Modern Physics

Contents:

Michelson Experiment

Special Relativity

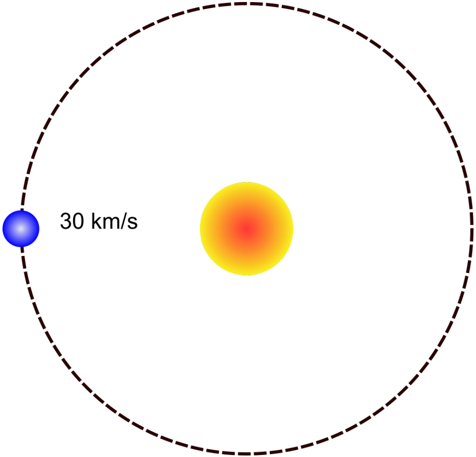
Michelson-Morley experiment 0

Stellar aberration (discovered and explained by James Bradley in 1727)

(Aberration.mp4)

Michelson-Morley experiment 1

Michelson's project: Measurement of the earth's speed relative to the ether



Michelson-Morley experiment 1 Comment 1

The aberration of light from stars was discovered by James Bradley in 1727.

The angle of observation of a star varies periodically over the course of the year.

With Newton's corpuscle theory of light, James Bradley was able to trace the effect back to the movement of the earth around the sun.

Since the speed of the earth is known, the speed of light can be determined from the aberration angle.

He discovered that the speed of light is the same for all the stars studied.

This astonishing result was confirmed more and more precisely in the following years.

Michelson-Morley experiment 1 Comment 2

The result is astonishing, since in Newton's corpuscular theory of light it can be expected that the speed of light also depends on the speed of the star.

At the beginning of the 19th century it was no longer possible to ignore the wave properties of light.

Similar to sound waves or waves on the water surface, the speed of light should be determined by the medium in which the waves propagate.

The hypothetical medium in which light waves propagate was called ether.

Unfortunately, it is not trivial to explain the aberration effect in a classical wave theory.

The inclination of the wave fronts and thus the direction of propagation of the wave is independent of the speed of an observer.

Michelson-Morley experiment 1 Comment 3

A possible solution to this problem was provided by Fresnel's ether theory in 1819.

The predictions of Fresnel's ether theory could be confirmed experimentally in first order in v / c.

With this success of Fresnel's ether theory, the wave theory of light was generally accepted.

Mickelson designed his famous experiment to test the ether theory in second order of v / c.

In contrast to measurements of the aberration of stars, which can be easily understood in a classical particle picture of light, Michelson tried to measure the influence of the earth's speed on the speed of light directly through the interference of waves.

Michelson-Morley experiment 1 Comment 4

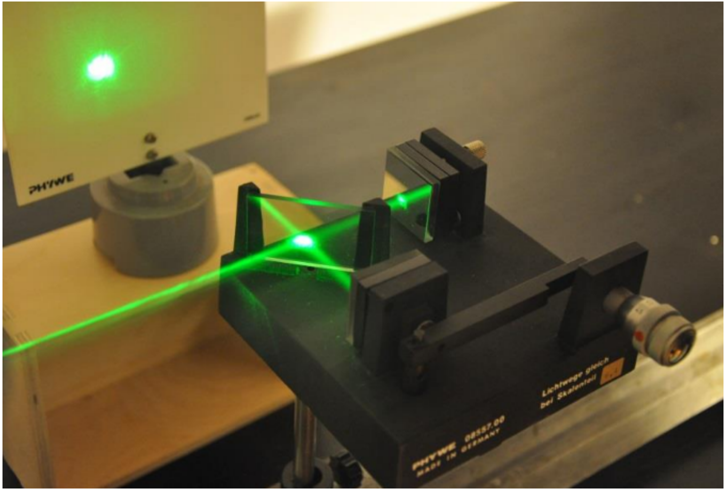
The Michelson-Morley experiment is an experiment designed to test a historical theory of the ether.

The Michelson-Morley experiment is not at all necessary to find or understand the theory of relativity.

However, the Michelson-Morley experiment nicely illustrates two important consequences of the Lorentz transform: time dilation and length contraction.

It is therefore worth taking a closer look at the historical experiment.

Michelson-Morley experiment 2



Michelson-Morley experiment 2 Comment

The central part of a Michelson interferometer is a beam splitter.

This is a glass plate that is coated on one side with a thin metal layer.

Part of the incident beam is reflected to one mirror and the remainder is transmitted to a second mirror.

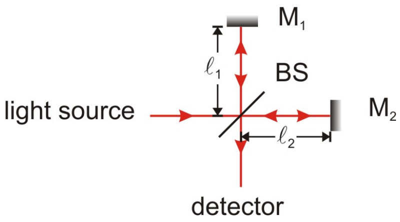
This process is repeated for the rays that are reflected by the two mirrors.

Now there are two rays that propagate towards the observation screen and interfere with each other.

Michelson-Morley experiment 3

Michelson Interferometer

- BS: beam splitter
- M1 and M2: mirror 1 and 2



(michelsonmorleystatic.mp4)

time on the way $\overline{BS - M_1 - BS}$

$$t_1 = \frac{2\ell_1}{c}$$

time on the way $\overline{BS - M_2 - BS}$

$$t_2 = \frac{2\ell_2}{c}$$

phase diffence between ray 1 and 2

$$\Delta\varphi = \omega(t_1 - t_2) = \frac{\omega}{c}(2\ell_1 - 2\ell_2) = \frac{2\pi}{\lambda}(2\ell_1 - 2\ell_2)$$

Michelson-Morley experiment 3 Comment

The sketch shows the schematic setup of the interferometer.

The phase difference between the two beams reaching the detector can easily be calculated.

