Lecture 16:

$(D_\mu \phi)^\dagger (D^\mu \phi)$ leads to gauge boson masses

$$m_W = \frac{v}{2} \cdot g \qquad (\approx 80 \text{ GeV})$$

$$m_Z = \frac{v}{2} \sqrt{g^2 + g'^2} \qquad (\approx 91 \text{ GeV})$$

$$m_A = 0$$

with $\quad W^\pm = \frac{1}{\sqrt{2}} (W_1 \mp i W_2)$

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}$$

Weinberg angle $\quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \approx 0.23$

Fermion-gauge-boson interactions

$$\mathcal{L}_{int} = \frac{-g}{\sqrt{2}} j^-_\mu W^{+\mu} + h.c.$$

$$- \frac{g}{\cos\theta_W} \left(j^3_\mu - \sin\theta_W \, j^{em}_\mu \right) Z^\mu$$

$$- e_0 \, j^{em}_\mu A^\mu$$

Couplings of Higgs boson to vector bosons

So far, we only considered ground state

$$\phi_g = \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

of the Higgs doublet

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v\left(1+\frac{h}{v}\right) \end{pmatrix}$$

leading to the mass terms

$$\frac{1}{2} m_Z^2 \, Z_\mu Z^\mu + \frac{1}{2} m_W^2 \left(W_{1\mu} W^{1\mu} + W_{2\mu} W^{2\mu} \right)$$

$$= \frac{1}{2} m_Z^2 \, Z_\mu Z^\mu + m_W^2 \, W_\mu^+ W^{-\mu}$$

of the vector bosons, with $m_Z = \frac{v}{2} \sqrt{g^2 + g'^2}$

$$m_W = \frac{v}{2} g$$

The couplings of the vector bosons to the Higgs boson can be obtained by replacing

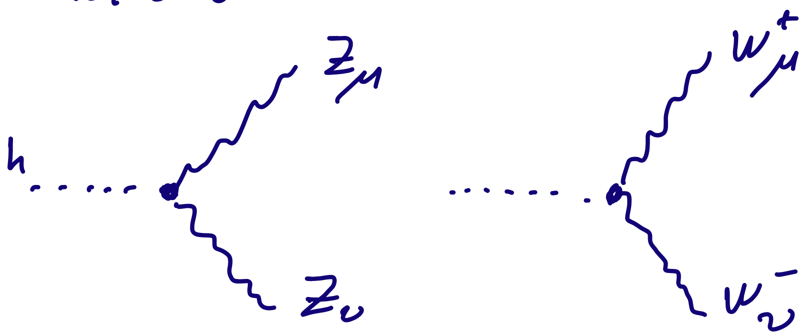
$$v \longrightarrow v\left(1+\frac{h}{v}\right)$$

in the mass terms:

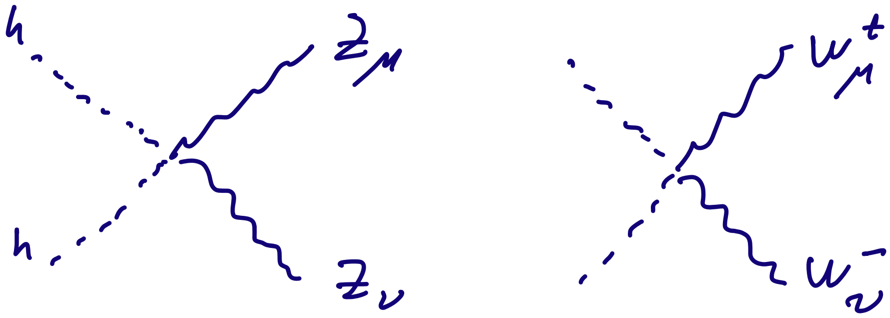
$$\frac{1}{2} m_V^2 \, V_\mu V^\mu \longrightarrow \frac{1}{2} m_V^2 \, V_\mu V^\mu \cdot \left(1+\frac{h}{v}\right)^2$$

$$= \frac{1}{2} m_V^2 \, V_\mu V^\mu \left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right)$$

→ interactions



$$i \, \frac{2 m_V^2}{v} \, g_{\mu\nu}$$



$$i \, \frac{2 m_V^2}{v^2} \, g_{\mu\nu}$$

IV.6 Fermion masses, Yukawa interactions & quark mixing

Mass terms of the form

$$m \bar{\psi} \psi = m \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right) = m \bar{\psi}_L \psi_R + h.c.$$

would violate the $SU(2)_L \otimes U(1)_Y$ symmetry of the SM, since left- and right-handed fermions transform under different representation.

