Modern Physics

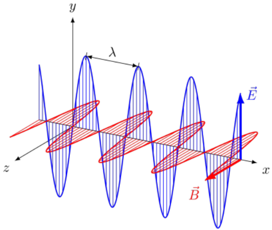
Contents:

Planck’s radiation law

Wave-particle dualism

- Thermal radiation
- Planck’s radiation law
- Photoelectric effect
- Laser
- Compton effect
- Pair production
- Matter waves
- Uncertainty relations

Planck's radiation law 1



$$\vec{E} = \vec{E}_0 \exp(i\vec{k}\vec{r} - \omega t)$$

$$\vec{B} = \vec{B}_0 \exp(i\vec{k}\vec{r} - \omega t)$$

Energy density of an electromagnetic wave

$$\frac{E}{V} = \frac{1}{2}\varepsilon_0 \vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2$$

Intensity of an electromagnetic wave (Poynting vector)

$$I = \frac{P}{A} = \left|\frac{\vec{E} \times \vec{B}}{\mu_0}\right|$$

Planck's radiation law 1 Comment

The figure shows the classical view of a plane electromagnetic wave according to the theory of Maxwell.

The energy density of an electromagnetic wave is given by the first underlined equation.

The intensity is given by the absolute value of the Poynting vector.

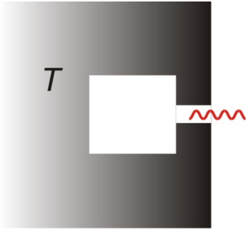
Both the energy density and the intensity of the wave is proportional to the square of the electric or magnetic field.

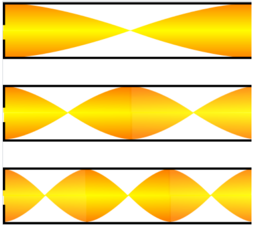
Maxwell's theory gives no hint about the temperature dependence of electromagnetic radiation.

Consequently, new ideas are needed to describe the temperature dependence of thermal radiation.

Planck's radiation law 2

Approach from Wilhelm Wien 1896:





standing waves in a cavity

$$\underline{L = n\frac{\lambda_n}{2} \quad \rightarrow \quad k_n = \frac{2\pi}{\lambda_n} = n\frac{\pi}{L}}$$

in three dimension

$$\boxed{\vec{k}_{n_1,n_2,n_3} = \frac{\pi}{L}\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}}$$

Planck's radiation law 2 Comment 1

In 1896 Wilhelm Wein made the first attempt to theoretically describe the thermal radiation of a black body.

He assumes that the electromagnetic spectrum in the cavity of the black body is formed by standing waves.

The left figure shows the model of a black body radiation source. In the following it is assumed that it is a cubic cavity with the edge length L.

The middle picture shows the standing waves in one spatial direction. If it is a metallic cavity, then the nodes and antinodes correspond to the magnetic field of the wave.

Planck’s radiation law 2 Comment 2

If it is a cavity in an insulating medium, then the nodes and antinodes correspond to the electric field of the standing wave.

The underlined formula gives the quantized wave number in one spatial direction and the formula outlined in red is the generalization for the three-dimensional case.

Planck's radiation law 3

Wilhelm Wien makes the following assumptions:

1. energy of the standing wave modes

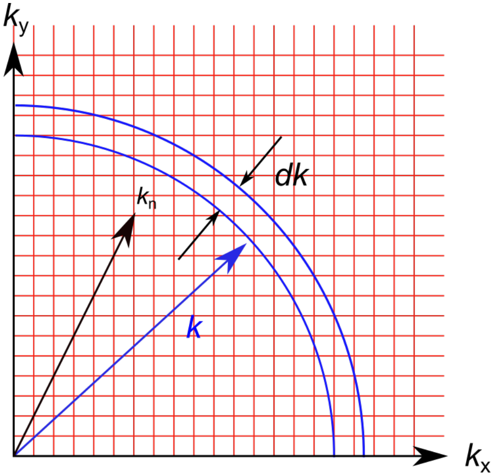
$$E_n = h\nu_n = \hbar\omega_n = \hbar c k_n$$

2. probability that a mode is thermally excited

$$w_n \propto \exp(-\hbar\omega_n/k_B T)$$

3. number of modes with wave number k_n in interval dk

$$dN = \frac{4\pi k^2 dk/8}{\pi^3/L^3}$$



Planck's radiation law 3 Comment 1

1. In Maxwell's electrodynamics, the energy of a wave is proportional to the square of the amplitude, i.e. to the square of the electric or magnetic field strength. The energy of the wave is independent of its frequency.