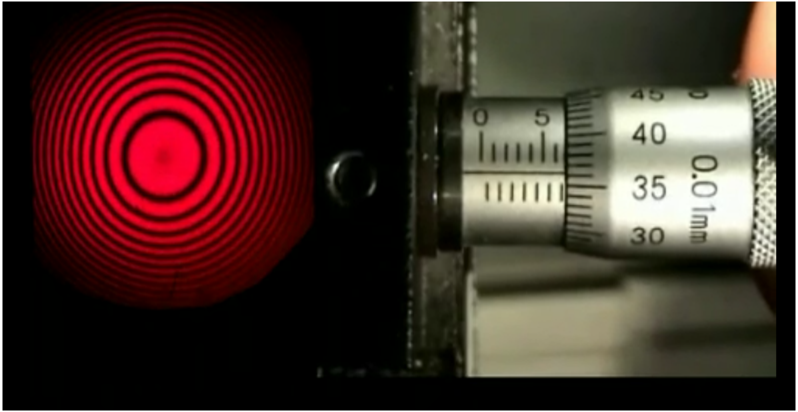
The time it takes for the light to travel the distance between the beam splitter and the two mirrors is given by the underlined formulas.

The phase difference results when the time difference between t_1 and t_2 is multiplied by the angular frequency of the light wave.

Alternatively, the path difference between the two beams that reach the detector can be multiplied by the wavenumber.

The animation shows the situation when the interferometer is resting in the ether.

Michelson-Morley experiment 4



(MichelsonInterferometerPathVariations.mp4)

Michelson-Morley experiment 4 — Comment

The video shows the interference pattern of a Michelson interferometer.

A slightly divergent Laser beam is used.

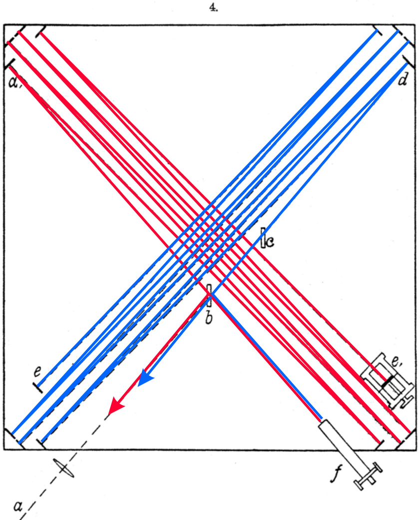
The interference pattern changes when the distance between a mirror and the beam splitter is changed.

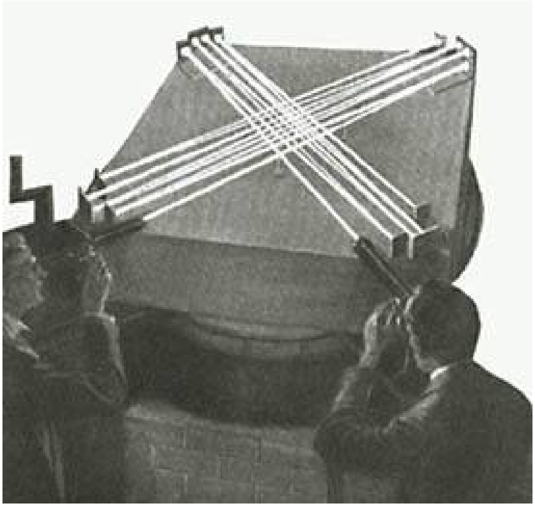
Michelson Morley Experiment (1881) 5

Michelson-Morley experiment 5 Comment

The picture shows the first Michelson interferometer, which was built in 1881.

Michelson Morley (1887) Experiment 6





Michelson-Morley experiment 6 Comment

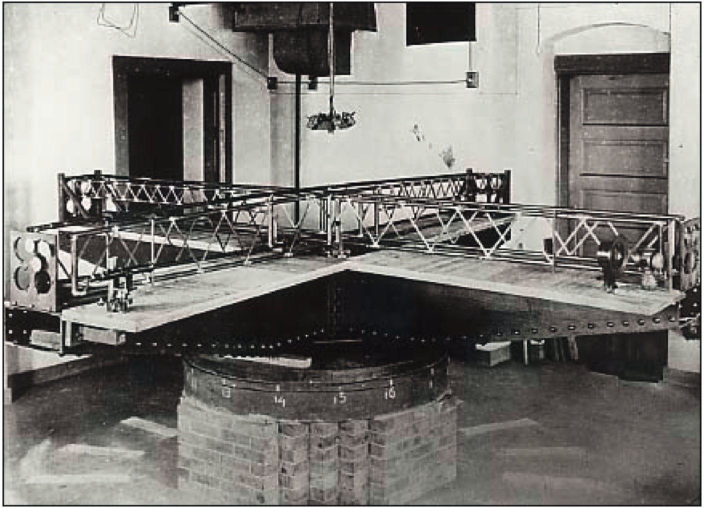
These pictures show the improved Michelson interferometer built in 1887.

The structure consists of a granite block that is placed in a mercury bath.

This structure greatly improves the mechanical stability.

The left picture shows that the light path has also been lengthened by at least a factor of ten.

