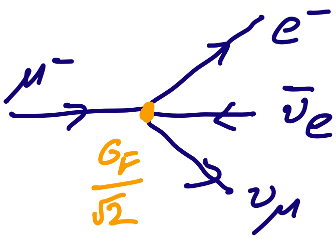
IV.3 Phenomenons leading to the development of the electroweak theory

We need an interaction to describe decay processes, such as

$$\mu^- \to e^- \, \bar{\nu}_e \, \nu_\mu$$
$$n \to p \, e^- \, \bar{\nu}_e$$

→ can be described by 4 fermion interactions, e.g.

$$\mathcal{L}_F = \frac{-G_F}{\sqrt{2}} \left(\bar{e} \gamma^\mu (1-\gamma_5) \nu_e \right) \left(\bar{\mu} \gamma_\mu (1-\gamma_5) \nu_\mu \right) + h.c.$$



→ Fermi theory

Problem: This Lagrangian also describes scattering processes, such as $e^- \bar{\nu}_e \to \mu^- \bar{\nu}_\mu$ with cross section $\sigma = \frac{G_F^2 \cdot s}{\pi}$

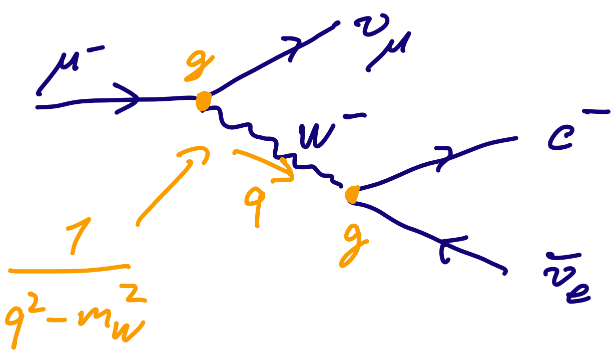
⇒ $\sigma \to \infty$ for large center-of-mass energies $\sqrt{s}$

⇒ violates unitarity of S-matrix!

Solution: 4 fermion interaction via exchange of vector bosons W with

↳ needs to couple to currents $\bar{\psi}\gamma^\mu(1-\gamma_5)\psi$

- $m_W > 0$
- el. charge ± 1
- only couples to left-handed fermions



For small energies ($q^2 \to 0$) this reproduces Fermi's theory:

$$\frac{g^2}{q^2 - m_W^2} \xrightarrow[q^2 \to 0]{} \frac{g^2}{m_W^2} = \text{const.} = \frac{G_F}{\sqrt{2}}$$

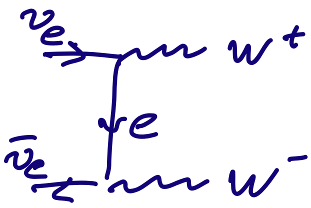
and we now get

$$\sigma(e^- \bar{\nu}_e \longrightarrow \mu^- \bar{\nu}_\mu) \sim \frac{G_F^2 \, m_W^2}{\alpha} = \text{const}$$

↑ for $\sqrt{s} \to \infty$

new unitarity problems:

