Modern Physics

Contents:

Modern Physics: Comment

The physical properties of atoms and solids are determined by electron waves.

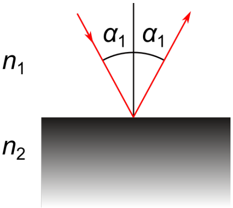
Therefore, it makes sense to remember both general wave properties and the basics of classic wave optics.

Classical wave optics

Reflection and Refraction

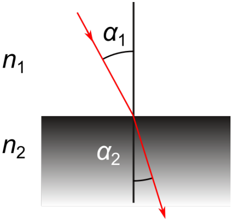
Classical wave optics

Reflection and Refraction 1



law of reflection

$$\alpha_1 = \alpha_{1'}$$



law of refraction

$$n_1 \sin\alpha_1 = n_2 \sin\alpha_2$$

law of refraction in the course of time

- 10^{th} century: Ibn Sahl, rediscovered by Willebrord van Roijen Snell in 1620
- 1660 Pierre de Fermat
- 1678 Christian Huygens
- 1886 Maxwell's electrodynamics

Reflection and Refraction 1 Comment 1

It has been known in ancient times that light is reflected and refracted on glasses.

The first mathematically correct description was given by Ibn Sahl in Persia in the 10^{th} century.

Even without the aid of mathematical formulas, lenses had been used in Europe as reading aids and magnifying glasses since the 13^{th} century.

The work of Ibn Sahl was rediscovered in 1620 by Willebrord van Roijen Snell.

That is why the law of refraction is often called Snellius' law of refraction.

The first correct physical justification for the law of reflection and refraction was worked out by Pierre de Fermat up to 1660.

Reflection and Refraction 1 Comment 2

Fermat was the first to use an extremal principle to derive a law of nature.

He postulates that light takes the path between two points for which the transit time is shortest.

Fermat had made a fundamental discovery, the deep meaning of which was only fully recognized more than 250 years later by Louis de Broglie.

Based on the extremal principle, de Broglie shows how the wave-particle dualism can be understood and thus founded modern physics.

Reflection and Refraction 1 Comment 3

In 1678 Christian Huygens derived the law of reflection and refraction for waves.

With his opinion that light is a wave phenomenon, he stood in sharp contradiction to Newton, who describes light in the context of his mechanics as a particle phenomenon.

However, the Newtonian corpuscles have nothing to do with the light particles - the photons - in modern quantum optics.

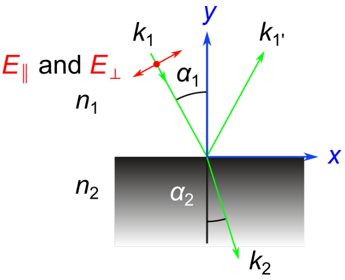
The wave-particle problem occupied the natural sciences intensively throughout the 18^{th} and 19^{th} centuries and was only solved in the context of classical wave theory by Albert Einstein’s special theory of relativity in 1905.

Reflection and Refraction 1 Comment 4

It has been known since Heinrich Hertz's experiments in 1886 that electromagnetic waves explain the phenomenon of light.

Classical optics can be fully understood in the context of Maxwell's theory of electrodynamics.

Reflection and Refraction 2



plane wave

$$\vec{E} = \vec{E}_0 \exp i(\vec{k}\vec{r} - \omega t)$$

the phase of all three waves must be the same at the interface for all position vectors $\vec{r}$ and times:

$$\vec{k}_1\vec{x} = \vec{k}_{1'}\vec{x} = \vec{k}_2\vec{x}$$

perpendicular incidence

$$r = \frac{E_r}{E_0} = \frac{n_1 - n_2}{n_1 + n_2}$$

- $\vec{k}_1$, $\vec{k}_{1'}$ and $\vec{k}_2$ lie in the same plane
- $k_{1,x} = k_{1',x} \quad \rightarrow \quad \alpha_1 = \alpha_{1'}$
- $k_{1,x} = k_{2,x} \quad \rightarrow \quad n_1 \sin\alpha_1 = n_2 \sin\alpha_2$

Reflection and Refraction 2 Comment 1

The figure illustrates the reflection and transmission for electromagnetic waves.

A plane wave falls from the top left onto an interface (e.g. air-glass).

The wave number vector $\vec{k}_1$ of this wave lies in the xy-plane.

The electric field of this wave can be divided into a component that vibrates in the xy plane and a component that vibrates perpendicular to the xy plane.

The vertical component is symbolized by a red point.

At the interface one has to demand that the phases of the incoming wave, the reflected wave and the transmitted wave are the same.

Reflection and Refraction 2 Comment 2

From this requirement it follows that the wave number vectors of the reflected waves $\vec{k}_{1'}$ and of the transmitted waves $\vec{k}_2$ must also lie in the xy-plane.

But it also follows from this that the x components of the wavenumber vectors must be the same. This results in the law of reflection and refraction.

Note: $\nu\lambda = c/n \quad \rightarrow \quad \lambda = \lambda_0/n \quad \rightarrow \quad k = 2\pi/\lambda = nk_0$.

Reflection and Refraction 2 Comment 3

The Maxwell equations can also be used to calculate the ratio of the electric field strengths of the three waves for both directions of polarization.

This results in complex formulas known as Fresnel's formulas.

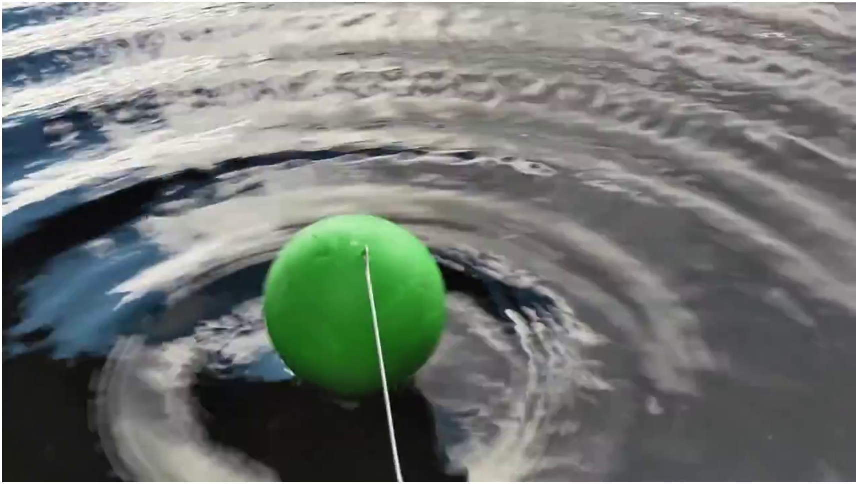
In the context of this lecture, only the special case of perpendicular incidence is required.

