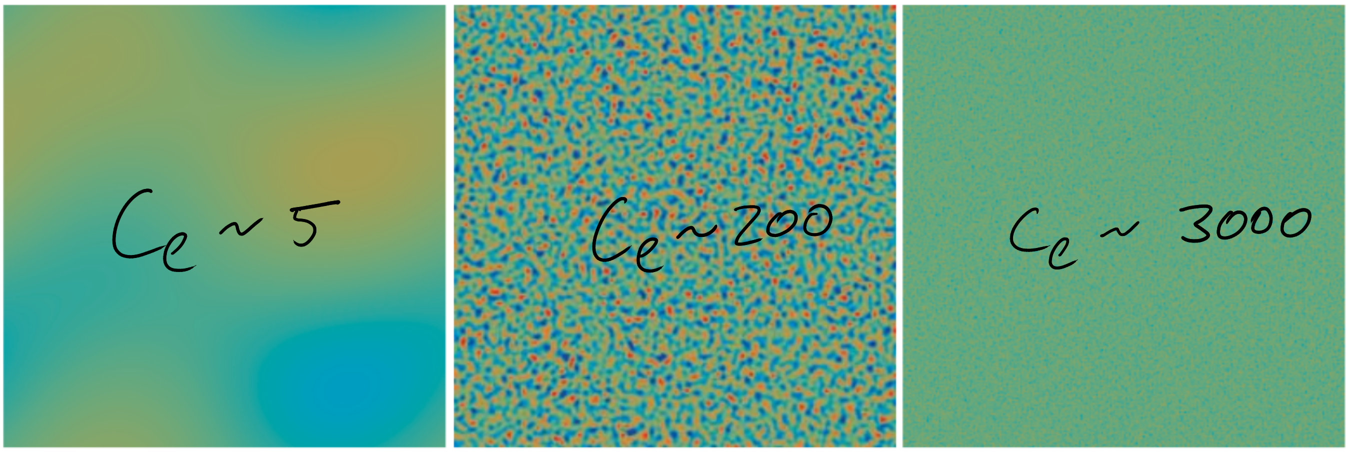
$$= T_0^2 \sum_\ell C_\ell \sum_m Y_{\ell m}(\vec{n})\, Y^*_{\ell m}(\vec{n}')$$

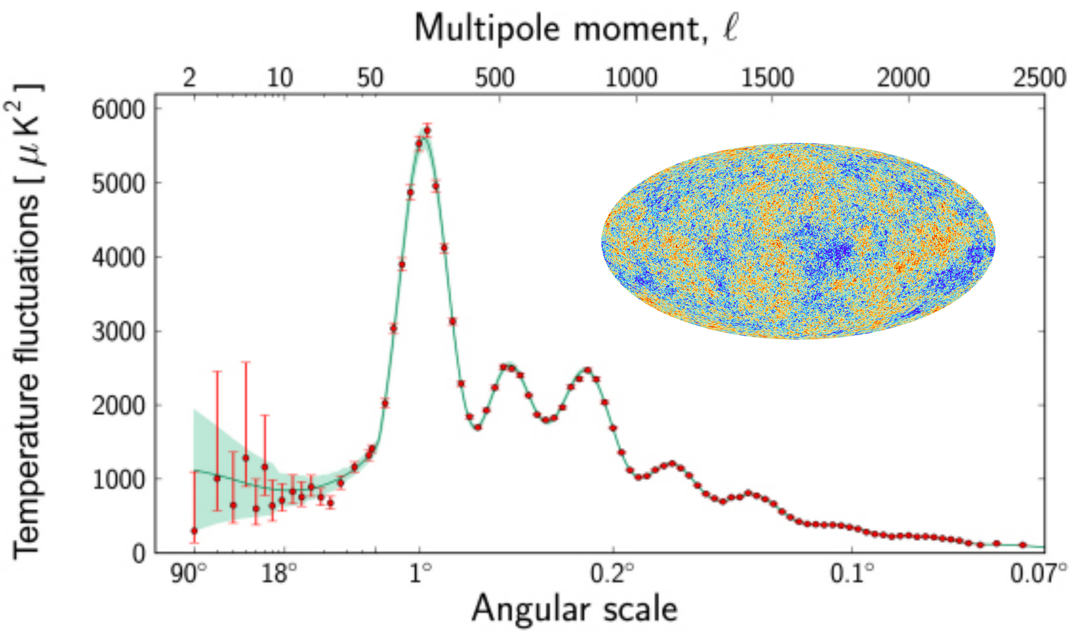
$$= T_0^2 \sum_\ell \frac{2\ell+1}{4\pi} C_\ell \, P_\ell(\cos\vartheta)$$

