Modern Physics WS 21/22 Sören Bieling

Problem Sheet 14

Ex. 14.1: Drude - Sommerfeld model

a) allowed $\vec{k}$-vectors: $\vec{k}_{n_1,n_2,n_3} = \frac{2\pi}{L}\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ volume around their tip: $\Delta k^3 = \left(\frac{2\pi}{L}\right)^3 = \frac{(2\pi)^3}{V}$

$V = L^3$

@ T=0 all occupied k states are enclosed by the Fermi sphere:

$V_F = \frac{4}{3}\pi k_F^3$

Each k state can be occupied twice (↑ & ↓):

$N = 2 \cdot \frac{V_F}{\Delta k^3} = 2 \cdot \frac{\frac{4}{3}\pi k_F^3 V}{(2\pi)^3} = \frac{1}{3}\frac{k_F^3}{\pi^2} \cdot V$

$k_F = \sqrt[3]{3\pi^2 \frac{N}{V}}$

b) Fermi Energy: $E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{\frac{2}{3}}$

c) K: $\rho = 0{,}86 \frac{g}{cm^3}$ $m_K = 39u$ single valence e^- / K atom $1u = 1{,}66 \cdot 10^{-27}$ kg

$\rho = \frac{M}{V} = \frac{m_K \cdot N}{V} \Leftrightarrow \rho_e = \frac{N}{V} = \frac{\rho}{m_K}$

$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{\rho}{m_K}\right)^{\frac{2}{3}} = 3{,}28 \cdot 10^{-19}\,J = 2{,}05\,eV$

d) starting from a) $N(k) = 2 \cdot \frac{\frac{4}{3}\pi k^3}{\frac{(2\pi)^3}{V}} = \frac{1}{3}\cdot\frac{k^3}{\pi^2}\cdot V$

with $E = \frac{\hbar^2 k^2}{2m}$

$\Leftrightarrow k = \frac{\sqrt{2mE}}{\hbar}$

$N(E) = \frac{1}{3}\frac{V}{\pi^2 \hbar^3}(2mE)^{\frac{3}{2}}$

$\frac{dN}{dE} = \cancel{\frac{1}{3}}\frac{V}{\pi^2 \hbar^3}\cancel{\frac{3}{2}}\sqrt{2mE}\cdot\cancel{2}m = \frac{\sqrt{2m^3 E}}{\pi^2 \hbar^3}\cdot V$

$D(E) = \frac{1}{V}\frac{dN}{dE} = \frac{\sqrt{2m^3 E}}{\pi^2 \hbar^3}$

There were 2 mistakes in problem sheet 14:

1.) $\rho = 0{,}86 \frac{g}{cm^3}$ (not $\frac{g}{m^3}$)

2.) d) $D(E) = \dots \frac{1}{\hbar^3} \dots$ (not $\frac{1}{\hbar^2}$)

Ex. 14.2: Heat capacity of the electron gas

a)

$$C_V^{e^- gas} = \left(\frac{\partial E}{\partial T}\right)_V = \frac{\pi^2}{2} \frac{N k_B^2}{E_F} T \qquad \text{with} \quad E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{\frac{2}{3}}$$

$$C_V^{id.gas} = \frac{3}{2} N k_B$$

with values for K from Ex. 14.1 c)

$$@ \; T = 300K \qquad \frac{C_V^{e^- gas}}{C_V^{id.gas}} = \frac{\frac{\pi^2}{\cancel{2}} \cancel{N k_B}}{\frac{3}{\cancel{2}} \cancel{N k_B}} \cdot \frac{k_B T}{E_F} = \frac{\pi^2}{3} \cdot \frac{k_B T}{E_F} \approx 4,2\,\%$$

$\Rightarrow$ About 4% contribute the conduction e^- to the total heat capacity at room temperature!

b)

$$C_V = C_V^{e^- gas} + C_V^{phonons} = \gamma T + \alpha T^3$$

$$@ \; T = T^* \qquad \alpha (T^*)^3 = \gamma T^* \qquad T^* = \sqrt{\frac{\gamma}{\alpha}}$$

$\Rightarrow$ Below T^* $C_V^{e^- gas} > C_V^{phonons}$