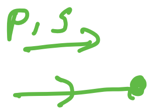
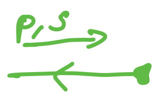
Lecture 9:

<u>Feynman rules</u> for QED

- incoming electron e^-: 

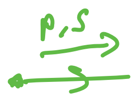
$u_s(\vec{p})$

  positron e^+: 

$\bar{v}_s(\vec{p})$

  photon γ: 

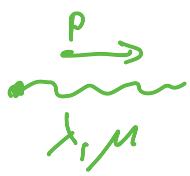
$\varepsilon_\mu(\vec{p}, \lambda)$

- outgoing electron e^-: 

$\bar{u}_s(\vec{p})$

  positron e^+: 

$v_s(\vec{p})$

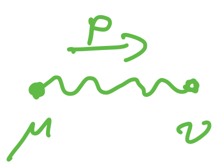
  photon γ: 

$\varepsilon^*_\mu(\vec{p}, \lambda)$

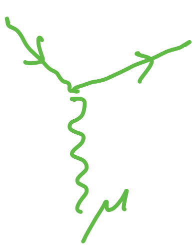
- propagators

  fermions (e^-, e^+) 

$$\frac{i}{\not{p} - m + i\varepsilon} = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$$

  photon γ 

$$\frac{-i\, g_{\mu\nu}}{p^2 + i\varepsilon}$$

- vertex 

$= -ie\,\gamma^\mu$

- ...

example $\quad e^- \; e^+ \longrightarrow \gamma \; \gamma$

2 diagrams

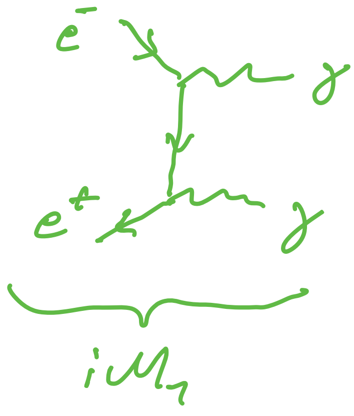
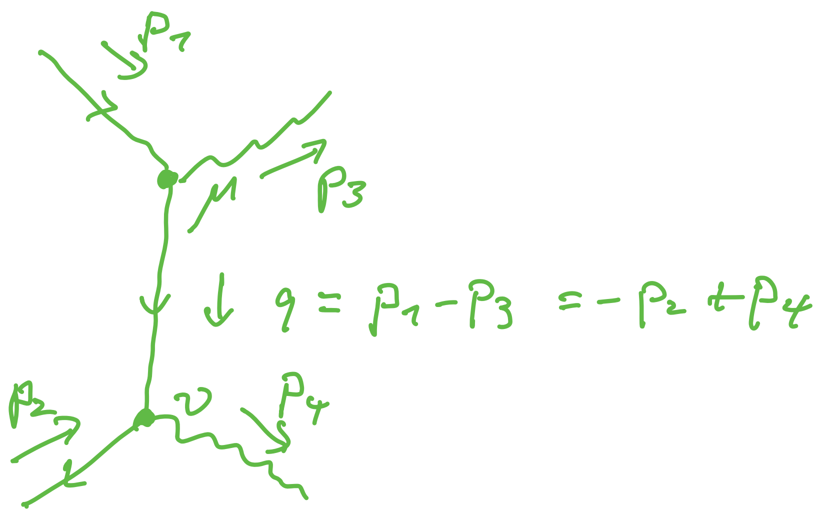


diagram 1:



$$i\mathcal{M}_1 = \bar{v}(p, s_2)\,(-ie\gamma^\nu)\,\frac{i}{\not{q} - m + i\varepsilon}\,(-ie\gamma^\mu)\,u(p_1, s)$$

$$\cdot\, \varepsilon^*_\mu(p_3, \lambda_3)\;\varepsilon^*_\nu(p_4, \lambda_4)$$

$$= -ie^2\;\frac{\bar{v}_2\,\not{\varepsilon}_4\,(\not{q} + m)\,\not{\varepsilon}_3\,u_1}{q^2 - m^2}$$

with $\quad \bar{v}_2 = \bar{v}(p_2, s_2) \quad u_1, \; \varepsilon_3, \varepsilon_4 = \ldots$

$i\mathcal{M}_2 \rightarrow$ exercise sheet

With $\mathcal{M}_1$ and $\mathcal{M}_2$ known, we can calculate the modulus square of the amplitude

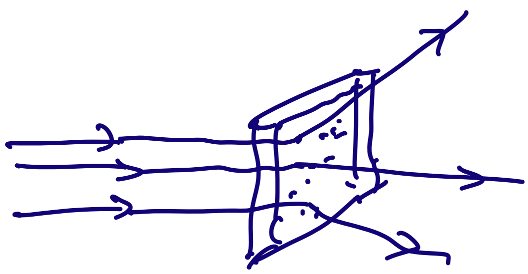
$$|\mathcal{M}|^2 = (\mathcal{M}_1 + \mathcal{M}_2)^2$$
$$= |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + \underbrace{\mathcal{M}_1\mathcal{M}_2^* + \mathcal{M}_2\mathcal{M}_1^*}_{=2\,\mathrm{Re}\,(\mathcal{M}_1\mathcal{M}_2^*)}$$

