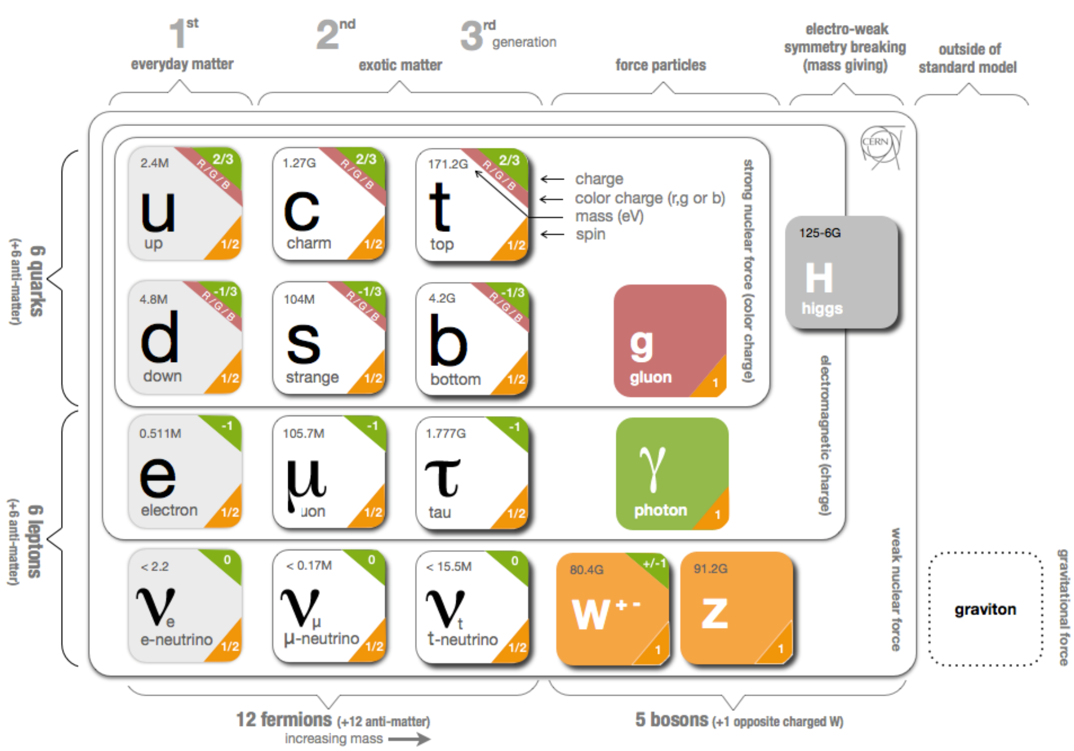
IV The Standard Model of particle physics

IV.1 Overview

All known elementary particles and their interactions (except gravity) are described by the Standard Model.

particle content:



gauge symmetry $\qquad SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

particles can be assigned to irreducible representations ($\hat{=}$ multiplets) of these groups:

$SU(3)_C$: color / strong interaction

-> sect. IV.2

- quarks have additional quantum number "color" with $N_C = 3$ possible states, transform under fundamental repr. of $SU(3)_C$
- 8 gluons : gauge bosons, transform under adjoint repr. of $SU(3)_C$
- all other particles transform under trivial repr.

  -> color-singlets, no strong interaction

$SU(2)_L$: weak interaction

- left-handed quarks and leptons (-> Weyl spinors) grouped into doublets

  $\begin{pmatrix} \text{up-type quark} \\ \text{down-type quark} \end{pmatrix}$, $\begin{pmatrix} \text{charged lepton} \\ \text{neutrino} \end{pmatrix}$

