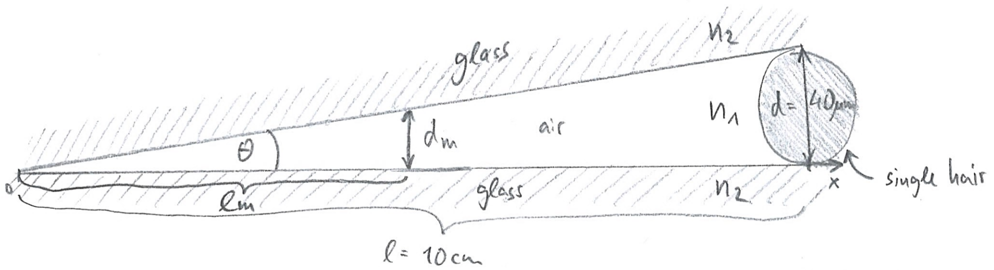
Modern Physics WS 21/22 Sören Bieling

Problem Sheet 2

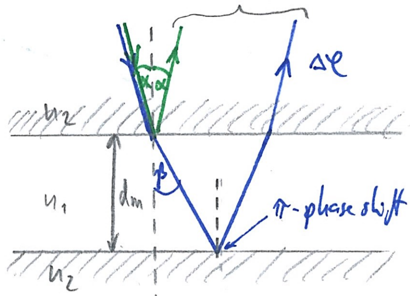
Ex. 2.1: Thin film interference at an aerial wedge

$\lambda = 550\,nm$

$n_2 > n_1$



$k_i = \frac{2\pi}{\lambda_i} = n_i \cdot \frac{2\pi}{\lambda}$



ray 1: zero additional phase shift

ray 2: π phase shift from reflection at $n_2 > n_1$

a) modulation of reflected intensity due to constructive & destructive interference

for $\alpha = 0°$: $\Delta\varphi = k_1 \cdot \Delta s = n_1 \cdot \frac{2\pi}{\lambda} \cdot 2d \pm \pi$

1.) constructive interference: $\Delta\varphi \overset{!}{=} m \cdot 2\pi$, $m \in \mathbb{Z}$

$$m \cdot 2\pi = n_1 \frac{2\pi}{\lambda} 2d + \pi \iff d_m = \frac{1}{2} \cdot \frac{\lambda}{n_1} \cdot \left(m \pm \frac{1}{2}\right)$$

2.) destruktive Interferenz

$\Delta\varphi \overset{!}{=} 2\pi\left(m + \frac{1}{2}\right)$, $m \in \mathbb{Z}$

$$2\pi\left(m + \frac{1}{2}\right) = n_1 \frac{2\pi}{\lambda} \cdot 2d + \pi \iff d_m = \frac{1}{2} \cdot \frac{\lambda}{n_1} \cdot m$$

b) distance of m^{th} dark fringe with respect to point of contact:

at point of contact: $d \to 0$ $\Delta\varphi \to \pi$ $\hat{=}$ dark fringe

$l = 10\,cm$, $d = 40\,\mu m$
$n_1 = 1$ (air), $\lambda = 550\,nm$

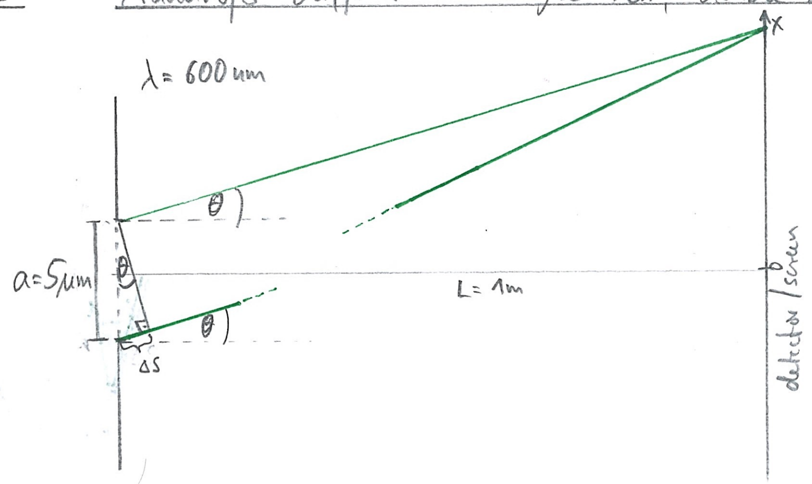
$$\frac{d}{l} = \frac{d_m}{l_m} = \tan\theta \approx \theta$$