$\cos\vartheta = \vec{n}\cdot\vec{n}'$

Legendre polynomial

Multipole ℓ $\leftrightarrow$ angular scale $\Delta\vartheta = \frac{180°}{\ell}$





Most striking feature: peak at $\ell \sim 200$

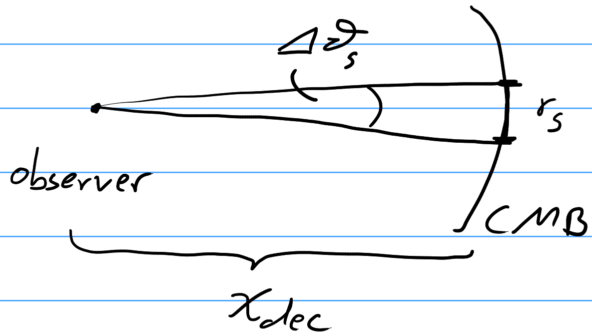
Corresponding angular scale: $\Delta\vartheta \sim \frac{180°}{\ell} \sim 1°$

Interpretation: Imprint of BAO

↳ Enhanced perturbation at scale

$$r_s = \int_0^{t_{dec}} c_s \frac{d\tilde{t}}{a(\tilde{t})}$$

To calculate corresponding angular scale, remember that photons move on straight lines in conformal coordinates



$$\Rightarrow \Delta\vartheta_s = \frac{r_s}{\chi_{dec}}$$

$$\chi_{dec} = \int_{t_{dec}}^{t_0} \frac{d\tilde{t}}{a(\tilde{t})} \approx \int_0^{t_0} \frac{d\tilde{t}}{a(\tilde{t})} = \eta_0$$

Very rough estimate:

$$c_s = \frac{1}{\sqrt{3}} = \text{const}$$

$$\Rightarrow \quad r_s = \frac{1}{\sqrt{3}} \eta_{dec} \quad \Rightarrow \quad \Delta\vartheta_s = \frac{1}{\sqrt{3}} \frac{\eta_{dec}}{\eta_0}$$

Assume MD until today $(\Omega_\Lambda = 0)$

$$\Rightarrow \quad \eta(t) = \frac{2}{\mathcal{H}} = \frac{2}{a(t)\, H(t)} = \frac{2}{a(t)\, H_0 \sqrt{\Omega_M \left(\frac{a_0}{a}\right)^3}}$$

