Generation of perturbations

Recap: Inflation = early period of accelerated expansion

= early period of shrinking comoving Hubble radius $\mathcal{H}^{-1}$

Consider perturbations in inflaton field

$$\phi(\eta, \vec{x}) = \bar{\phi}(\eta) + \frac{f(\eta, \vec{x})}{a(\eta)}$$

$$f_{\vec{k}}(\eta) = \int \frac{d^3x}{(2\pi)^3} f(\eta, \vec{x}) e^{-i\vec{k}\cdot\vec{x}}$$

For $k \gg \mathcal{H}$ perturbations can evolve:

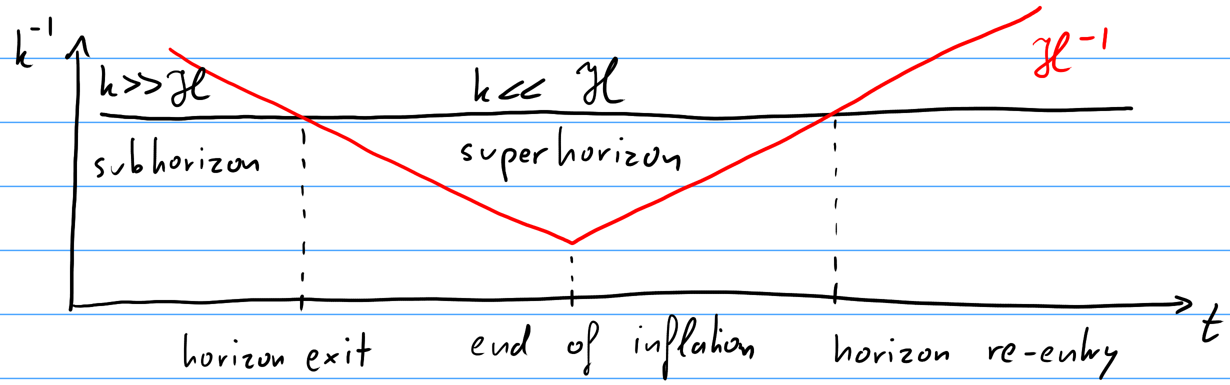
$$0 = f_{\vec{k}}'' + \left(k^2 - \frac{2}{\eta^2}\right) f_{\vec{k}} \overset{k^2 \gg \eta^2}{\approx} f_{\vec{k}}'' + k^2 f_{\vec{k}}$$

$\Rightarrow$ oscillations

As $\mathcal{H}$ grows, modes get pushed outside the horizon ($\mathcal{H}^{-1}$ shrinks below λ)

For $k \ll \mathcal{H}$ perturbations become frozen

$\Rightarrow$ Initial conditions for subsequent evolution



Where do perturbations in ϕ come from?

→ Quantum mechanics:

Zero-point fluctuations of harmonic oscillator

$$\langle 0 | \hat{x}^2 | 0 \rangle = \langle 0 | \hat{p}^2 | 0 \rangle = \frac{1}{2\omega} \qquad (\hbar = 1)$$

↳ Uncertainty principle

→ In subhorizon regime, each mode $f_{\vec{k}}$ behaves like indep. harmonic oscillator

$$\langle 0 | |f_{\vec{k}}|^2 | 0 \rangle = \frac{1}{2k} = |f_k(\eta)|^2$$

$$\Rightarrow \langle 0 | |f|^2 | 0 \rangle = \int d^3k \, d^3k' \, \delta(\vec{k} - \vec{k}') \, |f_k(\eta)|^2$$

$$= \int d \ln k \; 4\pi k^3 \, |f_k(\eta)|^2$$

$$= \int d \ln k \, (2\pi)^3 \, \underbrace{\Delta_f^2}_{\frac{k^2}{4\pi^2}}$$

$$\Rightarrow \langle 0 | |\delta\phi|^2 | 0 \rangle = \int d \ln k \, (2\pi)^3 \frac{1}{a^2} \Delta_\rho^2$$

$$= \int d \ln k \, (2\pi)^3 \Delta_{\delta\phi}^2$$

$$\hookrightarrow \frac{k^2}{4\pi^2 a^2}$$

At horizon crossing $k = \mathcal{H} = a \cdot H$

$$\Rightarrow \Delta_{\delta\phi}^2(k) \approx \frac{H^2}{4\pi^2}$$

Recall: Curvature perturbation $\mathcal{R} = -\Phi + \frac{\mathcal{H}}{\bar{\rho} + \bar{p}} \delta q$