For the ratio of the reflected field strength E_r to the incident field strength E_0, the formula framed in red results for the reflection coefficient r.

Coherence

Classical wave optics

Coherence 1



(InterferenzWasserwellen.mp4)

Coherence 1

Comment 1

Classic wave optics deal with interference and diffraction.

Diffraction means that waves can propagate around corners.

This is evident for water and sound waves, but not for light waves.

The video shows the interference of water waves.

Interference is caused by the superposition of waves of the same wavelength.

Interferences in one dimension lead to standing waves.

Coherence 1 Comment 2

As with standing waves, there are areas where the superimposed waves cancel each other out.

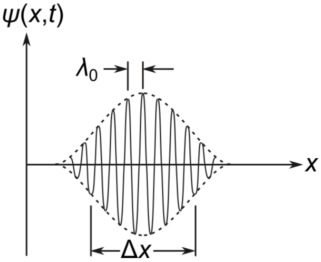
This is known as destructive interference.

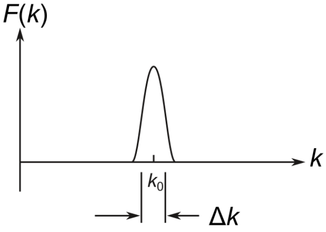
The amplitude is maximal for constructive interference.

Coherence 2

The description of waves by sine and cosine functions is an idealisation.

Waves have always a finite length $\rightarrow$ wave packets





Coherence 2 — Comment 1

In order to observe interference, light must be coherent.

The two spheres in the previous video have to excite the water surface with the same frequency so that an interference pattern can form.

In addition, the two balls must more or less simultaneously excite the surface of the water.

No interference pattern can form if the balls excite the water waves at very different times.

The excitation of the balls creates wave trains or wave packets of finite length.

An interference pattern can only form when the two wave packets overlap.

Coherence 2 Comment 2

This is evident with water waves as we can see the waves.

It is not obvious in visible light.

Instead of waves, we see colours..

The eye does not provide any information about the frequency or wavelength and the length of wave packets.

Unfortunately, the wave packets of natural light are very short, so it is not easy to observe interference.

It is therefore not obvious that visible light is a wave phenomena.

Around 1668 Newton was the first to observe interference in the form of Newton’s rings.

Coherence 2 Comment 3

He was also the first to use Newton's rings to assess the quality of lenses and spherical mirrors.

Newton made an attempt in 1672 to describe the phenomenon of interference in the context of a corpuscular theory of light.

However, this only succeeds with the modern quantum theory of electromagnetic waves.

The wave packets must be long compared to the dimensions of the experiment so that the wave packets overlap and form an interference pattern.

The nature of wave packets can already be seen in the videos of the first lecture.

Coherence 2 Comment 4

Waves are excited on a water surface by a falling stone and spread from the point of impact.

The water pulse generated by the stone quickly breaks down into its harmonic components due to dispersion.

The harmonic components propagate with the respective phase velocity, so that long water waves reach the shore faster than the waves with short wavelengths.

This experiment shows that each wave packet consists of a spectrum of harmonic waves with different wavelengths and wave numbers.

The left figure shows the wave packet.

The right figure shows the spectrum of wave numbers.

Coherence 2 Comment 5

The length of a wave packet is proportional to the inverse width of the spectrum, i.e. the length of the wave packets increases as the width of the spectrum decreases.

The wave packet still resembles a harmonic wave if the width of the spectrum is not too large.

The length of the wave packets is called the coherence length.

For an ideal sine or cosine wave, the width of the spectrum is zero and the wave is infinitely long.

This ideal case cannot be realised in nature.

Coherence 2

Comment 6

However, sine and cosine waves are used to describe experiments because of the simplicity of these functions.

Spectral lamps with narrow spectral lines emit light with long wave packets.

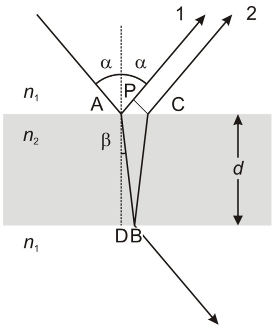
Lasers have been used since 1960 to produce light with long coherence lengths.

Thin films

Classical wave optics

Interference on thin films 1

Interference on dielectric thin films which are characterised by a refractive index.



Interference on thin films 1 — Comment 1

The sketch shows the reflection on a thin film.

The rays run parallel to the *k*-vectors of the waves.

The wave fronts are not shown in the sketch.

An incident ray is reflected at point A in ray 1.

A fraction of the wave penetrates the film and is reflected at point B.

A fraction of this beam is transmitted in beam 2 at point C.

Interference on thin films 1 — Comment 2

The distance between point A and P is generally different from the distance A, B, C.

Interference can be observed when the waves at point A and C come from the same wave packet.

Otherwise there is no fixed relationship between the phase of the two waves.

Since the wavelength of the wave packets of visible light is small, this type of interference can only be observed on thin layers.

Interference on thin films 1 — Comment 3

In principle there is also a reflected ray at point C, which is not shown in the sketch.

This ray can usually be neglected.

This is due to the reflection coefficient of the electromagnetic waves.

In general, the reflection coefficients are given by complicated formulas which depend on the polarisation of the wave.

The simple formula for the reflection coefficient, which was given in the first lecture, results only for perpendicular incidence.

Interference on thin films 2

$$r = \frac{k_1 - k_2}{k_1 + k_2} = \boxed{r = \frac{n_1 - n_2}{n_1 + n_2}}$$

With $\nu\lambda = v$ and $v = c/n$

$$\lambda = \frac{c}{n\nu} \quad \text{and} \quad k = \frac{2\pi}{\lambda} = 2\pi\frac{n\nu}{c} = n\frac{2\pi\nu}{c} = n\frac{2\pi}{\lambda_0} = nk_0$$

λ_0 denotes the vacuum wavelength and $k_0 = 2\pi/\lambda_0$ the wave number in vacuum.

E.g. for the interface air ($n_1 = 1$) glass ($n_2 = 1.5$) is $r = -0.2$ and only 4% (i.e. r^2 !!) of the intensity is reflected

Interference on thin films 2 Comment 1

The first formula was derived in the first lecture for the case of rope waves.

