Lectures 15:

Spontaneously broken gauge symmetries

$$\mathcal{L} = (D_\mu \phi)(D^\mu \phi)^* + \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

ground state $\quad \langle \phi \rangle = v = \sqrt{\frac{\mu^2}{2\lambda}}$

reparametrize $\quad \phi = \left( v + \frac{H(x)}{\sqrt{2}} \right) \cdot e^{\frac{i}{\sqrt{2}} \frac{\chi(x)}{v}}$

gauge transformation $\Rightarrow$ absorb $\chi(x)$ into vectorboson $A$

$\Rightarrow$ $A$ obtains mass

## IV.5 Electroweak Interactions and Symmetry Breaking

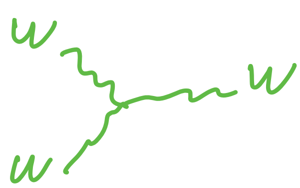
$$\underline{\mathcal{L}_{\text{Fermions}}} = \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} \, i \not{D} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L + \bar{e}_R \, i \not{D} e_R + \ldots$$

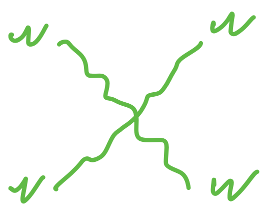
with $\quad i D_\mu = i \partial_\mu - g \vec{I} \cdot \vec{W}_\mu - \frac{g'}{2} Y B_\mu$

$\vec{I}$: weak isospin, $Y$: hypercharge

$$\underline{\mathcal{L}_{\text{gauge}}} = -\frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$\Rightarrow$ contains interactions




$$\underline{\mathcal{L}_{\text{Higgs}}} = (D_\mu \phi)^+ (D^\mu \phi) - V(\phi) \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$= (D_\mu \phi)^+ (D^\mu \phi) + \mu^2 \phi^+ \phi - \frac{\lambda}{4} (\phi^+ \phi)^2$$

potential $V(\phi)$ is minimal for $|\langle\phi\rangle|^2 = \frac{2\mu^2}{\lambda} =: \frac{v^2}{2}$

$\rightarrow$ choose ground state $\phi_g = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$\Rightarrow$ $SU(2)_L \otimes U(1)_Y$ broken to $U(1)_Q$

As in the previous section, we can reparametrize

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left(v + h(x) + i\chi(x)\right) \end{pmatrix}$$

The 3 would-be Goldstone bosons $\chi(x)$, $\phi^+(x)$ and $\phi^-(x) = (\phi^+(x))^+$ can be eliminated by a gauge transformation of the W bosons. It is therefore enough to consider

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

in the following.

Comment: Absorbing the would-be Goldstone bosons into the longitudinal degrees of freedom of the vector bosons is a gauge choice, known as unitary gauge.

Another class of popular gauge choices are $R_\xi$

gauges, where some modifications of the propagators, as well as diagrams with Goldstone bosons are required. The unitary gauge corresponds to the limit $\xi \to \infty$.

Inserting our parametrization of $\phi$ into the Higgs potential gives

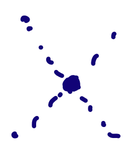
$$\mathcal{L}_{Higgs} \supset -V(\phi) = \frac{1}{2}\mu^2 (v+h)^2 - \frac{\lambda}{16}\cdot(v+h)^4$$

$$= -\mu^2 h^2 - \frac{\mu^2}{v} h^3 - \frac{\mu^2}{4v^2} h^4 + \text{const.}$$

$\rightarrow$ Higgs boson mass: $\quad m_h = \sqrt{2}\,\mu \quad (\approx 125\ \text{GeV})$

self-interactions of Higgs bosons

 $= -3i\,\dfrac{m_h^2}{v}$

 $= -3i\,\dfrac{m_h^2}{v^2}$

The masses of the vector bosons can be obtained from the kinetic term of $\phi$

$$(D_\mu \phi)^\dagger (D^\mu \phi) \supset \left| \left( -ig\,\vec{I}\vec{W}^\mu - ig'\frac{Y}{2}B^\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} \right|^2$$

$$\supset \left| \left( -ig \frac{\vec{\tau}}{2} \vec{W}^\mu - ig' \frac{1}{2} B^\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \frac{1}{2} \frac{v^2}{4} \begin{pmatrix} W_1^\mu \\ W_2^\mu \\ W_3^\mu \\ B^\mu \end{pmatrix}^T \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_1^\mu \\ W_2^\mu \\ W_3^\mu \\ B^\mu \end{pmatrix}$$

$W_3$ and $B$ are not mass eigenstates, matrix can be diagonalized by rotation

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}$$

with Weinberg angle $\theta_W$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \approx 0.23 \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$g \cdot \sin\theta_W = g' \cdot \cos\theta_W$