Michelson Morley (1905) Experiment 7

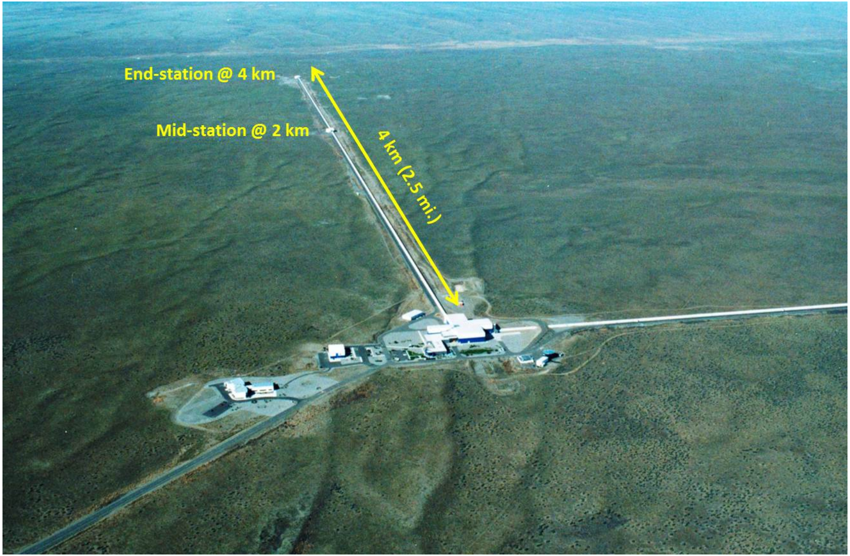


Michelson-Morley experiment 7 Comment

This picture shows the refined interferometer that was built in 1905.

The experiments by Michelson and Morley did not show the expected effect.

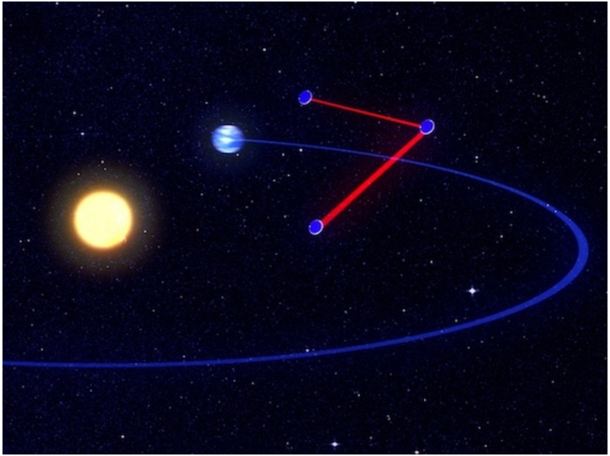
Michelson-Morley experiment: Ligo-Detector 8



Michelson-Morley experiment 8 Comment

Nowadays, Michelson interferometers are used to detect gravitational waves.

Michelson-Morley experiment 9



additional information: (https://www.elisascience.org/)

Michelson-Morley experiment 9 — Comment

Gravitational waves were successfully detected for the first time in 2015.

This opened a new window for the exploration of the cosmos that is not based on electromagnetic waves.

Today three gravitational wave detectors are working.

This allows the source of a gravitational wave to be localized and associated with electromagnetic signals.

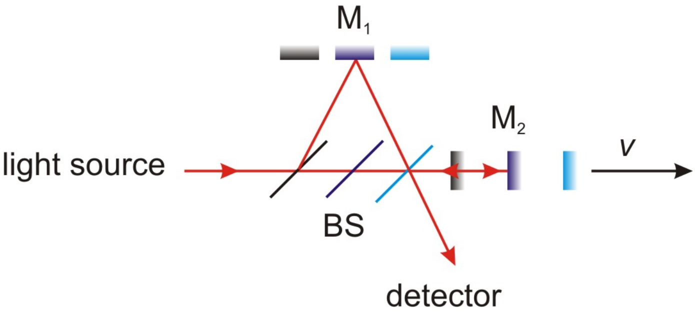
These gravitational wave detectors can be used to investigate sources of gravitational waves that are in our galaxy.

To study sources outside of our galaxy, much larger gravitational wave detectors are needed.

Therefore, a large interferometer in space is planned.

Michelson-Morley experiment 10

Michelson's idea: The interferometer moves with the earth through the "ether"



(michelsonmorleydinamic.mp4)

Michelson-Morley experiment 10 Comment

The figure illustrates Michelson's idea.

The interferometer moves through the ether at the speed of the earth.

The position of the interferometer is displayed three times in a row.

Due to the movement relative to the aether, the beam from the beam splitter to the mirror 1 has to be tilted a little forward.

Since the speed of light is much greater than the speed of the earth, the effect is very small and results automatically for a special ray in a slightly divergent bundle of light.

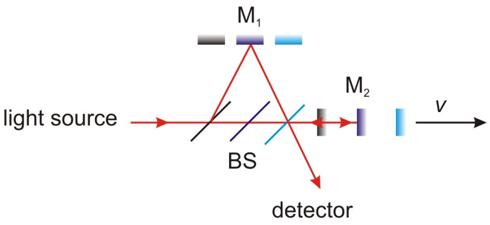
The video shows an animation of the expected effect.

Michelson-Morley experiment 11

a) time on path 1: $\overline{BS - M_1 - BS}$

$$\frac{t_1}{2} = \frac{\sqrt{\ell_1^2 + \left(\frac{vt_1}{2}\right)^2}}{c}$$

$$\rightarrow \boxed{t_1 = \frac{2\ell_1}{c} \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$



Michelson-Morley experiment 11 Comment

The time required on the way from the beam splitter to the first mirror can be calculated using the Pythagorean theorem.

The total time is twice the time from the beam splitter to the first mirror.

Michelson-Morley experiment 12

b) time on path 2: $\overline{BS - M_2 - BS}$

time $\overline{BS - M_2}$

$$t_{2\rightarrow} = \frac{\ell_2 + v\,t_{2\rightarrow}}{c} \quad \text{and} \quad t_{2\rightarrow} = \frac{\ell_2}{c - v} = \frac{\ell_2}{c}\frac{1}{1 - \frac{v}{c}}$$

time $\overline{M_2 - BS}$

$$t_{2\leftarrow} = \frac{\ell_2}{c}\frac{1}{1 + \frac{v}{c}}$$

and

$$t_{2\rightarrow} + t_{2\leftarrow} = \boxed{t_2 = \frac{2\ell_2}{c}\frac{1}{1 - \left(\frac{v}{c}\right)^2}} > t_1 \quad \text{when} \quad \ell_1 = \ell_2$$

$$t_1 = \frac{2\ell_1}{c}\frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Michelson-Morley experiment 12 Comment

For the necessary time on the way between the beam splitter and the second mirror, it must be taken into account that the mirror retracts from the beam splitter.

