MODERN PHYSICS

Winter Semester 22/23

Instructor: Prof. Bernd Pilawa (bernd.pilawa@kit.edu)

Exercise 11

§ Atoms & Solids §

Problem 1: Internal magnetic fields

From the point of view of an electron with orbital angular momentum $\vec{L}$ the positively charged nucleus revolves around the electron. The motion of the nucleus produces an internal magnetic field parallel to the electron's orbital angular momentum. The intrinsic spin magnetic moment $\vec{\mu}_s$ of the electron interacts with this internal magnetic field. The potential energy E_B of a magnetic moment $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by the following equation:

$$E_B = -\vec{\mu} \cdot \vec{B}. \tag{1.1}$$

Since $\vec{\mu}_s$ is proportional to the spin of the electron $\vec{S}$ and since $\vec{B}$ and $\vec{L}$ are proportional for a given orbit, we can deduce that the potential energy associated with spin-orbit coupling is of the form:

$$E_S \propto \vec{S} \cdot \vec{L} \tag{1.2}$$

a) Estimate the strength of the internal magnetic field due to the electron's orbital motion which results in the two sodium D-lines with $\lambda = 588.995\,\mathrm{nm}$ and $\lambda = 589.592\,\mathrm{nm}$.

b) Estimate the strength of the internal magnetic field due to the electron's orbital motion which results in the $\lambda = 766.41\,\mathrm{nm}$- and $\lambda = 769.90\,\mathrm{nm}$-lines observed in the $L = 1$ to $L = 0$ transition in potassium.

Problem 2: The Landé factor

The Landé factor g_J is the proportionality constant that relates the observed total magnetic moment μ of an electron, resulting from both spin and orbital angular momentum, to its total angular momentum $\vec{J}$:

$$\vec{\mu} = -g_J \mu_B \frac{\vec{J}}{\hbar}. \tag{2.1}$$

With the g-factor for the electron is taken to be approximately ≈ 2, we can express the Landé factor via the following equation:

$$g_J = \frac{3}{2} + \frac{s(s+1) - l(l+1)}{2j(i+1)} \tag{2.2}$$

a) Show in a vector diagram that $\vec{\mu}$ and $\vec{J}$ are not parallel.

b) Derive the Landé factor given in eq:g-factor by calculating the projection of $\vec{\mu}$ on the vector $\vec{J}$.

Hint: The required projection term is given by the following equation:

$$\frac{\vec{\mu} \cdot \vec{J}}{|\vec{J}|} = -\mu_B \frac{\vec{J} \cdot \vec{J} + \vec{J} \cdot \vec{S}}{\hbar \cdot |\vec{J}|} = -\mu_B \sqrt{j(j+1)} g_J. \tag{2.3}$$

Problem 3: Hexagonal lattice

The primitive translation vectors of the hexagonal Bravais lattice are given as:

$$\vec{a}_1 = \frac{\sqrt{3}a}{2}\vec{x} + \frac{a}{2}\vec{y}, \quad \vec{a}_2 = -\frac{\sqrt{3}a}{2}\vec{x} + \frac{a}{2}\vec{y}, \quad \vec{a}_3 = c\vec{z}. \tag{3.1}$$

a) Show that the volume of the primitive cell spanned by the above vectors is $\frac{\sqrt{3}a^2c}{2}$.

b) Show that the primitive translation vectors of the reciprocal lattice are the following:

$$\vec{b}_1 = \frac{2\pi}{\sqrt{3}a}\vec{x} + \frac{2\pi}{a}\vec{y}, \quad \vec{b}_2 = -\frac{2\pi}{\sqrt{3}a}\vec{x} + \frac{2\pi}{a}\vec{y}, \quad \vec{b}_3 = \frac{2\pi}{c}\vec{z}. \tag{3.2}$$

c) Describe and sketch the first Brillouin zone of the hexagonal lattice.

Problem 4: Laue & Bragg Condition

The crystal structure of a lattice can be determined by scattering of particles (photons, neutrons, electrons, etc.)

a) Write up the condition for constructive interference according to Bragg.

b) Write up the condition for constructive interference according to Laue.

c) Give the relation between the basis vectors of the Bravais lattice and the reciprocal lattice.

d) What is the geometrical meaning of a vector of the reciprocal lattice?