# MODERN PHYSICS

Winter Semester 23/24

Instructor: Prof. Bernd Pilawa (bernd.pilawa@kit.edu)

Exercise 3

## § Essentials of Thermodynamics & Special Relativity §

### Problem 1: Ideal gas and Maxwell velocity distribution

One liter of ideal Helium gas at a temperature $T_1 = 293\,\mathrm{K}$ and a pressure $p_1 = 1\,\mathrm{Bar}$ is heated to a temperature $T_2 = 523\,\mathrm{K}$, during the heating process the volume stays constant.

**(1)** Use the Maxwell-Boltzmann velocity distribution:

$$f(v) = \frac{4}{\sqrt{\pi}}\,(\frac{m}{2k_BT})^{\frac{3}{2}}\,v^2\,\exp(-\frac{mv^2}{2k_B\,T})$$

to derive an expression for the average of the squared velocities $\langle v^2\rangle$ and thus the average kinetic energy $\frac{1}{2}m\langle v^2\rangle$ of a Helium atom. Evaluate it at both temperatures.

$$Hint: \int_0^\infty x^4\,\exp\left(-a\,x^2\right)dx = \frac{3\sqrt{\pi}}{8a^{\frac{5}{2}}}$$

**(2)** Calculate the *rms* (root mean square) velocities ($\sqrt{\langle v^2\rangle}$) of the helium atoms at both temperatures.

**(3)** What is the final pressure of the helium gas?

### Problem 2: Special relativity

Muons are generated by cosmic rays in the upper atmosphere of the earth. In their own reference system (muons are at rest), they have a mean lifetime of $\tau = 2.2\,\mu s$ (proper time). Show that the muons which have been generated at a height of $h = 10\,\mathrm{km}$ and move at speed close to the speed of light with $v = 0.998\,c$ can reach the surface of the earth. Argue from the following two different perspectives:

**(1)** Point of view of the observer on earth (time dilation of the lifetime).

**(2)** Point of view of the muon (length contraction of the flight distance).

### Problem 3: Classic and relativistic Doppler effect

As an example for the classic or non-relativistic Doppler effect, we look at an ambulance driving by with 100 km/h and its siren on. In Germany this siren signal consists of an alternating perfect forth. Let's assume its base note to be a' (440 Hz). (The speed of sound in air is $c_s = 340\,\mathrm{m/s}$.)

**(1)** Calculate the siren's wavelength and frequency behind and in front of the ambulance.

**(2)** Now let's switch our ambulance for a jet airplane moving with velocity $v$ and emitting its engine noise. (Consider only one single frequency.) Make a sketch of the engine noise's wave fronts for the four cases of $v = 0, 0 < v < c_s$, $v = c_s$ and $v > c_s$.

**(3)** An analog effect of the first three cases is also observed for light. Explain the relativistic time dilation effect and derive the relativistic Doppler frequency shift by considering that the number of the observed wave crests must be the same in the moving and in the resting reference system.

**(4)** You are approaching a red traffic light ($\lambda = 633\,\mathrm{nm}$). How fast do you have to drive your car that the red light appears green ($\lambda = 540\,\mathrm{nm}$)?