The formula framed in red results when the wave number is replaced by the refractive index.

Since the phase velocity decreases with increasing refractive index, the wavelength becomes shorter with increasing refractive index.

Consequently, the wavenumber is proportional to the refractive index.

The reflection coefficient at the air / glass interface is -0.2.

The reflection coefficient refers to the amplitude of the electric field strength of the wave.

Interference on thin films 2 Comment 2

Since the reflected intensity is proportional to the square of the electric field, only 4% of the total intensity is reflected.

Although the exact value of the reflected intensity depends on the angle of incidence and the polarization, it is clear that most of the incident intensity is transmitted and not reflected.

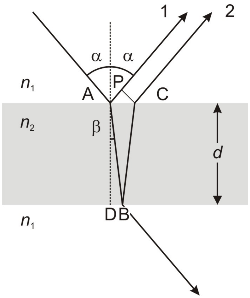
Most of the intensity is transmitted at points B and C.

The small reflected portion at point C can be neglected.

Therefore the intensity of ray 1 is roughly the same as the intensity of ray 2 and only these two rays need to be considered.

The intensity of all other rays is small and can be neglected.

Interference on thin films 3



For perpendicular incidence is the path difference $2d$ between ray 1 and 2. The additional phase of ray 2 is

$$\Delta\varphi_2 = k_2 \cdot 2d = n_2 k_0 \cdot 2d = n_2 \frac{2\pi}{\lambda_0} 2d$$

For $n_1 < n_2$ is $r < 0$ and $\Delta\varphi_1 = \pm\pi$

Interference on thin films 3 Comment

At normal incidence, the path difference between ray 1 and ray 2 is twice the layer thickness.

The phase difference determines the amplitude of the superposition of beams 1 and 2.

The spatial part of the phase results from the product of the wave vector and the position vector.

The additional phase of ray 2 results from the wave number k_2 in the slice multiplied by twice the slice thickness.

The reflection coefficient at the air / glass interface is negative.

This corresponds to an additional phase of $\pm\pi$ for ray 1.

Interference on thin films 4

For perpendicular incidence is the phase difference between ray 2 and 1

$$\Delta\varphi = \Delta\varphi_2 - \Delta\varphi_1 = n_2 \frac{2\pi}{\lambda_0} 2d \pm \pi$$

The condition for constructive interference is

$$\Delta\varphi = 2\pi m \quad \text{and} \quad m = 0, 1, 2 ...$$

The layer thickness is

$$d_m = \frac{1}{2n_2}\left(m\lambda_0 + \frac{\lambda_0}{2}\right) = (2m+1)\frac{\lambda_0}{4n_2}$$

and with $\lambda_2 = \lambda_0 / n_2$

$$d = \frac{\lambda_2}{4}, 3\frac{\lambda_2}{4} ...$$

Interference on thin films 4 Comment

The first formula framed in red gives the phase difference between ray 1 and 2 for perpendicular incidence.

It does not matter whether the refractive index of the layer is larger or smaller than that of the surrounding material.

The amplitude is maximal when the phase difference between ray 1 and 2 is a multiple of 2π.

The layer thickness can be calculated with the condition for constructive interference.

The layer thickness must be an odd multiple of a quarter of the wavelength within the layer.

Interference on thin films 5

Condition for destructive interference

$$\Delta\varphi = \pi m \quad \text{and} \quad m = 1, 3, 5....$$

The layer thickness is

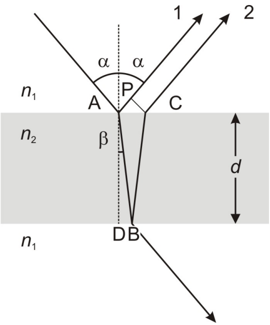
$$d_m = \frac{m\lambda_0}{2n_2} = m\frac{\lambda_2}{2} \quad \text{and} \quad m = 1, 2, 3, ...$$

Interference on thin films 5 Comment

Destructive interference occurs when the phase difference between beam 1 and 2 is an odd multiple of π.

The layer thickness has to be a multiple of half the wavelength within the layer.

Interference on thin films 6



The phase difference between ray 2 and 1 is

$$\Delta\varphi = \Delta\varphi_2 - \Delta\varphi_1 = \frac{2\pi}{\lambda_2}\overline{ABC} - \frac{2\pi}{\lambda_1}\overline{AP} \pm \pi = \frac{2\pi n_1}{\lambda_0} 2d\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\alpha} \pm \pi$$

Interference on thin films 6 Comment

If the beam falls obliquely on the thin layer, then the distance between points A and P and the distance between the points A and C must be calculated for ray 1 and ray 2, respectively.

The distance ABC must be multiplied with the wave number k_2 and the distance AP by the wave number k_1.

With a little geometry, the underlined formula results for the phase difference between ray 2 and ray 1.

Interference on thin films 7

The superposition of two waves with the same amplitude ψ_0 and the phase difference $\Delta\varphi$ is

$$\psi = \psi_1 + \psi_2 = \psi_0 e^{i\vec{k}\vec{r}} + \psi_0 e^{i(\vec{k}\vec{r}+\Delta\varphi)}$$
$$= \psi_0 e^{i\vec{k}\vec{r}} \left(1 + e^{i\Delta\varphi}\right)$$
$$= \psi_0 e^{i\vec{k}\vec{r}} e^{i\Delta\varphi/2} \left(e^{-i\Delta\varphi/2} + e^{i\Delta\varphi/2}\right)$$
$$= \psi_0 e^{i\vec{k}\vec{r}} e^{i\Delta\varphi/2} 2\cos(\Delta\varphi/2)$$

The intensity is given by

$$|\psi|^2 = \psi \cdot \psi^* = \boxed{I = 4I_0 \cos^2(\Delta\varphi/2)}$$

Interference on thin films 7 Comment

It is not difficult to calculate the sum of two superimposed waves.

For the sake of simplicity, the same amplitude is assumed for both waves.

The intensity results from the square of the magnitude of the complex wave function.

The variation between the maxima and minima of the intensity is determined by the square of the cosine function.

Interference on thin films 8

Interference on thin films 8

Comment

The picture shows the interference on soap bubbles.

The typical interference colours appear in daylight.