→ instead, generate masses via Yukawa couplings of fermions to Higgs doublet, e.g. for electron:

$$\mathcal{L}_{m_e} = -y_e \, \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}}_L \phi \, e_R + h.c. \qquad \phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\chi) \end{pmatrix}$$

$$\supset -y_e \frac{v}{\sqrt{2}} \left(\bar{e}_L e_R + h.c. \right) \rightarrow m_e = \frac{y_e v}{\sqrt{2}}$$

Yukawa coupling constant

$\mathcal{L}_{m_e}$ is invariant under the transformation

$$\mathcal{L}_{m_e} \xrightarrow{SU(2)_L \otimes U(1)_Y}$$

$$-y_e \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}}_L \left(e^{i\vec{\alpha}\frac{\vec{\tau}}{2}} e^{iY_L \beta} \right)^\dagger \left(e^{i\vec{\alpha}\frac{\vec{\tau}}{2}} e^{iY_\phi \beta} \right) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e^{iY_R \beta} e_R$$

$$= \mathcal{L}_{m_e}$$

since $\quad -Y_L + Y_\phi + Y_R = 1 + 1 - 2 = 0$

Similarly for down quark:

$$\mathcal{L}_{m_d} = -y_d \overline{\begin{pmatrix} u \\ d \end{pmatrix}}_L \phi \, d_R + h.c. \rightarrow m_d = \frac{y_d \cdot v}{\sqrt{2}}$$

invariant, since $\quad -Y_Q + Y_\phi + Y_d = -\frac{1}{3} + 1 - \frac{2}{3} = 0$

However, for up-quarks, we have

$$-Y_Q + Y_\phi + Y_u = -\frac{1}{3} + 1 + \frac{4}{3} \neq 0$$

$\Rightarrow$ different form required for $\mathcal{L}_{mu}$

There are 2 possibilities to form SU(2) singlets from 2 doublets χ_1, χ_2

1) $\chi_1^+ \chi_2$ and $\chi_2^+ \chi_1$

2) $\chi_1^T \varepsilon \chi_2$ and $\chi_2^T \varepsilon \chi_1$

or equivalently $\chi_1^+ \varepsilon \chi_2^*$, $\chi_2^+ \varepsilon \chi_1^*$

$$\varepsilon = i\tau_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \varepsilon^{-1} = -\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

proof for 2)

We consider the transform

$$\chi_1 \rightarrow U\chi_1 \qquad \chi_1^+ \rightarrow \chi_1^+ U^+$$

$$\chi_2 \rightarrow U\chi_2 \qquad \chi_2^+ \rightarrow \chi_2^+ U^+$$

with $U = e^{i\vec{\alpha}\frac{\vec{\tau}}{2}} = \sum_n \frac{1}{n!} \left(\frac{i\vec{\alpha}\vec{\tau}}{2}\right)^n$

$$\Rightarrow (U\chi_1)^T \varepsilon U\chi_2 = \chi_1^T \underbrace{U^T}_{=\varepsilon U^+ \varepsilon^{-1}} \varepsilon U\chi_2 = \chi_1^T \varepsilon \chi_2$$

