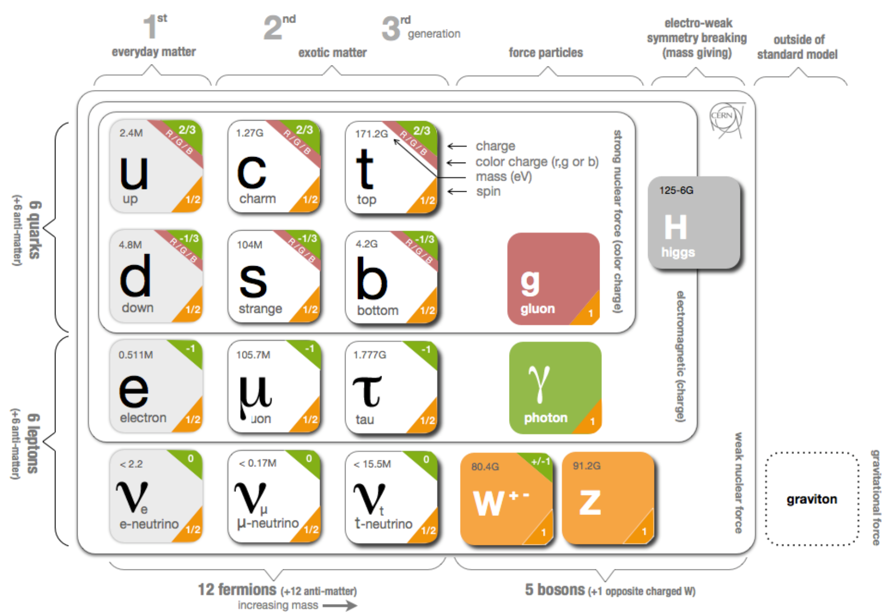
Lecture 12:

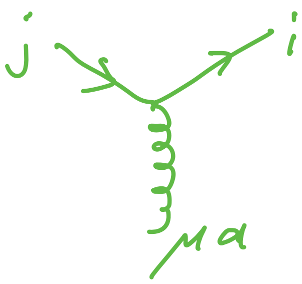
SM particles



QCD Feynman rules

$a \in \{1..8\}$

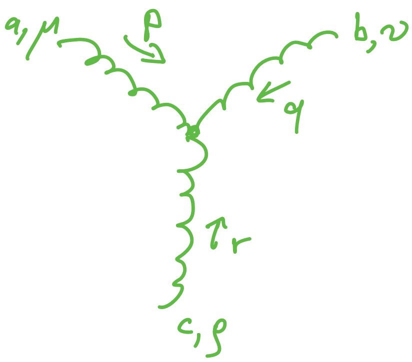
$\downarrow$



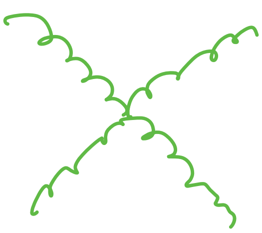
$$-i g_s \, t^a_{ij} \, \gamma^\mu$$

e.g. $t^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\nwarrow$ $i, j \in \{1, 2, 3\}$

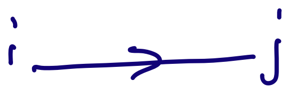


$$= g_s f^{abc} \left[(p-q)_\rho \, g_{\mu\nu} + (q-r)_\mu \, g_{\nu\rho} + (r-p)_\nu \, g_{\mu\rho} \right]$$

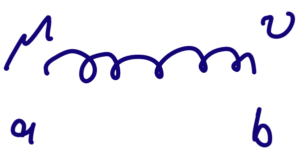


$= \ \ldots$

propagators



$$\delta^{ij} \frac{-i}{\not{p} - m + i\varepsilon}$$



$$\delta^{ab} \frac{-i}{p^2 + i\varepsilon} \left(g_{\mu\nu} - (1-\xi) \frac{p_\mu p_\nu}{p^2} \right)$$