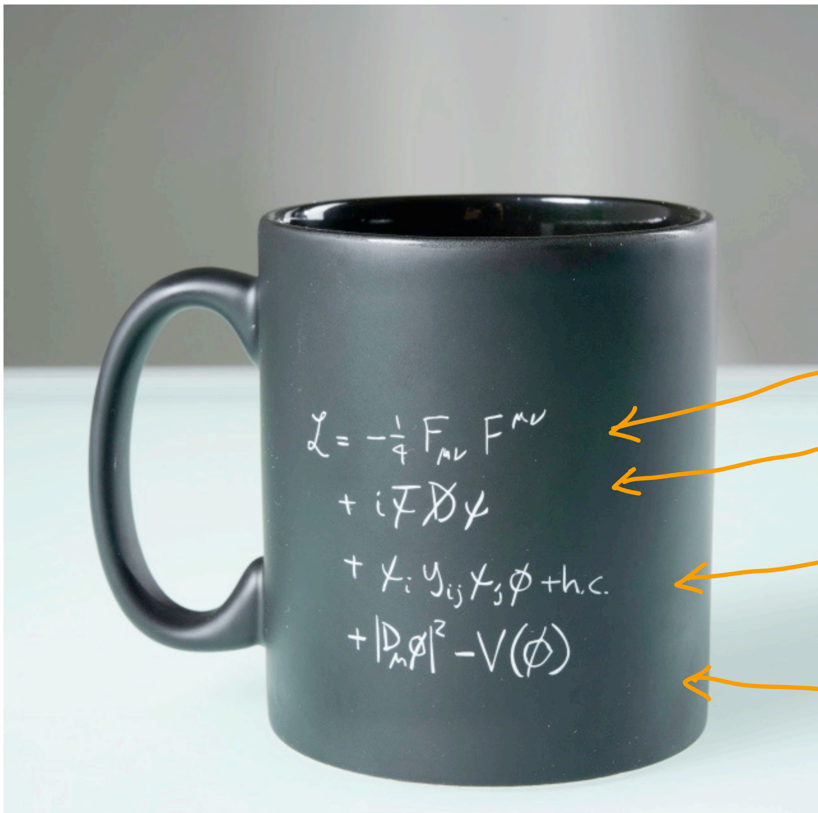
  transform under fundamental repr.

- 3 gauge bosons W^1, W^2, W^3 in adj. repr.

- right handed fermions in trivial repr.
  $\rightarrow$ no weak interaction

$U(1)_Y$: abelian interaction, couples gauge boson B to hypercharge Y of particles.

The Lagrangian of the SM can be written in a compact form:



gauge fields } gauge
fermion fields } interactions

Yukawa interactions, masses
$\rightarrow$ sect. IV.6.

Higgs field & potential
$\rightarrow$ electroweak symmetry breaking $\rightarrow$ IV. 4-5

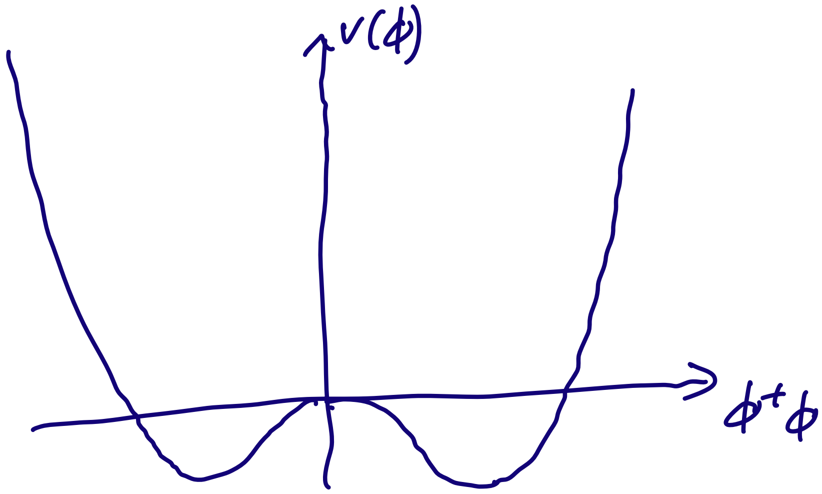
Higgs mechanism / electroweak symmetry breaking

In addition to fermion & gauge fields, we have a $SU(2)_L$ doublet of 2 complex scalar fields,

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi_0 \end{pmatrix}$$ ($\rightarrow$ 4 degrees of freedom)

with potential

$$V(\phi) = -\mu^2 \phi^+\phi + \lambda (\phi^+\phi)^2$$



$\rightarrow$ ground state is not at $\phi = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, but at

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$ (with some choice of coordinate system)

$\rightarrow$ Symmetry $SU(2)_L \otimes U(1)_Y$ of $\mathcal{L}$ broken by ground state, only $U(1)_Q$ symmetry remains ($\rightarrow$ QED)

The doublet ϕ can be reparametrized as expansion around ground state

$\Rightarrow$
- 1 d.o.f. interpreted as real scalar field H ($\rightarrow$ Higgs boson)
- 3 d.o.f combined with massless gauge bosons W^1, W^2, W^3, B

  $\Rightarrow$ 3 massive bosons W^+, W^-, Z

  + 1 massless photon γ (photon)