$\Rightarrow$ masses

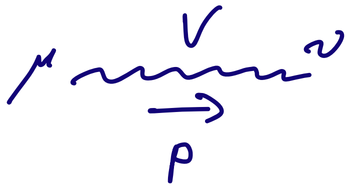
($\hat{=}$ eigenvalues of matrix)

$$m_1 = m_2 = m_W = \frac{v}{2} \cdot g \qquad (\approx 80 \text{ GeV})$$

$$m_Z = \frac{v}{2}\sqrt{g^2+g'^2} \qquad (\approx 91 \text{ GeV})$$

$$m_A = 0$$

propagators of $W$ and $Z$ (in unitary gauge)




$$\frac{-i}{p^2 - m_V^2}\left( g^{\mu\nu} - \frac{p^\mu p^\nu}{m_V^2} \right)$$

## Coupling of the gauge bosons to fermions

In the following, we assume that the hypercharges $Y_L$, $Y_R$, $Y_\phi$ of the left- and right-handed leptons and the Higgs doublet are not known. This also leads to the modifications

$$m_Z = \frac{v}{2}\sqrt{g^2 + Y_\phi^2\, g'^2}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + Y_\phi^2 g'^2}}\,, \qquad \sin\theta_W = \frac{g'\, Y_\phi}{\sqrt{g^2 + Y_\phi^2 g'^2}}$$

Lepton-gauge-boson couplings given by

$$\mathcal{L}_{\text{fermions}} = \overline{\begin{pmatrix}\nu_e \\ e\end{pmatrix}}_L \left( i\not\partial - \frac{g}{2}\vec{\tau}\cdot\vec{\not W} - \frac{g'}{2} Y_L \not B \right)\begin{pmatrix}\nu \\ e\end{pmatrix}_L$$

$$+ \bar{e}_R \left( i\not\partial - \frac{g'}{2} Y_R \not B \right) e_r + \dots$$

$$\supset -\frac{1}{2}\Big( g\,\bar{\nu}_{eL}\,(\not W_1 - i\not W_2)\, e_L + g\,\bar{e}_L\,(\not W_1 + i\not W_2)\,\nu_{eL}$$

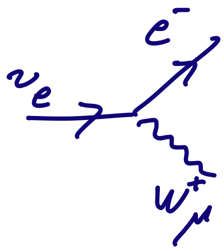
$$+ \bar{\nu}_{eL}\left(g\not W_3 + g' Y_L \not B\right)\nu_{eL} + \bar{e}_L\left(-g\not W_3 + g' Y_L \not B\right) e_L$$

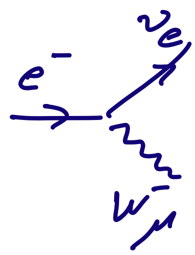
$$+ g' Y_R\, \bar{e}_R \not B\, e_R \Big)$$

→ $e - \nu_e - W$ coupling

$$-\frac{g}{\sqrt{2}}\, \bar{\nu}_{eL} \not{W}^+ e_L + h.c. = -\frac{g}{\sqrt{2}}\, \bar{\nu}_e W^+ \gamma^\mu \frac{1-\gamma_5}{2} e + h.c.$$

with $W^\pm = \frac{1}{\sqrt{2}} \left( W_1 \mp i W_2 \right)$




$$-\frac{ig}{\sqrt{2}} \gamma^\mu \frac{1-\gamma_5}{2}$$

remaining terms, expressed with fields $A, Z$:

$$\begin{pmatrix} B \\ W \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix}$$

$$A_\mu \Big[ \bar{\nu}_{eL} \gamma^\mu \nu_{eL} \cdot \left( -\frac{g}{2} \sin\theta_W - \frac{g'}{2} Y_L \cdot \cos\theta_W \right)$$

$$+ \bar{e}_L \gamma^\mu e_L \cdot \left( \frac{g}{2} \sin\theta_W - \frac{g'}{2} Y_L \cos\theta_W \right)$$

$$+ \bar{e}_R \gamma^\mu e_R \left[ -\frac{g'}{2} Y_R \cdot \cos\theta_W \right] \Big]$$

$$+ Z_\mu \left[ \ldots \right]$$

We now demand that

- neutrino - photon coupling vanishes

$$0 = -\frac{g}{2} \cdot \sin\theta_W - \frac{g'}{2} Y_L \cdot \cos\theta_W$$

$$= -\frac{1}{2}\,\frac{g g' \,(Y_\phi + Y_L)}{\sqrt{g^2 + g' Y_\phi^2}} \quad \Rightarrow \quad Y_\phi = - Y_L$$

- electron-photon coupling equal for $e_L$ and $e_R$, given by electric charge

$$\frac{g g'}{2\sqrt{g^2 + g'^2 \phi_Y^2}} \left( Y_\phi - Y_L \right) = \frac{g g'}{2\sqrt{g^2 + g' Y_\phi^2}} \cdot (-Y_R)$$

$$\Rightarrow \quad Y_R = Y_L - Y_\phi = 2 Y_L$$

→ We can choose $Y_L = -1$, $Y_R = -2$, $Y_\phi = 1$ as given above, and relate the couplings $g, g'$ to the electromagnetic coupling via

$$\bar{e}_L \not{A} e_L \Big( \frac{g}{2} \sin\theta_W + \underbrace{\frac{g'}{2}\cos\theta_W}_{= \frac{g}{2}\cdot \sin\theta_W} \Big) + \bar{e}_R \not{A} e_R \cdot \underbrace{g' \cos\theta_W}_{= g\cdot\sin\theta_W}$$

$$= \bar{e} \not{A} e \cdot g \cdot \sin\theta_W$$

→ We can identify the elementary charge

$$e_0 = g \cdot \sin\theta_W = g' \cdot \cos\theta_W$$

and express the coupling constants $g, g'$ with $e_0$ and $\sin\theta_W$.