For the way back, the beam splitter approaches and the time required is shortened.

The sum of both times results in the equation framed in red.

The comparison shows that the time t_2 is larger than the time t_1 if the distance between the mirrors and the beam splitter is the same for both mirrors.

This is the expected effect shown in the last video.

Michelson-Morley experiment 13

phase diffence between ray 2 and 1

$$\Delta\varphi = \omega(t_2 - t_1)$$

with $\ell_1 = \ell_2 = \ell$ and $\omega/c = k = 2\pi/\lambda$

$$\Delta\varphi = \omega(t_2 - t_1) = \underline{\frac{2\pi}{\lambda}2\ell\left(\frac{1}{1-\left(\frac{v}{c}\right)^2} - \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)}$$

- velocity of the earth $v_{\text{Earth}} \approx 3 \cdot 10^4\,\text{m/s}$
- speed of light $c \approx 3 \cdot 10^8\,\text{m/s}$

$$\boxed{\frac{v}{c} \approx 10^{-4}} \quad \rightarrow \quad \text{Taylor expansion}$$

Michelson-Morley experiment 13 — Comment

Here, too, the phase difference results when the time delay between the two beams is multiplied by the angular frequency of the wave.

The phase difference also results from the effective path difference multiplied by the wave number.

Since the speed of the earth around the sun is so much smaller than the speed of light, a Taylor expansion can be used.

Michelson-Morley experiment 14

Maximum phase difference of the moving interferometer

$$\Delta\varphi = \omega(t_2 - t_1) = \frac{2\pi}{\lambda} 2\ell \left(1 + \left(\frac{v}{c}\right)^2 - 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right) = \underline{2\pi \frac{\ell}{\lambda}\left(\frac{v}{c}\right)^2}$$

$$\boxed{\text{The factor } \ell/\lambda \text{ amplifies the small ratio of } (v/c)^2}$$

The interferogram should change when the moving interferometer is rotated. This should make it possible to determine the direction and magnitude of the earth's speed $\vec{v}$

The expected effect is not observed!

Michelson-Morley experiment 14 — Comment 1

Applying the Taylor expansion gives the underlined formula.

The small ratio between the speed of the earth and the speed of light should be measurable through the large ratio between the light path and wavelength.

The rotation of the earth changes the orientation of the interferometer in relation to the earth's speed, so that the interference pattern should change over the course of the day.

The more stable and larger the interferometer became over time, the clearer it became that the expected effect could not be observed.

With the special theory of relativity published in 1905, the ether theory became obsolete, but this did not mean that Michlson's interferometric work lost its meaning.

Michelson-Morley experiment 14 Comment 2

Albert Abraham Michelson was awarded the Nobel Prize in 1907 ”for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid”.

Einstein’s postulates

Special Relativity

Einstein's postulates

1. The laws of nature are the same in all inertial systems.
2. Observers always measure the value $c = 299\ 792\ 458$ m/s for the speed of light regardless of their speed.
3. There is no absolute state of rest

Einstein's postulates

Comment 1

The special theory of relativity is based on three postulates.

The first postulate says that all natural laws are the same in all inertial systems.

The second postulate says that the speed of light does not depend on the speed of the observer.

The third postulate says that there is no state of absolute rest.

Inertial reference systems were introduced by Newton.

An inertial frame is defined by the following conditions:

A body at rest always remains at rest in an inertial frame of reference, unless a force acts on the body.

Einstein’s postulates — Comment 2

A body moving at a certain speed will always move in a straight line at that speed, unless there is a force acting on the body.

The idea of an inertial frame of reference is a mathematical idealization that is never fully realized in nature.

E.g. a frame of reference system rotating with the earth is not an inertia reference system, due to the centrifugal force and Coriolis force.

Nevertheless it is possible to approach the idealisation of an inertial frame of reference locally and during short times.

The general theory of relativity drops this restriction and applies in general frames of reference.

Einstein's postulates Comment 3

The first postulate is a principle that has been used successfully since Newton's time.

The speed of light is a natural constant in Maxwell's theory of electrodynamics.

The second postulate says that Maxwell's electrodynamics correctly captures the essence of the speed of light.

Not Maxwell's theory of electrodynamics is an approximation, but Newton's theory of mechanics.

The second postulate demands that there must be a new theory of mechanics, which Einstein then develops within the framework of the special theory of relativity for inertial systems.

Einstein's postulates Comment 4

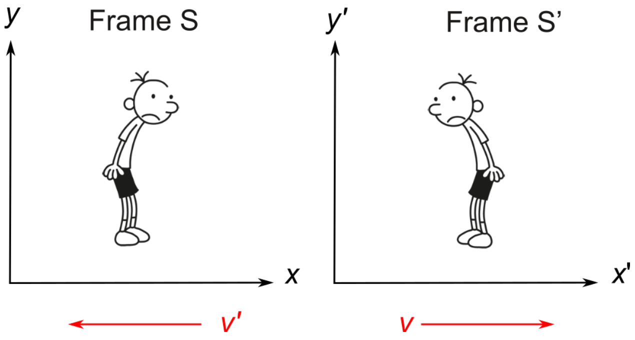
The third postulate confirms that all inertial systems are equivalent regardless of their relative speed.

The speed of light as a natural constant has the same numerical value in all inertial systems.

Lorentz Transformation

Special Relativity

Lorentz transformation 1



- Frame S’ moves to the right with respect to frame S at speed v
- Frame S moves to the left with respect to frame S’ at speed v'

Lorentz transformation 1 Comment

The sketch shows two inertial systems that move away from each other to the left and right.

