§ 12 The nature of dark matter

Recap: Growth of perturbations during RD essential for formation of non-linear structures

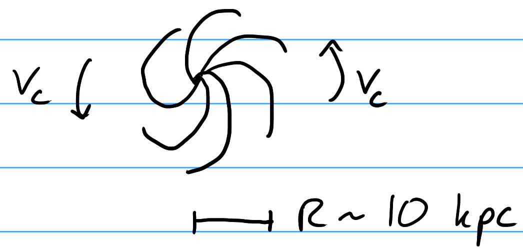
↳ Need non-relat. matter with no coupling to photons

⇒ cosmological evidence for DM

12.1 Astrophysical evidence for DM

Most famous example: Galactic rotation curves

Consider spiral galaxy:



Expected rotational velocity:

$$F_N = m \cdot a \quad \Rightarrow \quad \frac{G\, M(r)}{r^2} = \frac{v_c}{r}$$

with $\quad M(r) = 4\pi \int r^2 \rho(r)\, dr$

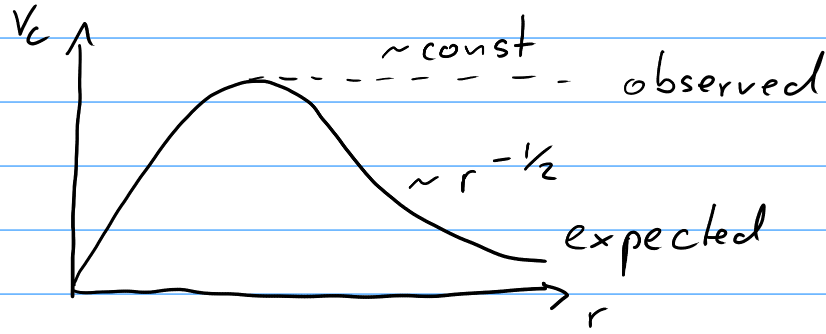
$$\Rightarrow \quad v_c(r) = \sqrt{\frac{G \cdot M(r)}{r}}$$

Can be measured via Doppler shift of starlight

Idea: Extend measurement beyond visible disc ($r > R$) using neutral hydrogen gas (21 cm line)

Expectation: For $r > R$: $M(r) \approx M(R)$

$$\Rightarrow v_c(r) = \sqrt{\frac{G\,M(R)}{r}} \sim r^{-1/2}$$



Observation: $v_c(r) \to$ const for $r > R$

$$\Rightarrow M(r) \sim r$$

$$\Rightarrow \rho(r) \sim \frac{1}{r^2}$$

↳ Evidence for DM halo surrounding galaxy

↳ Typically 5-10 times more massive than stellar disk

Similar observations on scale of galaxy clusters

↳ Confirms expectation that visible structures form inside potential wells generated by DM

↳ Use numerical simulations to study this process

12.2 N-body simulations

Idea:
- Assume initial conditions in linear regime ($P_m(k, a)$ for $a \ll a_0$)
- Describe perturbations through inhomogeneous distribution of simulation "particles"
- Evolve particles under influence of gravity
- Extract observables in non-linear regime (e.g. distribution of DM halos)

Modern simulations (e.g. Millenium XXL):

Cover scales between kpc and Gpc using over 10^{11} simulation particles

Challenge: Simulation of baryons

↳ Each particle has a mass of around $10^9 M_\odot$

⇒ Impossible to resolve star-formation, supernova explosions etc.

