Modern Physics

Winter Semester 23/24

Instructor: Prof. Bernd Pilawa (bernd.pilawa@kit.edu)

Exercise 5

§ Wave-Particle Dualism §

Problem 1: Ruby laser

The ruby laser is a solid state laser consisting of an Al_2O_3 rod doped with about 0.05% of Cr. On one end the rod has a highly reflecting mirror surface and a semi-transparent one on the other. A pulsed xenon flash bulb is used to excite the chromium atoms from a ground state $|1\rangle$ to an excited state $|3\rangle$ which then quickly decays into a metastable state $|2\rangle^*$. Neglecting the excited state lifetime, the three level system is characterized by the following rate equations:

$$\begin{aligned} \frac{dN_1}{dt} &= -W_p N_1 n_\gamma + \frac{N_2}{T_{21}} \\ \frac{dN_2}{dt} &= +W_p N_1 n_\gamma - \frac{N_2}{T_{21}} \\ N = N_1 + N_2 + N_3 &\approx N_1 + N_2, \end{aligned} \tag{1.1}$$

where W_p is the stimulated transition propability resulting from the pumping process, n_γ is the number of pump photons, N_i are the state populations and T_{ij} the corresponding state decay times.

a) Show that the population difference $\Delta N = N_1 - N_2$ in the steady state is given by the following equation:

$$\Delta N = N \cdot \frac{\frac{1}{T_{21}} - W_p n_\gamma}{\frac{1}{T_{21}} + W_p n_\gamma}. \tag{1.2}$$

b) Under what conditions can a population inversion $(N_1 - N_2 < 0)$ be achieved?

c) Why is the three-level system inefficient?

Problem 2: Compton effect

The Compton effect refers to the decrease in energy of a photon which is scattered inelastically by a charged particle. A.H. Compton used photons with a wavelength of $\lambda = 0.0711\,\text{nm}$ for his experiments (Molybdenum K_α-line).

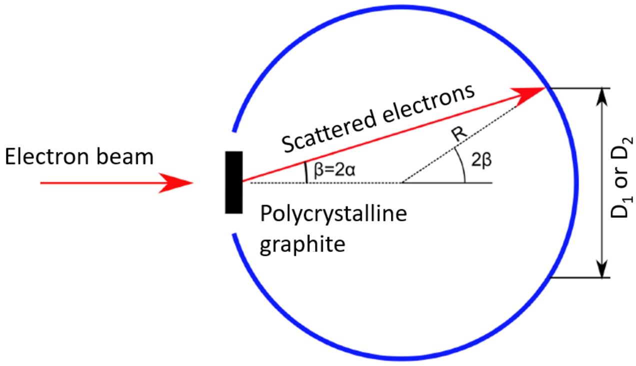
a) How large is the energy of these photons?

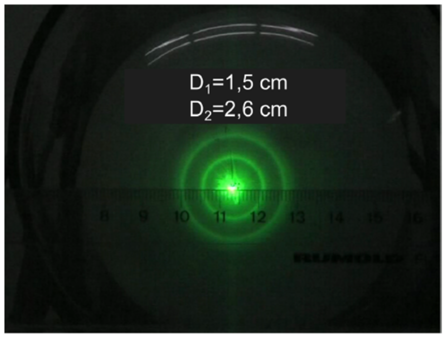
b) Calculate the wavelength of the photons which are scattered at free electrons under an angle of $\theta = 180°$ ($\Rightarrow$ in the direction of the source).

c) How large is the energy of one of these reflected photons?

Problem 3: Electron diffraction I: De Broglie wavelength

In an evacuated high vacuum chamber electrons are emitted from a hot cathode and accelerated through an electric potential difference. The resulting electron beam propagates through a polycrystalline graphite thin film. Behind the graphite layer, a hemispherical fluorescent screen with a radius of $R = 6.5\,\text{cm}$ (fig. left) is excited via the incident electrons and luminous effects in the form of fluorescent fringes can be observed.





a) Explain why the following pattern can be seen (fig. right).

b) Calculate the *de Broglie* wavelength of the electrons which are accelerated by a voltage of $U = 10\,\text{kV}$.

c) Determine the angles at which the electrons are scattered into the rings of the first two diffraction orders. The diameters of the first two diffraction rings are $D_1 = 1.5\,\text{cm}$ and $D_2 = 2.6\,\text{cm}$ (fig. right).

d) Calculate the distances of the lattice planes within the crystallite of the graphite layer which correspond to the diffraction pattern/rings.

Problem 4: Electron diffraction II: Double slit

The setup from problem 3 is now modified. The emitted electrons are accelerated through an electric potential difference of $U = 100\,\text{V}$ and transmitted through a sample with two vertical slits. The slits have a distance of $d = 1\,\mu\text{m}$ between each other and a width of $a_1 = 0.5\,\mu\text{m}$.

a) Calculate the *de Broglie* wavelength of the electrons.

b) Sketch the expected intensity pattern which is observed at a flat fluorescent screen at a distance of $l = 1\,\text{m}$ behind the double slit. What is the distance between the diffraction orders assuming incident plane waves?

c) What changes when the potential difference is halved? What is the result of using a sample with the same slit separation, but with a slit width of $a_2 = 0.25\,\mu\text{m}$?