Modern Physics WS 21/22

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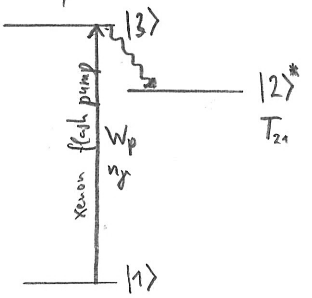
Problem Sheet 5

Ex. 5.1: Ruby Laser



a) steady state $\frac{dN_1}{dt} \overset{!}{=} 0 \overset{!}{=} \frac{dN_2}{dt}$ into eq. 1:

$$0 = -W_p n_\gamma N_1 + N_2/T_{21}$$

$$N_1 = N_2 \cdot \frac{1}{W_p n_\gamma T_{21}}$$

$$\Delta N = N_1 - N_2 = N_2\left(\frac{1}{W_p n_\gamma T_{21}} - 1\right) = N \frac{\frac{1}{W_p n_\gamma T_{21}} - 1}{\frac{1}{W_p n_\gamma T_{21}} + 1}$$

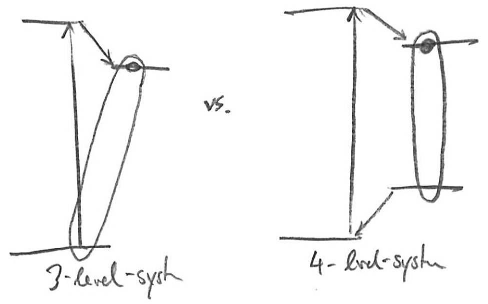
$$N \approx N_1 + N_2 = N_2\left(\frac{1}{W_p n_\gamma T_{21}} + 1\right) \Leftrightarrow N_2 \approx \frac{N}{\frac{1}{W_p n_\gamma T_{21}} + 1}$$

$$\Delta N = N \frac{\frac{1}{W_p n_\gamma T_{21}} - 1}{\frac{1}{W_p n_\gamma T_{21}} + 1} = \boxed{N \frac{\frac{1}{T_{21}} - W_p n_\gamma}{\frac{1}{T_{21}} + W_p n_\gamma}}$$

b) Under what conditions can a population inversion $\Delta N < 0$ be achieved?

$$\Delta N < 0 \leftrightarrow \underbrace{W_p n_\gamma}_{\text{pumping rate}} > \underbrace{\frac{1}{T_{21}}}_{\text{spont. emission rate}}$$

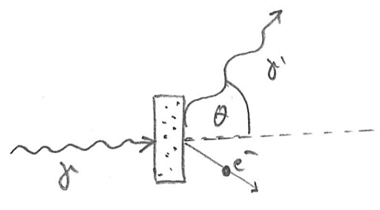
c) Why is the 3-level system inefficient?



For population inversion a large number of atoms $N_2 > \frac{N}{2}$ has to be in the excited state requiring a very high pumping power.
In the 4-level-system already a small number of atoms is sufficient to create population inversion requiring much smaller pumping power.

Ex. 5.2: Compton Effect

$\lambda = 0{,}0711$ nm



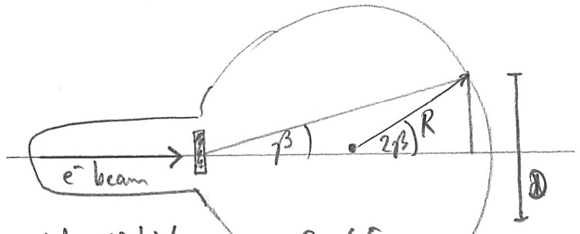
a) $E_\gamma = h\nu = \frac{hc}{\lambda} = 2{,}79 \cdot 10^{-15}\,\mathrm{J} = \underline{17{,}4\ \mathrm{keV}}$

b) $\lambda' - \lambda = \lambda_c (1 - \cos\theta)$ with $\lambda_c = \frac{h}{m_0 c} = 2{,}43$ pm