, since $i\tau_i^T = -i\varepsilon\tau_i\varepsilon^{-1}$

$\Rightarrow$ The term

$$\mathcal{L}_{m_u} = -y_u \overline{\begin{pmatrix} u \\ d \end{pmatrix}_L} \tilde{\Phi} \, u_R \quad + h.c.$$

$$\text{with } \tilde{\Phi} = \varepsilon \, \phi^* = \begin{pmatrix} \frac{1}{\sqrt{2}} (v + h + i\chi) \\ -\phi^- \end{pmatrix}$$

is invariant under $SU(2)_L$. The $U(1)_Y$ transforms of $\tilde{\Phi}$ is given by

$$\tilde{\Phi} \to e^{-iY_\phi \beta} \phi \qquad \left(\text{since } \phi \to e^{iY_\phi \beta} \phi\right)$$

and since $\quad -Y_Q - Y_\phi + Y_u = -\frac{1}{3} - 1 + \frac{4}{3} = 0,$

$\mathcal{L}_{m_u}$ is invariant under $U(1)_Y$

$\Rightarrow$ For the fermions of the 1. generation, the masses can be generated by $\mathcal{L}_{m_e} + \mathcal{L}_{m_d} + \mathcal{L}_{m_u}$.

Generalization to 3 generations

With the 3 generations of fermions in the SM, the most general form of the Yukawa interactions is given by

$$\mathcal{L}_{Yuk} = -Y^E_{ij} \bar{L}_i \Phi \, \ell_j - Y^D_{ij} \bar{Q}_i \Phi \, d_j - Y^U_{ij} \bar{Q}_i \tilde{\Phi} u_j + h.c.$$

where Y^E, Y^D, Y^U are complex 3×3 matrices and $L_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ $L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$ $L_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$

$\ell_1 = e_R$ $\ell_2 = \mu_R$ $\ell_3 = \tau_R$

$Q_1 = \begin{pmatrix} u \\ d' \end{pmatrix}_L$ $u_1 = u_R$, $u_2 = c_R$, ...
$d_1' = d_R'$, ...

naming convention: d_i' in this flavor basis
d_i in mass basis

matrices Y not diagonal

→ in this representation, the fermion fields are not mass eigenstates!

Transformation to mass eigenstates

We can apply unitary transforms of the fermion fields ($\hat{=}$ basis change / field reparametrization)

$$L_i \to U^L_{ij} L_j \qquad Q_i \to U^Q_{ij} Q_j$$

$$\ell_j \to U^\ell_{ij} \ell_j \qquad u_i \to U^u_{ij} u_j$$

$$d_j' \to U^d_{ij} d_j'$$

without affecting the discussion in sect. IV.5.

In particular, the transformations leave

$$\mathcal{L}_{Fermions} = \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} \, i\not{D} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L + \overline{e_R} \, i\not{D} e_R + \dots$$

invariant. Only the matrices Y are changed:

$$Y^E \longrightarrow U_L^+ \, Y^E \, U_\ell$$

$$Y^U \longrightarrow U_Q^+ \, Y^U \, U_u$$

$$Y^D \longrightarrow U_Q^+ \, Y^D \, U_d$$

and we can try to choose the transformations U such that the transformed matrices Y are diagonal

It is possible to choos U_L and U_ℓ such that

$$U_L^+ \, Y^E \, U_\ell = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \quad \text{with} \quad y_e, y_\mu, y_\tau \geq 0$$

$$\rightarrow m_e = \frac{y_e v}{\sqrt{2}}, \quad m_\mu, \quad m_\tau$$

Similarly, we choose U_Q, U_u, such that

$$U_Q^+ \, Y^U \, U_u = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad \text{with} \quad y_u, y_c, y_t \geq 0$$

$$\rightarrow m_u, m_c, m_t$$

But since this fixes U_Q, the remaining matrix

$$U_Q^+ \, Y^D \, U_d$$

cannot be diagonalized by choosing appropriate U_d.

$\rightarrow$ Need additional transformation

$$d_i' = V_{ij} \, d_j$$

of the down-type quarks, such that the resulting matrix

$$V^\dagger U_Q^\dagger \, y^D \, U_d V = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

can be diagonalized by an appropriate choice of the unitary matrices V and U_d.

The matrix V is called the CKM matrix

Nicola Cabibbo

Makoto Kobayashi $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ Nobel prize 2008

Toshihide Maskawa

and relates the flavor eigenstates d_i' to the mass eigenstates d_i.