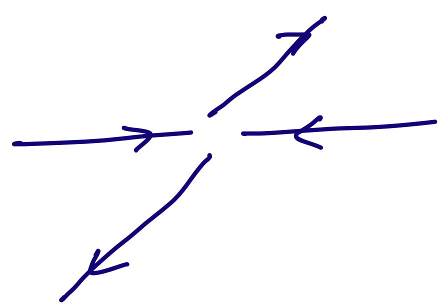
which enters the calculation of observables, e.g. cross sections.

III.4 Cross sections and decay width

Consider scattering of particles, either on

fixed target or each other





The reaction rate R (i.e. the number of scattering processes per time) can be written as

$$R = L \cdot \sigma$$

where the

- luminosity L depends only on experimental parameters, e.g.
  - number of incoming particles per time
  - surface area of the target, etc.

- cross section σ gives the transition rate of an initial state $|i\rangle$ to a final state $|f\rangle$, normalized to the incoming particle flux

$$\sigma = \frac{\text{transition rate}}{\text{incoming particle flux}}$$

Unit for cross sections: barn b

$$1\,b = 10^{-24}\,cm^2 \qquad (\,GeV^{-2} = 0{,}389 \cdot 10^{-3}\,b\,)$$

example: production of top-quark pairs at LHC

$\sigma_{t\bar{t}} \approx 1\,\text{nb} = 10^{-9}\,\text{b}$

LHC luminosity $\mathcal{L} \approx 10^{34}\,\text{cm}^{-2}\,\text{s}^{-1}$

$\Rightarrow \quad R = \sigma \cdot \mathcal{L} = 10^{-9} \cdot 10^{-24} \cdot 10^{34}\,\text{s}^{-1}$

$= 10$ events per second

Total number of events given by integratd luminosity:

$$\int \mathcal{L}\,dt = \mathcal{L} \cdot (\text{run-time of experiment})$$

LHC run 2 (2015-2018) $\int \mathcal{L} = 140\,\text{fb}^{-1}$
(run 3 currently in progress) $= 140 \cdot 10^{15}\,\text{b}^{-1}$

$N = \int \mathcal{L} \cdot \sigma = 140 \cdot 10^{15} \cdot 10^{-9}$

$= 140 \cdot 10^{6}$ $t\bar{t}$ pairs produced in run 2

Calculation of cross sections

For the scattering process

$$a_1(p_1) + a_2(p_2) \longrightarrow b_1(q_1) + \ldots\ b_n(q_n)$$

the differential cross section is given by

$$d\sigma = \underbrace{\frac{1}{2w(s, m_1^2, m_2^2)}}_{\text{flux factor}} \cdot \underbrace{\prod_{i=1}^{n} d\tilde{q}_i \ (2\pi)^4 \delta^4\Big(\sum_i q_i - p_1 - p_2\Big)}_{\text{phase space } d\Phi_n} \cdot \underbrace{\big|\langle b_1 .. b_n | \mathcal{M} | a_1 a_2 \rangle\big|^2}_{\text{matrix element}}$$

comments:

- the incoming particle flux is given by the flux factor

$$2w(s, m_1^2; m_2^2) = 4\sqrt{(p_1 p_2)^2 - m_1^2 m_2^2} = 2s$$

for $m_1 = m_2 = 0$

$$s = (p_1 + p_2)^2 = \cancel{p_1^2} + \cancel{p_2^2} + 2p_1 p_2$$

- $$d\Phi_n = \prod_{i=1}^{n} \frac{d^3\vec{q}_i}{(2\pi)^3 2E_i} \quad (2\pi)^4 \delta^4\Big(\sum_i q_i - p_1 - p_2\Big)$$

  is the Lorentz-invariant phase space measure. Integrating over the final state momenta $\vec{q}_i$ leads to the total cross section σ.

- for N identical particles in the final state, we need an additional factor of $\frac{1}{N!}$ to avoid double counting identical contributions in the phase-space integral

- If the particles in the initial and final state have spin, the spins are fixed in the states $|a\ldots\rangle$, $|b\ldots\rangle$ $\rightarrow$ polarized cross sections

In many cases we are not interested in (or we can't measure) the spin of the particles. To obtain the unpolarized cross section, we need to sum over the spin configurations of the final state particles and average over the spin in the initial state.

