Modern Physics

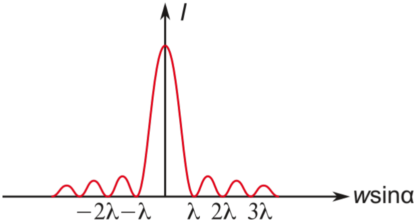
Contents:

Uncertainty relations

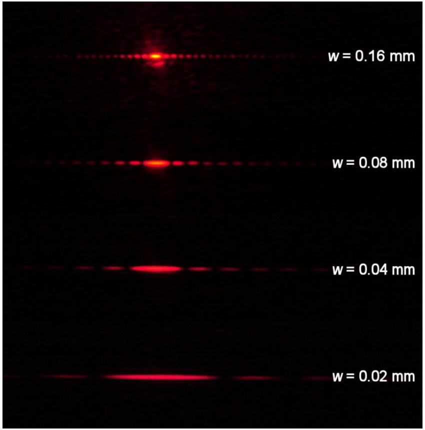
Wave-particle dualism

- Thermal radiation
- Planck's radiation law
- Photoelectric effect
- Laser
- Compton effect
- Pair production
- Matter waves
- Uncertainty relations

Uncertainty relations 1: Diffraction on a single slit



minima: $n\lambda = w \sin \alpha_n$



Uncertainty relations 1 Comment

The uncertainty relations are very useful for predicting the behavior of quantum systems.

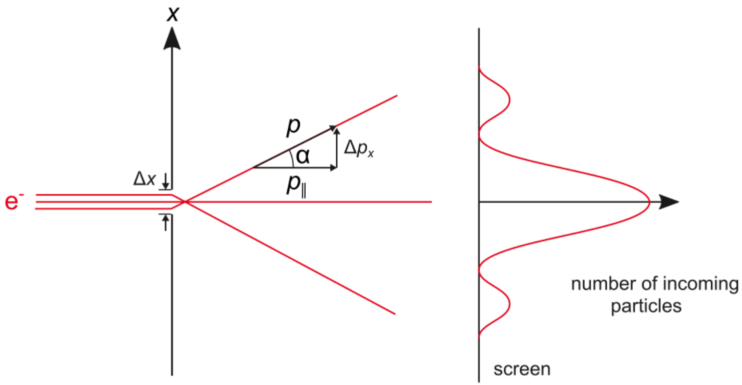
The strictly mathematical derivation of the uncertainty relations was given in 1927 by Werner Heisenberg.

The essence becomes apparent when the particle image is used to describe diffraction at a single slit.

The figures show the intensity after a single slit.

The distance between the minima increases as the width of the slit decreases.

Uncertainty relations 2: Diffraction on a single slit



the particles get a transversal momentum Δp_x after the slit

Uncertainty relations 2 — Comment

The momentum of the particles in front of the slit is parallel to the direction of the beam.

After the slit, some particles get a transverse momentum, so that the diffraction pattern of a single slit results when many particles are observed.

Uncertainty relations 3

Estimate of the transversal momentum with the 1st diffraction minimum

$$\Delta x \sin \alpha_1 = \lambda \quad \text{und} \quad \underline{\sin \alpha_1 = \frac{\Delta p_x}{p}}$$

and with the de Broglie wavelength $\lambda = h/p$

$$\Delta x \sin \alpha_1 = \lambda = \frac{h}{p} = \Delta x \frac{\Delta p_x}{p} \qquad \text{und} \qquad \underline{\Delta x \Delta p_x = h}$$

the exact Heisenberg's uncertainty relation is

$$\boxed{\Delta x \Delta p_x \geq \frac{\hbar}{2}}$$

Uncertainty relations 3 Comment 1

The first diffraction minimum can be used to estimate the relationship between the width of the slit and the uncertainty of the transverse momentum Δp_x.

The first equation gives the diffraction condition for the 1st minimum.

The underlined equation gives the sinus of this deflection angle as the quotient of the corresponding Δp_x for α_1 and the total momentum of the electron.

The second underlined equation results when the de Broglie wavelength is used.

The slit width determines the transverse momentum Δp_x of the particles.

Uncertainty relations 3 — Comment 2

With the slit, the particles are localized along the x-direction.

The width of the slit determines the uncertainty of the localization.

The smaller Δx , the larger Δp_x becomes.

The equation outlined in red shows Heisenberg's exact result.

If the particle can be localized in the range Δx, the smallest possible uncertainty of the momentum p_x is $h/4\pi$.

Uncertainty relations 4

uncertainty relation of energy: with $\Delta E = c\Delta p$ and $\Delta t = \Delta x / c$

$$\boxed{\Delta t \Delta E \geq \frac{\hbar}{2}}$$

uncertainty relation of angular momentum: with $\Delta L = r\Delta p$ and $\Delta \varphi = \Delta x / r$

$$\boxed{\Delta \varphi \Delta L \geq \frac{\hbar}{2}}$$

Uncertainty relations 4 Comment 1

There are also uncertainty relations for other physical quantities.

The time-energy uncertainty and the angle-angular momentum uncertainty are particularly important.

Here, too, a strict derivation of the uncertainty relations would go beyond the scope of this lecture.

The formulas given are intended to make the uncertainty relations at least plausible.

A photon can be considered for the time-energy uncertainty relation. The photon moves with the speed of light c and the energy-momentum relation is $E = cp$.

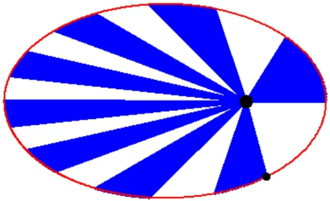
Uncertainty relations 4 — Comment 2

For the angular momentum uncertainty relation, a particle moving on a circular path with radius r can be considered.

The angular momentum is $L = rp$ and the variation of the angle is $\Delta\varphi = \Delta x/r$. Here, Δx denotes a certain distance on the circumference of the circle.

Uncertainty relations 5: Consequences

Planets move in orbital planes around the sun



uncertainty relations of a quantum particle $\Delta z \to 0$ $\Delta p_z \to \infty$ or $\Delta p_z \to 0$ $\Delta z \to \infty$.