The spectrum of thermal radiation shows that the energy of the waves must depend on the frequency. Wilhelm Wien makes the simplest possible assumption: the energy of the waves is proportional to the frequency.

This means that the energy of the standing wave in the cavity is proportional to the frequency ω_n.

Since Max Planck explained how this frequency dependence of the energy comes about, the proportionality constant has since been called Planck's constant h.

Planck's radiation law 3 Comment 2

2. In analogy to Maxwell's velocity distribution, Wilhelm Wien assumes that the probability that a wave is excited is proportional to the Boltzmann factor.

3. The energy of the radiation in a cavity is thus on the one hand proportional to the Boltzmann factor and on the other hand proportional to the number of available modes of the standing waves.

The sketch shows how the number of modes of the standing waves can be determined.

The vectors $\vec{k}_{n_1,n_2,n_3}$ form a grid, which is indicated by the red lines.

Planck's radiation law 3 Comment 3

Each cell of the grid has the volume π^3/L^3.

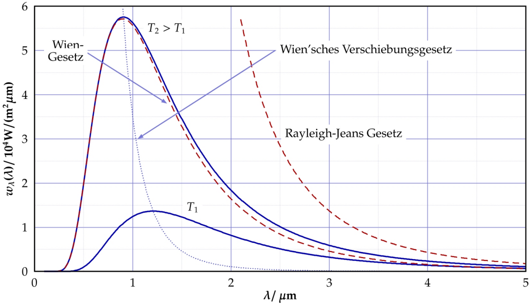
The formula underlined in red gives the number of $\vec{k}_{n_1,n_2,n_3}$ vectors whose endpoints lie in a spherical shell with the radius k and the thickness dk.

The factor 1/8 takes into account that the wave number vectors of the standing wave modes only have positive components, so that only one eighth of a spherical shell has to be taken into account.

Planck's radiation law 4

Wien's radiation law of 1896 gives the radiation intensity dI in the wavenumber interval $(k, k + dk)$

$$dI \propto \frac{4\pi k^2 dk}{(2\pi)^3} \cdot \hbar ck \cdot \exp(-\frac{\hbar ck}{k_B T}) \qquad \propto \frac{d\lambda}{\lambda^5} \exp(-\frac{ch}{\lambda k_B T})$$



Planck's radiation law 4 Comment 1

With these assumptions Wilhelm Wien calculates the energy of the radiation in the cavity of the black body.

The electromagnetic energy in the cavity can be observed through the intensity of the radiation leaving the cavity through the small opening. The intensity is proportional to the energy of the radiation in the cavity.

The first factor gives the number of modes in the wave number interval $k, k + dk$. The second factor gives the energy of the modes and the third factor determines the probability with which these modes are thermally excited.

The intensity is measured experimentally as a function of the wavelength λ and the formula underlined in red can be used to fit the measured spectrum. dλ gives the resolution of the spectrometer used.

Planck's radiation law 4 Comment 2

To do this, the wave number is expressed by the wavelength $k = 2\pi/\lambda$ and dk by $|dk| = 2\pi\,\mathrm{d}\lambda/\lambda^2$.

The figure shows the exact result of Planck's radiation formula for two temperatures and the result of Wien's theory as a dashed line.

Wien's radiation formula almost perfectly describes the spectrum of thermal radiation.

Wilhelm Wien was awarded the Nobel Prize in 1911 "for his discoveries regarding the laws of thermal radiation".

In order to check whether Wien's radiation formula is exact, Otto Lummer and Ernst Pringsheim carried out precision measurements of thermal radiation at the physical-technical Reichsanstalt (today PTB) in 1896.

Planck's radiation law 4 Comment 3

They showed that Wein's law of radiation is an approximation and not exact.

In 1900 Max Planck published his radiation formula, which perfectly describes the measurements of Otto Lummer and Ernst Pringsheim.

Also in 1900, John W. S. Rayleigh published a radiation formula in which the Boltzmann factor is omitted and it is assumed that the energy of the modes is simply proportional to the thermal energy $k_B T$.

This is based on the assumption that in thermal equilibrium the energy of all components of a physical system should be a multiple of $k_B T$.

The dashed line shows that this approach cannot describe the radiation spectrum.