Can show that $\mathcal{R} = -H \frac{\delta\phi}{\dot{\bar{\phi}}}$

$$\Rightarrow \Delta_{\mathcal{R}}^2 = \frac{H^2}{\dot{\bar{\phi}}^2} \Delta_{\delta\phi}^2 = \frac{4\pi}{M_p^2 \varepsilon} \frac{H^2}{4\pi^2}$$

$$\Rightarrow \boxed{\Delta_{\mathcal{R}}^2 = \frac{1}{\pi\varepsilon} \frac{H^2}{M_p^2}}$$

$\hookrightarrow$ Slow-roll inflation: $H, \varepsilon \approx$ const

$\Rightarrow \Delta_{\mathcal{R}}^2 \approx$ const

$\Rightarrow$ Scale-invariant perturbations (Harrison-Zeldovich spectrum)

More accurate calculation: - H changes slowly during inflation

- Modes that cross horizon at different times give slightly different contribution

$$\Rightarrow \Delta_{\mathcal{R}}^2 \sim \left(\frac{k}{k_*}\right)^{n_s - 1}$$

$$\text{with } n_s - 1 = \frac{d \ln \Delta_{\mathcal{R}}^2}{H\,dt} = \left(\frac{2\dot{H}}{H^2} - \frac{\dot{\varepsilon}}{\varepsilon H}\right) = -2\varepsilon - \eta$$

Firm prediction of slow-roll inflation:
n_s should be slightly smaller than unity

Observation: $n_s = 0.9603 \pm 0.0073$

$\Rightarrow$ Strong evidence for inflation

$\Rightarrow$ Can constrain inflationary potential using observations

Gravitational waves from inflation

Inflation also creates tensor perturbations

$$h_{ij}(\eta, \vec{x}) = \sum_{\alpha = +,\times} h_\alpha(\eta, \vec{x})\, e_{ij}^\alpha$$

Define $f_{\alpha,\vec{k}} = \frac{a M_p}{\sqrt{32\pi}} h_{\alpha,\vec{k}}(\eta)$

$\Rightarrow \Delta^2_{f_{\alpha,k}} = \frac{k^2}{4\pi^2}$ (as before)

$$\Rightarrow \Delta_h^2 = 2\Delta_{h_\alpha}^2 = 2 \frac{32\pi}{a^2 M_p^2} \cdot \frac{k^2}{4\pi^2}$$