The electrons of an atom cannot move in orbital planes!

Uncertainty relations 5 Comment

It has been known since Johannes Kepler (1571-1630) that planets move in orbital planes around the sun.

The figure illustrates the 2nd Kepler law: A line connecting a planet and the sun sweeps over the same areas at the same time intervals.

Newton showed that this is a direct consequence of conservation of angular momentum.

If the plane of the orbit is within the xy-plane, then the uncertainty of Δz is zero and consequently the momentum uncertainty Δp_z is infinitely large.

On the other hand, the position uncertainty along the z-axis is infinitely large when the momentum in the z-direction is zero.

Therefore, the movement of a quantum particle cannot be restricted to one plane.

Uncertainty relations 6: Consequences

Estimation of the kinetic energy of a proton in the atomic nucleus

nuclear diameter $d \approx 2 \cdot 10^{-15}$ m

$$\Delta p \geq \frac{\hbar}{2d} = \frac{h}{4\pi d}$$

energy uncertainty

$$\Delta E \geq \frac{\Delta p^2}{2m_\mathrm{P}} = \frac{c^2 \Delta p^2}{2m_\mathrm{P} c^2} = \frac{(3 \cdot 10^8\,\mathrm{m/s})^2 (4.14 \cdot 10^{-15})^2\,(\mathrm{eVs})^2}{2 \cdot 16\pi^2\,(2 \cdot 10^{-15})^2\,\mathrm{m}^2\,940 \cdot 10^6\,\mathrm{eV}}$$
$$= 5.2\,\mathrm{MeV}$$

For comparison: the binding energy of the electron of the hydrogen atom is 13.6 eV

$\rightarrow$ nuclear forces are very much stronger than the Coulomb force

Uncertainty relations 6 Comment 1

Protons and neutrons make up the atomic nucleus.

The diameter of the nuclei is between 10^{-15} m for helium and $16 \cdot 10^{-15}$ m for uranium.

The core diameter indicates the positional uncertainty of the nucleons (i.e. protons and neutrons) and the momentum of the nucleons can be estimated using the uncertainty relation.

The formula $E_{\mathrm{kin}} = p^2/2m$ of classical physics can be used to estimate the kinetic energy of nucleons, since the energy uncertainty ΔE is very much smaller than the rest energy of a proton ($m_{\mathrm{p}}c^2 = 940 \cdot 10^6$ eV).

Uncertainty relations 6 Comment 2

The kinetic energy of the nucleons is large compared to the binding energy of the electrons.

It is obvious that the nuclear forces that bind the nucleons are much stronger than the Coulomb force that determines the properties of the electron cloud.

Atoms

Modern Physics

Contents:

Atoms

Early atomic physics 1 — Introduction

The idea that atoms form the smallest units of matter comes from ancient Greece.

It was not until the beginning of the 19th century that this idea was seriously studied.

With the ideal gas laws, and in particular Avogadro's law of 1811 (i.e. the pressure and volume of a gas are determined only by the number of particles in the gas) it became possible to measure the relative masses of chemical compounds.

Since there are innumerable chemical compounds, it was not until 1869 that Dmitri Mendeleev and in the same year Lothar Meyer formulated the first version of the periodic table of the elements.

The early versions of the periodic system enabled the discovery of a large number of new, unknown elements.

Early atomic physics 2

Introduction

The noble gases in particular were discovered.

The noble gases were completely absent from the first version of the periodic table.

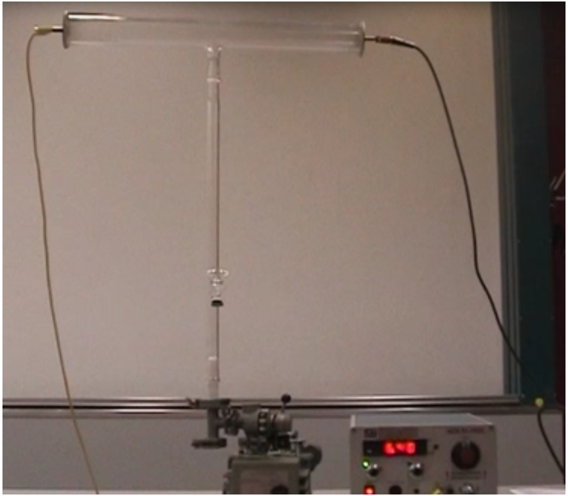
Based on the atomic mass, it had been assumed since the beginning of the 19th century that the atoms are composed of the lightest element, i.e. hydrogen.

Electron

Early atomic physics

- The electron and the elementary charge
- The nucleus
- The spectrum of the hydrogen atom
- Bohr's model of the hydrogen atom
- The spectra of the alkali metals
- X-ray spectra
- The Frank-Hertz experiment

The electron and the elementary charge 1



(Gasentadungsroehre.mp4)

The electron and the elementary charge 1 Comment

The starting point for the physical investigation of atoms and molecules were experiments with gas discharge tubes.

A gas discharge tube is a tube with electrodes at the ends to which a very high voltage is applied.

In the video, the tube is filled with air and you can see that the air starts to emit light during evacuation.

The emission spectrum is an important source of information.

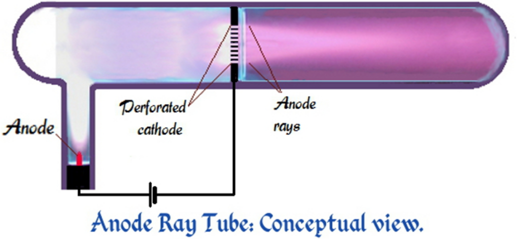
The study of the hydrogen spectrum by Johannes Balmer in 1885 is famous.

In 1896, Pieter Zeeman discovered that the spectral lines of atoms split and shift when a magnetic field is applied.

This discovery is significant because Hendrik Lorentz was able to conclude in 1899 that the spectral lines of atoms and molecules are caused by electrons.