gauge parameter, ξ-dependence cancels in physical observables

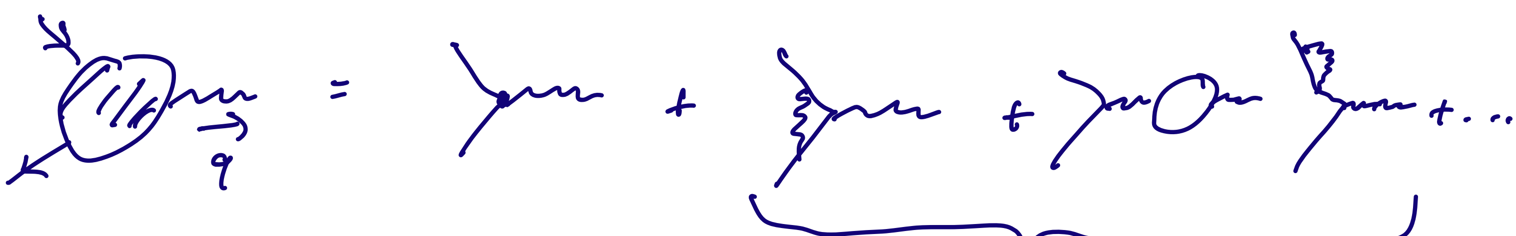
→ can set $\xi = 1$

[ghost

$$\delta^{ab} \frac{i}{p^2 + i\varepsilon}$$]

running coupling, asymtotic freedom, confinement

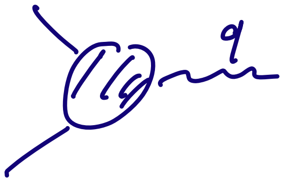
electron-photon vertex in QED, including higher orders in pert. theory



e

e^3

The higher order corrections depend on the momentum transfer q. To improve the convergence of the pert. expansion, we can absorb this q-dependence into a "renormalized coupling constant $e_r(Q)$" such that



$$\Big|_{q=Q} = -i e_r(Q)\, \gamma^\mu$$

$\rightarrow$ "running coupling" $e_r(Q)$

(or equivalently $\alpha(Q) = \dfrac{e_r^2(Q)}{4\pi}$)

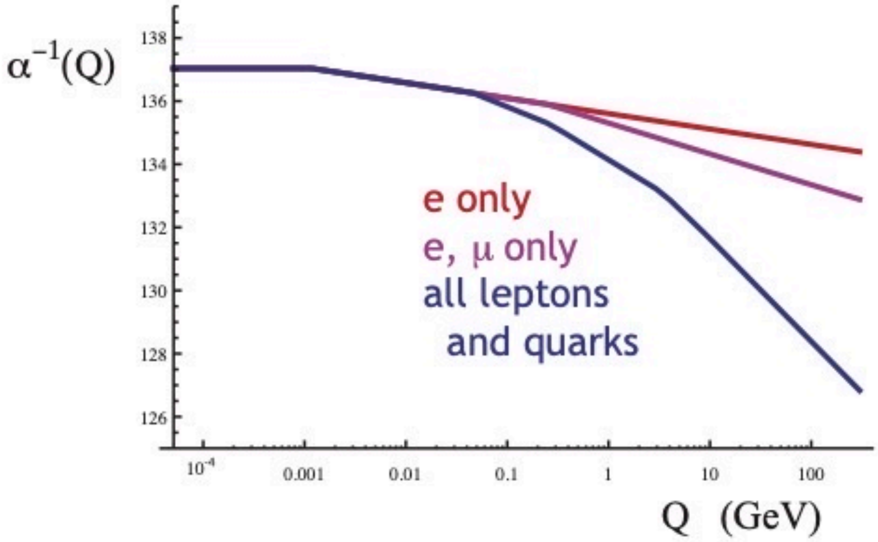
comment: renormalization is also required to absorb divergent contributions appearing in diagrams with loops.