$\lambda' = \lambda + 2\lambda_c = \underline{75{,}96\ \mathrm{pm}}$ $\cos(180°) = -1$

c) $E_{\gamma'} = \frac{hc}{\lambda'} = 2{,}62 \cdot 10^{-15}\,\mathrm{J} = \underline{\underline{16{,}3\ \mathrm{keV}}}$

Ex. 5.3: Electron diffraction I: De Broglie wavelength

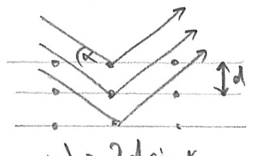


$U = 10\,kV$ $R = 6{,}5\,cm$

a) Explanation of fluorescent interference rings:

electron beam → matter wave
graphite → crystal lattice } Bragg diffraction

$\Rightarrow$ interference $\left\{ \begin{matrix} \text{constructive} \\ + \\ \text{destructive} \end{matrix} \right\}$



$n\lambda = 2d \sin\alpha_n$

graphite = polycrystalline = all crystal orientations present
↓
interference rings

b)

$$E_{kin} = \frac{1}{2} m_e v^2 = \frac{p^2}{2 m_e} \Leftrightarrow p = \sqrt{2 m_e e U}$$

↑

$E_{kin} = e \cdot U$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2 m_e e U}} = \underline{12{,}3\,pm}$$

c) angles of 1. & 2. diffraction order? ($D_1 = 1{,}5\,cm$ $D_2 = 2{,}6\,cm$)

$$\sin(2\beta) = \frac{D/2}{R}$$

$$\beta_1 = \frac{1}{2} \arcsin\left(\frac{D_1}{2R}\right) = 3{,}31° \quad (0{,}058\,rad)$$

$$\beta_2 = \frac{1}{2} \arcsin\left(\frac{D_2}{2R}\right) = 5{,}77° \quad (0{,}101\,rad)$$

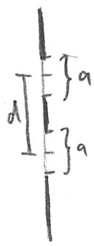
d) Lattice plane distance via Bragg formula: $n \cdot \lambda = 2d \sin\alpha_n$ with $2\alpha = \frac{1}{2}\beta$

$$d = \frac{n \cdot \lambda}{2 \sin\alpha_n} = \frac{n\lambda}{2 \sin(\frac{1}{2}\beta_n)}$$

$n=1$, β_1: $d_1 = \underline{\underline{2{,}13\,Å}}$

$n=1$ β_2: $d_2 = \underline{\underline{1{,}22\,Å}}$

Ex. 5.4: Electron diffraction II: Double slit



exp. setup from Ex. 5.3 but with $U = 100\,V$ & graphite → double slit $\left\{ \begin{matrix} d = 1\,\mu m \\ a = 0{,}5\,\mu m \end{matrix} \right\}$

$d = 2a$

a) analog to Ex. 5.3 b): $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2 m_e e U}} = 123\,pm$

b) recall Ex. 2.2 c)

$$I = I_0 \underbrace{\cos^2\left(\frac{\pi d}{\lambda} \sin\theta\right)}_{\text{double slit}} \cdot \underbrace{\left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta}\right]^2}_{\text{single slit}}$$

Minima (SS) @ $x_m = l \frac{m\lambda}{a} = m \cdot 246\,\mu m$

Maxima (DS) @ $x_n = l \cdot \frac{n\lambda}{d} = l \cdot \frac{n\lambda}{2a} = n \cdot \underline{123\,\mu m}$

↳ minima of single slit:

$m \cdot \lambda = a \sin(\theta_m)$ with $m = \pm 1, \pm 2, \ldots$

↳ maxima of double slit:

$n \cdot \lambda = d \sin(\theta_n)$ with $n = 0, \pm 1, \pm 2$

Detector screen at $l = 1\,m$ away:

$$\theta_i \approx \tan\theta_i = \frac{x}{l}$$

$x_1 = 123\,\mu m$ ↑

distance between diffraction orders

c) ① $U \to \frac{1}{2} U$: $\lambda \to \sqrt{2}\lambda$: $x_n \to \sqrt{2} x_n$

② $a \to \frac{1}{2} a$ ($d = 4a$): $x_m \to 2 x_m$

maxima with $\pm 4, \pm 8, \ldots \cdot x_1$ are suppressed