Applying the transform V in the Lagrangian

$$\mathcal{L}_{Fermion} = \overline{\begin{pmatrix} u \\ d' \end{pmatrix}}_L \, i\not{D} \begin{pmatrix} u \\ d' \end{pmatrix} + \ldots$$

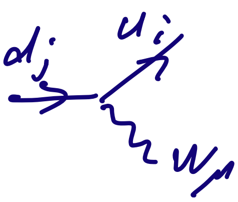
$$= \overline{\begin{pmatrix} u \\ Vd \end{pmatrix}_L} \, i\not{\partial} \begin{pmatrix} u \\ Vd \end{pmatrix} + \ldots$$

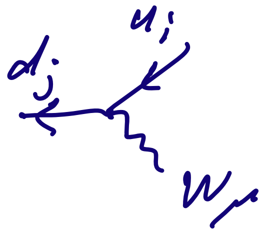
transformation V enters the quark-W-boson interaction

$$\mathcal{L}_{int,\, qqW} = \frac{-g}{\sqrt{2}} J^-_\mu W^{+\mu} + h.c.$$

with $j^-_\mu = \bar{u}_{iL} \gamma_\mu d_{jL} V_{ij}$

(as given in sect. IV.5)





$-i \frac{g}{\sqrt{2}} \gamma_\mu P_L V_{ij}$ $\qquad$ $-i \frac{g}{\sqrt{2}} \gamma_\mu P_L V^*_{ij}$

$\Rightarrow$ Since $V^*_{ij} \neq V_{ij}$ this interaction is different for quarks and anti-quarks!

($\Rightarrow$ CP violation, TTP 1 & 2)

General form of the CKM matrix

For n quark families, the matrix V is a unitary $n \times n$ matrix

$\Rightarrow$ would expect n^2 $\big(= \underbrace{2n^2}_{V \in \mathbb{C}^{n\times n}} - \underbrace{n^2}_{\text{conditions } V^\dagger V = V V^\dagger = \mathbb{1}} \big)$

free parameters

The number of parameters can be further reduced by absorbing $2n-1$ phase factors into a redefinition

$$u_i \to e^{i\alpha_i} u_i \quad , \quad d_i \to e^{i\beta_i} d_i$$

of the quark fields, with the constraint that the Lagrangian is real.

$\Rightarrow$ $n^2 - (2n-1) = (n-1)^2$ free parameters,

can be represented by $\frac{1}{2} n(n-1)$ angles

$\frac{1}{2}(n-1)(n-2)$ phases

$\Rightarrow$

n	angles	phases
2	1	0
3	3	1

For $\underline{n=2}$, the CKM-Matrix is a orthogonal matrix

$$V_{2\text{-fam}} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

with the $\underline{\text{Cabibbo angle } \theta_C}$.

Since $V^*_{2\text{-fam}} = V_{2\text{-fam}}$, CP-violation is not possible with only 2 quark families.

With the $\underline{n=3}$ families of the SM, the standard representation of V is given by

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 0 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_c & s_c & 0 \\ -s_c & c_c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with the 3 angles

$$s_c = \sin\theta_c = 0.2245$$

$$s_{23} = \sin\theta_{23} = 0.042$$

$$s_{13} = \sin\theta_{13} = 0.0035$$

and the phase $\qquad \delta = 1.2$

The unitary triangle(s)

The unitarity condition $V^\dagger V = V V^\dagger = \mathbb{1}$ of the CKM matrix enforces relationships between the entries of

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & \ddots & \\ V_{td} & & \end{pmatrix}$$

e.g.