The electron and the elementary charge 2

Eugen Goldstein (1886)



The electron and the elementary charge 2 Comment

Eugen Goldstein modified the simple discharge tube in 1886.

He used grids as electrodes and discovered that there were rays behind the grids, which were then called canal rays.

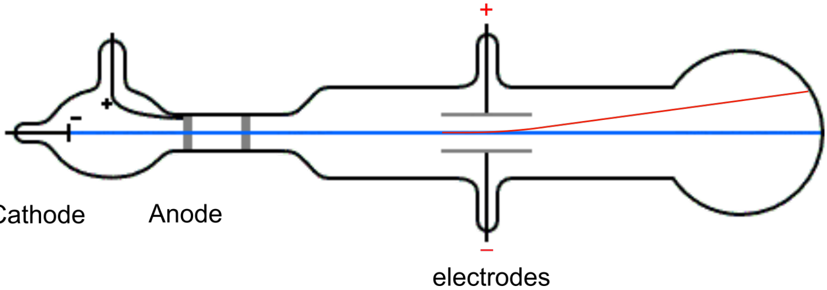
In the figure, a grid is used as the cathode.

The channel rays must consist of positive charges and are called anode rays.

When the grid is positively charged there are also rays called cathode rays.

The electron and the elementary charge 3

The cathode ray tube of Joseph John Thomson (1897)



$\vec{F} = m\vec{a} = q\vec{E}$ acceleration $\boxed{\vec{a} = \frac{q}{m}\vec{E}}$

The electron and the elementary charge 3 Comment 1

The experiments of Sir Joseph John Thomson, in which he discovered the electron in 1897, were decisive.

Thomson expanded the simple canal tube by installing two more electrodes and a magnetic field in the space behind the electrode so that the beams could be deflected by an electric and magnetic field.

The underlined equation gives Newton’s equation of motion and it turns out that the acceleration of the charges depends on the ratio of the charge to the mass of the particles.

A corresponding result is obtained in the case of the Lorentz force.

The picture shows the cathode ray tube with which Thomson discovered the electron.

The electron and the elementary charge 3 — Comment 2

The blue line shows the beam when no voltage is applied to the deflection electrodes.

The red line shows the case with the voltage applied so that the beam is deflected.

When comparing his measurements with anode and cathode ray tubes, Thomson found that the mass of the particles that make up the cathode ray must be much smaller than the mass of the particles of the anode ray.

He also found that the ratio q/m is always the same for the cathode ray, while different ratios q/m can occur with the anode ray.

The electron and the elementary charge 3 Comment 3

Since the mass of the electrons is more than a factor of 1000 smaller than the mass of the atoms and molecules, the behaviour of the cathode rays differs from the anode rays not only due to the charge of the particles.

Thomson soon realized that he had discovered the first subatomic particle.

A year later, in 1898, Wilhelm Wien discovered the proton in a similar experiment.

In his reflections on the Zeeman effect in 1899, Hendrik Lorentz came to the conclusion that the light emission of the atoms must be caused by electrons.

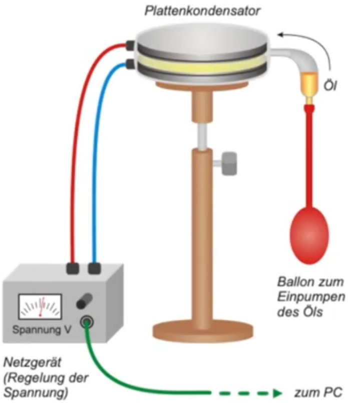
The electron and the elementary charge 3 Comment 4

In 1902 Hendrik Lorentz and Pieter Zeeman were awarded the Nobel Prize “in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena”.

In 1906 Joseph John Thomson was awarded the Nobel Prize “for his theoretical and experimental investigations on the conduction of electricity by gases”.

The electron and the elementary charge 4

The Millikan experiment (1910)



(MillikanVersuch.mp4)

The electron and the elementary charge 4 Comment 1

With the anode and cathode rays only the ratio q/m could be determined, since the mass of the particles was unknown at that time.

Robert Millikan and Harvey Fletcher therefore introduced a new method of charge measurement in 1910.

Instead of the unknown particles of the anode and cathode rays, they used macroscopic oil droplets.

The diameter of the droplets and thus the mass of the droplets can be determined with a microscope.

The illustration shows the experimental setup. Oil droplets are blown through a nozzle between the charged plates of a capacitor.

The electron and the elementary charge 4 Comment 2

There the oil droplets can be observed with a microscope.

The oil droplets carry an electrical charge, which is created by natural radioactivity or the disintegration of the oil.

If there is no voltage applied on the plates, the droplets slowly sink to the bottom.

When a voltage is applied, the speed of the droplets changes due to the Coulomb force.

The evaluation of the measurements shows that the charge is quantized in units of the elementary charge.

The numerical value of the elementary charge can also be determined fairly precisely.

The electron and the elementary charge 4 Comment 3

The work of Robert Millikan and Harvey Fletcher made mass spectroscopy possible.

Since the numerical value of the elementary charge is known, the mass of atoms and molecules can be determined in channel beam experiments.

In 1923 Robert Millikan was awarded the Nobel Prize Robert “for his work on the elementary charge of electricity and on the photoelectric effect”.

Nucleus

Early atomic physics

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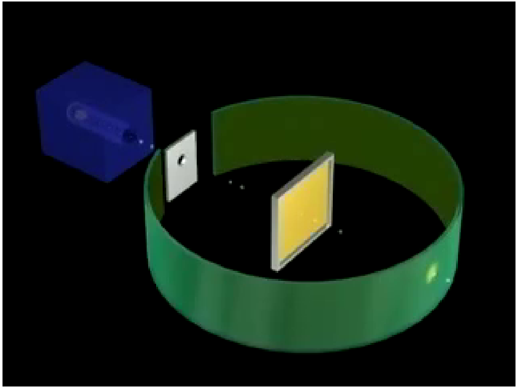
The nucleus 1

Rutherford scattering (1909-13): scattering of α-particles on a thin gold foil

→ most α-particles are not at all scattered

(RutherfordExperiment.mp4)



The nucleus 1 Comment

The animation illustrates Rutherford’s scattering experiment.

