Modern Physics

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Exercise 6

§ Wave-Particle Dualism & Atoms §

Problem 1: Heisenberg's uncertainty principle

Due to the wave nature of matter (e.g. electrons, atoms, molecules ...), it is impossible to know both its location and momentum along one axis with arbitrary high precision. The theoretical limit for the product of the uncertainty of position Δx and the uncertainty of momentum Δp is given by Heisenberg's uncertainty principle:

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}. \tag{1.1}$$

a) Use the relation to estimate the minimal uncertainty in velocity of an electron in a hydrogen atom. Assume that the radius of the hydrogen atom is 0.1 nm. Do you expect relativistic effects in the motion of the electron?

b) Show that the diffraction of a particle from a single slit complies with the uncertainty principle. Derive the uncertainty in momentum from the width of the main diffraction peak.

c) Calculate the minimal uncertainty in velocity of a grain of sand (mass $m = 1\,\text{mg}$) that is observed in a light microscope. Assume that the uncertainty of position is given by the wavelength of visible light. How long would you have to wait in order to observe a motion along a distance of 1 µm?

Problem 2: Franck-Hertz experiment

In an evacuated glass tube electrons are accelerated between a hot cathode and an anode by applying a variable voltage U_0. Just before the anode, a retarding voltage of about 0.5 V is applied to a mesh in order to repel the electrons of lower energy. Inside the glass tube, there is a small amount of neon vapor. With increasing acceleration voltage an increasing number of light emitting stripes/fringes between cathode and mesh can be observed. The anode current is characterized by a number of maxima and minima.

a) Explain the variation of the anode current as a function of the acceleration voltage.

b) The maxima are separated by a voltage of about $U_0 = 19\,\text{V}$. Calculate the energy difference between the excited energy levels of the neon atom.

c) The red-colored light emitting stripes correspond to a wavelength of about $\lambda = 632.8\,\text{nm}$. What is the corresponding photon energy?

Problem 3: Bohr's atomic model

Using Bohr's quantum conditions, calculate the first five energy levels of the He^+-ion. The attractive force between the He-nucleus and the single electron of the ion is given by Coulomb's law with Z = 2:

$$F = \frac{Ze^2}{4\pi\epsilon_0 r^2}. \tag{3.1}$$

Problem 4: Schrödinger's equation: Potential step

One beam of particles with kinetic energy E_p comes from the left side and hits a potential step (dotted, see figure below) with a height of $V_0 < E_p$.

a) Write down the Schrödinger equation for the particle beam for the left side and the right side of the potential step.

b) With the assumption, that the wave function as well as its derivative have to be steady, derive the reflection coefficient r (reflectivity R).

c) For what value of $\frac{E_p}{V_0}$ the reflectivity R equals to 0.5.?