- $\nu\bar{\nu} \rightarrow W_L^+ W_L^-$ ← longitudinal polarizations

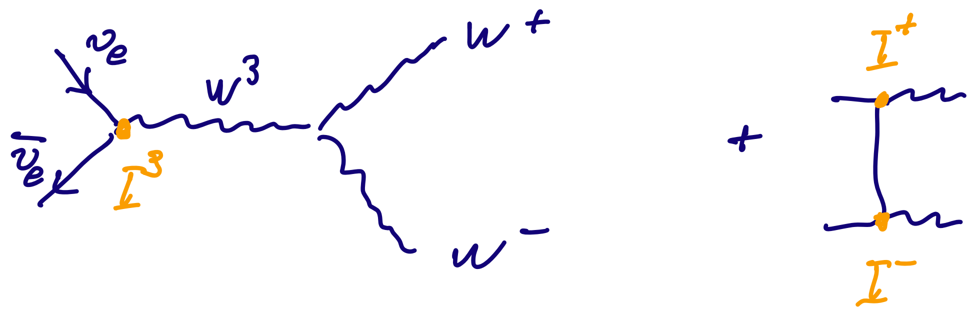


$$\sigma \sim \frac{g^4 s}{m_W^4}$$

→ unitarity violation

solution: introduce additional neutral boson W^3 $(\sim Z + \gamma)$

→ additional diagram



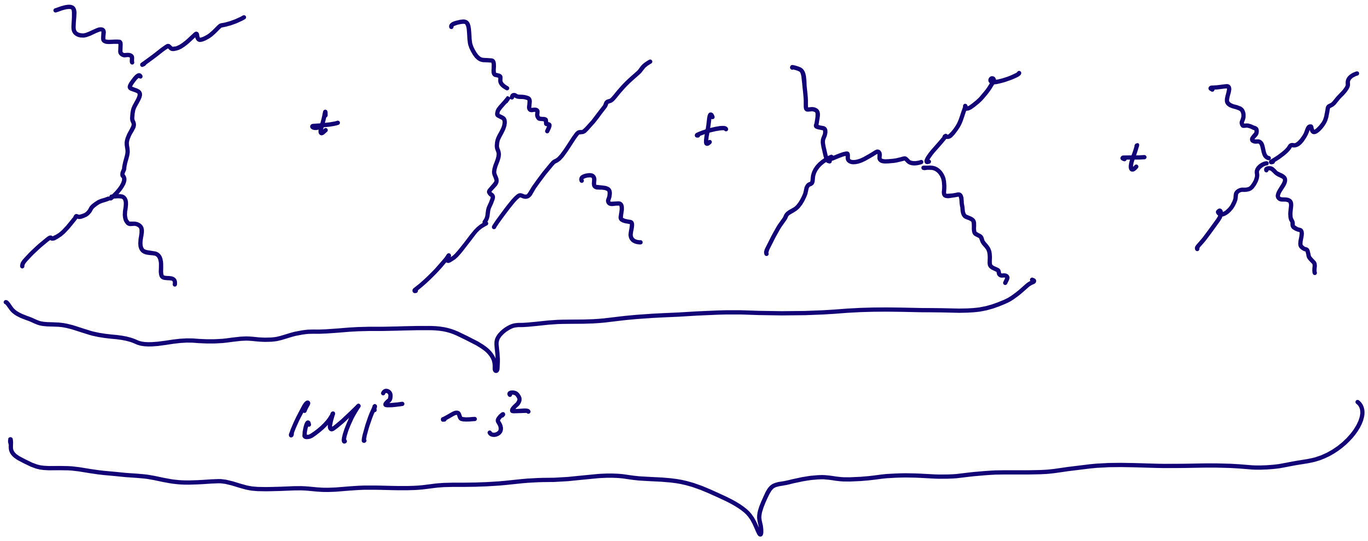
restores unitarity if additional relation for coupling to fermions applies

$$[I^i, I^j] = i\epsilon_{ijk} I^k$$

→ weak isospin I

non-abelian gauge theory with $SU(2)$ symmetry

- $W_L W_L \rightarrow W_L W_L$ scattering

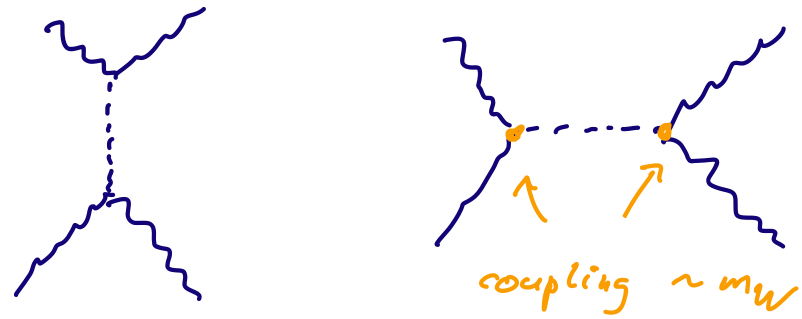


$|\mathcal{M}|^2 \sim s^2$

$|\mathcal{M}|^2 \sim s$

unitarity can be restored with additional scalar bosons

$\rightarrow$



$\Rightarrow$ The theory of weak interactions should have
- 3 massive vector bosons W^+, W^-, W^3
- $SU(2)$ symmetry $\Rightarrow$ non-abelian gauge theory
- scalar particle(s)

However, mass terms $m^2 A_\mu A^\mu$ of vector bosons would break the gauge symmetry.

