MODERN PHYSICS

Winter Semester 23/24

Instructor: Prof. Bernd Pilawa (bernd.pilawa@kit.edu)

Exercise 9

§ Atoms §

Problem 1: Normal Zeeman effect of the red Cadmium line

The red Cadmium line of wavelength $\lambda = 643.8\,\mathrm{nm}$ results from the transition $5^1\mathrm{P} \leftrightarrow 5^1\mathrm{D}$.

a) Calculate the energy difference between the 5^1D- and the 5^1P-levels.

b) Calculate $\Delta\lambda$ when a magnetic field is applied.

c) Explain the splitting of the red Cadmium line.

d) The line width of the red Cadmium line is 1.2 pm. How large the smallest magnetic field strength has to be so that a line splitting can be observed?

Problem 2: Stern-Gerlach experiment with Hydrogen atoms

In a Stern-Gerlach experiment hydrogen atoms are prepared in their ground state and propagate in the x-direction with a speed of $v_x = 14.5\,\mathrm{km\,s^{-1}}$. The beam passes a region with a magnetic field gradient in the z-direction $\frac{dB}{dz} = 600\,\mathrm{T\,m^{-1}}$. The force F_z acting on the hydrogen atoms in the magnetic field gradient is given by

$$F_z = \mu_z \cdot \frac{dB_z}{dz}, \tag{2.1}$$

where μ_z is the z-component of the magnetic moment of the atoms.

a) Determine the maximum acceleration of the hydrogen atoms.

b) What is the maximum distance between the two lines observed in the detection plane? Assume that the magnetic field is confined to a region of a width $\Delta x \approx 75\,\mathrm{cm}$ in the direction of the beam. Beyond this region the atoms travel a distance of 1.25 m to the detection plane.

c) What is the maximum distance if silver atoms at a speed of $v_x = 250\,\mathrm{m\,s^{-1}}$ are used in the same beam experiment?

Problem 3: The eigenvalue problem

The eigenvalue problem is of importance in quantum mechanics as the eigenvalues correspond to the values that can be observed in a measurement. For an operator $\hat{\mathbf{A}}$ and a vector $\vec{\phi}$ (that is not equal to the zero vector) the eigenvalue problem is defined by the following equation:

$$\hat{\mathbf{A}} \left|\phi\right\rangle = a \left|\phi\right\rangle, \tag{3.1}$$

where $a \in \mathbb{C}$. Then the Ket-vector $|\phi\rangle$ is an eigenvector of $\hat{\mathbf{A}}$ with the eigenvalue a. The conjugated operator $\hat{\mathbf{A}}^\dagger$ is defined by $\langle\psi|\,\hat{\mathbf{A}}^\dagger\,|\xi\rangle = \langle\xi|\,\hat{\mathbf{A}}\,|\psi\rangle^*$. An hermitian operator is defined by $\hat{\mathbf{A}} = \hat{\mathbf{A}}^\dagger$.

a) Show that $(\hat{\mathbf{A}}\hat{\mathbf{B}})^\dagger = \hat{\mathbf{B}}^\dagger\hat{\mathbf{A}}^\dagger$.

b) Show that hermitian operators have real eigenvalues.