⇒ Need effective description of "sub-grid" physics

↳ hydrodynamical simulations

Output (example): Halo mass function

↳ Number of DM halos with mass $M_h > M$ as function of M

Observations make it possible to constrain DM models

Example: Warm dark matter

Assume that DM is not perfectly cold

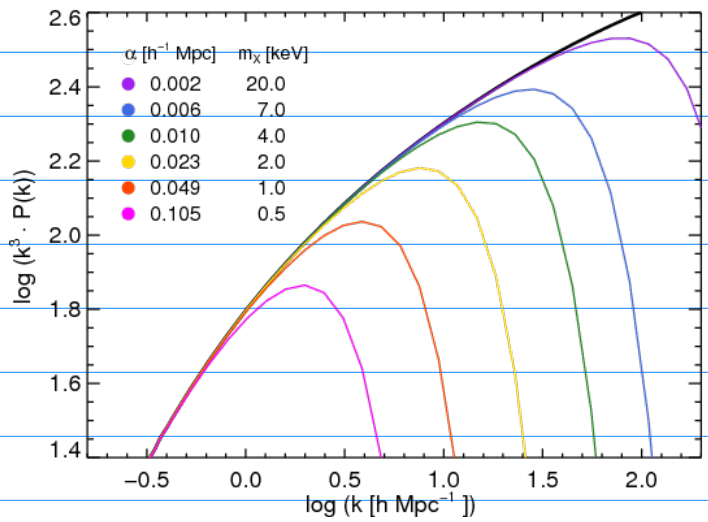
⇒ Pressure $P > 0$

⇒ Sound speed $c_s > 0$

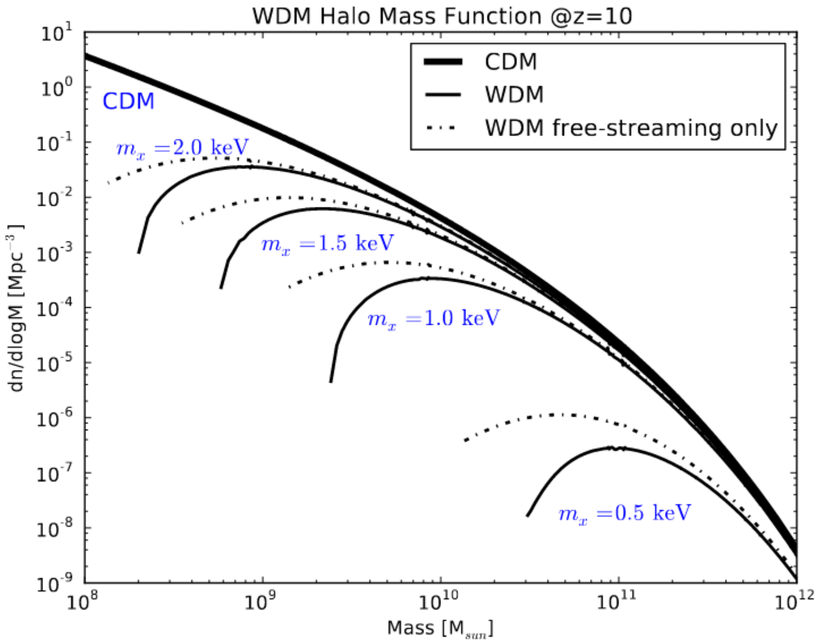
⇒ Jeans length $k_J < \infty$

⇒ No growth of structures with $k > k_J$

⇒ Suppression of $P_m(k)$ for large k



⇒ Suppression of small-scale structures



Interpretation: Relativistic particles escape from overdensities ("free-streaming")

12.3 Properties of DM

- DM is stable
- DM is electrically neutral
- DM is non-baryonic (BBN + CMB)
- DM is dissipationless + collisionless
- DM is non-relativistic ("cold")

structure formation

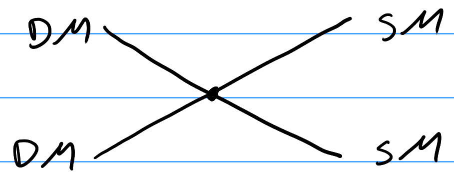
$\Rightarrow$ No SM particle has these properties

↳ New particle beyond the SM?

↳ How can such a particle be produced in the early universe?

12.4 Dark matter production

Model-independent approach: Consider general interaction DM + DM $\leftrightarrow$ SM + SM



Define annihilation rate $\Gamma_{ann}^{eq} = \langle \sigma_{ann} v \rangle n_{DM}^{eq}$

Boltzmann eq.: $\frac{x}{Y_{DM}^{eq}} \frac{dY_{DM}}{dx} = -\frac{\Gamma_{ann}^{eq}}{H}\left(\left(\frac{Y_{DM}}{Y_{DM}^{eq}}\right)^2 - 1\right)$

with $x = \frac{m_{DM}}{T}$, $Y_{DM} = \frac{n_{DM}}{s}$

Assume that for high temperatures $(x \ll 1)$ the interaction rate is large $(\Gamma_{ann}^{eq} \gg H)$