$\Rightarrow$ We need another mechanism to generate the vector boson masses. In the SM, this is realized via spontaneous symmetry breaking in the Higgs mechanism (1964, Nobel prize 2013 for P. Higgs, F. Englert)

The theory of electroweak interactions was formulated in 1967 by S. Glasgow, A. Salam, S. Weinberg
→ GWS theory (Nobel prize 1979)

experimental confirmation:

- 1973: observation of neutral current interaction
- 1983: direct detection of W and Z bosons

IV.4 Spontaneous symmetry breaking

of a complex scalar field ϕ

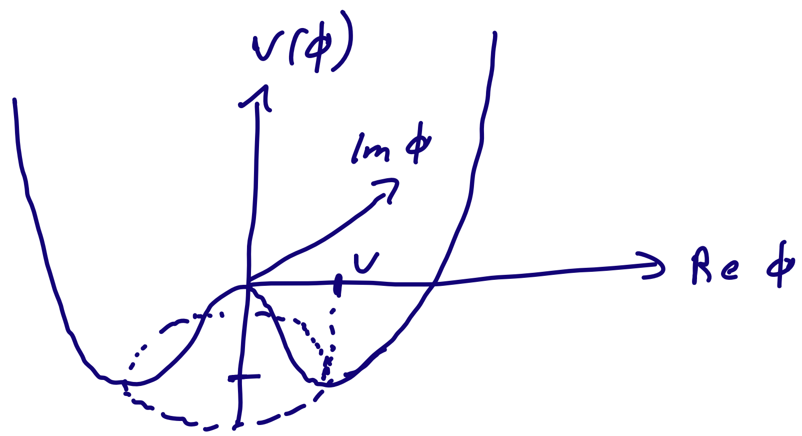
We consider the Lagrangian

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) \underbrace{+ \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2}_{V(\phi)}$$

with $\mu^2 > 0$, $\lambda > 0$

which is invariant under the (global) $U(1)$ symmetry $\phi \to e^{i\alpha} \phi$.

The potential has the form of a mexican hat:



with a local maximum at $\phi = 0$ and a local minimum on the circle

$$|\phi| = v = \sqrt{\frac{\mu^2}{2\lambda}}$$

"vacuum expectation value (vev)"

$$\frac{\partial V}{\partial \phi^*} = -\mu^2 \phi + 2\lambda \phi (\phi^* \phi)$$
$$= \phi \cdot \left(|\phi|^2 - \frac{\mu^2}{2\lambda} \right) \cdot 2\lambda = 0$$

$\Rightarrow$ The ground state of the theory is not at $\langle \phi \rangle = 0$, but at some point with $|\langle \phi \rangle| = v$.

Initially, all points on the circle $|\phi| = v$ are valid ground states, but the system will settle on one of these points as the "true" minimum, which breaks the $U(1)$ symmetry of the theory

$\rightarrow$ spontaneous symmetry breaking

In the following we choose $\phi = v$ as the true minimum and reparametrize the field ϕ as expansion around the minimum:

$$\phi = v + \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$$

with 2 real scalar fields (and $\langle\varphi_1\rangle = \langle\varphi_2\rangle = 0$)

$$\Rightarrow V(\phi) = -\mu^2 \phi^*\phi + \lambda(\phi^*\phi)^2$$

$$= \phi^*\phi\left(-\mu^2 + \lambda\phi^*\phi\right)$$

with $\phi^*\phi = v^2 + \frac{1}{2}\varphi_1^2 + \frac{1}{2}\varphi_2^2 + \sqrt{2}\,v\,\varphi_1$