α-particles are the nuclei of helium atoms.

For most α-particles, the gold foil is no obstacle.

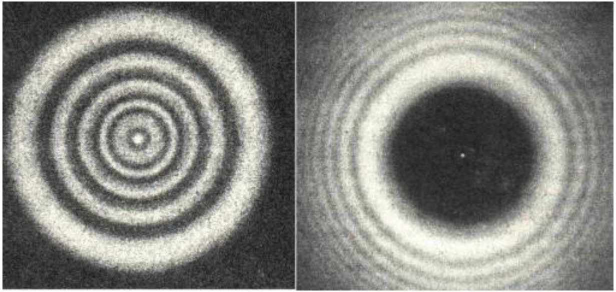
Only a few α-particles are scattered.

From the number of scattered α-particles as a function of the scattering angle, Rutherford was able to show that the α-particles were scattered at quasi point-like atomic nuclei.

The diameter of the atomic nuclei is so small that it could not be determined with this experiment.

The nucleus 2

diffraction on a hole and on a pinhead



(FresnelbeugungFreiburg.mp4)

Diffraction experiments with electrons on the nucleus $d \approx 10^{-14}$ m

$$\lambda = \frac{h}{p} = \frac{hc}{eU} = \frac{12{,}42 \cdot 10^{-7}\,\mathrm{Vm}}{U} \quad \rightarrow \quad \lambda << 10^{-14}\,\mathrm{m} \rightarrow U >> 124\,\mathrm{MV}$$

The nucleus 2 Comment 1

If you want to determine the diameter of an atomic nucleus, you have to make use of the wave character of the particles.

The pictures demonstrate the optical analogue. The left picture shows the scattering of photons at a pinhole. The picture on the right shows the scattering of photons at the head of a pin.

The video shows an experiment with an expanded laser beam. The light is collected on a screen behind the objects (pin head, edge and two pinholes).

With the pin head you can also see that there are maxima and minima of the intensity due to the interference of diffracted light waves in the vicinity of the pin but also in the shadow area.

The nucleus 2 — Comment 2

If you want to observe the diffraction pattern of an atomic nucleus instead of the head of a pin, you can use electron waves, for example, whose wavelength is smaller than the diameter of the atomic nucleus.

Since the core diameter is in the range of 10^{-14} m, high-energy relativistic electrons must be used.

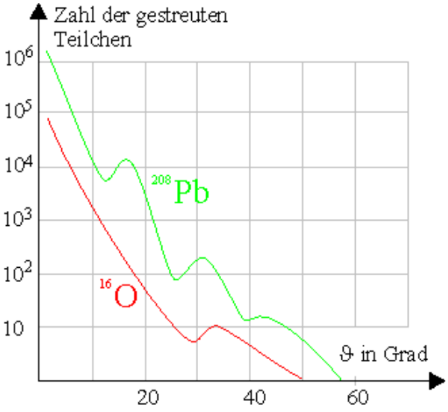
The rest energy of the electron can be neglected and the energy of the electron is given by the product of the speed of light and momentum, i.e. $E = cp$

If the electron is accelerated with the voltage U, then the kinetic energy results from the product of the voltage and the elementary charge, i.e. $E_{\text{kin}} = eU$ and the formula for the de Broglie wavelength results in the formula underlined in red for the wavelength of relativistic electrons.

If the wavelength is to be smaller than 10^{-14} m, the voltage must exceed 100 MV.

The nucleus 3

elastic electron scattering



the kinetic energy of the electrons is 500 MeV

The nucleus 3 Comment 1

The figure shows the experimental results of elastic electron scattering on the nuclei of oxygen and lead.

The acceleration voltage is 500 MV and the kinetic energy of the electron therefore very much larger than the rest energy of $m_0c^2 = 500$ keV.

In the forward direction there is a very large number of electrons, since most of the electrons penetrate the sample, as in the Rutherford experiment, without being deflected by the nuclei.

Outside the electron beam, the interference of the diffracted electron waves is observed.

In case of oxygen, the diffraction minimum of 1st order can be resolved.

The nucleus 3

Comment 2

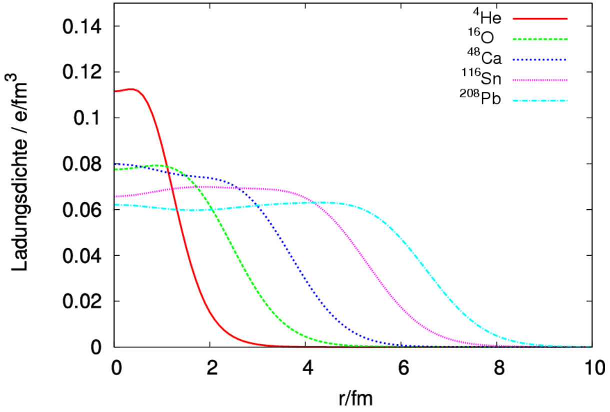
In the case of lead, three diffraction maxima can be observed.

As with the gap or the pinhole, the minima shift to larger deflection angles when the core radius is reduced.

Even without a detailed evaluation, the figure shows that the core radius of lead is larger than the core radius of oxygen.

The nucleus 4

nuclear charge density



$\rightarrow$nuclear radius $\boxed{\approx 10^{-15}\,\text{m}}$ (atomic radius $\approx 10^{-10}$ m)

The nucleus 4 Comment

The distribution of the charge density in the atomic nucleus can be determined from such measurements of elastic electron diffraction.

The figure shows the distribution of the nuclear charge density for different nuclei.

The core radius is in the range of a few femtometers, i.e. a few 10^{-15} m.

The radius of the electron shell can be derived from measurements of the crystal structure.

The atomic radius is about five orders of magnitude larger than the radius of the atomic nucleus.

If you take the radius of the Adenauerring (1.1 km) for comparison, then the atomic nucleus corresponds to a marble ($\approx$1 cm) lying in the tower of the castle.