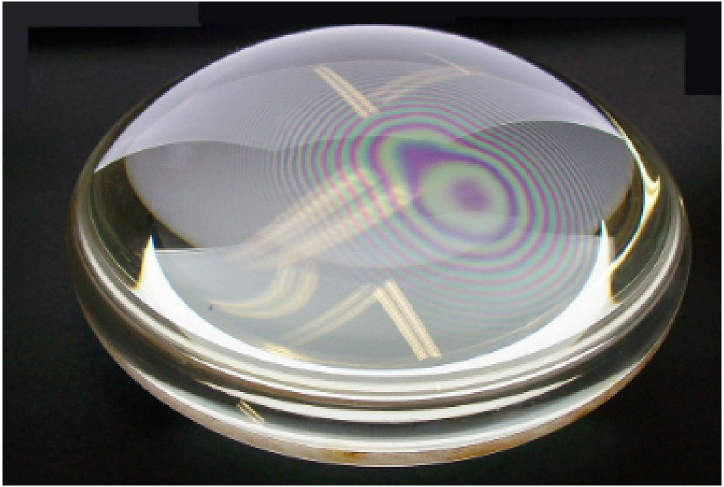
Some wavelengths of the white spectrum are eliminated by destructive interference.

The effect depends on the thickness of the soap bubble and the angle of incidence.

Usually these colours can also be observed on thin oil films on water.

Interference on thin films 9

Newton rings Interference on a thin air layer



Interference on thin films 9 Comment

The picture shows the Newton rings.

A lens is placed on a glass plate.

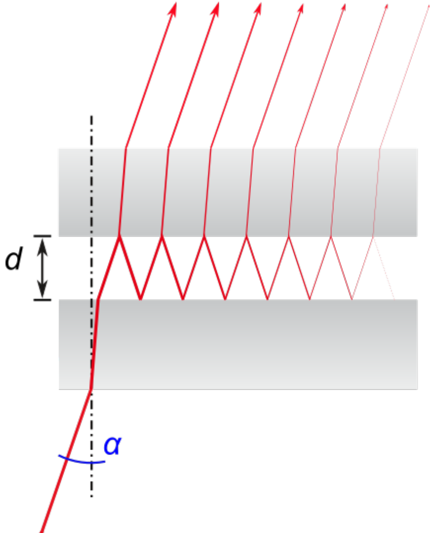
The interference is due to the small gap between the lens and the glass plate.

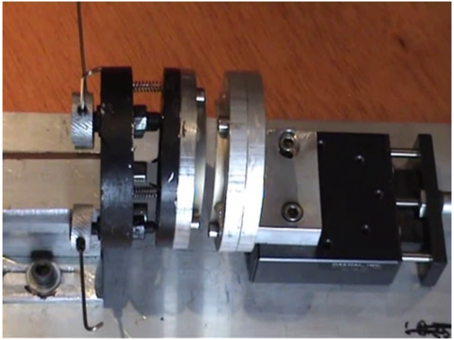
The thickness of the glass plate and the lens is usually very much greater than the coherence length, so that no interference can occur either in the lens or in the glass plate.

Fabry-Perot

Classical wave optics

Fabry-Perot Interferometer 1





(FabryPerotInterferometer.mp4)

Fabry-Perot Interferometer 1 — Comment 1

The Fabry-Perot interferometer is a useful device for resolving the fine structure of spectral lines.

It consists of two mirrored glass plates aligned in parallel.

Only a small fraction of the intensity can pass through the mirrors.

The left figure shows the path of a beam that enters the interferometer at an angle α.

Light entering the interferometer is trapped in the gap between the two glass plates.

Only a small fraction can leave the interferometer.

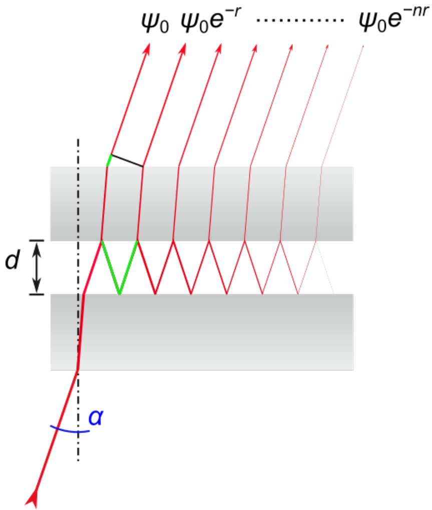
Fabry-Perot Interferometer 1 Comment 2

The light rays leaving the interferometer can interfere if the coherence length of the light is large enough.

In contrast to the interference on thin layers, very sharp interference maxima can be observed.

The video shows the construction of a simple Fabry-Perot interferometer.

Fabry-Perot Interferometer 2



Phase difference between neighbouring rays

$$\Delta\varphi = \frac{2\pi}{\lambda} 2d\cos\alpha$$

Condition for constructive interference

$$\Delta\varphi = 2\pi m \quad \rightarrow \quad m\lambda = 2d\cos\alpha_m$$

Half width at half height of the maxima

$$\Delta\varphi_{1/2} = r$$

Fabry-Perot Interferometer 2 Comment 1

Unlike the interference in thin films, the phase difference between adjacent beams occurs entirely in air.

The refractive indices $n_{1,2}$ in the formula $\Delta\varphi = \frac{2\pi n_1}{\lambda_0} 2d\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\alpha}$ are 1, so that the formula for the phase difference framed in red results.

The underlined formula for the path difference between two adjacent maxima results from the condition for constructive interference.

The path difference is proportional to the cosine of the angle of incidence.

The number m becomes smaller as the angle to the normal of the glass plate increases.

Fabry-Perot Interferometer 2 — Comment 2

The figure on the left shows the beam path in the interferometer again.

The amplitude of the rays leaving the interferometer is gradually reduced by a factor expressed by an exponential function.

The number *r* in the exponent indicates half the width of the interference maxima at half the intensity.

The reduction of the amplitude should be as small as possible in order to achieve high resolution.

Therefore, the light should be trapped within the gap of the interferometer as long as possible.

Fabry-Perot Interferometer 2 — Comment 3

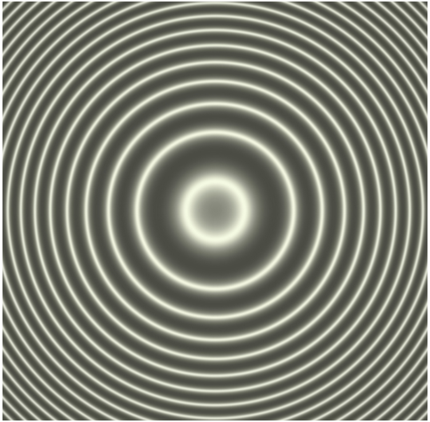
If the reduction factor is small enough, the number of interfering rays is limited by the coherence length of the light examined.