$\mu^2 = 2\lambda v^2$

$$= \lambda\left(\frac{1}{2}\varphi_1^2 + \frac{1}{2}\varphi_2^2 + \sqrt{2}\,v\,\varphi_1\right)^2 - \lambda v^4$$

$\hookrightarrow$ constants can be dropped in $\mathcal{L}$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial\varphi_1)^2 + \frac{1}{2}(\partial\varphi_2)^2$$

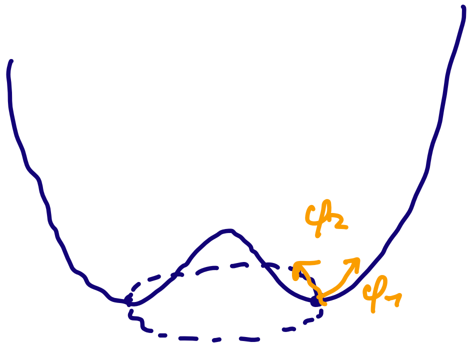
$$\underbrace{-2\lambda v^2\varphi_1^2}_{\hat{=}\frac{1}{2}m_1^2\varphi_1^2} \underbrace{-\sqrt{2}\lambda v\varphi_1(\varphi_1^2+\varphi_2^2) - \frac{\lambda}{4}(\varphi_1^2+\varphi_2^2)^2}_{\text{interactions}}$$

$\Rightarrow$ φ_1, φ_2 correspond to particlos with masses

$m_1 = 2\sqrt{\lambda}\,v$

$m_2 = 0$

The massless particles resulting from symmetry breaking are called <u>Goldstone bosons</u> and correspond to the degrees of freedom along the degenerate minimum of the potential



$\Rightarrow$ **Goldstone theorem**

For every spontaneously broken degree of freedom of a symmetry, there exists a massless Goldstone boson.

$\Rightarrow$ Number of Goldstone bosons: $N-M$

with N: Number of generators of the symmetry group of the full Lagrangian
($\hat{=}$ dimension of the algebra)

M: Number of generators of the remaining symmetry group after symmetry breaking

here: $N=1$ $\quad M=0$

Spontaneously broken gauge symmetries / Higgs mechanism

We now modify the above Lagrangian to include a gauge interaction:

$$\mathcal{L} = (D_\mu \phi)(D^\mu \phi)^* + \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

with $D_\mu = \partial_\mu - igA_\mu$ and the local $U(1)$ symmetry

$$\phi \rightarrow e^{ig\Lambda(x)}\phi \qquad A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x)$$

The ground state is again at $\langle\phi\rangle = v = \sqrt{\frac{\mu^2}{2\lambda}}$ and we can reparametrize the field ϕ as

$$\phi = \left(v + \frac{H(x)}{\sqrt{2}}\right) \cdot e^{\frac{i}{\sqrt{2}}\frac{\chi(x)}{v}}$$

This parametrization will simplify the following calculation.

with 2 real scalar fields H and χ, such that $\langle H\rangle = \langle\chi\rangle = 0$.

$$V(\phi) = -\mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2$$

$\mu^2 = 2\lambda v^2$

$$= 2\lambda v^2 H^2 + \sqrt{2}\lambda v H^3 + \frac{1}{4}H^4 - \underbrace{\lambda v^4}_{\rightarrow \text{ ignored}}$$

$$D_\mu \phi = (\partial_\mu - igA_\mu)\phi$$

$$= e^{\frac{i}{\sqrt{2}}\frac{\chi(x)}{v}} \left(\partial_\mu - igA_\mu + \frac{i}{\sqrt{2}v}\partial_\mu\chi\right)\left(v + \frac{H}{\sqrt{2}}\right)$$