The Q dependence of the coupling $e_r(Q) =: e(Q)$ is given by the differential equation

$$\frac{d}{d(\log Q)}\, e(Q) = \beta(e(Q))$$

$$= + \frac{e^3}{12\pi^2} + \mathcal{O}(e^5) \qquad > 0$$

(considering only electrons)



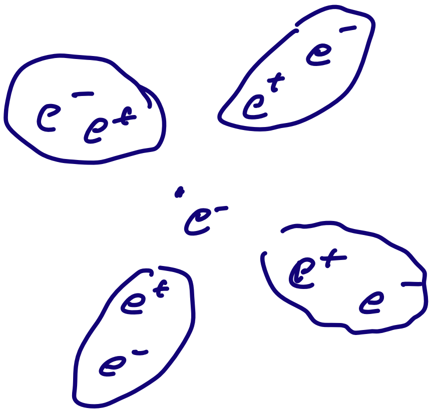
Peskin

Fig. 11.1: Dependence of $\alpha^{-1}(Q)$ on the momentum transfer Q predicted by the vacuum polarization effect. The three curves show the vacuum polarization effect from electrons only, from electrons and muons, and from all leptons and quarks. The effect of each particle f turns on for $Q > 2m_f$.

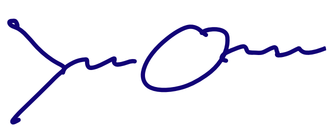
$\rightarrow \quad \alpha(m_e) \approx \frac{1}{137} \qquad \alpha(M_Z \approx 91\ \text{GeV}) \approx \frac{1}{129}$

$\rightarrow$ the electromagnetic coupling increases with energy

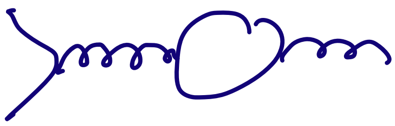
interpretation: electric charge is shielded by virtual e^+e^- pairs $\rightarrow$ smaller charge seen at large distances ($\hat{=}$ low energies)

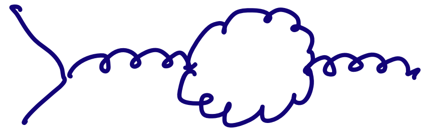


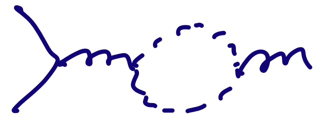
$\sim$



Repeating the same calculation for QCD, we have fermionic and bosonic contributions, e.g.







$$\Rightarrow \frac{d}{d\log Q} g_s(Q) = \beta(g_s(Q))$$

number of quark flavors

$$= -\left(\frac{11}{3} C_A + \frac{4}{3} n_f T_f\right) \frac{g_s^3}{16\pi^2} + \mathcal{O}(g_s^5)$$

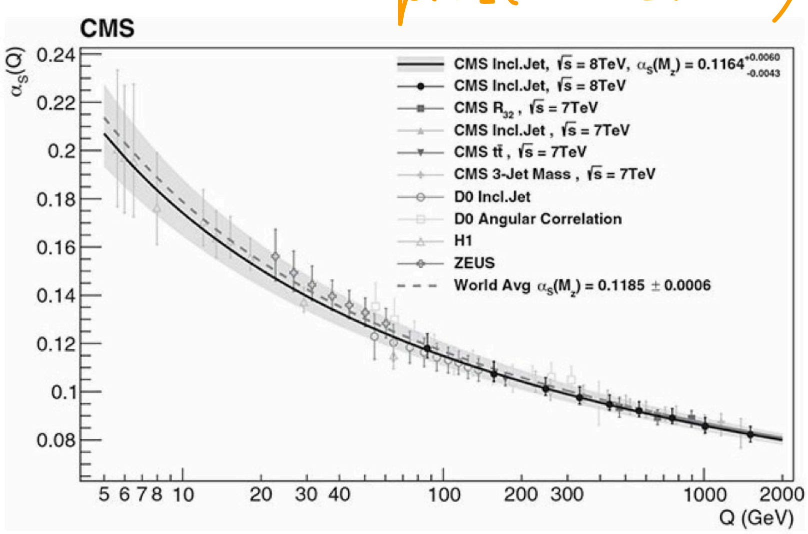
$$= -11 + \frac{2}{3} n_f < 0$$

6

$\Rightarrow$ the strong coupling decreases with energy

$\Rightarrow$ asymptotic freedom

(Gross, Wilczek, Politzer 1973,
Nobel prize 2004)