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

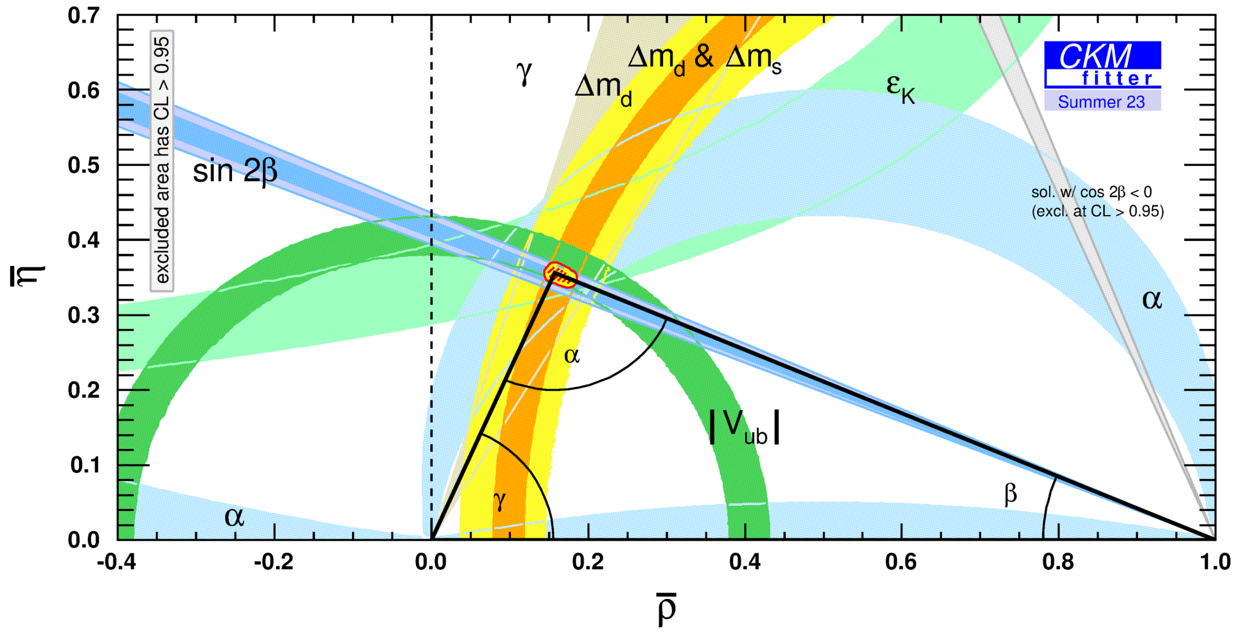
$\Rightarrow$ sum of 3 complex numbers $= 0$

$\hat{=}$ sum of 3 vectors in complex plane, forming triangle

$\Rightarrow$ 1 unitarity triangle for each of these relations.

typically represented as

$$1 + \underbrace{\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}}_{= -\bar{\rho} - i\bar{\eta}} + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} = 0$$



→ angles and side lengths of the triangle determined by various flavor observables.

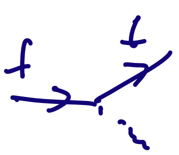
→ any inconsistency in the determination of the vertex at $\bar\rho + i\bar\eta$ would be a hint towards physics beyond the SM.

Higgs couplings to fermions

can be obtained via replacement $m_f \to m_f\left(1+\frac{h}{v}\right)$ in mass terms

$$-m_f \bar\psi_f \psi_f \quad \to \quad -m_f \bar\psi_f \psi_f \cdot \left(1+\frac{h}{v}\right)$$

(in analogy to calculation at end of sect. IV.5)



$-i\,\frac{m_f}{v}$

IV.7 The SM Lagrangian

To summarize the previous sections, we collect all terms of the SM Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{Fermions} + \mathcal{L}_{Higgs} + \mathcal{L}_{gauge} + \mathcal{L}_{Yukawa}$$

$$\Big(+ \underbrace{\mathcal{L}_{gauge\text{-}fix} + \mathcal{L}_{ghost}}_{\rightarrow \text{ needed for quantization}} \Big)$$

with

$$\mathcal{L}_{Fermions} = \sum_{\psi} \bar{\psi}\, i \not{D}\, \psi$$

$$\psi \in \begin{pmatrix} \nu \\ e \end{pmatrix}_{iL},\ e_{iR},\ \underbrace{\begin{pmatrix} u \\ d' \end{pmatrix}_{iL},\ u_{iR},\ d'_{iR}}_{\times 3 \text{ color degrees}}$$

$$i\not{D} = i\partial_\mu - g\,\vec{I}\,\vec{W}_\mu - \frac{g'}{2} Y B_\mu - g_s t^a G^a_\mu$$

