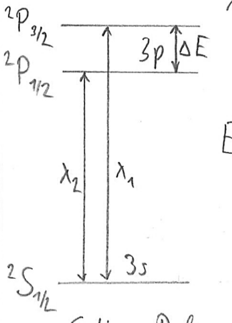
Modern Physics WS 21/22 Sören Bieling

Problem Sheet 11

Ex. 11.1: Internal magnetic fields

a) $\lambda_1 = 588{,}995\ \text{nm}$

$\lambda_2 = 589{,}592\ \text{nm}$



Sodium D lines

$$E = h\cdot\nu = \frac{hc}{\lambda} \rightarrow \Delta E_\lambda = -\frac{hc}{\lambda^2}\Delta\lambda$$

$$= 2{,}13\ \text{meV}$$

$\Delta\lambda = \lambda_1 - \lambda_2 = -0{,}597\ \text{nm}$

$\lambda = \frac{\lambda_1+\lambda_2}{2} = 589{,}294\ \text{nm}$

$$E_B = -\vec{\mu}\cdot\vec{B} = 2\cdot\mu_B\cdot(\pm 1/2)\cdot|\vec{B}| \quad \text{with} \quad \vec{\mu}_s = -g_s\mu_B\frac{\vec{S}}{\hbar}$$

$g_s \approx 2$

$\vec{S} \rightarrow S_z = \pm 1/2\ \hbar$

($\vec{B}$-field dictates quantization axis)

$$\Delta E_B = 2\cdot E_B = 2\mu_B\cdot B \stackrel{!}{=} \Delta E_\lambda$$

$$B = \frac{\Delta E_\lambda}{2\mu_B} = \underline{\underline{18{,}4\ \text{T}}}$$

$\mu_B = \frac{e}{2m_e}\hbar = 9{,}274\cdot 10^{-24}\ \frac{J}{T}$

b) instead of Na (Z=11) (Sodium) $\rightarrow$ K (Z=19) (Potassium): repeat a) but with

$\lambda_1 = 766{,}41\ \text{nm}$

$\lambda_2 = 769{,}90\ \text{nm}$

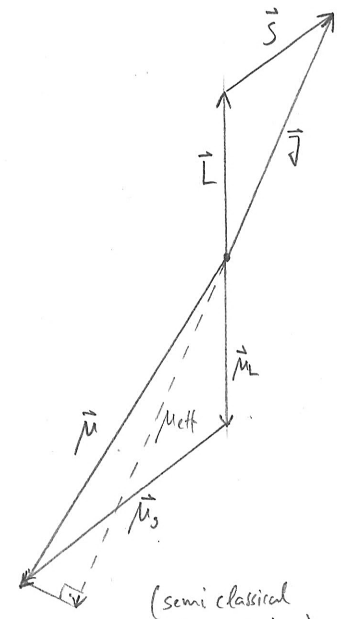
$\Delta\lambda = \lambda_1 - \lambda_2 = -3{,}49\ \text{nm}$

$\bar{\lambda} = \frac{\lambda_1+\lambda_2}{2} = 768{,}155\ \text{nm}$

$$\Delta E_\lambda = -\frac{hc}{\bar{\lambda}^2}\Delta\lambda = 7{,}33\ \text{meV} \qquad B = \frac{\Delta E_\lambda}{2\mu_B} = \underline{\underline{63{,}3\ \text{T}}}$$

Ex. 11.2: The Landé factor

a)



(semi classical vector projection)

$$\vec{\mu} = -g_J\mu_B\frac{\vec{J}}{\hbar} \quad \text{with} \quad g_J = \frac{3}{2} + \frac{s(s+1)-l(l+1)}{2j(j+1)}$$