$$l_m = l \cdot \frac{d_m}{d} = \frac{l}{d} \cdot \frac{1}{2} \cdot \frac{\lambda}{n_1} \cdot m \approx 686\,\mu m \cdot m$$

c) $\Delta m = 1 \to \Delta l_m = 686\,\mu m$

d) $n_1 = 1$ (air) $\longrightarrow n_1 = 1{,}333$ @ $\lambda = 550\,nm, 20°$ $\Delta l_m \to 516\,\mu m$

Ex. 2.2: Fraunhofer diffraction: single slit, double slit & optical grating



From lecture:

$$\Psi = \Psi_0 \frac{1}{a} \int_{x=0}^{a} e^{i\Delta\varphi(x)} dx$$

intensity

$$I = |\Psi|^2 = I_0 \left(\frac{\sin(\Delta\varphi/2)}{\Delta\varphi/2}\right)^2 = I_0 \operatorname{sinc}^2(*)$$

$$* = \frac{\pi a}{\lambda} \cdot \sin\theta$$

a) θ of 1st & 2nd minima: $\sin\theta = \frac{\Delta s}{a}$

$$\Delta\varphi_{SS} = k \cdot \Delta s = \frac{2\pi}{\lambda} \cdot a \sin\theta \underset{\substack{\uparrow \\ \sin\theta \approx \theta}}{\approx} \frac{2\pi}{\lambda} \cdot a \cdot \theta$$

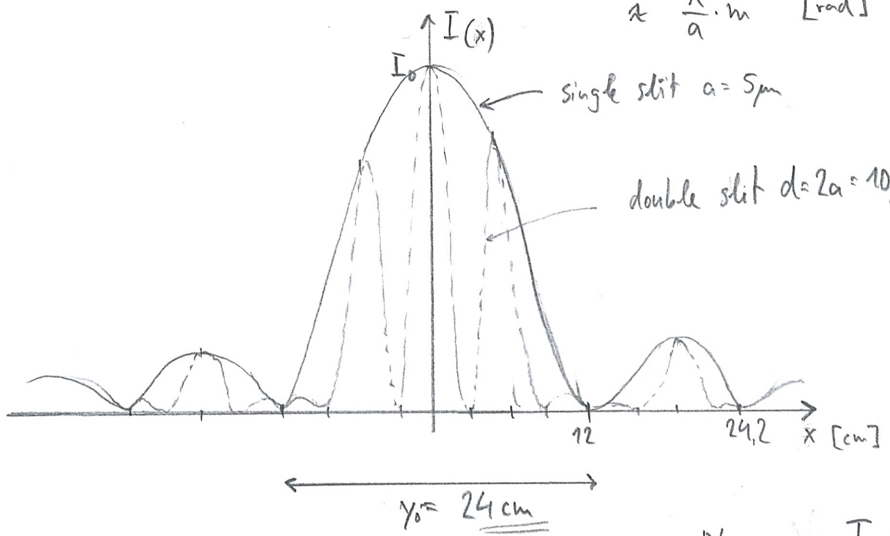
intensity has minima for $\Delta\varphi = 2\pi \cdot m \quad m = \pm1, \pm2, \pm3, \ldots$

$$\cancel{2\pi} \cdot m = \frac{\cancel{2\pi}}{\lambda} \cdot a \cdot \sin\theta \qquad \theta_{min} = \arcsin\left(\frac{\lambda}{a} \cdot m\right) \approx \frac{\lambda}{a} \cdot m \quad [rad]$$