$\rightarrow$ interactions

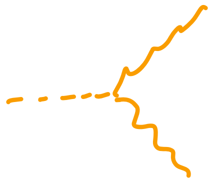
W, Z, A, g

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) \qquad V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

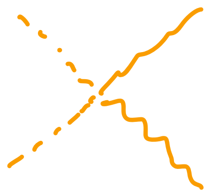
$$\rightarrow \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad v^2 = \frac{4\mu^2}{\lambda}$$

$\rightarrow$ m_Z, m_W, m_h $\qquad \begin{pmatrix} B \\ W_3 \end{pmatrix} \xrightarrow{\theta_W} \begin{pmatrix} A \\ Z \end{pmatrix}$

interactions :



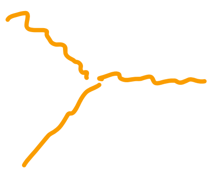
$h W^+ W^-$, $h Z Z$



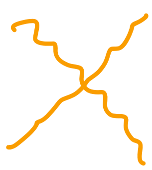
$h h W^+ W^-$, $h h Z Z$

$$\mathcal{L}_{gauge} = -\frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

interactions



ggg $\qquad W^+W^-A$, W^+W^-Z



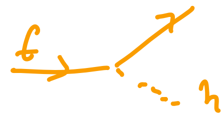
$gggg$ $\qquad W^+W^-AA$, W^+W^-AZ

W^+W^-ZZ, $W^+W^-W^+W^-$

$$\mathcal{L}_{Yukawa} = -Y^E_{ij} \overline{L}_i \Phi \ell_{jR} - Y^D_{ij} \overline{Q}_i \Phi d_{jR} - Y^U_{ij} \overline{Q}_i \tilde{\Phi} u_R + h.c.$$

$\rightarrow$ fermion masses m_f

quark mixin $d' = V d$

interactions 

$\Rightarrow$ SM has 18 free parameters:

- 3 coupling constants g, g', g_s
- μ, λ in $V(\phi)$
- 9 fermion masses m_f
- 3 angles + 1 phase in CKM matrix

EW parameters g, g' and $v = \frac{2\mu}{\sqrt{\lambda}}$ typically fixed by specifying e.g. G_F, M_Z, M_W

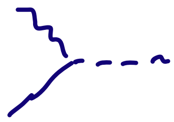
V. Higgs phenomenology

The Higgs mechanism

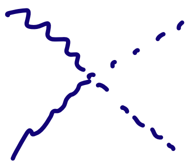
- plays crucial role in SM
- developed in 1960s
- predicts Higgs boson
- Higgs boson was for a long time last missing SM particle
- besides its mass m_h, all properties predicted by SM:
  - scalar boson
  - parity 1
  - all couplings fixed by masses of SM particles

coupling to gauge bosons

$V = W, Z$

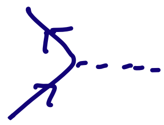


$$g_{HVV} = \frac{2M_V^2}{v}$$



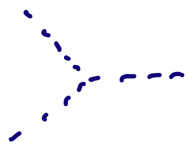
$$g_{HHVV} = \frac{2M_V^2}{v^2}$$

Yukawa coupl.



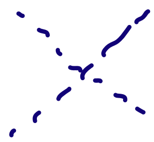
$$g_{Hff} = \frac{m_f}{v}$$

trilinear coupl.



$$\lambda_{HHH} = 3\,\frac{m_h^2}{v}$$

quartic coupl.



$$\lambda_{H^4} = 3\,\frac{m_h^2}{v^2}$$

With the discovery of Higgs boson on 4. 7. 2012, all particles of the SM have been observed and the last missing parameter

$$m_h \approx 125 \text{ GeV}$$

of the SM has been fixed.

Since all properties of Higgs boson are fixed by SM, detailed experimental measurements of these properties provide strong check of SM, and in particular of the mechanism of EW symmetry breaking.

$\Rightarrow$ Precise theoretical predictions for Higgs production and decays required.

V.7 Higgs decays

Higgs bosons mainly decay into pair of heavy particles X,

$$H \to XX$$

for which decay kinematically allowed. $(\to m_h > 2m_X)$