The nucleus 5

Linear accelerator
from Stanford University

The nucleus 5 Comment

These experiments were largely carried out and directed by Robert Hofstadter in the 1950s at Stanford University.

The left figure shows the elongated hall of the linear accelerator of the university and on the right a photo of the experiment hall with the target (left yellow box), the deflection magnets (blue structures) and the detector (right yellow box).

The accelerator has been continuously developed so that the electrons currently reach an energy of 50 GeV.

Robert Hofstadter was awarded the Nobel Prize in 1961 “for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons”.

The nucleus 6

A nucleus contains Z protons with the charge (Z is the atomic number)

$$\boxed{q_P = +e}$$

and N neutrons which carry no electric charge

$$\underline{\text{mass of proton} \approx \text{mass of neutron}}$$

The notation for nuclei is

$$\boxed{{}^{A}_{Z}\text{nuclide}}$$

$A = Z + N$ is the mass number

Isotops denote nuclei with the same Z but different values of A e.g.

^{12}C, ^{13}C and ^{14}C

The nucleus 6 Comment 1

The atomic number Z determines the electronic properties of an atom and can be determined directly from the characteristic X-ray spectrum of the element.

In 1920 Rutherford concluded from a comparison of the atomic number Z with the mass of the nucleus that the nucleus must contain neutral particles in addition to the positively charged protons.

These neutral particles are called neutrons.

The nucleus 6 Comment 2

However, direct detection of the neutron was difficult and was only achieved by Rutherford’s student James Chadwick in 1932.

James Chadwick was awarded the Nobel Prize as early as 1935 “for the discovery of the neutron”

The neutron is slightly heavier than the proton and is only stable in the atomic nucleus.

As a free particle, the neutron uses its rest energy to transform itself into the lighter proton and an electron with a lifespan of 880 seconds.

With the discovery of the neutron, the basic structure of an atomic nucleus was understood.

The nucleus 6 Comment 3

An atomic nucleus consists of Z protons.

Z denotes the atomic number.

There are also N neutrons.

The sum of the atomic number and the number of neutrons is called the mass number $A = Z + N$.

Nuclei are denoted by the chemical name of the element.

The mass number is given as the index at the top left.

The atomic number is sometimes given as an index at the bottom left.

The nucleus 6 Comment 4

Since the nuclear charge is already clearly determined by the name of the element, this index is often omitted.

Most elements can be formed with different numbers of neutrons.

The different nuclei of an element are called isotopes.

Spectrum Hydrogen Atom

Early atomic physics

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- The Frank-Hertz experiment

The spectrum of the hydrogen atom 1

Johannes Balmer (1885): spectral lines of hydrogen in the visible range of the spectrum (WasserstoffSpektrallampe.mp4)

$$\lambda_{\mathrm{n}} = A\frac{n^2}{n^2-4} \quad \text{and} \quad n = 3, 4, 5 \ldots$$

With $A = 364.568$ nm one finds the energy of the photons $\Delta E_n = hc/\lambda_n$

$$\Delta E_n = \frac{hc}{A}\left(1-\frac{4}{n^2}\right) = \frac{4hc}{A}\left(\frac{1}{2^2}-\frac{1}{n^2}\right) = R_y\left(\frac{1}{2^2}-\frac{1}{n^2}\right) = 13.6\,\mathrm{eV}\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$$

The spectrum of the hydrogen atom 1 Comment 1

The video shows a discharge tube that is filled with hydrogen gas.

If a high voltage is applied, the H_2 molecules disintegrate through electron impact and atomic hydrogen is formed.

Excited hydrogen atoms give off their energy through spontaneous emission of photons and return to the ground state.

The figure shows the spectral lines in the visible range of the electromagnetic spectrum.

In 1885 Johannes Balmer found the simple formula underlined in red with which the wavelengths of the spectral lines of hydrogen in the visible spectral range can be calculated.

The spectrum of the hydrogen atom 1 — Comment 2

The comparison with the experimental data determines the constant A:
$A = 364.568$ nm.

The energy of the transitions can be calculated from the wavelength using Planck's law $E = h\nu = hc/\lambda$.

The constant $R_y = \frac{4hc}{A} = 13.6$ eV is called the Rydberg energy.

The spectrum of the hydrogen atom 2

Theodore Lyman (1906): hydrogen spectral lines in the ultraviolet range

$$\lambda_n = \frac{A}{4}\frac{n^2}{n^2-1} \quad \text{and} \quad n = 2, 3, 4, 5 \ldots$$

With Balmer's constant $A = 364.568$ nm and Planck's law $\Delta E_n = hc/\lambda_n$

$$\Delta E_n = 13.6\,\text{eV}\left(1 - \frac{1}{n^2}\right)$$

The spectrum of the hydrogen atom 2 Comment

In 1906 Theodore Lyman observed the spectral lines of the hydrogen atoms in the ultraviolet region of the spectrum.

The wavelength of the spectral lines can be calculated with a small modification of the Balmer formula.

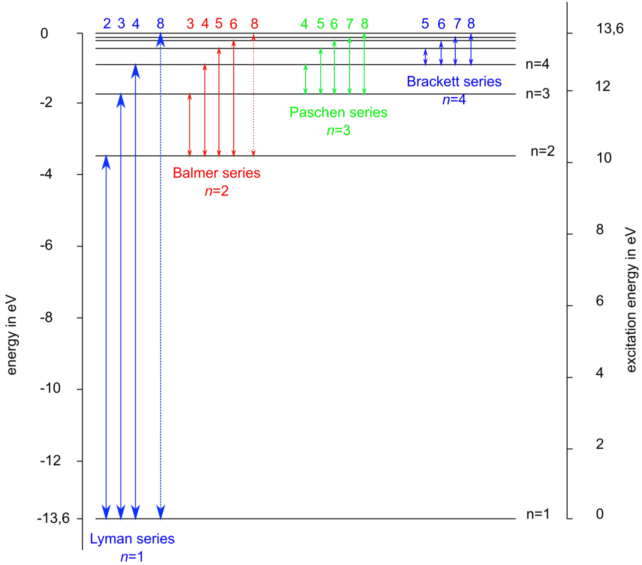
If the transition energy is calculated from the wavelength using Planck's law, the formula below, underlined in red, results.

