Modern Physics

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Exercise 1

§ Classical Wave Phenomena §

Problem 1: Wave Equation

(1) What is the general three-dimensional wave equation? & What changes when it is reduced to just one dimension?

(2) Show explicitly that the following functions $\psi(x,t)$ fulfill the one-dimensional wave equation:

- $\psi(x,t) = A\sin(k\,x - \omega\,t)$
- $\psi(x,t) = A\sin(k\,x)\,\cos(\omega\,t)$

(3) Make a sketch for both waves from 2) at the different points of times for $\psi(x,0)$ and $\psi(x,t=2s)$ under the assumption that $k = \frac{\pi}{3}\,\mathrm{cm}^{-1}$ and $\omega = \frac{\pi}{4}\mathrm{s}^{-1}$. What is the wavelength, frequency and period of both waves? & What is their direction of propagation?

Problem 2: Fermat's Principle

The principle of least time, named after the French mathematician *Pierre de Fermat*, implies that the propagation of light between two points follows the path which requires the minimum amount of time. Use this principle to derive the following laws in geometrical optics:

(1) Law of reflection.

(2) Law of refraction.

Problem 3: Standing Sound Waves

A loudspeaker box is next to a pipe, which ends are both open. The pipe has a length $l = 2\,\mathrm{m}$.

(1) Determine the wave function of the pressure wave, which can propagate along the pipe (x-direction). Use the complex notation.

(2) At which frequencies of the loudspeaker box resonances occur within the pipe, when considering that the speed of sound in air is $340\,\mathrm{m.s}^{-1}$?

(3) Make a sketch of the pressure development along the pipe for the case of the smallest resonance frequency.

(4) Make a sketch of the displacement development along the pipe for the case of the smallest resonance frequency.

(5) What happens, when one of the ends of the pipe is closed? Make a sketch of the displacement development for the first three resonance frequencies.