The Fabry-Perot interferometer only works with a large coherence length, i.e. with narrow spectral lines.

The condition for constructive interference also shows that the resolution of the interferometer is greatest for the smallest angle α_m.

Fabry-Perot Interferometer 3

Condition for constructive interference: $m\lambda = 2d\cos\alpha_m$



Fabry-Perot Interferometer 3 Comment

The figure shows the simulated output signal of a Fabry-Perot Interferometer.

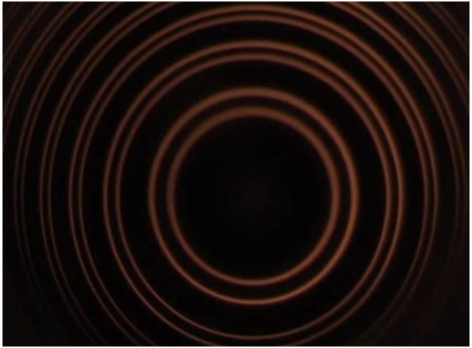
The order of the interference fringes decreases with increasing radius of the rings.

The radius of the rings also depends on the wavelength.

The radius of the rings decreases with increasing wavelength.

Fabry-Perot Interferometer 4: Splitting of the sodium D-lines

$$D_1 : \lambda = 589.5924\,\text{nm and } D_2 : \lambda = 588.9951\,\text{nm}$$



Fabry-Perot Interferometer 4 — Comment

The video shows the splitting of the yellow sodium line into two components.

The interference maxima show a double line structure.

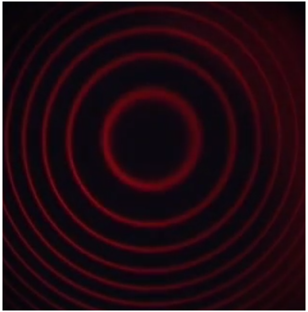
The greatest effect is observed for the two inner rings, which correspond to the highest *m* number for the path difference between adjacent rays.

Since the angle α decreases with increasing wavelength (compare $m\lambda = 2d\cos\alpha_m$) the inner ring corresponds to the D_1-line ($\lambda = 589.5924\,\text{nm}$) and the outer ring to the D_2-line ($\lambda = 588.9951\,\text{nm}$).

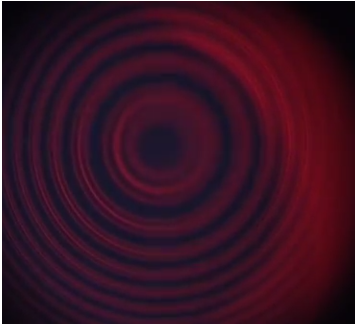
Fabry-Perot Interferometer 5

Splitting of the red cadmium line when a magnetic field is applied

without a magnetic field



with a magnetic field



(ZeemanCdLinie.mp4)

Fabry-Perot Interferometer 5 Comment

The video shows the splitting of the red cadmium line into three components when a magnetic field is applied.

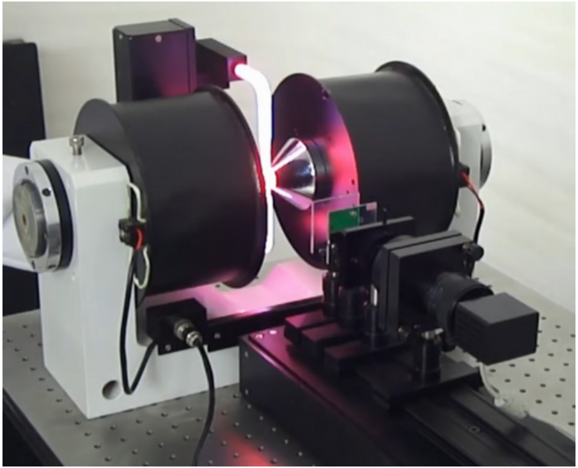
The splitting of the spectral lines in a magnetic field was discovered by Pieter Zeeman in 1896 and is called the Zeeman effect in honor of Pieter Zeeman.

Hendrik Lorentz concluded in 1899 from an analysis of the Zeeman effect that the spectral lines of atoms are caused by electrons.

Pieter Zeeman and Hendrik Lorentz were awarded the Nobel Prize in 1902 “in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena”

Fabry-Perot Interferometer 6

Splitting of the green Hg line in the magnetic field



(ZeemanGrueneHgLinie.mp4)

Fabry-Perot Interferometer 6 Kommentar

The figure shows the experimental setup.

The spectral lamp, which is filled with a gas of mercury atoms, stands between the poles of an electromagnet.

In front of it is the Fabry-Perot interferometer with the small camera for recording the interference image.

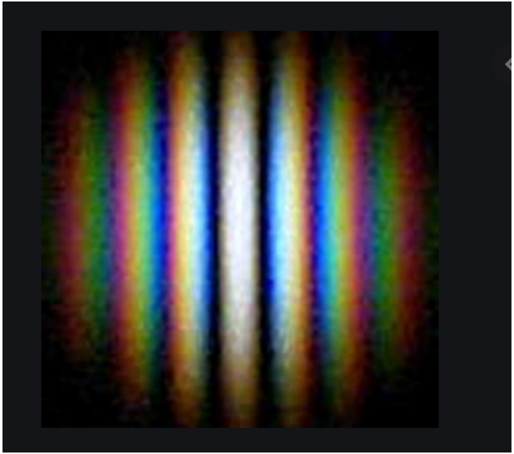
The video shows the experiment and the splitting of the interference fringes into three components when the magnetic field is switched on.

Double slit

Classical wave optics

Diffraction at the double slit 1

Thomas Young 1802



Diffraction at the double slit 1 Comment 1

Thomas Young's double slit experiment has been widely accepted as evidence that light is a wave phenomenon.

The picture shows the result of the experiment.

Daylight falls through a hole in a shutter onto a double slit and colored stripes can be seen on the wall behind the double slit.

This simple experiment created one of the greatest problems in physics in the 19^{th} century.

In 1725, the famous English astronomer James Bradley discovered the aberration of starlight. He found that the angle of observation of stars varies periodically over the course of the year.

Diffraction at the double slit 1 Comment 2

With Newton's corpuscular theory of light, he was able to trace this effect back to the movement of the earth around the sun.