This situation can be approximated by two cars moving away from each other on a flat road at constant speed.

Lorentz transformation 2

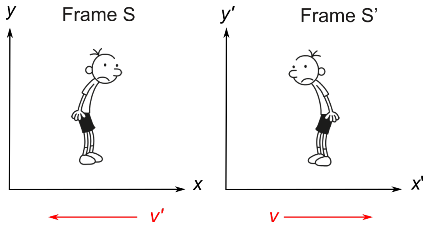
Galilei transformation: the time is independent of the frame of reference, i.e.

$$\boxed{t = t'}$$

$$x = x' + vt' \qquad x' = x - vt$$

$$y = y' \qquad y' = y$$

$$z = z' \qquad z' = z$$



Lorentz transformation 2 Comment 1

The time is measured in both cars with identical stopwatches.

The stopwatches are set to zero when the coordinates of both frames completely overlap.

In this way, the same time is measured in both vehicles or reference systems.

Lorentz transformation 2 — Comment 2

The value of a coordinate x' in the right frame S' results in the left frame S when the product of speed and time is added.

For an observer in the coordinate system S, the coordinates of a point (x', y', z') in the coordinate system S' are given by the equations on the left.

The value of a coordinate x in the left frame S results in the right frame S' if the product of speed and time is subtracted, i.e. for an observer in the coordinate system S', the coordinates of a point (x, y, z) in the coordinate system S are given by the equations on the right.

The coordinates y and z do not change and are no longer mentioned in the following.

Lorentz transformation 2 Comment 3

This coordinate transformation is called the Galileo transformation.

Newtonian mechanics is based on the Galileo transformation.

Newton's equation of motion $F = ma$ is the same in all inertial systems if the Galilei transformation is applied.

Newtonian mechanics thus fulfills Einstein's first postulate.

The laws of nature are the same in all inertial frames of reference.

Lorentz transformation 3

The speed of light does not change in the two coordinate systems due to the relative speed between frame S and frame S '

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2}}$$

and

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \psi'}{\partial t'^2} = \frac{\partial^2 \psi'}{\partial x'^2}}$$

The wave function ψ denotes either the electric or the magnetic field of an electromagnetic wave

ct and *x* must be transformed similar

Lorentz transformation 3 — Comment 1

Einstein’s first postulate does not hold true when the Galilei transform is used to transform Maxwell’s equations of electrodynamics.

In a vacuum, Maxwell’s equations are reduced to simple wave equations for the electric and magnetic field.

The electromagnetic waves propagate at the speed of light.

If the Galileo transform is used, the speed of light would depend on the speed of the coordinate system.

This result is correct if, for example, sound waves are considered.

Finding the correct transformation equations for the Maxwell equations is a mathematical problem that was solved by Hendrik Lorentz in 1895.

Lorentz transformation 3 Comment 2

Hence the transformation is called the Lorentz transformation.

In the wave equations framed in red, the wave functions ψ and ψ' indicate the electric and magnetic field of a plane electromagnetic wave in the coordinate system S and S', which propagates along the x-axis.

Since the wave equations must be the same in both reference systems, the product of time and the speed of light ct must be transformed in the same way as the coordinate x.

It is a direct consequence of the Lorentz transformation that time is no longer absolute, but depends on the frame of reference.

The clocks in the two coordinate systems show different times, even if the clocks in the two coordinate systems were synchronized at the beginning of the experiment.

Lorentz transformation 3 — Comment 3

In addition, the electric and magnetic fields are different in the two coordinate systems.

This is obvious if, for example, a charge is considered that rests in the coordinate system S'.

In this case there is only an electric field in the coordinate system S', but no magnetic field.

According to Ampere's law, there is also a magnetic field in the coordinate system S, since the charge moves in this coordinate system.

The Lorentz transformation does not only have to be applied to the coordinates, but also to the electric and magnetic fields.

Lorentz transformation 3 Comment 4

The transformation of electromagnetic fields is more complicated and is not covered in this lecture.

Lorentz transformation 4

ansatz:

$$\begin{aligned} x &= x' + vt' &\rightarrow\quad x &= \gamma\left(x' + \frac{v}{c}ct'\right) \\ y &= y' &\rightarrow\quad y &= y' \\ z &= z' &\rightarrow\quad z &= z' \\ t &= t' &\rightarrow\quad ct &= \gamma\left(ct' + \frac{v}{c}x'\right) \end{aligned}$$

With the wave equation one gets for the factor γ

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

Lorentz transformation 4 Comment 1

Starting from the Galileo transformation, the Lorentz transformation can easily be guessed.

The Galileo transformation must be extended in such a way that the coordinate *x* and the product *ct* are treated the same.

In the formula for the coordinate *x*, the product of speed and time is expanded to include the speed of light.

It should also be noted that the time in the S' coordinate system is no longer the time in the S coordinate system.

The formula for transforming the time is extended analogously to the formula for transforming the coordinate *x*.

Lorentz transformation 4 Comment 2

Only the coordinates x' and ct' have to be swapped.

When these formulas are used to transform the wave equation, it turns out that an additional factor γ is needed.

The formula outlined in red shows γ.

The wave equation does not change form when the x and ct coordinates are transformed with these formulas.

This can easily be checked if a scalar wave function is used .

The complicated transformation of the electromagnetic fields is not required in this lecture.

Lorentz transformation 5

Observer in coordinate system S. The coordinate system S' moves to the right

$$\boxed{\begin{aligned} ct &= \gamma(ct' + \frac{v}{c}x') \\ x &= \gamma(x' + \frac{v}{c}ct') \\ y &= y' \\ z &= z' \end{aligned}}$$

Observer in coordinate system S'. The coordinate system S moves to the left

$$\boxed{\begin{aligned} ct' &= \gamma(ct - \frac{v}{c}x) \\ x' &= \gamma(x - \frac{v}{c}ct) \\ y' &= y \\ z' &= z \end{aligned}}$$

Lorentz transformation 5 — Comment 1

The first set of formulas framed in red gives the Lorentz transformation for a coordinate system S, in which the observer is located, and the coordinate system S', which moves to the right.