The $\partial_\mu \chi$ term can be absorbed into A_μ with the gauge transformation

$$A_\mu \rightarrow A_\mu + \partial_\mu\left(\frac{\chi}{\sqrt{2}gv}\right)$$

$$D_\mu \phi = e^{\frac{i}{\sqrt{2}} \frac{\chi}{v}} \left(\partial_\mu - i g A_\mu \right) \left(v + \frac{H}{\sqrt{2}} \right)$$

$$\Rightarrow (D_\mu \phi)(D^\mu \phi)^* = \frac{1}{2} (\partial_\mu H)(\partial^\mu H) + g^2 A_\mu A^\mu \left(v + \frac{H}{\sqrt{2}} \right)^2$$

$$\Rightarrow \mathcal{L} = \frac{1}{2} (\partial_\mu H)(\partial^\mu H) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overbrace{g^2 v^2}^{=\frac{1}{2} m_A^2} A_\mu A^\mu - \overbrace{2\lambda v^2}^{=\frac{1}{2} m_H^2} H^2$$

$$\underbrace{+ g^2 A_\mu A^\mu \left(\sqrt{2}\, v H + \frac{H^2}{2} \right) - \sqrt{2}\, \lambda v H^3 - \frac{\lambda}{4} H^4}_{\text{interactions}}$$

The contributions of the field $\chi(x)$ have been completely absorbed into the gauge field A, which obtains a mass $m_A = \sqrt{2}\, g v$. Instead of a massless Goldstone boson, this so called "would-be Goldstone boson" contributes the longitudinal degree of freedom of A.

The real scalar particle H with mass $m_H = 2\sqrt{\lambda}\, v$ is called "Higgs boson".

Field content of the theory:

before symmetry breaking		after symmetry breaking	
complex scalar ϕ	(2 d.o.f.)	real scalar H	(1 d.o.f.)
massless vector A	(2 d.o.f.)	massive vector A	(3 d.o.f.)

For a broken gauge symmetry with n scalar fields, we obtain

$(N-M)$ massive gauge bosons

$n-(N-M)$ scalar Higgs bosons,

where N, M are again the number of generators before and after symmetry breaking, respectively.

IV.5 Electroweak Interactions and Symmetry Breaking

Kinetic terms of the fermions, e.g. electron and electron neutrino (mass terms will be discussed in IV.6)

$$\mathcal{L}_{kin} = \bar{e}\, i \not{\partial} e + \bar{\nu}_e\, i \not{\partial} \nu_e$$

can be split into left- and right-handed contributions ($\psi_{L,R} = \frac{1}{2}(\mathbb{1} \mp \gamma_5)\psi$, resp. Weyl spinors)

$$\mathcal{L}_{kin} = \bar{e}_L i\not{\partial} e_L + \bar{e}_R i\not{\partial} e_R + \bar{\nu}_{e,L} i\not{\partial} \nu_{eL}$$

$$= \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} i\not{\partial} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L + \bar{e}_R i\not{\partial} e_R$$

weak isospin doublet

$\mathcal{L}_{kin}$ is invariant under global transformations of weak isospin

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \longrightarrow e^{-ig\vec{\alpha}\vec{\tau}} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

$$e_R \longrightarrow e_R$$

→ conserved charge: total weak isospin $\vec{I}^2$

→ $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$: isospin doublet $I = \frac{1}{2}$ $\quad I_3(\nu_e) = +\frac{1}{2}$, $\quad I_3(e^-) = -\frac{1}{2}$

e_R : isospin singlet $I = I_3 = 0$

additional $U(1)$ symmetry

→ hypercharge Y, will be related to electrical charge via

$$Q = I_3 + \frac{Y}{2} \qquad \text{(Gell-Mann-Nishijima relation)}$$

$$\Rightarrow \quad Y(e_L) = Y(\nu_e) = -1$$

$$Y(e_R) = -2$$

$\leftarrow$ needs to be the same such that invariant under simultaneous $SU(2)$ and $U(1)$ transforms.