Larkoski

Fig. 8.3 A collection of various measurements of the strong coupling α_s at energy scales Q ranging from 5 GeV to nearly 2 TeV. The solid line is the predicted running from the QCD β-function with the input value $\alpha_s(m_Z) = 0.1164$. From V. Khachatryan *et al.* [CMS Collaboration], "Measurement and QCD analysis of double-differential inclusive jet cross sections in pp collisions at $\sqrt{s} = 8$ TeV and cross section ratios to 2.76 and 7 TeV," J. High Energy Phys. **1703**, 156 (2017) [arXiv:1609.05331 [hep-ex]].

$\Rightarrow$ • small couplings at large energies, e.g.

$$\alpha_s(M_Z \approx 91\ \text{GeV}) \approx 0{,}118$$

$\rightarrow$ can apply perturbation theory

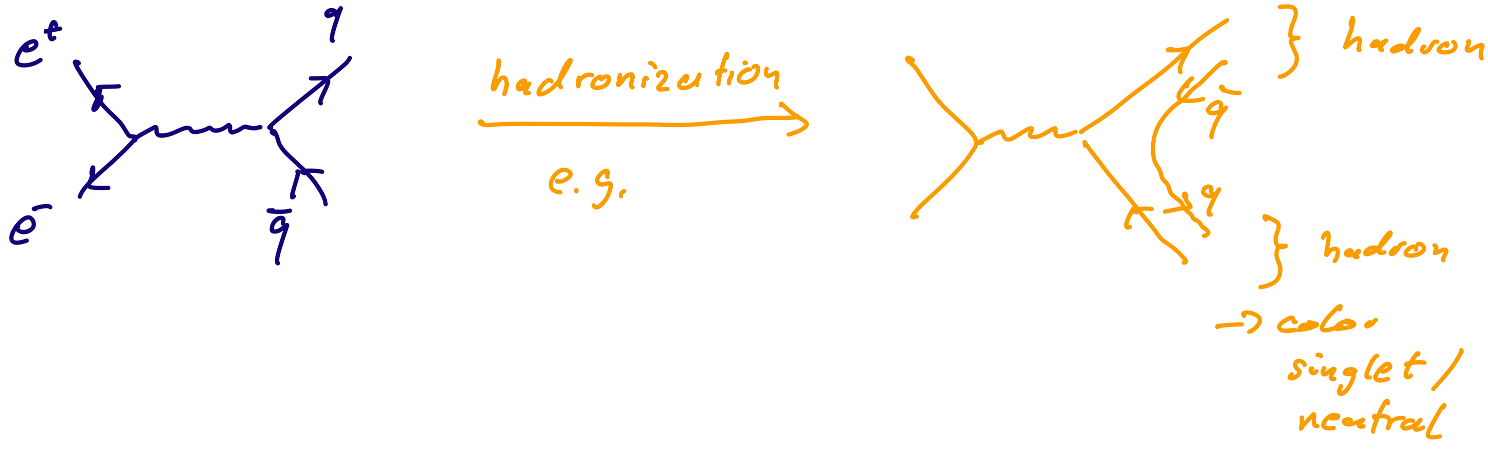
• large couplings $\alpha_s \gtrsim 1$ for $Q \lesssim 1$ GeV

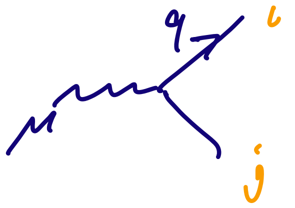
$\rightarrow$ pert. theory not applicable,

$\rightarrow$ bound states of quarks and gluons, forming color-neutral hadrons ("confinement")

Hadronic cross section in e^+e^- collisions

similar to $e^+e^- \rightarrow \mu^+\mu^-$, in e^+e^- collisions also quark-anti-quark pairs can be produced, forming hadrons



photon quark vertex: 

$\sim \quad i e_q \, \delta^{ij}$ ← color

calculating $\sum{}' |\mathcal{M}|^2$ for processes with quarks and gluons:

