Karlsruhe Institute of Technology

SS 2022

Mathematical Methods of Theoretical Physics

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Exercise Sheet 7

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Exercise 1: Perturbation theory for algebraic equations (6 points)

(a) Find the roots of the equation

$$x^2 + x + 6\epsilon = 0 \tag{1.1}$$

using second-order perturbation theory and compare your result against the exact solution for $\epsilon = 0.01$ and $\epsilon = 0.001$.

(b) Consider the equation

$$x^2 - 2(1+\epsilon)x + 1 - \epsilon = 0\,. \tag{1.2}$$

Try to make a power series ansatz in ϵ for x to find perturbative solutions for the roots. Compare this to the exact solutions. Why does the naive approach fail?

(c) Consider the equation

$$\epsilon x^3 + x^2 - 2x + 1 = 0 \tag{1.3}$$

and find the first two terms of the behaviour of the roots of this polynomial in the limit $\epsilon \to 0$. *Hint: It may help to use the method of dominant balance to identify the required rescaling.*

Exercise 2: Perturbation theory for differential equations (5 points)

Consider the initial-value problem

$$y''(x) + (1 - \epsilon\, x)y(x) = 0\,, \qquad y(0) = 1\,, \qquad y'(0) = 0\,. \tag{2.1}$$

(a) Find the second-order perturbative solution to Eq. (2.1) for $\epsilon \ll 1$.

(b) Plot the perturbative solution for $\epsilon = 0$ and for $\epsilon = 1/40$.

(c) *(optional)* Find the exact solution of Eq. (2.1) using a computer algebra system and plot it for different values of ϵ. Compare the behaviour of the perturbative and the exact solution.