The coupling $g$ can also be related to Fermi's constant $G_F = \frac{\sqrt{2}}{8}\frac{g^2}{m_W^2} = \frac{1}{\sqrt{2}\, v^2}$

$\uparrow$
$m_W = \frac{g}{2} v$

$\Rightarrow$ Higgs vacuum expectation value

$$v = \frac{1}{\sqrt{\sqrt{2}\, G_F}} \approx 246 \text{ GeV}$$

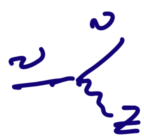
is the characteristic energy scale of EW symmetry breaking

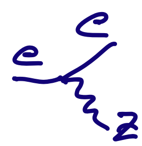
Summary of gauge-boson couplings to electrons and electron neutrinos

$\rightarrow$ interaction Lagrangian

$$\mathcal{L}_{int} = \frac{-g}{\sqrt{2}} j^{-}_{\mu} W^{+\mu} + h.c.$$


$$- \frac{g}{\cos\theta_W} \left( j^{3}_{\mu} - \sin\theta_W \, j^{em}_{\mu} \right) Z^{\mu}$$




$$- e_0 \, j^{em}_{\mu} A^{\mu}$$


with the currents

$$j^{-}_{\mu} = \bar{\nu}_{eL} \gamma_{\mu} e_L$$

$$j^{3}_{\mu} = \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}}_L \gamma_{\mu} \frac{\tau_3}{2} \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

$$j^{em}_{\mu} = -\bar{e} \gamma_{\mu} e$$

For the leptons of the 2. and 3. generation, we obtain the same interaction Lagrangian $\mathcal{L}_{int}$ with the replacements

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \longrightarrow \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

Minor modifications of currents necessary for gauge couplings to quarks:

$$j_\mu^- = \bar{u}_{iL} \gamma_\mu d_{jL} V_{ij}$$

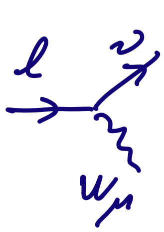
with $u_1 = u$, $u_2 = c$, $u_3 = t$
$d_1 = d$, $d_2 = s$, $d_3 = b$
and quark mixing matrix $V_{ij}$
→ sect. IV. 6

$$j_\mu^3 = \overline{\begin{pmatrix} u_i \\ d_i \end{pmatrix}}_L \gamma_\mu \frac{\tau_3}{2} \begin{pmatrix} u_i \\ d_i \end{pmatrix}$$

$$j_\mu^{em} = Q_i e_0 \bar{q}_i \gamma_\mu q_i$$

with charge $Q_i = Q(q_i)$

→ Feynman rules



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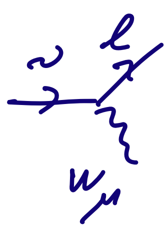
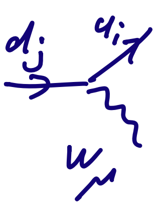
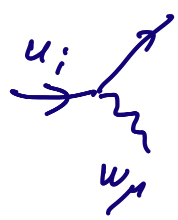



$$= -i \frac{g}{\sqrt{2}} \gamma_\mu P_L$$

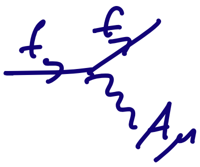
$$P_L = \frac{1-\gamma_5}{2}$$



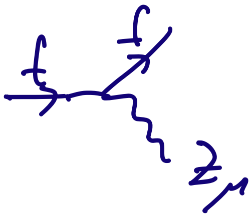

$$= -i \frac{g}{\sqrt{2}} \gamma_\mu P_L V_{ij}$$




$$= -i \frac{g}{\sqrt{2}} \gamma_\mu P_L V_{ij}^*$$




$$= -ie Q_f \gamma_\mu$$




$$= -\frac{ig}{\cos\theta_W}\,\gamma_\mu\left(g_V^f - g_A^f\,\gamma_5\right)$$

with
$$g_V^f = \frac{I_3}{2} - Q_f \cdot \sin^2\theta_W$$
$$g_A^f = \frac{I_3}{2}$$

comment on conventions:

In the literature, one often finds different conventions for e.g. the definition of the covariant derivative $D_\mu$, affecting e.g. the Feynman rules.

In the paper arXiv: 1209.6213, these signs are represented by parameters $\eta = \pm 1$, allowing for an easy conversion of the different conventions.

The discussion given here follows the convention

$$\eta = \eta' = \eta_G = \eta_Z = \eta_e = 1$$