$s=0 \rightarrow g_L = 1$

$l=0 \rightarrow g_s = 2$

$$\vec{\mu} = -\mu_B\frac{(\vec{L}+2\vec{S})}{\hbar} \qquad \underbrace{\vec{L}+\vec{S}}_{\vec{J}} + \vec{S}$$

b)
$$\mu_{eff} = \frac{\vec{\mu}\cdot\vec{J}}{|\vec{J}|} = -\mu_B\frac{(\vec{L}+2\vec{S})}{\hbar}\cdot\frac{\vec{J}}{|\vec{J}|} = -\mu_B\frac{\vec{J}\cdot\vec{J}+\vec{S}\cdot\vec{J}}{\hbar|\vec{J}|} = -\mu_B\sqrt{j(j+1)}\,g_J$$

$$\stackrel{*}{=} \cancel{-}\cancel{\mu_B}\frac{|\vec{J}|^2+\frac{1}{2}(|\vec{J}|^2+|\vec{S}|^2-|\vec{L}|^2)}{\cancel{\hbar}|\vec{J}|} = \cancel{-}\cancel{\mu_B}\frac{|\vec{J}|}{\cancel{\hbar}}g_J$$

$$g_J = +\frac{3}{2}\frac{\cancel{|\vec{J}|^2}}{\cancel{|\vec{J}|^2}} + \frac{|\vec{S}|^2-|\vec{L}|^2}{2|\vec{J}|^2}$$

$$= \frac{3}{2} + \underline{\underline{\frac{s(s+1)-l(l+1)}{2j(j+1)}}}$$

$*\quad \vec{L} = \vec{J}-\vec{S} \quad |()^2$

$\vec{L}\cdot\vec{L} = \vec{J}\cdot\vec{J} - 2\cdot\vec{J}\cdot\vec{S} + \vec{S}\cdot\vec{S}$

$\vec{J}\cdot\vec{S} = \frac{1}{2}(|\vec{J}|^2+|\vec{S}|^2-|\vec{L}|^2)$

$|\vec{J}|^2 = j(j+1)\hbar^2$

$|\vec{L}|^2 = l(l+1)\hbar^2$

$|\vec{S}|^2 = s(s+1)\hbar^2$

Ex. 11.3: Hexagonal lattice

$$\vec{a}_1 = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} a \qquad \vec{a}_2 = \begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} a \qquad \vec{a}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} c$$

a) Volume of unit cell given by: $V_{cell} = \vec{a}_1 (\vec{a}_2 \times \vec{a}_3)$ "Spatprodukt"

$$V_{cell} = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} \cdot \left[\begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right] a^2 c$$

$$= \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1/2 - 0 \\ 0 - (-\sqrt{3}/2) \\ 0 \end{pmatrix} a^2 c = \left(\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \right) a^2 c = \underline{\underline{\frac{\sqrt{3}}{2} a^2 c}}$$

b) The primitive translation vectors of reciprocal space are given by all cyclic permutations of:

$$\vec{b}_1 = \frac{2\pi}{V_{cell}} (\vec{a}_2 \times \vec{a}_3) = \frac{2\pi \cancel{ac}}{\frac{\sqrt{3}}{2} a^{\cancel{2}} \cancel{c}} \left[\begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right] = \frac{2\pi}{\sqrt{3}/2\, a} \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \\ 0 \end{pmatrix} = \underline{\underline{\frac{2\pi}{a} \begin{pmatrix} 1/\sqrt{3} \\ 1 \\ 0 \end{pmatrix}}}$$