Similarly for 2. & 3. generation, quarks, scalar Higgs doublet

	fields	I	I_3	Y	Q
left-h. leptons	$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	$\frac{1}{2}$	$+1/2$ $-1/2$	-1	0 -1
right-h. leptons	$\ell = e_R, \mu_R, \tau_R$		0	-2	-1
left-h. quarks	$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$	$\frac{1}{2}$	$+1/2$ $-1/2$	$1/3$	$2/3$ $-1/3$
right-h. up-type q.	$u = u_R, c_R, t_R$		0	$4/3$	$2/3$
right-h. down-type q.	$d = d_R, s_R, b_R$		0	$-2/3$	$-1/3$
complex scalar Higgs doublet	$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\frac{1}{2}$	$+1/2$ $-1/2$	1	1 0

Instead of global symmetries, we demand
<u>local gauge invariance</u>:

- 3 gauge bosons $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ for $SU(2)_L$

• 1 gauge boson B_μ

covariant derivative

$$iD_\mu = i\partial_\mu - g\,\vec{I}\cdot\vec{W}_\mu - \frac{g'}{2}\,Y\,B_\mu$$

with g, g' : coupling constants (one for each symmetry)

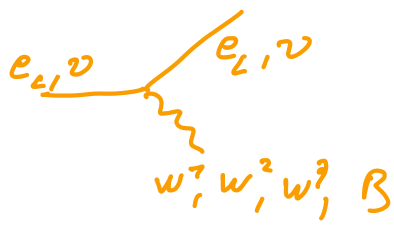
$\vec{I}, Y$ depend on multiplet it is acting on

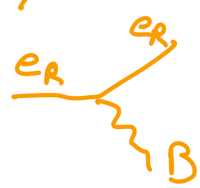
$\vec{I} = \frac{\vec{\sigma}}{2}$ for doublet, $\vec{I} = 0$ for singlet

e.g. $$iD_\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = \left(i\partial_\mu - \frac{\vec{\sigma}}{2}\vec{W}_\mu - g'\,\frac{-1}{2}\,B_\mu \right) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

$$iD_\mu\, e_R = \left(i\partial_\mu - g'\,\frac{(-2)}{2}\,B_\mu \right) e_R$$

$$\underline{\mathcal{L}_{\text{Fermions}}} = \underbrace{\overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L}\, i\not{D} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L}_{\text{contains couplings}} + \underbrace{\bar{e}_R\, i\not{D}\, e_r}_{\text{only contains}} + \ldots$$



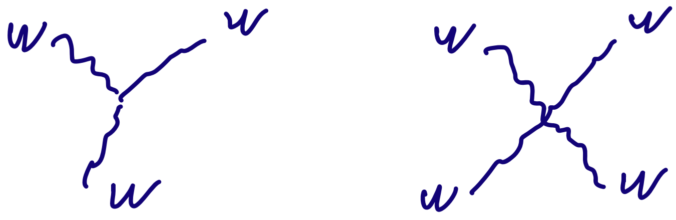


kinetic terms of gauge fields:

$$\underline{\mathcal{L}_{\text{gauge}}} = -\frac{1}{4}\,W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4}\,B_{\mu\nu} B^{\mu\nu}$$

with $$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$
$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\,\epsilon^{ijk}\,W^j_\mu W^k_\nu$$

→ self interactions of W-bosons



The electroweak $SU(2)_L \otimes U(1)$ symmetry is broken by the vacuum expectation value of the Higgs doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^+ (D^\mu \phi) - V(\phi)$$

$$= (D_\mu \phi)^+ (D^\mu \phi) + \mu^2 \phi^+ \phi - \frac{\lambda}{4} (\phi^+ \phi)^2$$

The potential $V(\phi)$ is minimal for

$$|\langle \phi \rangle|^2 = \frac{2\mu^2}{\lambda} =: \frac{v^2}{2}$$

→ choose $\phi_g = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ as ground state

→ breaks 3 d.o.f. of $SU(2)_L$

Are there remaining symmetries?

ϕ_g not invariant under $U(1)_Y$ transformations, since

$$e^{i\alpha Y}\phi_g = e^{i\alpha 1}\phi_g \neq \phi_g$$

but a $U(1)$ symmetry associated to the electric charge $Q = I_3 + \frac{Y}{2}$ remains: $e^{i\alpha Q}\phi_g = e^{i\alpha 0}\phi_g = \phi_g$

↳ i.e. a specific direction in the space spanned by I_1, I_2, I_3, Y

$\Rightarrow$ $SU(2)_L \otimes U(1)_Y$ is broken to the $U(1)_Q$ symmetry of electromagnetism.