The second set of formulas framed in red gives the Lorentz transformation for a coordinate system S', in which the observer is located, and the coordinate system S, which moves to the left.

Since the Lorentz transformation leaves the speed of light unchanged, the Lorentz transformation according to Einstein's postulates is the correct transformation between inertial systems.

The Maxwell equations are invariant when the Lorentz transform is applied.

Lorentz transformation 5 Comment 2

Newton’s equation of motion is not invariant when the Lorentz transform is applied.

Einstein’s postulates state that Newtonian mechanics is an approximation that is only valid if the speeds involved are much smaller than the speed of light.

For macroscopic objects on earth, this limitation is generally not a problem.

But in cosmic space and in the microcosm of atoms, molecules and solids the speeds are often comparable to the speed of light.

The main challenge of the theory of relativity was therefore the reformulation of mechanics.

Lorentz transformation 5 Comment 3

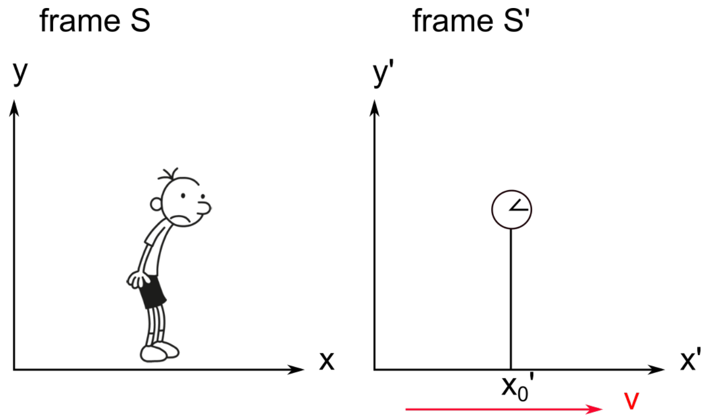
As long as no accelerations are involved, this problem can be solved within the framework of the special theory of relativity.

General relativity solves the problem when accelerations and large distances are important, which is usually the case with cosmological problems.

Time dilation

Special Relativity

Time dilation and length contraction: time dilation 1



With $t = \gamma(t' + \frac{v}{c^2} x_0{}')$ follows for time intervals $\boxed{\Delta t = \gamma \Delta t'}$

Time dilation and length contraction 1 Comment 1

Due to the Lorentz transformation, time is no longer an absolute quantity that can be defined for the entire cosmic space.

A distinction must be made between the time of a clock that is resting in relation to an observer and the time of a clock that is moving in relation to an observer.

The sketch shows an observer in the coordinate system S.

The person in the coordinate system S observes a clock that is moving away with the speed v and the coordinate system S’.

There is one fundamental problem with the sketch, however.

Time dilation and length contraction 1 Comment 2

The information between the watch and the viewer is always transmitted by electromagnetic waves.

I will completely ignore this aspect in this section.

The effects caused by the finite transit time of the light waves are discussed in the section: „Visible“ effects due to the Lorentz transformation.

In this section only the direct consequences of the Lorentz transformation are discussed.

With the underlined formula of the Lorentz transformation one finds the equation framed in red, which relates the corresponding time intervals in the coordinate systems S and S ’.

Time dilation and length contraction 1 Comment 3

Please note that the location of the clock x_0' in the coordinate system S' does not change.

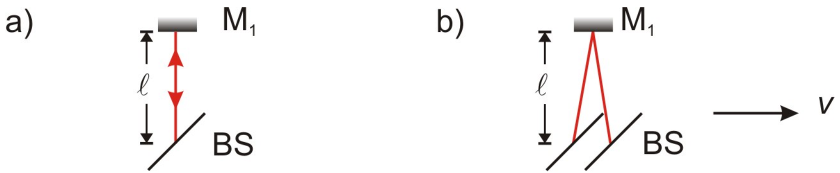
If the interval between the two times t_1 and t_2 is calculated, the position of the clock disappears and only the difference between the two times remains.

The time in the coordinate system S' in which the clock rests is called proper time.

With $\gamma = 1/\sqrt{1 - \mathrm{v}^2/\mathrm{c}^2}$ is Δt always larger than the proper time $\Delta t'$.

Time dilation and length contraction 2

time dilation and the Michelson interferometer



a) The interferometer at rest relative to an observer

necessary time for a light pulse $$\boxed{t_0 = \frac{2\ell}{c}}$$

Time dilation and length contraction 2 Comment

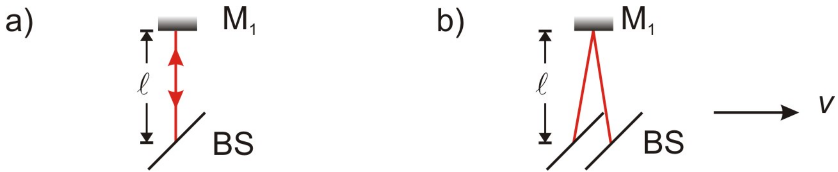
The effect of time dilation can be illustrated with the Michelson interferometer.

The time that the light needs to pass the distance between the beam splitter and the mirror 1 defines the proper time when the interferometer is at rest with respect to an observer.

Figure a) shows this situation.

The formula outlined in red indicates the proper time.

Time dilation and length contraction 3



b) the interferometer moves with respect to an observer

necessary time for a light pulse

$$\boxed{t(v) = \frac{2\ell}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = t_0\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}$$

t_0: proper time and $t(v)$: time of the moving clock $\rightarrow$ time dilation

Time dilation and length contraction 3 Comment

Figure b) illustrates the situation when the interferometer moves relative to an observer.