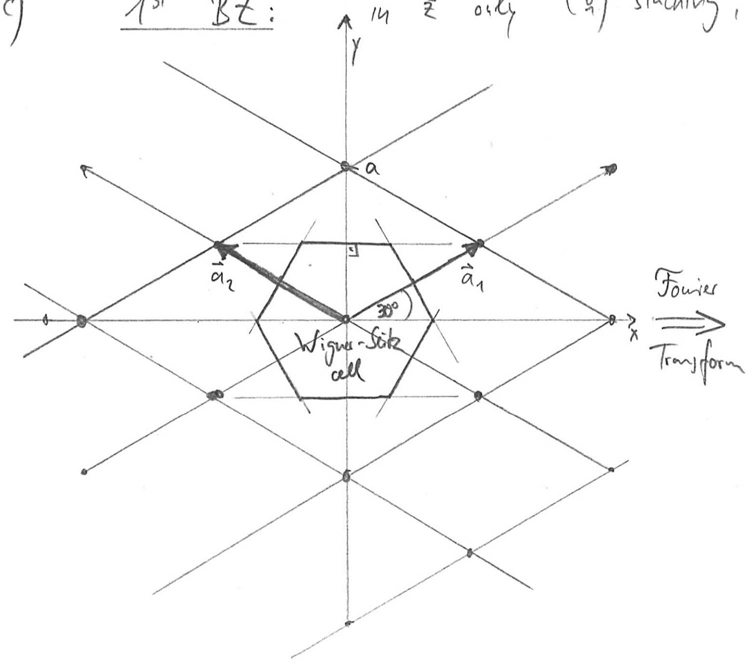
$$\vec{b}_2 = \frac{2\pi}{V_{cell}} (\vec{a}_3 \times \vec{a}_1) = \frac{2\pi \cancel{ac}}{\frac{\sqrt{3}}{2} a^{\cancel{2}} \cancel{c}} \left[\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} \right] = \frac{2\pi}{\sqrt{3}/2\, a} \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \\ 0 \end{pmatrix} = \underline{\underline{\frac{2\pi}{a} \begin{pmatrix} -1/\sqrt{3} \\ 1 \\ 0 \end{pmatrix}}}$$

$$\vec{b}_3 = \frac{2\pi}{V_{cell}} (\vec{a}_1 \times \vec{a}_2) = \frac{2\pi \cancel{a^2}}{\frac{\sqrt{3}}{2} \cancel{a^2} c} \left[\begin{pmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} \times \begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} \right] = \frac{2\pi}{\frac{\sqrt{3}}{2} c} \begin{pmatrix} 0 \\ 0 \\ \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \end{pmatrix} = \underline{\underline{\frac{2\pi}{c} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}}$$

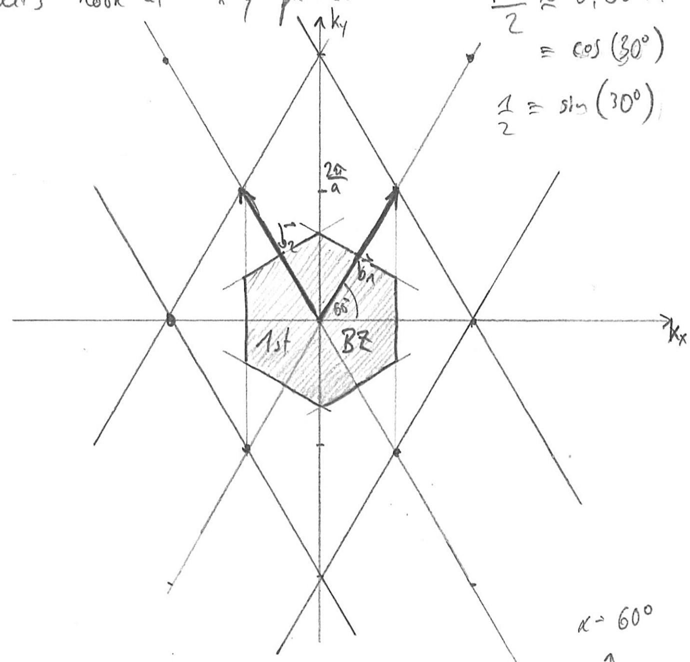
c) 1st BZ: in $\vec{z}$ only $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ stacking, so let's look at x-y plane:

$\frac{\sqrt{3}}{2} \approx 0{,}866\ldots = \cos(30°)$

$\frac{1}{2} = \sin(30°)$



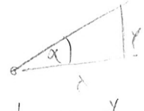
Fourier $\Longrightarrow$ Transform



$\alpha = 60°$

$\uparrow$

$\begin{pmatrix} 1/\sqrt{3} \\ 1 \\ 0 \end{pmatrix}$



$\tan\alpha = \frac{y}{x}$