MODERN PHYSICS

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Exercise 14

§ Solids §

Problem 1: Free electron model (Drude-Sommerfeld model)

In the free electron model for metallic solids it is assumed that the weakly bound valence electrons of the metal atoms are no longer bound to particular atoms, but can move freely throughout the whole solid. Therefore, the valence electrons can be approximated as free electrons in a 3D potential well of infinite depth and sides of length L. The energy E of the electron states is thus given using the positive integer quantum numbers n_1, n_2 and n_3:

$$E_{n_1,n_2,n_3} = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 (n_1 + n_2 + n_3) = \frac{\hbar^2}{2m}\vec{k}^2_{n_1,n_2,n_3}. \tag{1.1}$$

a) Since L is very large for a macroscopic solid, a continuum of energy states can be assumed. Show that the number of energy states N can be expressed using the volume of the Fermi sphere with maximum wave number k_F as radius.

b) Show that the Fermi energy at $T = 0\,\mathrm{K}$ is given by the following formula:

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{\frac{2}{3}}. \tag{1.2}$$

c) The metal potassium has a density of $\rho = 0.86\,\mathrm{g/cm^3}$ and an atomic mass of $m = 39$. Calculate the Fermi energy using the assumption that every potassium atom contributes a single electron to the free electron gas.

d) Show that the density of states for the free electron gas is given by the following formula:

$$D(E) = \frac{1}{V}\frac{dN}{dE} = \frac{\sqrt{2m^3}}{\pi^2\hbar^3}\sqrt{E}. \tag{1.3}$$

Problem 2: Heat capacity of the electron gas

The specific heat of the conduction electrons can be derived analogously to the phonon specific heat (c.f. problem 13.2) by considering the density of states of electrons in three dimensions and the Fermi-Dirac distribution function and yields to be linearly proportional to the solid's temperature:

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{\pi^2}{2}\frac{Nk_B^2}{E_F}T, \tag{2.1}$$

with the Fermi energy given in eq. (1.2).

a) In the classical model for an ideal gas of N particles the specific heat is $C_V = \frac{3}{2}Nk_B$. Estimate the proportion of electrons that contribute to the specific heat at $T = 300\,\mathrm{K}$.

b) At low temperatures the total specific heat of a metal can be expressed by

$$C_V = C_V^{\text{electrons}} + C_V^{\text{phonons}} = \gamma T + \alpha T^3 \tag{2.2}$$

Find an expression for the temperature T^* at which the contribution of phonons and electrons is equal. Which contribution is larger below T^*?