$\Rightarrow$ DM is in equilibrium $(Y_{DM} = Y_{DM}^{eq})$

For $x > 1$, Y_{DM}^{eq} becomes exponentially suppressed:

$$Y_{DM}^{eq} \sim e^{-x} \Rightarrow \Gamma_{ann}^{eq} \sim e^{-x}$$

$\Rightarrow$ Thermal equilib. cannot be maintained

$\Rightarrow$ DM decouples when $\Gamma_{ann}^{eq} = H$

$\hookrightarrow$ Freeze-out temperature T_f

$$n_{DM} \overset{\text{non-rel.}}{\sim} (m_{DM} T_f)^{3/2} e^{-m_{DM}/T_f} \sim m_{DM}^3 x_f^{-3/2} e^{-x_f}$$

$$H \sim \sqrt{g_*} \frac{T_f^2}{M_P} \sim \sqrt{g_*} \frac{m_{DM}^2}{M_P x_f^2}$$

$$\Rightarrow m_{DM}^3 x_f^{-3/2} e^{-x_f} = \frac{m_{DM}^2}{M_P x_f^2} \Rightarrow \sqrt{x_f}\, e^{-x_f} = \frac{\sqrt{g_*}}{m_{DM} M_P \langle \sigma v \rangle}$$

To first approximation $\frac{dY_{DM}}{dx} = 0$ for $x > x_f$

$$\Rightarrow \quad Y_{DM}(x_0) = Y_{DM}(x_f) = Y_{DM}^{eq}(x_f)$$

$$Y_{DM}^{eq}(x_f) = \frac{n_{DM}^{eq}(x_f)}{\underset{\uparrow\, g_* T_f^3}{s(x_f)}} \sim \frac{x_f^{3/2}}{g_*} e^{-x_f} \sim \frac{x_f}{\sqrt{g_*}\, m_{DM} M_p \langle\sigma v\rangle}$$

$$\Omega_{DM} = \frac{m_{DM}\, Y_{DM}(x_0)\, s_0}{\rho_c} \sim \frac{s_0}{\rho_c M_p} \frac{x_f}{\sqrt{g_*}\langle\sigma v\rangle}$$

To estimate x_f, assume $m_{DM} \sim 1\,\text{TeV}$

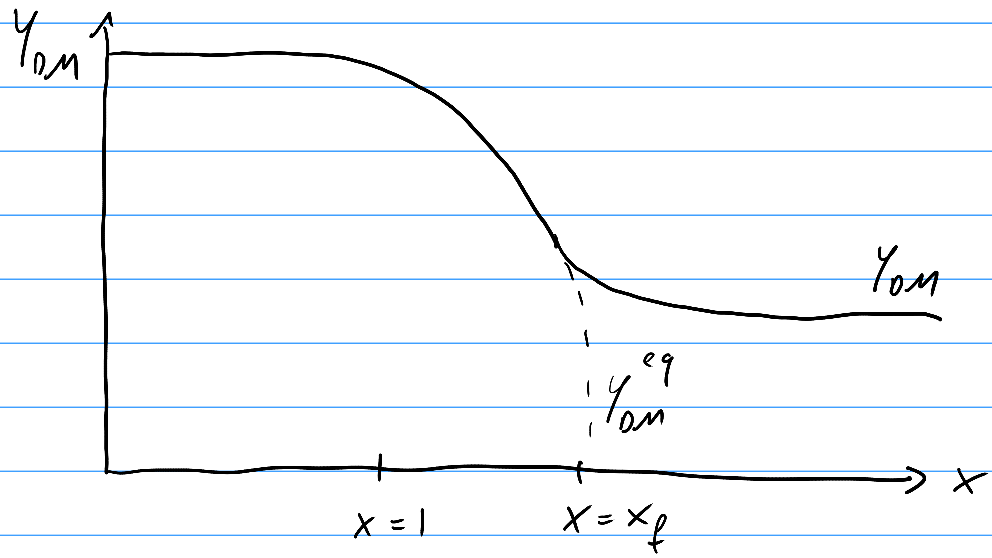
$$\langle\sigma v\rangle \sim \frac{1}{(1\,\text{TeV})^2}$$

$$\sqrt{g_*} \sim 10$$

$$\Rightarrow \quad \sqrt{x_f}\, e^{-x_f} \sim 10^{-15} \quad \Rightarrow \quad x_f \sim 30$$

↳ Freeze-out happens when DM is highly non-relativistic (cold thermal relic)

↳ Perfect for structure formation



Substituting numbers:

$$\boxed{\Omega_{DM} h^2 \approx \frac{3 \cdot 10^{-27}\ \frac{cm^3}{s}}{\langle \sigma v \rangle}}$$