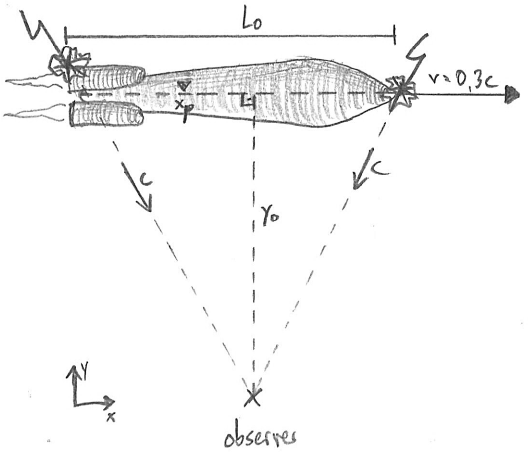
Modern Physics WS 21/22 Sören Bieling

Problem Sheet 4

Ex. 4.1: Simultanity & relativity



a) position observer relative to spaceship

i) when the flashes appear: @ t_1

simultaneous flashes → observer perpendicular to center/midpoint of spaceship $d_{c,1} = y_0$

Length L_0 is contracted in the observers frame of reference by:

$$L(v) = \frac{1}{\gamma} L_0 = \sqrt{1-\left(\frac{v}{c}\right)^2}\, L_0 = 0{,}954 \cdot L_0$$

So the time each flash travels is:

$$t_{Flash} = \frac{1}{c}\sqrt{\left(\frac{1}{2}L(v)\right)^2 + y_0^2}$$

In that time the spaceship travels a further: $\Delta l = v \cdot t_{Flash}$, so

ii) when the flashes get perceived: @ t_2 distance to center $d_{c,2} = \sqrt{(v \cdot t_{Flash})^2 + y_0^2}$

b) Lorentz transformation: $(x, y, z, t) \longleftrightarrow (x', y', z', t)$ (moving with v)

$$ct = \gamma\left(ct' + \frac{v}{c}x'\right) \qquad \text{with } \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$x = \gamma\left(x' + \frac{v}{c}ct'\right)$$

$$y = y' \quad z = z' \quad \Big\} \text{ invariance of transversal coordinates}$$

c) To recap from a) in the reference system of the resting observer two flashes appear simultaneously at $t = 0\,s$ from the positions $x_1 = 0$ (rear) and $x_2 = L(v)$ (front).

Seen from the reference frame of the spaceship the observer moves with $-v$ away.

$$\rightarrow \quad ct' = \gamma\left(ct - \frac{v}{c}x\right) \qquad t_1' = \gamma\left(t - \frac{v}{c^2}x_1\right) = 0 = t_1$$

$$t_2' = \gamma\left(t - \frac{v}{c^2}x_2\right) = -\frac{v}{c^2}\gamma L(v) = -\frac{0{,}3}{c}\cdot L_0$$

⇒ This means when the pilot is in the center of the spaceship, he/she will not see both flashes simultaneously.

but at $t_{rear} = \frac{1/2\, L_0}{c}$ and $t_{front} = \frac{1/2\, L_0}{c} - \frac{0{,}3\, L_0}{c}$

d) variable pilot position x_p between rear $x = 0$ and front $x = L_0$

$$t_{rear} = \frac{x_p}{c} \qquad t_{Front} = \frac{L_0 - x_p}{c} - \frac{0{,}3 L_0}{c} \qquad t_{rear} \stackrel{!}{=} t_{Front}$$

$$x_p = L_0 - x_p - 0{,}3 L_0$$

$$2x_p = 0{,}7 L_0 \quad \longleftrightarrow \quad x_p = 0{,}35\, L_0$$

Ex. 4.2: Thermal Radiation

$T = 273\,K + 33\,K$

a) use Wien's displacement law $\lambda_{max} = 2{,}9\,mm\cdot K \cdot \frac{1}{T} = 9{,}5\,\mu m$

$r_{SE} = 1{,}5\cdot 10^{11}\,m$

$E_0 = 1{,}361\,\frac{kW}{m^2}$

$r_s = 6{,}96\cdot 10^{8}\,m$

b) Total power radiated by the sun: $P_{total} = 4\pi r_{SE}^2 \cdot E_0$

Intensity at surface of sun is: $I_{sun} = \frac{P_{total}}{4\pi r_s^2} = \left(\frac{r_{SE}}{r_s}\right)^2 \cdot E_0$

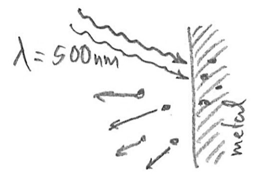
Stefan Boltzmann law for sun: $P_{total} = \sigma \cdot 4\pi r_s^2 \cdot T^4$

$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2} \approx 5{,}67\cdot 10^{-8}\,\frac{W}{m^2 K}$

$$T^4 = \frac{P_{total}}{\sigma\cdot 4\pi r_s^2} = \frac{1}{\sigma}\left(\frac{r_{SE}}{r_s}\right)^2 E_0 = \frac{1}{\sigma} I_{sun} \quad\leftrightarrow\quad T_{sun} = \sqrt[4]{\left(\frac{r_{SE}}{r_s}\right)^2 \frac{E_0}{\sigma}} = 5778\,K$$

with Wien's displacement law: $\lambda_{max} = 2{,}9\,mm\cdot K\cdot \frac{1}{T_{sun}} = 502\,nm$

Ex. 4.3: Photoelectric Effect



$c = \lambda \cdot \nu$

$v_{max} = 2{,}6\cdot 10^5\,m/s$

$m_e = 9{,}11\cdot 10^{-31}\,kg$

a) calculate W_A (work fct.)

$$\mathcal{E}_{kin,e} = h\nu - W_A \quad\Leftrightarrow\quad W_A = h\nu - \mathcal{E}_{kin,e}$$
$$= h\frac{c}{\lambda} - \frac{1}{2}mv^2$$
$$= 3{,}66\cdot 10^{-19}\,J$$
$$= 2{,}29\,eV$$

↳ so probably sodium (Na)

b) max. λ for which e^- emission is still possible

$\mathcal{E}_{kin,e} = 0 \rightarrow h\nu_{min} = W_A = \frac{hc}{\lambda_{max}} \quad\leftrightarrow\quad \lambda_{max} = \frac{hc}{W_A} = 542\,nm$

Ex. 4.4: Spontaneous & Stimulated radiation

$T = 300\,K$

$$\frac{\dot{n}_{spon}}{\dot{n}_{stim}} = e^{\left(\frac{h\nu}{k_B T}\right)} - 1$$

a) visible = (380 nm - 750 nm) mid: λ = 565 nm

$$\frac{\dot{n}_{spon}}{\dot{n}_{stim}} = e^{\left(\frac{hc}{\lambda k_B T}\right)} - 1 = 7{,}32\cdot 10^{36}$$

⇒ Radiation dominated by spont. emission!!

b) microwave λ (30 cm - 1 mm) e.g. λ = 3 mm

$$\frac{\dot{n}_{spon}}{\dot{n}_{stim}} = e^{\left(\frac{hc}{\lambda k_B T}\right)} - 1 = 1{,}6\,\%$$

⇒ Radiation is dominated by stim. emission!