- sum over color d.o.f. of external particles
- factor $\begin{Bmatrix} 1/3 \\ 1/8 \end{Bmatrix}$ for each $\begin{Bmatrix} \text{quark} \\ \text{gluon} \end{Bmatrix}$ in initial state

R ratio of inclusive cross section

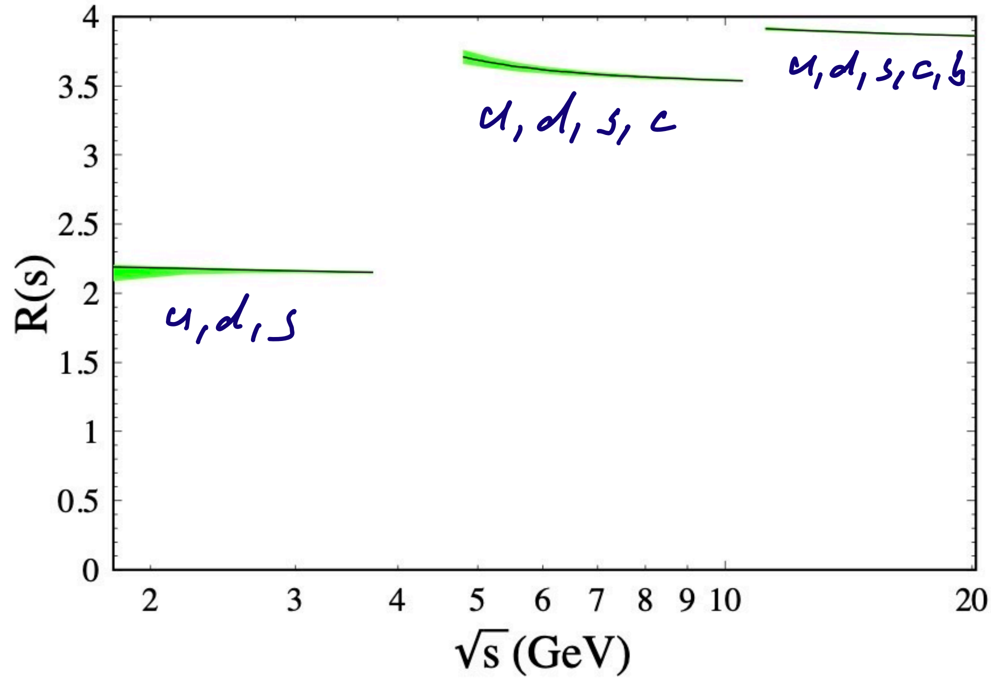
$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\approx \sum_q \frac{\sigma(e^+e^- \to q\bar{q})}{(e^+e^- \to \mu^+\mu^-)} = \sum_q e_q^2 \cdot \theta(\sqrt{s} - 2m_q) \cdot N_C$$

↑ sum over all q with $2m_q < \sqrt{s}$

e.g. for $m_b < \frac{\sqrt{s}}{2} < m_t$:

