Problem 1 (4 Points)

K^+ mesons are produced in a height of 10 km above ground. The half-life time of K^+ mesons is $\tau_{1/2} = 8.6 \cdot 10^{-9}$ s.

a) Calculate the minimum velocity of the K^+ mesons, for which at least half of the particles are expected to reach the ground.
b) What is the time of flight t_E of the K^+ mesons in the reference frame of the earth?
c) How is the time of flight t_E related to the half-life time $\tau_{1/2}$ of the K^+ mesons?

Problem 2 (4 Points)

The kinetic energy of cold neutrons is 0.1 meV.

a) Calculate the speed of these neutrons.
b) Calculate the de Broglie wavelength of these neutrons.
c) Spherical viruses form a crystal with the lattice constant $d = 1.0 \cdot 10^{-8}$ m. Calculate for these neutrons the angle under which the 1st order diffraction maximum can be observed with respect to the incident neutron beam.
d) Calculate the length of the corresponding vector of the reciprocal lattice.

Problem 3 (4 Points)

During the decay ^{22}Na $\rightarrow$ ^{22}Ne one photon with the energy 1.28 MeV is emitted.

a) Calculate the wavelength of the corresponding electromagnetic wave.
b) Assume that the photons are scattered from electrons at rest. Calculate the energies of the scattered photons.
c) Calculate i) the smallest photon energy and ii) the highest kinetic energy of the electron after scattering.

Problem 4 (4 Points)

On a hot day (30° C) a black car is exposed to the plain sun. The intensity of the sun is 700 W/m^2.

a) What is the temperature of the car in thermal equilibrium when only half of its surface is exposed to the sun? Assume that the car behaves like a black body and neglect the effect of thermal conduction.
b) Sketch the radiation spectrum of a black body.
c) How large is the wavelength of the maximum of the spectrum for the car in thermal equilibrium?
d) Repeat the calculation for a car which reflects 30 % of the incident radiation. How large is the equilibrium temperature of this car?

Problem 5 (4 Points)

a) Calculate for the hydrogen atom the energy of the ground state, the first, and the second excited state, ignoring the spin of the electron.
b) Sketch the energy level scheme of these states, denote the states and indicate the allowed electric dipole transitions. Give the corresponding selection rule.
c) Sketch the energy level scheme of the first excited state including now the effect of the spin. Use the spectroscopic notation to denote the energy states.
d) What is the reason for the splitting of the ground state of the hydrogen atom in two energy levels? Sketch the splitting and denote the energy levels.

Please note reverse side.

Problem 6 (4 Points)

There are two electrons in the neutral Helium atom. The excited states with lowest energy result from the excitation of one electron into hydrogen-like orbitals while the other electron stays in the ground state.

a) Sketch the expected energy level scheme and denote the energy levels, when the excited electron is lifted into a state with the quantum number $\mathrm{n} = 3$. Ignore spin-orbit coupling.
b) Give reasons for the energy difference between states of different angular momenta L.
c) Explain the energy difference between spin singlet and triplet states.
d) Give the states of problem a), which can be excited from the groundstate by electric dipole transitions.

Problem 7 (4 Points)

a) The vectors of a primitive unit cell of the bcc lattice with the lattice constant a are $\vec{a}_1 = \frac{a}{2}(\vec{x}-\vec{y}+\vec{z})$, $\vec{a}_2 = \frac{a}{2}(\vec{y}-\vec{z}+\vec{x})$ and $\vec{a}_3 = \frac{a}{2}(\vec{z}-\vec{x}+\vec{y})$. $\vec{x}$, $\vec{y}$ and $\vec{z}$ denote orthogonal unit vectors. What is the volume of the unit cell?
b) Give the primitive vectors of the corresponding reciprocal lattice.
c) The vectors of a primitive unit cell of the fcc lattice with the lattice constant a are $\vec{a}_1 = \frac{a}{2}(\vec{x}+\vec{y})$, $\vec{a}_2 = \frac{a}{2}(\vec{y}+\vec{z})$ and $\vec{a}_3 = \frac{a}{2}(\vec{z}+\vec{x})$. What is the volume of the unit cell?
d) Give the primitive vectors of the corresponding reciprocal lattice.

Problem 8 (4 Points)

a) Calculate the energy of a free electron described by a simple plane wave in a region of constant potential energy V.
b) Calculate the density of conduction electrons of potassium. Hint: The mass of the potassium atom is $39u$ and there is one conduction electron per atom.
c) Calculate the Fermi wavenumber k_F of potassium.
d) Calculate the Fermi energy of potassium.

Required physical constants:

Speed of light:	$c = 3 \cdot 10^8\,\mathrm{m/s}$
Mass of the electron:	$m_\mathrm{e} = 500\,\mathrm{keV}/c^2$
Mass of the neutron:	$m_\mathrm{n} = 939\,\mathrm{MeV}/c^2$
Planck's constant:	$h = 4.14 \cdot 10^{-15}\,\mathrm{eVs}$
Elementary charge:	$e = 1.6 \cdot 10^{-19}\,\mathrm{As}$
Compton wavelength:	$\lambda_\mathrm{C} = 2.426 \cdot 10^{-12}\,\mathrm{m}$
Stefan-Boltzmann's constant:	$\sigma = 5.67 \cdot 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$
Wien's constant:	$b = 2.9\,\mathrm{mm} \cdot \mathrm{K}$
Boltzmann constant:	$k_\mathrm{B} = 8.6 \cdot 10^{-5}\mathrm{eV/K}$
Rydberg unit of energy:	$R = 13.6\,\mathrm{eV}$
Density of potassium:	$\rho_\mathrm{K} = 0.86\,\mathrm{g/cm^3}$
Atomic mass unit:	$u = 1.66 \cdot 10^{-27}\,\mathrm{kg}$