$\Rightarrow$ replace $|\langle f|\mathcal{M}|i\rangle|^2$ with

$$\sum{}' |\langle f|\mathcal{M}|i\rangle|^2 = \frac{1}{s_1 s_2} \sum_{\text{spins}} |\langle f|\mathcal{M}|i\rangle|^2$$

where s_1, s_2 are the number of spin states of the initial state particles:

$s=1$ scalars

$s=2$ spinors, e^+, e^-, q massless vector bosons γ, g

$s>3$ masive vector bosonns W, Z

Calculation of decay widths

For the decay process of a particle A

$$A \longrightarrow b_1 + \dots + b_n$$

the partial decay width is given by

$$\Gamma_{A\to b_1 \dots b_n} = \frac{1}{2E_A} \int d\Phi_n \; |\langle b_1 \dots b_n | \mathcal{M} | A \rangle|^2$$

(with spin sum/average and factor $\frac{1}{N!}$ for identical particles as before)

Usually, particles can have multiple decay channels, e.g.

$$H \to \gamma\gamma \,, \qquad H \to b\bar{b}$$

$$H \to Z Z^* \to 4 \text{ leptons}$$

$\Rightarrow$ total decay width

$$\Gamma_A = \sum_{\substack{\text{decay} \\ \text{channels } b}} \Gamma_{A \to b}$$

the lifetime of the particle is then given by

$$\tau_A = \frac{\hbar}{\Gamma_A}$$

III.5 The process $e^+ e^- \to \mu^+ \mu^-$

The muon μ^-

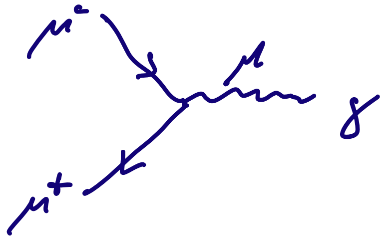
is the charged lepton of the 2nd lepton family in the SM

- properties (quantum numbers, interactions...) identical to electron, except for mass

- mass $m_\mu = 105 \text{ MeV} \approx 200\, m_e$

  $(m_e = 511 \text{ keV})$

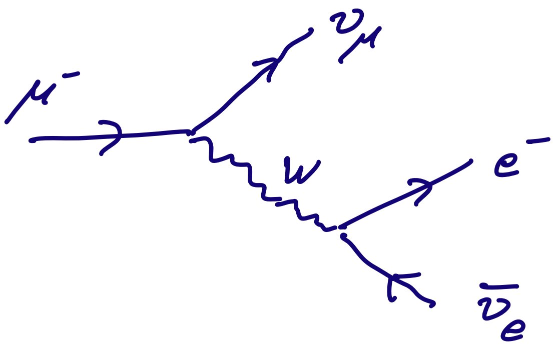
- interaction with photon



$= -ie\gamma^\mu$

- instable, decays into electron e^-, muon neutrino ν_μ and electron anti-neutrino $\bar{\nu}_e$

$$\mu^- \longrightarrow e^- + \nu_\mu + \bar{\nu}_e$$



Fermi constant
$G_F = 1{,}16637 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$

decay width $\quad \Gamma_\mu = \dfrac{G_F^2 \, m_\mu^5}{192 \, \pi^3} = 3 \cdot 10^{-19}\,\mathrm{GeV}$

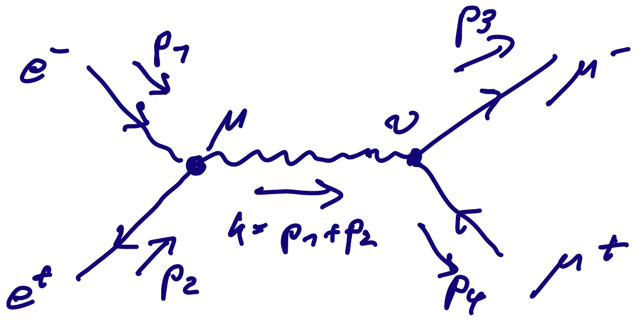
$\Rightarrow$ lifetime $\quad \tau_\mu = \dfrac{\hbar}{\Gamma_\mu} = 2{,}19 \cdot 10^{-6}\,\mathrm{s}$

- mainly produced in decays of pions (e.g. in atmosphere)

$$\pi^- \to \mu^- + \bar{\nu}_\mu \quad , \quad \pi^+ \longrightarrow \mu^+ + \nu_\mu$$

Calculation of the cross section of $e^+e^- \to \mu^+\mu^-$

At leading order (LO) only one diagram



using abbreviations $u_1 = u(p_1, s_1)$ etc.
for the spinors, the matrix element is

$$iM = \bar{v}_2 (-ie\gamma^\mu) u_1 \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon} \bar{u}_3 (-ie\gamma^\nu) v_4$$

only relevant for diagrams with loops

$$\Rightarrow M = e^2 \; \bar{v}_2 \gamma^\mu u_1 \frac{g_{\mu\nu}}{k^2} \bar{u}_3 \gamma^\nu v_4$$