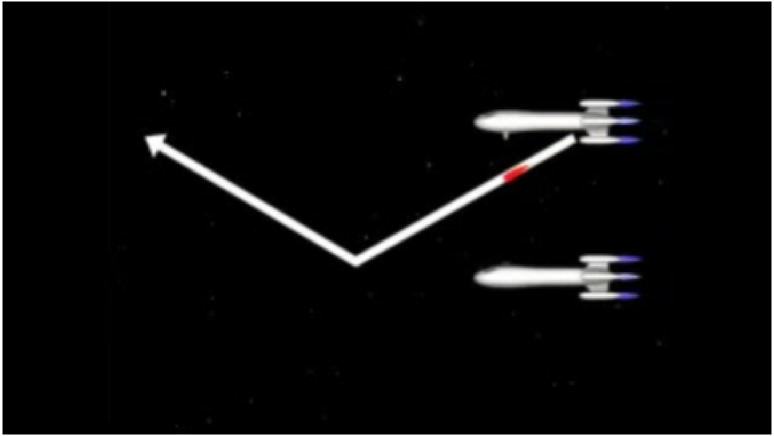
The light pulse between the beam splitter and the mirror M_1 has to travel a longer distance and therefore needs more time (this is due to the aberration effect, which will be discussed in the next lecture).

The time required can be calculated using the formula outlined in red.

The same formula was obtained with the Lorentz transformation.

Clocks that move relative to an observer run slower for the observer.

Time dilation and length contraction 4



(TimeDilationTwoSpaceships.mp4)

Time dilation and length contraction 4 — Comment

The video illustrates the effect.

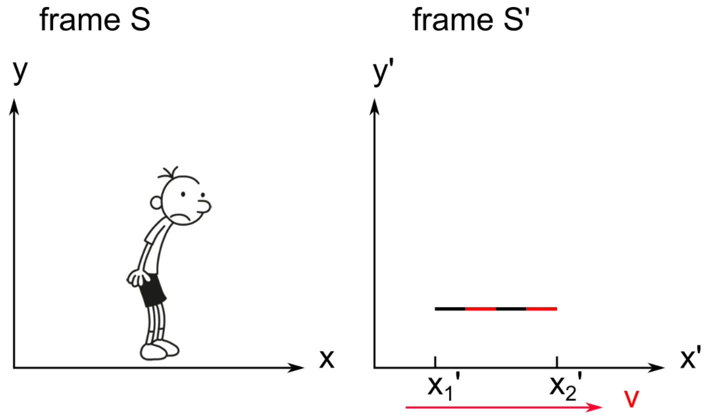
The video suggests that the effect of time dilation affects both the pilots of the spaceships and the observer on the asteroid.

Time dilation is not just the math of a complicated theory, it affects living organisms.

Since time is sacred to many people, many people felt challenged in their beliefs and tried to contradict the special theory of relativity.

In this context, the so-called twin paradox has become very popular.

Time dilation and length contraction: length contraction 5



With $x' = \gamma(x - \frac{v}{c}ct)$ is $\Delta x' = \gamma \Delta x$ and $\boxed{\Delta x = \gamma^{-1}\Delta x' = \Delta x' \sqrt{1 - \left(\frac{v}{c}\right)^2}}$

Time dilation and length contraction 5 Comment 1

The length contraction is closely related to the time dilation.

The figure shows an observer in the coordinate system S who is looking at a scale in the coordinate system S'.

If the observer wants to determine the length of the ruler, then he must determine the position of the ruler ends at the same time.

Now again, only the effect of the Lorentz transformation is considered and the question of how the measurement can actually be realized is excluded.

The Lorentz transformation underlined in red is used to calculate the effect of length contraction.

Time dilation and length contraction 5 — Comment 2

If the distance between the end points of the ruler is measured at the same time, the time-dependent terms cancel each other out and the formula outlined in red results.

$\Delta x'$ is the proper length, since the ruler rests in the coordinate system S'.

The length in the coordinate system S of the observer is smaller than the length in the coordinate system S'.

The effect is called length contraction.

Time dilation and length contraction 6

length contraction and the Michelson interferometer

the time t_1 between the beam splitter and mirror 1 is

$$t_1 = \frac{2\ell}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

the time t_2 between the beam splitter and mirror 2 is

$$t_2 = \frac{2\ell}{c}\frac{1}{1-\left(\frac{v}{c}\right)^2}$$

In order for the two times to be the same, the light path in the forward direction must be shortened

$$\boxed{\ell(v) = \ell\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Time dilation and length contraction 6 Comment

Here, too, the comparison with the Michelson interferometer is instructive.

The first formula specifies the time that the light needs between the beam splitter and the mirror M_1.

The second formula specifies the time that the light needs between the beam splitter and the mirror M_2.

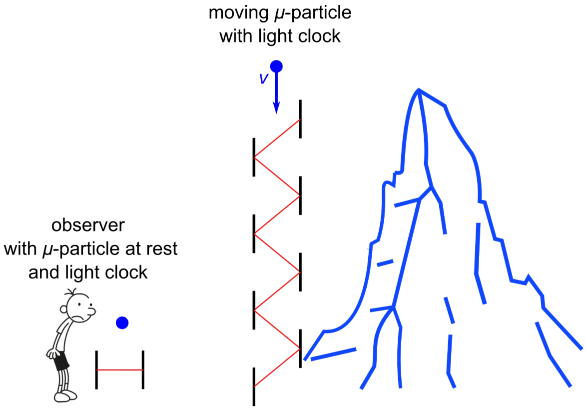
Since the interference between the two partial beams cannot depend on the relative speed between the interferometer and an observer, the two times t_1 and the time t_2 must be the same.