Knowing the speed of the earth around the sun, Bradley was also able to determine the speed of light.

The speed of light determined by these astronomical measurements agrees well with the terrestrial measurements.

Amazingly, it turned out that the speed of light is the same for all observed stars.

However, in the context of Newton's corpuscular theory of light, the speed of light should also depend on the speed of the stars.

It was known that stars move at different and high speeds.

Diffraction at the double slit 1 Comment 3

In the context of a wave theory of light it was assumed that light waves propagate in a medium called the ether.

Therefore the speed of light waves is of course independent of the speed of the stars.

In the context of a wave theory, however, it is by no means obvious that the alignment of the wave fronts and thus the direction of propagation should depend on the speed of the earth.

This problem was solved by Augustin-Jean Fresnel in1818.

He assumed that the ether is partly carried along by the earth and thus explained the stellar aberration within the wave theory of light.

Diffraction at the double slit 1 Comment 4

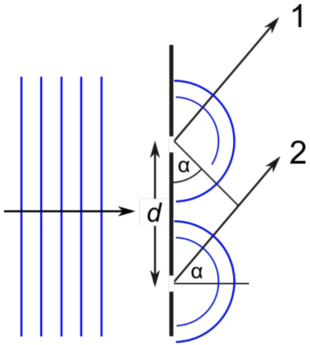
Through Fresnel’s ether theory, the wave theory of light was generally accepted.

However, Fresnel’s theory did not pass all experimental tests.

In the course of the 19th century the ether theory became more and more complicated and contradictory until the problems were solved in 1905 by Albert Einstein within the framework of the special theory of relativity.

Diffraction at the double slit 2

A plane wave comes in from the left and cylindrical waves leave the slits.



Diffraction at the double slit 2 Comment

The figure outlines the situation with a double slit.

The blue lines indicate the wave fronts.

The phase is constant along a wave front.

A plane wave meets the double slit from the left.

After the slits, the light spreads evenly in all directions.

This is indicated by the blue semicircles.

Cylindrical wave fronts leave the slits.

In small areas, the cylindrical wave fronts can be approximated by plane waves, which in turn can be assigned a beam direction.

Diffraction at the double slit 3

Diffraction at the double slit 3 Comment

The illustration shows the superposition of water waves.

The waves interfere and there are directions of contructive and destructive interference.

The parallel beams 1 and 2 can be used when the interference is observed far from the double slit or when a lens is used to focus parallel beams.

This experimental situation is called Fraunhofer diffraction.

Inclined rays must be used to describe diffraction near the slits.

This experimental situation is called Fresnel diffraction.

Diffraction at the double slit 4

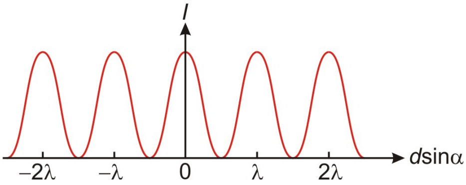
The condition for constructive interference of parallel rays is

$$d \sin \alpha_m = m\lambda$$

and the condition for destructive interference

$$d \sin \alpha_m = \left(m + \frac{1}{2}\right)\lambda$$

with $m = 0, \pm 1, \pm 2 \ldots.$



Intensity after a double slit

Diffraction at the double slit 4 Comment 1

The path difference between ray 1 and 2 results from the product of the distance between the slits multiplied with the sine of the deflection angle.

Constructive interference occurs when the path difference is a multiple of the wavelength.

Destructive interference occurs when half a wavelength is added.

The condition for constructive interference depends on the wavelength.

This is why colored strips can be observed when the gaps are illuminated with white light.

Diffraction at the double slit 4 Comment 2

For red light, the maxima are observed for larger deflection angles than for blue light, since the wavelength of red light is greater than the wavelength of blue light.

The intensity behind a double slit is determined by the square of the cosine function.

This is similar to the interference on thin films.

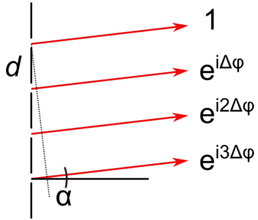
The figure shows the intensity as a function of the path difference between rays 1 and 2.

Grids

Classical wave optics

Diffraction on a grating 1

Grid with four slits



$$\Delta\varphi = \frac{2\pi}{\lambda} d \cdot \sin\alpha$$

The condition for the main maxima is the same as for the double slit

$$\boxed{d \sin\alpha_m = m\lambda}$$ and $$m = 0, \pm 1, \pm 2, \ldots$$

Diffraction on a grating 1 — Comment 1

Before discussing the case of a grid with many slits, I consider the simple case of a lattice with four slits.

The double slit is supplemented by two additional slits.

As with the double slit, parallel rays are considered that emanate from the slits in different directions.

The phase difference between two adjacent beams results from the same formula that also applies to the double slit.

The condition for constructive interference is also the same as for the double slit.

The path difference between two adjacent beams is a multiple of the wavelength for the main maxima of the intensity.

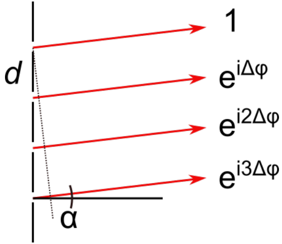
Diffraction on a grating 1 — Comment 2

In the sketch of the grating, the additional phase factor is given for each ray.

The additional phase difference between two neighboring rays must be a multiple of 2π for the main maxima.

Diffraction on a grating 2

Minima of the intensity



The phase factors add up to zero if the phase difference between two neighboring rays is $\Delta\varphi = \pi/2$ or the path difference is $\lambda/4$, i.e.

$$1 + e^{i\pi/2} + e^{i2\pi/2} + e^{i3\pi/2} = 1 + i - 1 - i = 0$$

Diffraction on a grating 2

Comment

The phase factor between two neighboring rays is equal to $\pi/2$ if the path difference between two neighboring rays is a quarter of a wavelength.

The four phase factors are 1, i, -1 and $-$i, so that when the four waves overlap, the amplitudes add to zero.

If the path difference is $\lambda/2$, the phase factors are $+1$, -1, $+1$ and -1 so that the amplitudes add up to zero when the four waves overlap.

If the path difference between two neighboring rays is three quarters of the wavelength, the phase factors are 1, $-$i, -1 and $+$i and the amplitudes of the four waves add up to zero when the waves are superimposed.

With four slits there are three intensity minima between two main maxima.