The infrared range of the spectrum was examined by Friedrich Paschen in 1908.

The wavelength of the spectral lines can also be calculated with a small modification of the Balmer formula.

Obviously, the transition energy results from the difference between two energy terms.

The spectrum of the hydrogen atom 3



$$\boxed{E_n = -13.6\,\text{eV}\,\frac{1}{n^2}}$$

e.g. for the Paschen series

$$\Delta E_n = E_n - E_3$$

$$\Delta E_n = -13.6\,\text{eV}\left(\frac{1}{n^2} - \frac{1}{3^2}\right)$$

and $n = 4, 5, 6, \ldots$

The spectrum of the hydrogen atom 3 Comment 1

With the energy of the photons, it is easy to find the underlying energy level scheme of the hydrogen atom.

The formula outlined in red indicates the quantized energy of the electron in the hydrogen atom.

This makes it easy to calculate the respective transition energy.

The transition energy of the Lyman series is given by $\Delta E_n = E_n - E_1$, the transition energy of the Balmer series is given by $\Delta E_n = E_n - E_2$ and the Paschen series by $\Delta E_n = E_n - E_3$.

The figure shows the energy levels scheme of the hydrogen atom with the various series of spectral lines.

The ground state with the quantum number $n = 1$ has the lowest energy.

The spectrum of the hydrogen atom 3 Comment 2

For $n \to \infty$ the ionization energy results, which is 13.6 eV above the ground state.

The right scale of the figure gives the excitation energy above the ground state.

If the ionization energy is assigned the value zero, then the left-hand scale of the figure results.

The energy of the ground state is then -13.6 eV.

If the excitation energy is increased above 13.6 eV, the electron is no longer bound to the proton.

The blue double arrows show the transitions of the Lyman series in the ultraviolet range of the spectrum.

The spectrum of the hydrogen atom 3 Comment 3

The red double arrows show the transitions of the Balmer series in the visible range of the spectrum.

The green double arrows show the transitions of the Paschen series in the infrared range of the spectrum.

The second group of blue double arrows is the Brackett series, which is also in the infrared range of the spectrum and was first detected in 1922.

Bohr’s model

Early atomic physics

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- The Frank-Hertz experiment

Bohr's model of the hydrogen atom 1

What was known in 1913 about atoms?

1. Lorentz (1889): electromagnetic radiation of atoms is due to electrons.
2. Planck (1900): the energy of electromagnetic radiation is quantised:

$$E = h\nu = hc/\lambda$$

3. Balmer (1885), Lyman (1906), Paschen (1908): the binding energy of electrons in hydrogen atoms is given by the formula

$$E_n = -13.6\,\text{eV}\frac{1}{n^2} \quad \text{and} \quad n = 1, 2, 3\ldots$$

4. Rutherford (1913): The mass and the positive charge are concentrated in the atomic nucleus.

Bohr's model of the hydrogen atom 1 Comment

The slide summarizes the state of knowledge from 1913.

Nils Bohr used this information to formulate a first theoretical model of the atom.

Bohr's model is very simple and yet allows precise predictions of atomic properties.

Bohr's model of the hydrogen atom 2

Bohr's postulates (1913)

1. electrons move in circular orbits around the nucleus.

2. Stable orbits are only possible when the angular momentum $L = rmv$ is quantized according to

$$\boxed{L = n\hbar} \qquad \text{and} \qquad n = 1, 2, \ldots$$

3. Photons are emitted or absorbed in a transition between the stable orbits. The energy of the photons equals

$$\boxed{E = h\nu = |E_n - E_m|} \qquad \text{and} \qquad E_n \neq E_m$$

Bohr's model of the hydrogen atom 2 Comment 1

Since the wave-particle dualism was not yet established in 1913, Bohr assumes that the electrons behave similarly to planets orbiting the atomic nucleus.

That is Bohr's first postulate.

Since the energy of the electrons is quantized, there must be special circular orbits for which the angular momentum assumes the value $n\hbar$.

That is Bohr's second postulate.

Since electrons moving on circular orbits emit electromagnetic waves, radiation for the special circular orbits must be forbidden for whatever reason.

Bohr's model of the hydrogen atom 2 Comment 2

Electromagnetic waves are only emitted or absorbed when the electron changes stable orbits. In accordance with Max Planck's ideas, energy quanta $\hbar\omega$ are emitted or absorbed.

That is Bohr's third postulate.

Bohr's model of the hydrogen atom 3

circular orbit:

Coulomb force = centrifugal force

i.e.

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r}$$

small calculation

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r} \quad \rightarrow \quad \frac{e^2}{4\pi\varepsilon_0} = \frac{r^2 m \cdot mv^2}{mr} \quad \rightarrow \quad r = \frac{4\pi\varepsilon_0}{e^2 m} r^2 m^2 v^2$$

radius of the circular orbit

$$\boxed{r = \frac{4\pi\varepsilon_0}{e^2 m} L^2} \quad \text{with the angular momentum} \quad L = r\,mv$$

Bohr’s model of the hydrogen atom 3 Comment

On the circular orbit, the centripetal force is caused by the Coulomb force between the atomic nucleus and the electron or in other words: the centrifugal force of the circular motion is compensated by the Coulomb force.

After a little calculation, the equation outlined in red results.

The radius of the circular path is proportional to the square of the orbital angular momentum.

Bohr's model of the hydrogen atom 4

energy of the electron

$$\underline{E = E_{kin} + E_{pot}}$$

kinetic energy

$$E_{kin} = \frac{1}{2} m v^2$$

potential energy between two charges Q_1 and Q_2

$$\boxed{E_{pot} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r}}$$

with the charge of the proton $Q_1 = e$ and the charge of the electron $Q_2 = -e$

$$\boxed{E_{pot} = -\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r}}$$

Bohr’s model of the hydrogen atom 4 Comment 1

The energy of the electron results from the sum of kinetic energy and potential energy.