$$= \frac{2}{a_0 H_0} \sqrt{\frac{a(t)}{a_0 \Omega_M}}$$

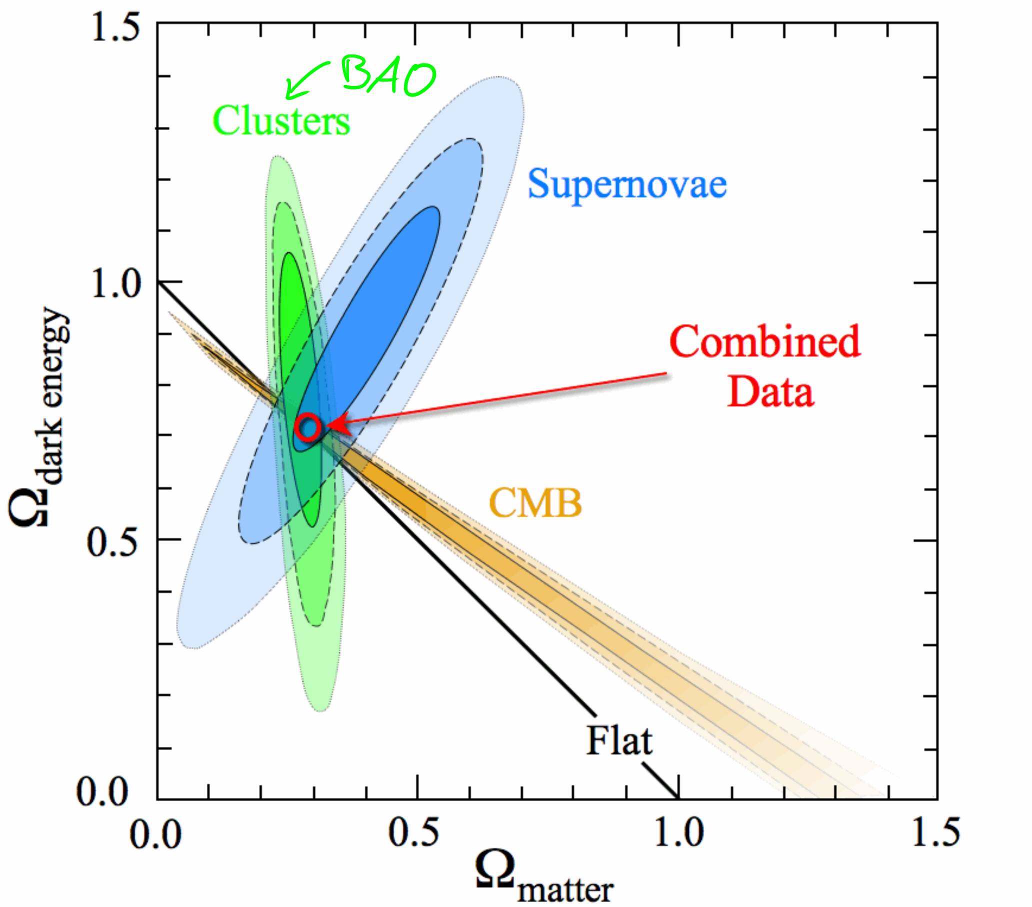
$$\Rightarrow \quad \Delta\vartheta_s = \frac{1}{\sqrt{3}} \sqrt{\frac{a_{dec}}{a_0}} = \frac{1}{\sqrt{3(1+z_{dec})}} = 0.017$$

$$\approx 1^\circ$$

Easy to extend calculation to include $c_s(t)$, $\Omega_\Lambda \neq 0$, $\Omega_{curv} \neq 0$, ...

E.g. for non-zero curvature

$$\Delta\vartheta = \frac{r_s}{S_k(\chi_{dec})} \quad \Rightarrow \quad \text{Strong constraint on } \Omega_{curv}$$



Huge success of ΛCDM!

Would like to understand also magnitude of peaks

Naive guess: Temperature anisotropies correspond to δT at decoupling

But: Need to include perturbations also in photon redshifting

Perturbed photon geodesics

Recap: $\dfrac{dp^0}{d\lambda} = -\Gamma^0_{\alpha\beta}\, p^\alpha p^\beta$ with $p^\mu = \dfrac{dx^\mu}{d\lambda}$

Homogeneous universe: $\dfrac{1}{p}\dfrac{dp}{d\eta} = -\dfrac{1}{a}\dfrac{da}{d\eta} \Rightarrow p \sim \dfrac{1}{a}$