$$\overset{k=aH}{=} \frac{16}{\pi} \frac{H^2}{M_p^2}$$

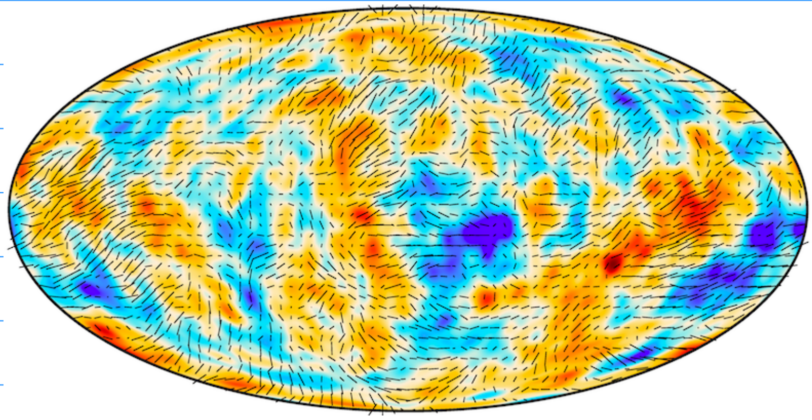
↳ independent of ε

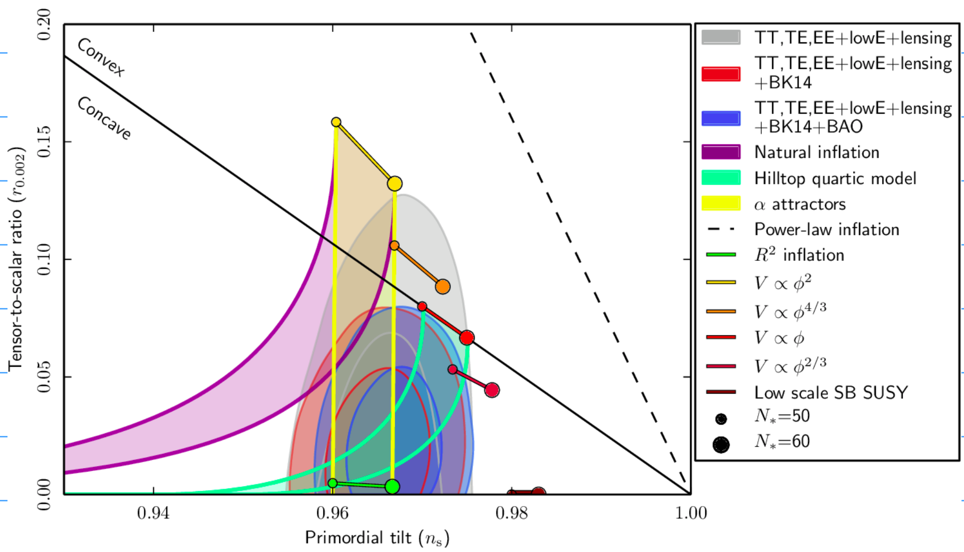
Write $\Delta_h^2(k) = A_t \left(\frac{k}{k_*}\right)^{n_t} \Rightarrow n_t = -2\varepsilon$

Define $\boxed{r = \frac{A_t}{A_s} = 16\varepsilon}$ (tensor-to-scalar ratio)

In the future it may be possible to constrain r with GW observatories

Already now: Constraints on r from CMB polarization (B modes)





Planck: $r < 0.1$ (95% CL)

Recall: $\varepsilon = \varepsilon_V$, $\eta = 4\varepsilon_V - 2\eta_V$

$$\Rightarrow \quad n_s - 1 = -6\varepsilon_V + 2\eta_V \approx -0.04$$

$$16\varepsilon_V < 0.1 \quad \Rightarrow \quad \eta_V \lesssim 0$$

$\Rightarrow$ Inflationary potential must be concave ($V'' < 0$)

$\Rightarrow$ $V(\phi) = \frac{1}{2} m^2 \phi^2$ excluded by data

Reheating

- Inflation ends when $\varepsilon > 1$

  => Kinetic energy no longer negligible

  => Inflaton quickly "rolls" towards potential minimum and starts oscillating

Recap ($V = \frac{1}{2} m \phi^2$): $\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$

Ansatz: $\phi(t) = A(t)\cos(mt)$

=> $3H\,A(t) + 2A'(t) = 0$

=> $A(t) \sim a(t)^{-3/2}$

=> $\rho_\phi \sim A(t)^2 \sim a^{-3}$

=> Transition from ΛD to MD

- Energy stored in oscillations can be transferred to SM particles ("inflaton decay")

  => Transition from MD to RD

- Estimate reheating temperature T_R:

$$\frac{\pi^2}{30} g_* T_R^4 \approx \rho_{\phi,MD} \leq \rho_{\phi,\Lambda D} \approx \frac{3}{8\pi} M_p^2 H^2$$

From $r < 0.1$ and $A_s \approx 2 \cdot 10^{-9}$ we get

$$A_t \sim \frac{16}{\pi} \frac{H^2}{M_p^2} < 2 \cdot 10^{-10}$$

$$\Rightarrow g_* T_R^4 < 10^{-11} M_p^4$$

Assuming $g_* \geq g_{*,SM} = 106.75$

$$\Rightarrow T_R < 7 \cdot 10^{15} \text{ GeV}$$ (model-independent)

Much smaller values of T_R possible if inflaton decay is slow