The formula outlined in red gives the potential energy of two point charges Q_1 and Q_2.

If both charges have the same sign, then the potential energy increases when the distance between the charges is reduced.

This corresponds to a spring that is compressed.

If the compressed spring is released, the potential energy of the spring leads to an accelerated movement.

Bohr's model of the hydrogen atom 4 Comment 2

In the case of the two point charges, the stored potential energy is converted into kinetic energy of the two charges.

The charge of the atomic nucleus and the electron have opposite signs.

The potential energy increases when the distance between the atomic nucleus and the electron is increased.

If the electron is pulled away from the atomic nucleus and then released, it rushes towards the atomic nucleus.

If the electron has a momentum at right angles to this movement, the electron generally follows an elliptical trajectory around the atomic nucleus.

Bohr’s model of the hydrogen atom 4 Comment 3

The electron rushes towards the nucleus and misses it because it is deflected to the side by the transverse momentum.

In the further course of the movement, the kinetic energy of the electron is reduced and the potential energy increases again, etc.

The second formula outlined in red gives the potential energy of an electron in the Coulomb field of a proton.

Bohr's model of the hydrogen atom 5

with the balance between

Coulomb force = centrifugal force

i.e.

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r} \quad \Big| \times r$$

results

$$-E_{pot} = 2E_{kin}$$

energy of the electron

$$E = E_{kin} + E_{pot} = -\frac{1}{2}E_{pot} + E_{pot} = \frac{1}{2}E_{pot} \propto \frac{1}{r} \propto \frac{1}{n^2}$$

Bohr's model of the hydrogen atom 5 Comment

In the case of circular motion, the equilibrium between Coulomb force and centrifugal force shows that the kinetic energy is proportional to the potential energy.

This results in the equation underlined in red and the total energy of the electron is proportional to the potential energy.

The total energy of the electron is therefore inversely proportional to the radius of the circular path.

From the experimental results of Balmer, Lyman and Paschen it is known that the binding energy of the electron is proportional to the inverse square of the quantum number *n*.

Bohr's model of the hydrogen atom 6

With $r = \frac{4\pi\varepsilon_0}{e^2 m} L^2$ is the energy

$$E = \frac{1}{2} E_{pot} = -\frac{1}{8\pi\varepsilon_0} \frac{e^2}{r} = \underline{-\frac{1}{(8\pi\varepsilon_0)(4\pi\varepsilon_0)} \frac{e^4 m}{L^2}} = E_n = -13.6\,\text{eV} \frac{1}{n^2}$$

therefore

$$\underline{L \propto n}$$

or

$$L = n \cdot \sqrt{\frac{e^4 m}{32\varepsilon_0^2 \pi^2 13.6\,\text{eV}}} = n \cdot \frac{4.1 \cdot 10^{-15}\,\text{eVs}}{2\pi} = n\hbar$$

and

$$\boxed{L = n\hbar} \qquad \text{and} \qquad n = 1, 2, \ldots$$

Bohr’s model of the hydrogen atom 6 Comment

With the formula for the radius of the circular orbit, the underlined equation results and it is obvious that the orbital angular momentum must be proportional to the quantum number n.

With the numerical values it follows that the proportionality constant must be $\hbar$.

Bohr’s model of the hydrogen atom 7

the radius of the orbit is ($r = \frac{4\pi\varepsilon_0}{e^2 m} L^2$)

$$r_n = \frac{4\pi\varepsilon_0 \hbar^2}{e^2 m} \cdot n^2$$

Bohr’s radius

$$a_B = \frac{4\pi\varepsilon_0 \hbar^2}{e^2 m} \quad \rightarrow \quad \boxed{a_B = 0.529 \cdot 10^{-10}\ \mathrm{m}}$$

and

$$\boxed{r_n = a_B \cdot n^2}$$

Bohr’s model of the hydrogen atom7 Comment

With the quantized orbital angular momentum, the radius of the Bohr orbits result.

The formula underlined in red gives the radius of the Bohr orbits.

The radius of the orbit is proportional to the quantum number *n*.

The constant of proportionality results from the numerical values.

The constant of proportionality is called the Bohr radius.

The Bohr radius defines the relevant length scale in atomic physics.

The formulas outlined in red give the Bohr radius and the radius of the Bohr orbits.

Bohr’s model of the hydrogen atom 8

generalisation for nuclei with the atomic number Z:

orbital radius

$$\boxed{r_n = \frac{a_B}{Z} n^2}$$

energy

$$\boxed{E_n = -13.6\,\mathrm{eV}\frac{Z^2}{n^2}}$$

Bohr's model of the hydrogen atom 8 Comment

The two equations framed in red give the generalization of the radius and the energy if the nuclear charge is Ze.

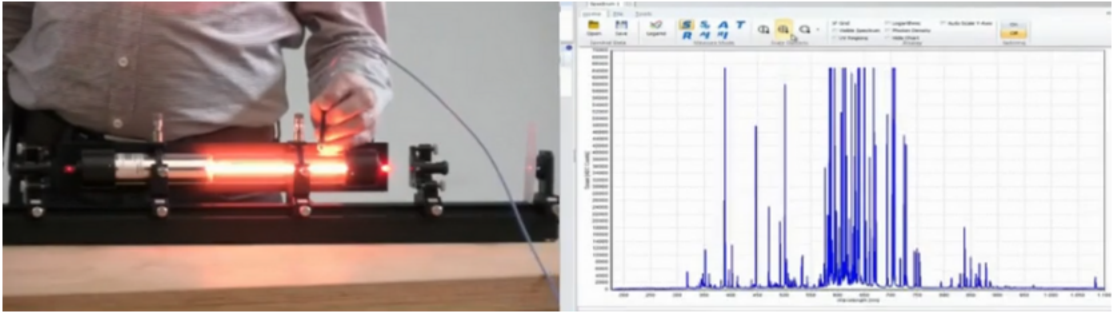
The radius of the orbit decreases with increasing atomic number and the energy $|E_n|$ increases with the square of the atomic number.