Including perturbations: $\dfrac{1}{p}\dfrac{dp}{d\eta} = -\dfrac{1}{a}\dfrac{da}{d\eta} - \hat{p}^i \dfrac{\partial \psi}{\partial x^i} + \dfrac{\partial \Phi}{\partial \eta}$

long calculation

unit vector parallel to $\vec{p}$

Defining $\dfrac{d\Psi}{d\eta} = \dfrac{\partial \Psi}{\partial \eta} + \hat{p}^i \dfrac{\partial \Psi}{\partial x^i}$ and

using $\dfrac{1}{p}\dfrac{dp}{d\eta} + \dfrac{1}{a}\dfrac{da}{d\eta} = \dfrac{d}{d\eta}\log(ap)$

$$\Rightarrow \boxed{\frac{d}{d\eta}\log(ap) = -\frac{d\Psi}{d\eta} + \frac{\partial(\Psi + \Phi)}{\partial \eta}}$$

↳ Photons lose energy (redshift) as they escape from overdensity

Integration along the line of sight

$$\log(ap)_0 = \log(ap)_{dec} + (\Psi_{dec} - \Psi_0) + \int_{\eta_{dec}}^{\eta_0} d\eta \frac{\partial}{\partial \eta}(\Psi + \Phi)$$

same for all photons

Using $\quad a p \sim a \bar{T}\left(1 + \frac{\delta T}{\bar{T}}\right)$

$$\Rightarrow \log(ap)_0 = \log(a_0 \bar{T}_0) + a_0 \bar{T}_0 \left.\frac{\delta T}{\bar{T}}\right|_0$$

$$\log(ap)_{dec} = \log(a_{dec}\bar{T}_{dec}) + a_{dec}\bar{T}_{dec} \underbrace{\left.\frac{\delta T}{\bar{T}}\right|_{dec}}_{\frac{1}{4}(\delta_\gamma)_{dec}}$$

$\rho \sim T^4$

$$\Rightarrow \boxed{\left.\frac{\delta T}{\bar{T}}\right|_0 = \underbrace{\left(\frac{1}{4}\delta_\gamma + \Psi\right)_{dec}}_{\text{Sachs-Wolfe effect}} + \underbrace{\int_{\eta_{dec}}^{\eta_0} d\eta \frac{\partial}{\partial \eta}(\Psi + \Phi)}_{\text{Integrated Sachs-Wolfe effect}}}$$

Sachs-Wolfe effect
→ photons lose energy if produced inside overdensity ($\Psi < 0$)

Integrated Sachs-Wolfe effect
→ photon energy affected by overdensities along its path

↳ For adiabatic perturbations

$$\delta_\gamma \approx \frac{4}{3}\delta_m \approx -\frac{8}{3}\Phi = -\frac{8}{3}\Psi$$

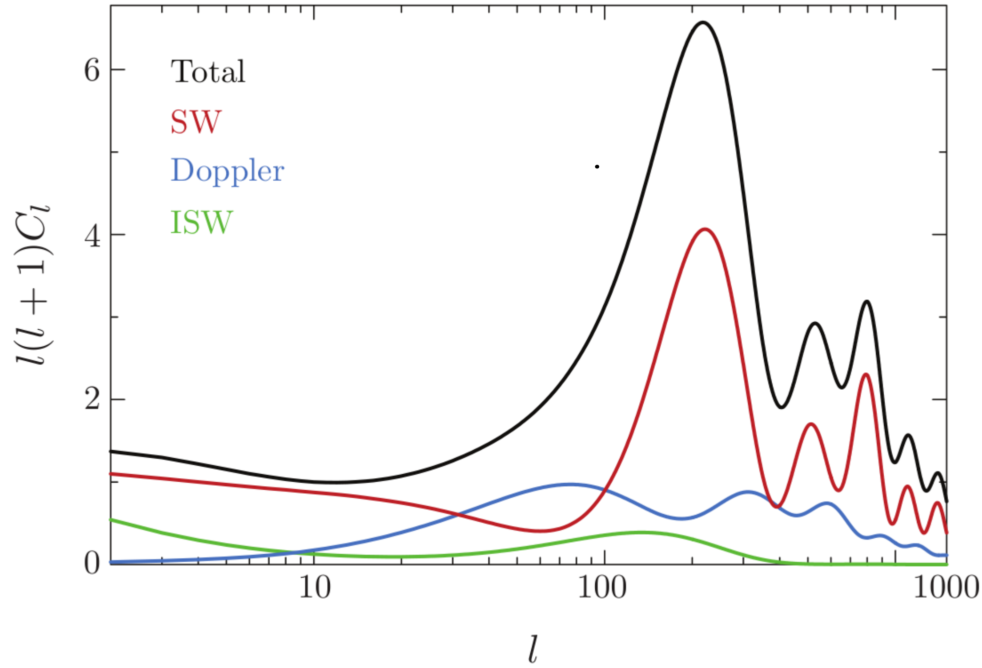
$$\Rightarrow \frac{1}{4}\delta_\gamma + \Psi \approx \frac{1}{3}\Psi \begin{cases} < 0 & \text{for overdensity} \\ > 0 & \text{for underdensity} \end{cases}$$