Diffraction on a grating 3: General rules for a grid with N slits

Principal maxima result when the path difference between neighboring rays is an integral multiple of λ

Minima arise when the path difference between two neighboring rays is a multiple of λ/N and not a multiple of λ

The deflection angles for the minima are

$$d \sin \alpha_{m'} = \frac{m'}{N}\lambda$$

with $m' = \pm 1, \pm 2, \ldots$ and m' is not a multiple of N

Diffraction on a grating 3 Comment

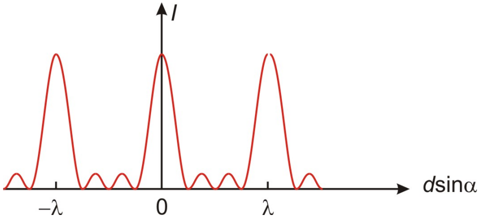
The general rules for the principal maxima and the minima of a diffraction grating are as follows:

For the main maxima, the path difference between two neighboring rays is a multiple of the wavelength λ.

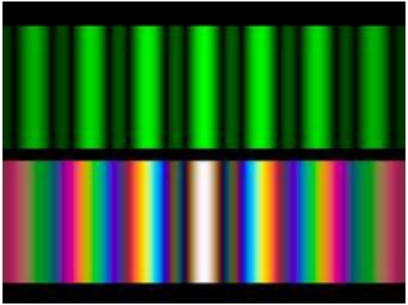
For the minima of the intensity, the path difference between two neighboring rays is a multiple of the wavelength divided by the number N of the slits, as long as this condition does not lead to the condition for the maxima.

Diffraction on a grating 4

Intensity for a grid with 4 slits



Intensity of a grid with 3 slits



Diffraction on a grating 4 Comment

The figure on the left shows the intensity as a function of the path difference of neighboring rays after a grating with four slits.

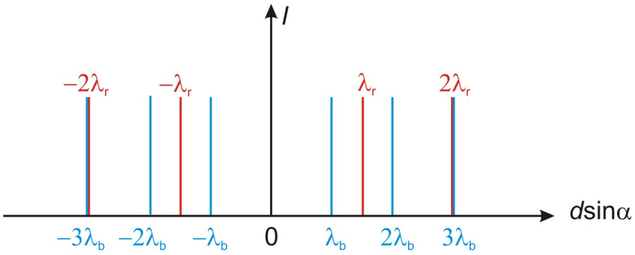
The right figure shows the diffraction on a grid with three slits.

The two minima between the main maxima are clearly visible when almost monochromatic light is used.

When white light is used, the diffraction pattern of the various wavelength overlap.

Diffraction on a grating 5

Diffraction gratings can be used for spectroscopy



The highest spectral resolution of a grid

$$\frac{\lambda}{\Delta\lambda} = Nm$$

Diffraction on a grating 5 Comment 1

Since the width of the main maxima becomes smaller with an increasing number of slits, diffraction gratings can be used for spectroscopy.

The possible resolution of a grid increases with the number of slits.

The figure shows schematically the intensity after a grating with many slits.

The number of slits is so large that only the intensity of the main maxima has to be taken into account.

The intensity is plotted as a function of the path difference between two adjacent rays for two different colors (red and blue).

Diffraction on a grating 5 Comment 2

The path difference between the main maxima increases with the order of diffraction.

The second order path difference is e.g. twice as large as the first order path difference.

Therefore the resolution of a grating is also proportional to the diffraction order.

The total resolution of a diffraction grating is therefore determined by the product of the diffraction order m and the number of slits N.

The formula outlined in red indicates the resolution of a diffraction grating.

The figure also shows that different diffraction order can overlap.

In order to avoid this effect, filters must be used that limit the spectral range.

Diffraction on a grating 5 Comment 3

In order to achieve the maximum resolution of a diffraction grating, all slits of the grating must be illuminated.

The light must be parallel in front of the grating.

This is achieved by projecting the light source onto an entrance slit.

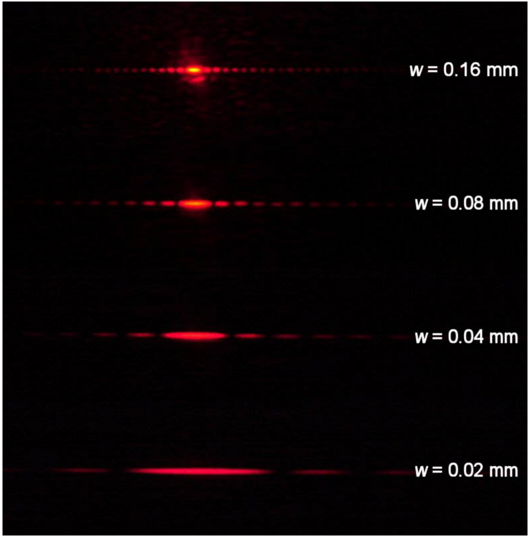
The entrance slit is at the focal point of a second lens that guides the light onto the grating.

Since the entrance slit must always have a finite slit width, this always reduces the ideal resolution of a diffraction grating.

Single silt

Classical wave optics

Diffraction at a single slit 1



Diffraction at a single slit 1 Comment

Similar to the double slit, minima and maxima of the intensity can be observed when light falls on a single slit.

The figure shows the intensity after a single slit that is illuminated with a laser.

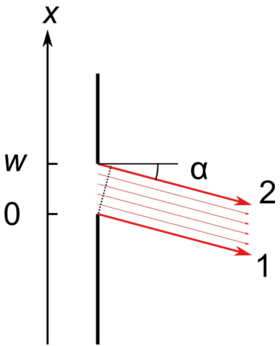
The vertical length of the strips is therefore short.

The width of the slits is reduced from 0.16 mm to 0.02 mm.

The maxima of the diffraction pattern become wider and the distance between the minima increases as the width of the slit is reduced.

Diffraction at a single slit 2

A single slit illuminated by a plane wave



The phase $\Delta\varphi(x)$ of the partial rays depends on the location x

$$\Delta\varphi(x) = \frac{2\pi}{\lambda} x \sin\alpha$$

Diffraction at a single slit 2 Comment 1

The sketch shows a slit perpendicular to the plane of projection.

The coordinate $x = 0$ is assigned to the lower end of the slit and $x = w$ to the upper end.

A coordinate can be assigned to each point within the slit.

A partial beam starts from every point within the slit.

Parallel partial beams can be considered if the diffraction pattern is observed far away from the slit.