Due to the length contraction, no movement-related effect can be observed with the Michelson interferometer.

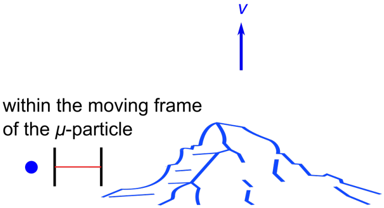
Hall and Rossi

Special Relativity

Experiment of Hall and Rossi



time dilation



length contraction

Experiment of Hall and Rossi — Comment 1

The special theory of relativity was very important for the development of quantum mechanics.

In 1927 Dirac used the special theory of relativity to derive the Schrödinger equation of an electron.

This enabled him to almost completely explain the optical spectrum of the hydrogen atom.

Despite the enormous importance of the special theory of relativity, it was not until 1941 that Hall and Rossi succeeded in experimentally testing the fundamental effects of time dilation and length contraction.

Hall and Rossi carried out their experiments with μ-particles, which were discovered in 1936.

Experiment of Hall and Rossi — Comment 2

The lifetime of a μ-particle at rest is around 2 μs.

After a certain time, the μ-particle decays into an electron and another particle called the electron neutrino.

To measure the lifespan, the μ-particles in the matter are stopped and the electrons are counted, which are created by the decay of the μ-particles with high kinetic energy.

The number of counts decreases exponentially according to the lifetime of the μ-particle.

For Hall and Rossi's experiment it is important that the number and the speed of the μ-particles can be measured.

Experiment of Hall and Rossi — Comment 3

The μ-particles are generated by cosmic radiation in the earth's atmosphere and move at high speeds towards the ground.

The number of μ-particles with a certain speed was measured on a mountain and at sea level.

To better understand what is going on, one can imagine that each μ-particle is connected to a light clock.

The same number of μ-particles is observed on the mountain and at the base of the mountain.

The μ-particles on the mountain move towards the surface of the earth at a certain speed.

The μ-particles examined at the foot of the mountain are at rest.

Experiment of Hall and Rossi — Comment 4

The figure on the left shows that a light pulse can travel back and forth between the two mirrors of the light clock faster with the resting μ-particles than with the moving particles.

The clock of the moving μ-particles runs slower than the clock of the resting μ-particles.

At the foot of the mountain there are more moving μ-particles than one would expect based on the observation of the resting μ-particles.

The lifetime of the μ-particles increases due to the movement of the particles.

This effect is known as time dilation.

Experiment of Hall and Rossi Comment 5

The figure on the right illustrates the situation when the observer moves with the μ-particles.

The height of the mountain is reduced in the frame of reference of the μ-particle by length contraction.

This shortens the time that the μ-particles need to get from the top to the base of the mountain.

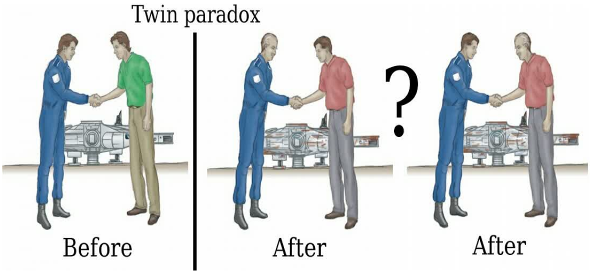
For this reason, a larger number of μ-particles reaches the foot of the mountain than can be expected from observing the μ-particles at rest.

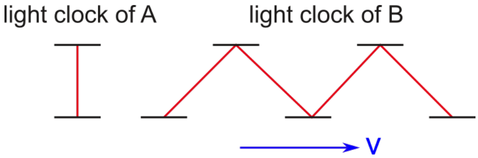
The square root behavior of time dilation and length contraction can be observed if the effect is measured for different velocities of the μ-particles.

Twin paradox

Special Relativity

Twin paradox





Twin Paradox — Comment 1

The figures illustrate the twin paradox.

Twin A stays on Earth while twin B embarks on a journey to a distant star.

When twin A watches twin B’s clock, he finds that twin B’s clock is running slower than his own.

Twin A therefore expects that his brother will be younger than himself when he returns to earth.

The light clocks in the picture below illustrate the effect.

The paradox assumes that the situation is symmetrical for both twins.

Therefore, when twin B returns to Earth, he expects his brother to have aged less than himself.

Twin Paradox

Comment 2

The paradox takes the symmetry of the frames of reference of twin A and twin B for granted.

But that is not true.

The frame of reference of twin A is an inertial system during the entire journey of twin B, while there must be accelerations in the frame of reference of twin B, since otherwise he could not return to earth.

The reference system of twin B cannot be an inertial system during the entire journey.

The defined properties of an inertial system can be used to objectively test whether a reference system is an inertial system or not.

Twin Paradox

Comment 3

It is therefore not surprising that the situations of twin A and twin B are different.

By exchanging radio signals, the twins can check each other's watch.

Therefore, it will come as no surprise to either twin A or twin B that twin A will be older than twin B.

The effect was tested in 1971 with a famous experiment.

Atomic clocks were used in the Hafele-Keating experiment.

The clocks traveled the earth twice in airplanes and were compared to an atomic clock in the United States Naval Observatory.

Twin Paradox Comment 4

The evaluation of the experiment is complex, since the general theory of relativity must also be used to analyze the experimental data.

Revision

Summary in questions

1. What are Einstein's postulates?
2. Sketch the Michelson interferometer.
3. What should be observed with the Michelson interferometer?
4. What is meant by proper time and proper length?
5. Write down the formulas for the time dilation and the length contraction and explain the quantities involved.
6. Illustrate the effect of time dilation with a light clock.
7. Describe the experiment by Hall and Rossi.
8. Write down the Galileo transformation and develop the Lorentz transformation from it.