Modern Physics

Winter Semester 22/23

Instructor: Prof. Bernd Pilawa (bernd.pilawa@kit.edu)

Exercise 10

§ Demo Final Exam §

Physical constants:

Speed of light: $c = 3 \times 10^8\,\mathrm{m/s}$
Planck's constant: $h = 6.626 \times 10^{-34}\,\mathrm{Js}$
Elementary charge: $e = 1.602 \times 10^{-19}\,\mathrm{C}$
Mass of the electron: $m_e = 511\,\mathrm{keV/c^2} = 9.11 \times 10^{-31}\,\mathrm{kg}$
Compton wavelength of the electron: $\lambda_{C,e} = \frac{h}{m_e c} = 2.426\,\mathrm{pm}$
Boltzmann constant: $k_B = 1.381 \times 10^{-23}\,\mathrm{J/K}$
Stefan-Boltzmann constant: $\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2} = 5.67 \times 10^{-8}\,\mathrm{W/m^2K^4}$
Atomic mass unit: $u = 1.66 \times 10^{-27}\,\mathrm{kg}$
Rydberg unit of energy: $R = 13.6\,\mathrm{eV}$

Problem 1:

a) What is the meaning of the acronym LASER?

b) What is the difference between normal light and the light of a laser?

c) Sketch the setup of a HeNe-laser and explain how the light of the laser is generated.

d) The light of the HeNe-laser is polarised. Why?

Problem 2:

An antenna on board of a satellite emits one short pulse per second. The satellite is moving relative to a receiver with the velocity of 90% of the speed of light.

a) Calculate the frequency of pulses arriving at the receiver.

b) A second antenna is placed 2 m behind the first antenna in the direction of the motion of the satellite. Calculate the time interval between two pulses recorded by the receiver, when the pulses are emitted simultaneously by the two antennae on board of the satellite.

c) The mass of the satellite at rest is 1 kg. Calculate the energy which is at least necessary to accelerate the satellite from zero to 90% of the speed of light.

d) Calculate the time which is necessary for a power station with an output power of 1 GW to provide this amount of energy.

Problem 3:

a) Albert Einstein explained Planck's law by three fundamental processes. Explain these processes.

b) Make a sketch of Planck's law, i.e. plot the spectral radiance as a function of the wavelength λ.

c) Write up and explain the Stefan-Boltzmann's law.

d) What is a black body?

e) A sphere with a radius $r = 10\,\text{cm}$ approximating a black body is exposed to the sun ($150\,\text{W}\,\text{m}^{-2}$) and absorbs the whole arriving power of the sun. Calculate the temperature of the sphere in thermal equilibrium when the temperature of the surrounding is $T = 300\,\text{K}$.

Problem 4:

a) Write up the Schrödinger equation for an electron moving within a region with constant potential energy V. The total energy of the electron is $E > V$.

b) Calculate the wave function and the momentum of an electron moving along the x-axis.

c) The electron moves within a region with $V = V_1 < E$ in the direction of increasing x-values and hits a barrier where the potential energy changes from V_1 to $V_2 < E$. How large is the probability that the electron is reflected?
Hint: The wave function at the interface of the barrier is both continuous and continuously differentiable.

Problem 5:

Spherical coordinates (r, ϑ, φ) are used to solve the Schrödinger equation of a single electron in the radially symmetric electric field of a nucleus.

a) Sketch and write down the equation for the effective potential energy $\Phi_{\mathit{eff}}(r)$ of the electron.

b) Which quantum numbers determine the radial $R(r)$ and angular part $\mathcal{Y}(\vartheta, \varphi)$ of the wave function? Write down their range of values they can take as well as the eigenvalue equation of their corresponding operators.

c) Which additional quantum numbers are still necessary to fully characterize the quantum state of the electron? Write down their range as well as the eigenvalue equation of their corresponding operators.

d) Sketch the radial wave functions of the first excited state of the electron.

Problem 6:

The orbital angular momentum $\vec{l}$ of the electron and its spin $\vec{s}$ couple to the total angular momentum $\vec{j}$ according to $\xi \vec{l} \cdot \vec{s}$.

a) Explain the origin for the coupling of $\vec{l}$ and $\vec{s}$.

b) Give the sign of the coupling constant ξ.

c) Calculate the energy difference between the energy of the total angular momentum state with the highest and smallest j-value, respectively.

d) Calculate the potential energy of an electron in an homogeneous magnetic field $\vec{B}$ in terms of the quantum numbers of $\vec{j}$.