⇒ overdensity ↔ cold spot in CMB

Third effect: $\left.\frac{\delta T}{\bar{T}}\right|_0 = (\hat{n} \cdot \vec{v}_e)_{dec}$ ⇒ Doppler effect

$\vec{v}_e$: electron velocity

$\hat{n}$: unit vector along line of sight



↳ Height of peaks depends on strength of BAOs, i.e. on Ω_{rad}, Ω_b and Ω_m

	Parameter	Plik best fit	Plik [1]	CamSpec [2]	$([2]-[1])/\sigma_1$	Combined
ΛCDM parameters	$\Omega_b h^2$	0.022383	0.02237 ± 0.00015	0.02229 ± 0.00015	-0.5	0.02233 ± 0.00015
	$\Omega_c h^2$	0.12011	0.1200 ± 0.0012	0.1197 ± 0.0012	-0.3	0.1198 ± 0.0012
	$100\theta_{MC}$	1.040909	1.04092 ± 0.00031	1.04087 ± 0.00031	-0.2	1.04089 ± 0.00031
	τ	0.0543	0.0544 ± 0.0073	$0.0536^{+0.0069}_{-0.0077}$	-0.1	0.0540 ± 0.0074
	$\ln(10^{10}A_s)$	3.0448	3.044 ± 0.014	3.041 ± 0.015	-0.3	3.043 ± 0.014
	n_s	0.96605	0.9649 ± 0.0042	0.9656 ± 0.0042	$+0.2$	0.9652 ± 0.0042
derived parameters	$\Omega_m h^2$	0.14314	0.1430 ± 0.0011	0.1426 ± 0.0011	-0.3	0.1428 ± 0.0011
	H_0 [km s^{-1}Mpc^{-1}] . . .	67.32	67.36 ± 0.54	67.39 ± 0.54	$+0.1$	67.37 ± 0.54
	Ω_m	0.3158	0.3153 ± 0.0073	0.3142 ± 0.0074	-0.2	0.3147 ± 0.0074
	Age [Gyr]	13.7971	13.797 ± 0.023	13.805 ± 0.023	$+0.4$	13.801 ± 0.024
	σ_8	0.8120	0.8111 ± 0.0060	0.8091 ± 0.0060	-0.3	0.8101 ± 0.0061
	$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$ . .	0.8331	0.832 ± 0.013	0.828 ± 0.013	-0.3	0.830 ± 0.013
	z_{re}	7.68	7.67 ± 0.73	7.61 ± 0.75	-0.1	7.64 ± 0.74
	$100\theta_*$	1.041085	1.04110 ± 0.00031	1.04106 ± 0.00031	-0.1	1.04108 ± 0.00031
	r_{drag} [Mpc]	147.049	147.09 ± 0.26	147.26 ± 0.28	$+0.6$	147.18 ± 0.29

Note: ϑ_{MC} used instead of H_0 for technical reasons