- Yukawa interactions

  $y_{ij} \bar{\psi}_i \phi \psi_j$

  $\Rightarrow$ fermion masses & quark mixing, interactions with Higgs bosons

IV 2. Quantum chromodynamics (QCD)

here only basics and qualitative discussion

Lagrangian & Interactions

$$\mathcal{L}_{QCD} = \sum_f \sum_{i,j=1}^{3} \bar{q}_f^{\,i} \left(i \not{D} - m_f \right) q_f^{\,i} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \mathcal{L}_{GF} + \mathcal{L}_{gh}$$

m_f: generated by Higgs mechanism → sect. IV.4

$\mathcal{L}_{GF}$: gauge fixing

$\mathcal{L}_{gh}$: ghosts

with quark fields q_f^i ← color $i \in \{1,2,3\}$, ← flavor $f \in \{u,d,s,c,b,t\}$

and covariant derivative

$$D^\mu_{ij} = \delta_{ij}\, \partial^\mu - i g_s\, t^a_{ij} A^{a\mu}$$

g_s: strong coupling constant

t^a_{ij}: SU(3) generator in fundamental repr. → $t^a = \frac{\lambda^a}{2}$ with Gellmann matrices λ^a

$\mathcal{L}_{QCD}$ is invariant under local SU(3) transformations in color space

$$q_f^i \longrightarrow U_{ij}\, q_f^j \qquad \text{with} \quad U = e^{i g_s \theta^a(x)\, t^a}$$

(→ sect. II.5)

Gauge fixing and ghost fields

Similar to QED, we can choose e.g. Lorenz gauge $\partial_\mu A^{\mu a} = 0$ to eliminate 1 of the 2 unphysical polarizations of the gluon field. This can be done by adding a gauge fixing term

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}\left(\partial_\mu A^{\mu a}\right)^2$$

to the Lagrangian.

In contrast to QED, it is not enough to demand only physical polarizations in external state (Gupta-Bleuler formalism, sect. III.7) in non-abelian gauge theories.

Consistent quantization usually done using path integral formalism (→ TTP 2), requires introduction of unphysical ghost fields to subtract the remaining degree of freedom. These Faddeev-Popov ghosts don't appear as external state of physical processes, but only as virtual particles in higher-order corrections.

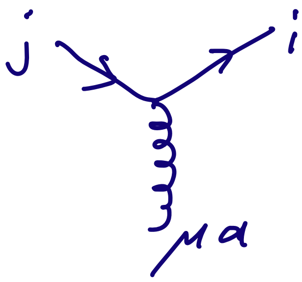
Interactions

quark - gluon interaction given by

$\bar\psi_i \, D^\mu_{ij} \, \psi_j$ with $D^\mu = \partial^\mu - i g_s \, t^a A^{\mu a}$

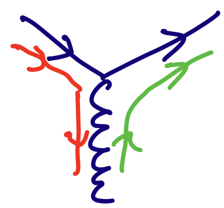
$\rightarrow \; -i g_s \, t^a_{ij} \; \bar\psi_i \gamma^\mu \psi_j A^a_\mu$

$\rightarrow$ Feynman rule



$-i g_s \, t^a_{ij} \, \gamma^\mu$

transformation in color space

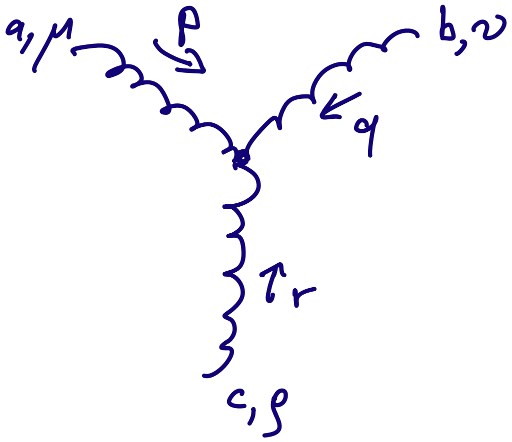


gluon self interactions given by

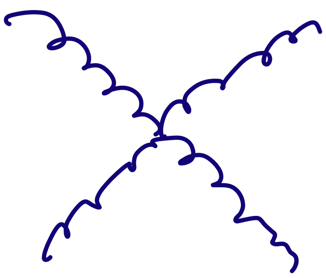
$-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$

with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \underbrace{\partial_\nu A^a_\mu}_{\text{momentum } p_\nu \text{ of field } A^a_\mu} + g_s f^{abc} A^b_\mu A^c_\nu$ ($\rightarrow$ sect II.5)

$\rightarrow$ cubic and quartic terms in gluon field A

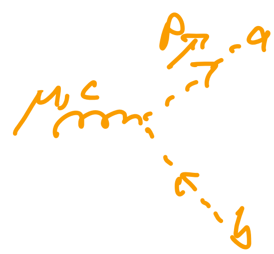


$$= g_s f^{abc}\left[(p-q)_\rho\, g_{\mu\nu} + (q-r)_\mu\, g_{\nu\rho} + (r-p)_\nu\, g_{\mu\rho}\right]$$



$= \ldots$

additional ghost-gluon vertex



$= -g_s f^{abc} p^\mu$

→ e.g. in corrections to 2-poin function,

, unphysical polarizations of virtual gluons are subtracted by diagram