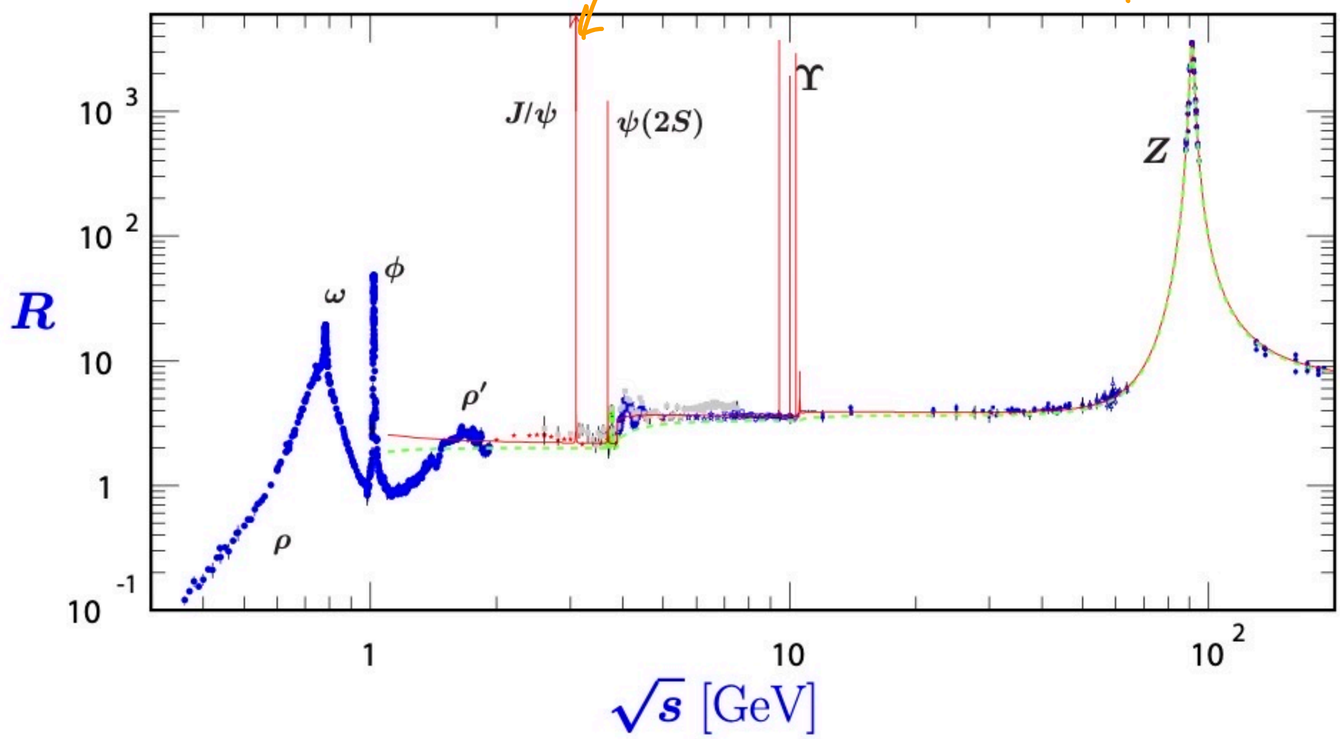
$$R(s) = 3 \cdot \Big(\underbrace{\frac{4}{9}}_{u} + \underbrace{\frac{1}{9}}_{d} + \underbrace{\frac{1}{9}}_{s} + \underbrace{\frac{4}{9}}_{c} + \underbrace{\frac{1}{9}}_{b} \Big) = \frac{11}{3}$$



arXiv: 0212294

measurement

bound states, e.g.
$J/\psi \sim c\bar{c}$
possible below $2 m_q$ threshold



particle data group

Hadrons

Quarks and gluons don't exist as free particles (due to confinement) and instead form color-neutral bound states, called hadrons.

2 types:

- $q\bar{q}$ → mesons
  "color + anti-color"
- qqq → baryons
  qqq "color singlet of 3 quarks"

The quark flavors $u, d, s \ldots$ only differ by their mass and electroweak interactions, but not their QCD interactions.

⇒ Bound states of e.g. u and d quarks are invariant under the $SU(2)$ transformations

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow U \begin{pmatrix} u \\ d \end{pmatrix}$$

In analogy to spin:

- define isospin $\vec{I}$ with $I_3 |u\rangle = \frac{1}{2} |u\rangle$
  $I_3 |d\rangle = -\frac{1}{2} |d\rangle$

- $[H_{QCD}, \vec{I}] = 0$ (QCD invariant under isospin transformations)

  $\rightarrow$ $\vec{I}^2$, I_3 good quantum numbers to describe hadrons

- decomposition of product states into irreducible representations analogous to angular momentum addition.

  $\rightarrow$ Multiplets of hadrons with similar properties.