Some partial beams are marked by red arrows in the sketch.

The path difference to partial beam 1 is proportional to the coordinate x

Diffraction at a single slit 2 Comment 2

The formula underlined in red indicates the phase difference of a partial beam, which originates from the location with the coordinate x, to partial beam 1.

The path difference must be multiplied by the wave number $2\pi/\lambda$.

Diffraction at a single slit 3

Integration over the partial beams

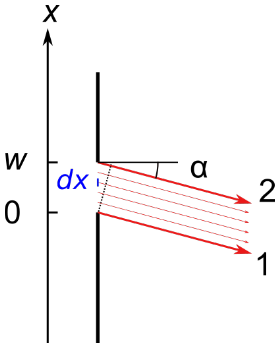
$$\psi = \psi_0 \frac{1}{w} \int_{x=0}^{w} e^{i\Delta\varphi(x)} dx = \psi_0 \frac{1}{w} \left. \frac{e^{i\Delta\varphi(x)}}{i\frac{2\pi}{\lambda}\sin\alpha} \right|_{x=0}^{w}$$

and with $\Delta\varphi = \Delta\varphi(w) = \frac{2\pi}{\lambda} w \sin\alpha$

$$\psi = \psi_0 \frac{e^{i\Delta\varphi} - 1}{i\Delta\varphi} = \psi_0 e^{i\Delta\varphi/2} \frac{e^{i\Delta\varphi/2} - e^{-i\Delta\varphi/2}}{i\Delta\varphi}$$

is the intensity $\psi\psi^*$ behind the single slit

$$I = I_0 \left(\frac{\sin \Delta\varphi/2}{\Delta\varphi/2} \right)^2 \quad \text{and} \quad I_0 = \psi_0 \psi_0^*$$



Diffraction at a single slit 3 Comment

The amplitude of each partial beam is proportional to the differential *dx*.

The wave behind the gap results when all partial beams are integrated.

The factor $1/w$ is used to normalize the integral.

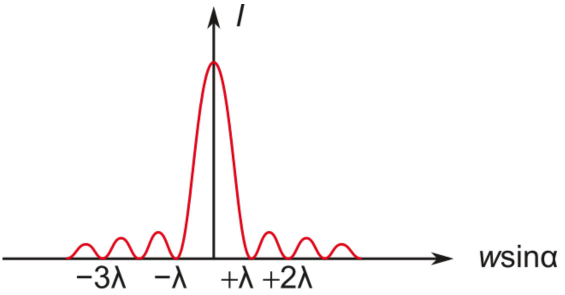
The calculation results in the formula outlined in red for the intensity behind the slit.

The phase difference $\Delta\varphi$ denotes the phase difference between the partial beams 2 and 1.

Diffraction at a single slit 4

Condition for destructive interference

$$m\pi = \frac{\Delta\varphi}{2} = \frac{\pi}{\lambda} w \sin\alpha \quad \rightarrow \quad \boxed{m\lambda = w\sin\alpha} \quad m = \pm 1, \pm 2 \ldots$$



Diffraction at a single slit 4 Comment

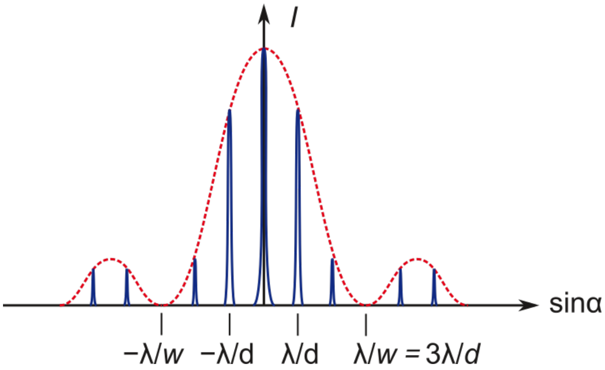
For the diffraction minima, the phase difference between ray 1 and 2 must be a multiple of π, and the path difference between ray 1 and 2 is a multiple of the wavelength.

In contrast to this, maxima result for a grating when the path difference is a multiple of the wavelength.

The figure shows a sketch of the intensity behind a single slit as a function of the path difference between rays 1 and 2.

Diffraction at a single slit 5

The effect of the single slits on a diffraction grating



Intensity behind a diffraction grating with $w = d/3$

Diffraction at a single slit 5 — Comment 1

The slits in a grid always have a finite width.

The intensity behind a grating results from the product of the intensity of the grating and the single slit.

The figure shows the intensity behind a diffraction grating with many slits.

The intensity between the main maxima is negligibly small.

The intensity of the main maxima is modulated by the intensity function of the individual slit.

In the sketch it is assumed that the slit width is one third of the slit distance.

Diffraction at a single slit 5 — Comment 2

Therefore, the principal maxima of the third, sixth, etc. order are suppressed by the condition for destructive interference of the single slit.

The intensity function of the single slit can be modified in such a way that the main intensity is shifted from the zero order to another diffraction order.

This is called blazing.

If the intensity function of the individual slits is changed, the intensity of the main maxima changes, but not the respective deflection angle.

Due to blazing, the intensity is directed to a selected diffraction order.

Revision

Summary in questions

1. What is the coherence length?
2. When is light called coherent?
3. What is the interference condition for constructive interference for a thin film in air and perpendicular incidence of light?
4. Write down the reflection coefficient for the electric field at normal incidence of light on a dielectric interface.
5. What is the Fraunhofer type of observation?
6. Sketch a Fabry-Perot interferometer.
7. What determines the resolution of the Fabry-Perot interferometer?
8. Sketch the interference pattern of a Fabry-Perot interferometer when it is illuminated with a monochromatic light wave.

Summary in questions 2

9. What is the interference condition for constructive interference with a double slit?
10. Sketch the intensity after a double slit as a function of the path difference of the rays.
11. What is the interference condition for constructive interference for a grid with N slits?
12. What is the condition for destructive interference for a grid with N slits? item Sketch the intensity of a grid with 5 slits as a function of the path difference of neighboring partial beams.
13. How big is the spectral resolution $\lambda/\Delta\lambda$ for a grid with N slits.

Summary in questions

14. Why does the resolution depend on the diffraction order?
15. Sketch the intensity after a single slit as a function of the path difference of the peripheral rays.
16. Sketch the intensity after a grid with 4 slits as a function of the path difference of neighboring rays, if the slit openings are a quarter of the slit distance.