$\theta_1 = 6{,}89°$ or $0{,}120$ rad

$\theta_2 = 13{,}89°$ or $0{,}242$ rad $\approx 0{,}24$ rad

b)

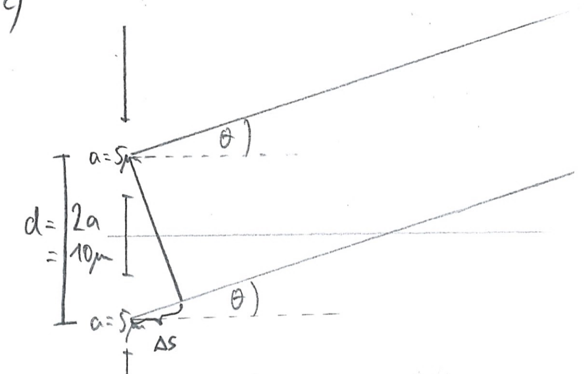


$\cancel{\tan\theta} = \frac{x}{L} \approx \theta$

$x_1 = 12$ cm

$x_2 = 24{,}2$ cm

c)



$\Delta s = d \sin\theta$

$\Delta\varphi_{DS} = k\Delta s = \frac{2\pi}{\lambda} d\sin\theta$

Now: $I = I_{Double\,slit} \cdot I_{single\,slit}$

$$= I_0 \cos^2(\Delta\varphi_{DS}/2) \cdot \operatorname{sinc}^2(\Delta\varphi_{SS}/2)$$

$$= I_0 \underbrace{\cos^2\left(\frac{\pi d}{\lambda} \sin\theta\right)}_{\text{double slit}} \cdot \underbrace{\left[\frac{\sin\left(\frac{\pi a}{\lambda} \cdot \sin\theta\right)}{\frac{\pi a}{\lambda} \sin\theta}\right]^2}_{\text{single slit}}$$

$$\overset{d=2a}{=} I \cos^2\left(\frac{2\pi a}{\lambda} \sin\theta\right) \cdot \left[\frac{\sin\left(\frac{\pi a}{\lambda} \sin\theta\right)}{\left(\frac{\pi a}{\lambda} \sin\theta\right)}\right]^2$$

maxima at: $\frac{\cancel{\pi} d}{\lambda} \sin\theta \overset{!}{=} m \cdot \cancel{\pi} \quad m \in \mathbb{Z}$

$\sin\theta_m = \frac{\lambda}{d} \cdot m$

$\theta_0 = 0°$

$\theta_1 = 3{,}44°$

$\theta_2 = 6{,}89°$

$\theta_3 = 10{,}37°$

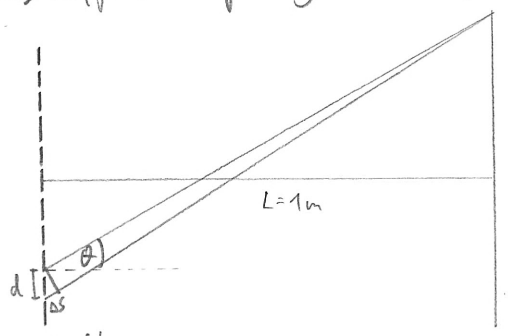
Diffraction pattern of double slit originates from 2 wave interference / superposition:

$$\Psi_{DS} = \Psi_1 + \Psi_2 = \Psi_0 e^{ikr} + \Psi_0 e^{ikr + \Delta\varphi} \rightarrow I_{DS} = |\Psi|^2 = I_0 \cos^2\left(\frac{\Delta\varphi}{2}\right)$$