  E.g. for mesons $q\bar{q}'$ with $q, q' \in \{u, d\}$

$$2 \otimes 2 = 3 \oplus 1$$

pions

$$\left.\begin{array}{l} \pi^+ \hat{=} u\bar{d} \\ \pi^- \hat{=} d\bar{u} \end{array}\right\} m_{\pi^\pm} = 135 \text{ MeV}$$

$$\pi^0 \hat{=} \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \quad m_{\pi^0} = 140 \text{ MeV}$$

spin 0

- The full wave function

$$\psi = \psi(x)\,\psi(\text{spin})\,\psi(\text{flavor})\,\psi(\text{color})$$

of hadrons has to be symmetric ($\rightarrow$ bosons) or anti-symmetric ($\rightarrow$ fermions)

We can also extend the doublet $\begin{pmatrix} u \\ d \end{pmatrix}$ to a triplet $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$ with (approximate) $SU(3)$ symmetry

quark masses not degenerate

$m_d \approx 2\ \mathrm{MeV}$ $\quad m_u \approx 5\ \mathrm{MeV}$ $\quad m_s = 90\ \mathrm{MeV}$,

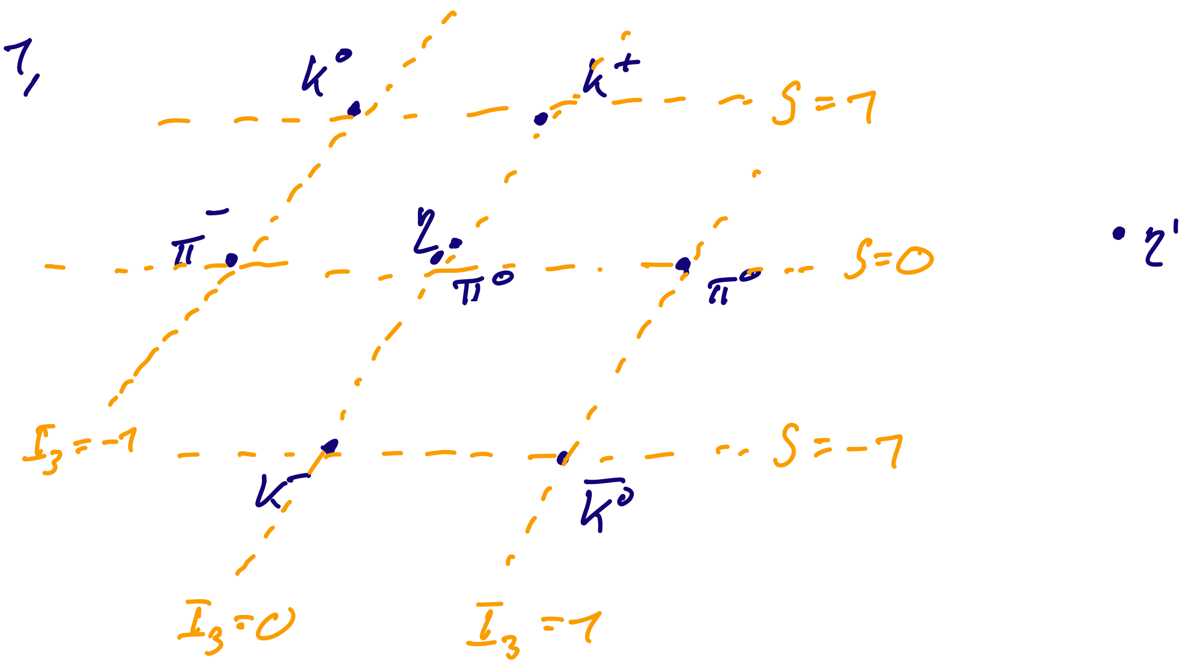
but α_s still large at energy scale m_s

$SU(3)$ has 2 generators that commute with every other generator $\rightarrow$ can choose e.g. I_3 and strangeness S to characterize hadrons.

Meson multiplets for $q\bar{q}'$ states wit $q, q' \in \{u, d, s\}$

$$3 \otimes \bar{3} = 8 \oplus 1$$

e.g. for $J=1$,



Similarly for baryonic states:

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

$\to$ octets, e.g. with $J = \frac{1}{2}$

decuplet, e.g. with $J = \frac{3}{2}$