Introduction to Theoretical Particle Physics
Exercise Sheet A
Winter term 2023/24
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Exercise A.1: Scalar symmetry breaking — 8 P

Consider a theory with N real scalar fields governed by a Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^T(\partial^\mu \Phi) - V(\Phi^T\Phi)\,, \tag{1.1}$$

where Φ is a vector, $\Phi = (\varphi_1, \ldots, \varphi_N)$. Let the potential be

$$V(\Phi^T\Phi) = -\frac{\mu^2}{2}(\Phi^T\Phi) + \frac{\lambda}{4}(\Phi^T\Phi)^2\,, \tag{1.2}$$

where μ and λ are positive constants. This potential is manifestly symmetric under the $SO(N)$ group.

(a) Let $R = (\Phi^T\Phi)$. Find the minimum, R_{vac}, of the potential in Eq. (1.2).

Consider a possible pattern of symmetry breaking where one of the scalar fields obtains a non-zero expectation value. In this case one can write

$$\varphi_1 = v + \chi_1\,, \qquad \varphi_i = \chi_i \text{ for } i \in \{2, \ldots, N\}\,. \tag{1.3}$$

where v is a constant and χ_i describe small excitations around the minimum of the potential.

(b) Express v in terms of R_{vac}.

(c) Rewrite the Lagrangian in terms of the fields χ_i. What is the symmetry of the Lagrangian after the symmetry breaking? How many Goldstone bosons do you expect? How many massive scalar fields are present?

Now think of a different pattern of symmetry breaking, where two fields obtain a non-zero expectation value. In this case one can write

$$\varphi_{1,2} = v_{1,2} + \chi_{1,2}\,, \qquad \varphi_i = \chi_i \text{ for } i \in \{3, \ldots, N\} \tag{1.4}$$

where v_1 and v_2 are constants and, again, χ_i describe small excitations around the minimum of the potential.

(d) Express v_1 and v_2 in terms of R_{vac} and a mixing angle ϑ.

(e) Explain why, despite the fact that two fields obtain a non-zero expectation value, the above symmetry breaking pattern is equivalent to the previous one. Confirm this by rewriting the Lagrangian in terms of the fields χ_i and by diagonalising the mass matrix.

Exercise A.2: Singlet Higgs Model **12 P**

A common way to extend the Standard Model (SM) of particle physics consists in adding more Higgs bosons. The simplest way to do this is to add an additional scalar boson to the SM $SU(2)$ Higgs doublet. In this exercise, the interactions of the SM Higgs boson are investigated. Then the effect of an additional scalar boson will be studied. Since we are adding a single scalar (a singlet), this extension of the SM is called the singlet Higgs model.

Consider the part of the SM Lagrangian responsible for symmetry breaking,

$$\mathcal{L}_{\mathrm{EWSB}} = -\frac{\lambda}{4}\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)^2 , \tag{2.1}$$

where the Higgs field $\varphi(x)$ is an $SU(2)$ doublet. After spontaneous symmetry breaking took place, the Higgs field is expanded around its vacuum expectation value (vev) v:

$$\varphi(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} , \tag{2.2}$$

where $h(x)$ is a real scalar.

(a) Use Eq. (2.2) to expand the potential in Eq. (2.1) in terms of the Higgs boson h. Determine the mass of the Higgs boson in terms of λ and v.

(b) After symmetry breaking the terms in Eq. (2.1) with cubic or higher powers of h describe self-interactions of the Higgs boson. List the different interactions given by Eq. (2.1) after expanding in h and determine the respective coupling in terms of λ and v.

We now extend the SM by adding a real scalar S. The new scalar couples to the SM Higgs boson through a modified potential $V(\varphi, S)$. The remaining SM Lagrangian stays unchanged. In particular, the new scalar S does not couple to the fermions and gauge bosons of the SM. The new Lagrangian term is given by

$$\mathcal{L}_{\mathrm{extended}} = -V(\varphi, S) = -\frac{\lambda}{4}\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)^2 - \frac{a_1}{2}\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) S - \frac{a_2}{2} S^2 , \tag{2.3}$$

where φ is the SM $SU(2)$ Higgs doublet, S is a scalar boson and a_1 and a_2 are coupling constants.

(c) The vev's of φ and S are required to be extrema of the potential $V(\varphi, S)$. Writing $\varphi^\dagger\varphi = r_\varphi^2$, this yields the conditions

$$\left.\frac{\partial V(r_\varphi, S)}{\partial r_\varphi}\right|_{\substack{r_\varphi = r_{\varphi,\mathrm{vac}} \\ S = S_{\mathrm{vac}}}} \stackrel{!}{=} 0 , \tag{2.4}$$

$$\left.\frac{\partial V(r_\varphi, S)}{\partial S}\right|_{\substack{r_\varphi = r_{\varphi,\mathrm{vac}} \\ S = S_{\mathrm{vac}}}} \stackrel{!}{=} 0 . \tag{2.5}$$

Show that

$$r_{\varphi,\mathrm{vac}}^2 = \frac{v^2}{2} , \qquad S_{\mathrm{vac}} = 0 , \tag{2.6}$$

is a solution to these conditions.

(d) After expanding the potential Eq. (2.3) around the vev's of φ and S as

$$\varphi(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} , \qquad S(x) = S_{\text{vac}} + s(x) = s(x) \ , \tag{2.7}$$

the mass terms for the field excitations $h(x)$ and $s(x)$ can be written as

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \left(M_H^2 h^2 + M_S^2 s^2 + M_{HS}^2 hs \right) \ . \tag{2.8}$$

Calculate the masses M_H^2, M_S^2 and M_{HS}^2 using Eq. (2.3).

(e) Which additional vertices do you find now compared to exercise b)? List them and their coupling strength in terms of the introduced constants.

(f) In order to get rid of the mixed term $\propto hs$ in $\mathcal{L}_{\text{mass}}$, we introduce linear combinations of $h(x)$ and $s(x)$ which depend on a mixing angle ϑ:

$$h_1(x) = \cos(\vartheta)h(x) + \sin(\vartheta)s(x) \ , \tag{2.9}$$
$$h_2(x) = -\sin(\vartheta)h(x) + \cos(\vartheta)s(x) \ . \tag{2.10}$$

Determine the angle ϑ_0 as a function of λ, v, a_1 and a_2 such that the mixed term in $\mathcal{L}_{\text{mass}}$ vanishes.

(g) Compare the vertices you find after reparametrizing $\mathcal{L}_{\text{extended}}$ in terms of $h_1(x)$ and $h_2(x)$ with your results of exercise e). You need not specify the coupling constants. Are there more or less vertices?