Modern Physics WS 21/22 Sören Bieling

Ex. 2.2: (cont'd)

d) → diffraction grating instead of single/double slit



Diffraction pattern of optical grating originates as superposition of N single slits:

$$\Psi = \Psi_0 \sum_{n=1}^{N} e^{i(n-1)\Delta\varphi}$$

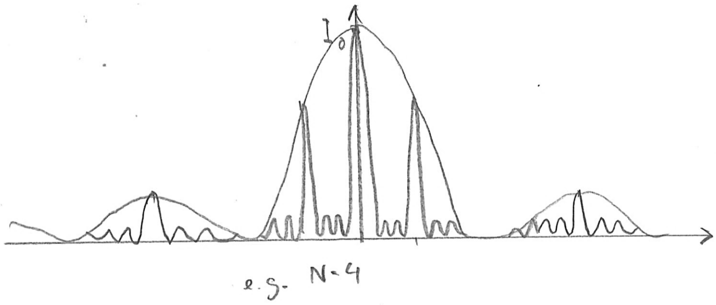
$$I_{grating} = |\Psi|^2 = I_0 \left[\frac{\sin(N\cdot\Delta\varphi/2)}{\sin(\Delta\varphi/2)}\right]^2$$

$$\Delta\varphi = k\,\Delta S = \frac{2\pi}{\lambda}\cdot d\sin\theta$$

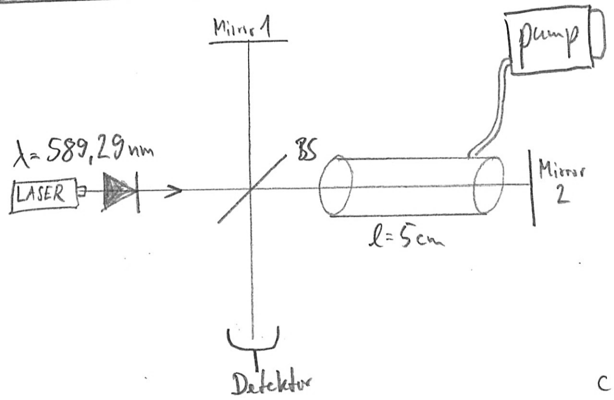
$$I = I_{grating}\cdot I_{\substack{single\\slit}} = I_0 \underbrace{\left(\frac{\sin(N\frac{\pi}{\lambda}d\sin\theta)}{\sin(\frac{\pi}{\lambda}d\sin\theta)}\right)^2}_{\text{opt. grating}} \cdot \underbrace{\left(\frac{\sin(\frac{\pi}{\lambda}a\sin\theta)}{\frac{\pi}{\lambda}a\sin\theta}\right)^2}_{\text{single slit}}$$

opt. grating ↓ N−1 Minima, N−2 side maxima

single slit ↓ envelope



Ex. 2.3: Michelson interferometer



a) pipe evacuated

$$\#\lambda = N_0 = \frac{l}{\lambda_0} = 84847{,}87\ \lambda_s$$

b) pipe vented

$$\#\lambda = N_{air} = \frac{l}{\lambda_n} = \frac{l}{\lambda_0}\cdot n_{air} = N_0\cdot n_{air}$$

c) phase of tube $\varphi = \underset{\substack{\uparrow\\\text{passed twice}}}{2}kl = 2\frac{2\pi}{\lambda_n}\cdot l$

$$\Delta\varphi_{\substack{vac\\\downarrow\\vented}} = \varphi_{air} - \varphi_0 = 2\cdot\frac{2\pi}{\lambda_0}\cdot l\,(n_{air}-1)$$

From bright ring/fring to next bright ring/fringe is 2π phase differ.

$$49{,}6\cdot\cancel{2\pi} = \cancel{2\pi}\,2\frac{l}{\lambda_0}(n_{air}-1)$$

$$n_{air} = 1 + \frac{49{,}6}{2}\frac{\lambda_0}{l} = 1{,}0002923$$