Alkali Spectra

Atoms

The spectra of the alkali metals 1

The optical spectrum of the helium and neon atoms



The spectra of the alkali metals 1 Comment 1

As an example, the figure shows the emission spectrum of the helium and neon atoms in the discharge tube of a He-Ne laser.

The spectrum is complicated so that no information about the energy levels of the helium and neon atoms can be obtained.

Bohr’s model of the atom does not help with this spectrum.

For most atoms, the spectra cannot be evaluated without the help of the Schrödinger equation.

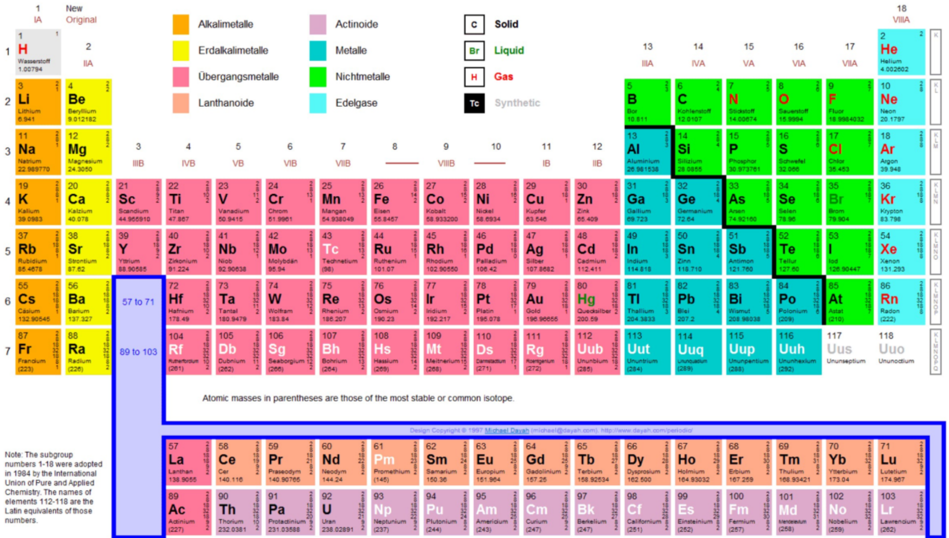
The optical spectra of the alkali metals are an exception.

The spectra of the alkali metals 1 Comment 2

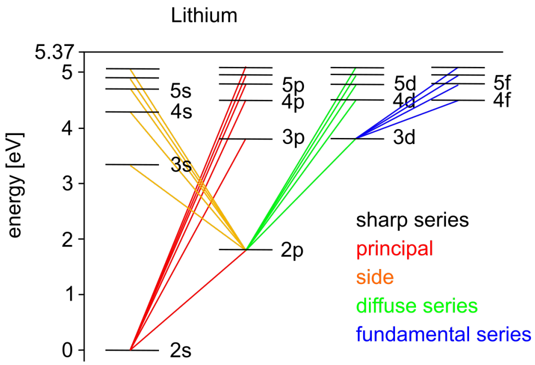
The optical spectra of the alkali metals are similar to the spectrum of the hydrogen atom, i.e. the transition energy between the energy levels can be described by the difference between two energy terms.

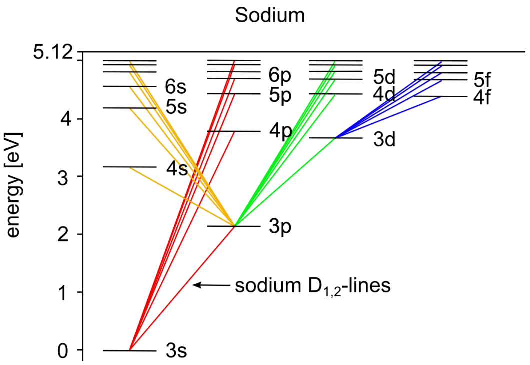
Similar to the hydrogen atom, the spectra of the alkali metals contain series that can be traced back to the excitation of one valence electron.

The spectra of the alkali metals



The spectra of the alkali metals 2





The spectra of the alkali metals 2 — Comment 1

The figure shows the energy level schemes of lithium and sodium as an example.

First there are the sharp series (George Liveing and James Dewar 1890), which are characterized by sharp spectral lines.

There is one principal series that can be observed in absorption and there are side series that can only be observed in emission.

The principal series is marked in red. The side series with the greatest transition energies is marked in orange.

Similar to the hydrogen atom, an energy level scheme can be set up using the spectral lines.

The spectra of the alkali metals 2 Comment 2

In absorption, the transitions of the principal series start from a level which is marked with the letter 's'.

The transitions lead to energy levels marked with the letter 'p' (for principal series).

The spectral lines of the side series, which can only be observed in emission, start at the energy levels marked with the letter 's' (for side series) and end at the energy levels marked with the letter 'p'.

Similar to the hydrogen atom, the energy levels can be assigned a quantum number and the transition energy results from the formula $\Delta E = E(n) - E(m)$ (Rydberg Schuster law 1896).

The functions $E(n)$ are called energy terms. The energy terms of the alkali metals are generalizations of the energy terms of the hydrogen atom.

The spectra of the alkali metals 2 Comment 3

In addition, the diffuse series was discovered together with the sharp series.

In the figures, the transitions of the diffuse series with the largest transition energies are marked in green.

On emission, the transitions start at energy levels marked with the letter ‘d’ (for diffuse series).

In 1907 Arno Bergmann discovered a fourth series. The corresponding energy levels are marked with the letter ‘f’.

Revision

Summary in questions

1. Give the uncertainty relations of momentum, energy, and angular momentum.
2. Give reasons for the fact that atomic orbitals differ so much from the orbits of planets.
3. Calculate the wavelength when electrons are accelerated with a voltage of 500 MV.
4. Give the general notation for the description of nuclei.
5. Calculate the shortest wavelength of the Lyman-, the Balmer, and the Paschen series.
6. Give Bohr's postulates.
7. Give the numerical value of Bohr's radius and the general formula for the radius