Planck’s radiation law 4 Comment 4

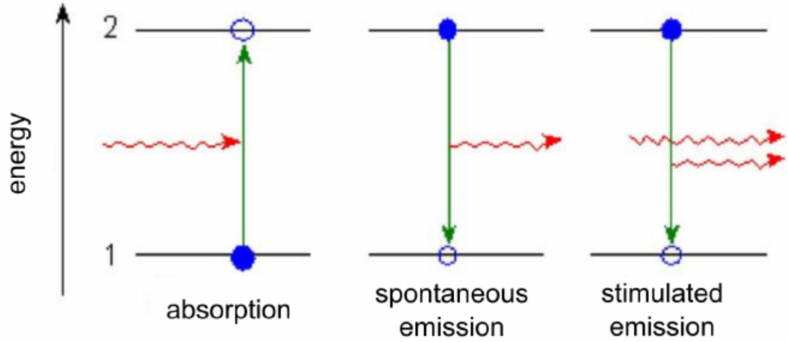
The Rayleigh formula was revised by Sir James Jeans and republished in 1905, but this did not improve the theory.

The Rayleigh-Jeans-law results for very large wavelengths as a limit case of Planck’s radiation formula and has thus achieved a certain popularity.

Planck's radiation law 5

Three processes are important

- absorption
- spontaneous emission
- stimulated emission



Planck’s radiation law 5 Comment 1

The success of Wien’s radiation law is based on the assumption that the energy of the modes in the cavity is proportional to the frequency of the mode.

Although there was no quantum physics in 1900, Max Planck speculatively assumes that matter consists of quantized oscillators which can only absorb and emit electromagnetic radiation in units of $\hbar\omega$.

On the one hand, these hypothetical oscillators take energy from the electromagnetic waves in units of $\hbar\omega$ and, on the other hand, supply the waves with energy in units of $\hbar\omega$.

With this idea, Max Planck gives a physical explanation for Wilhelm Wien’s assumption.

Max Planck can trace the temperature dependence of thermal radiation back to the thermal occupation of the energy levels of the quantized oscillators.

Planck’s radiation law 5 Comment 2

Albert Einstein took up the idea of Max Planck when investigating the photoelectric effect in 1905 and recognized that light is a stream of energy quanta that are “localized in spatial points, move without division and can only be absorbed and generated as a whole”.

With this Albert Einstein discovered the light particles that have since been called “photons”.

Planck’s radiation law 5 Comment 3

In 1913 Ernest Rutherford discovered the atomic nucleus and Niels Bohr formulated the first quantized model of the atom that is explicitly based on the findings of Max Planck and Albert Einstein.

Max Planck was awarded the Nobel Prize for the year 1918 “in recognition of the merit he had made in the development of physics through the discovery of energy quanta”.

Albert Einstein was awarded the Nobel Prize in 1921 “for his contributions to theoretical physics, especially for his discovery of the law of the photoelectric effect”.

Planck's radiation law 5 Comment 4

In 1916 Albert Einstein derived Planck's radiation law on the basis of Bohr's 1913 atomic model.

Einstein assumes that photons are absorbed and emitted during the atomic transition from a Bohr orbit with the energy E_1 to a Bohr orbit with the energy E_2.

In the figure, the energy of two Bohr orbits are indicated by horizontal lines.

When a photon is absorbed, the electron changes from orbit 1 to orbit 2.

The energy of orbit 2 corresponds to the sum of the energy of orbit 1 and the energy of a photon. The probability that the photon is absorbed is maximal when $E_2 - E_1 = \hbar\omega$ holds.

In the case of spontaneous emission, the electron in orbit 2 emits for some reason (e.g. vacuum fluctuations) a photon with energy $\hbar\omega$ and changes to orbit 1.

Planck’s radiation law 5 Comment 5

In the case of stimulated emission, a photon with energy $\hbar\omega$ hits an atom and thereby triggers the transition from orbit 2 to orbit 1.

An additional photon with energy $\hbar\omega$ is emitted.

In the figure, the photon that triggers the emission is indicated by an additional red curled arrow.

After the stimulated emission there are two identical photons.

These two photons have the same energy and correspond to waves with the same wave vector.

Planck's radiation law 6

spontaneous emission:

For $E_2 > E_1$, the occupation number N_2 is reduced by the spontaneous emission of a photon with the energy $E_\gamma = h\nu = hc/\lambda = E_2 - E_1$

$$\frac{dN_2^{spon}(\lambda)}{dt} = -A_{21}(\lambda)N_2$$

A_{21} is the Einstein coefficient, which describes the transition rate due to the spontaneous emission of photons

Planck's radiation law 6 Comment

N_2 denotes the occupation number of state 2 of the quantum system.

The occupation number of state 2 is reduced by the spontaneous emission of photons.

Einstein formulated the underlined equation.

The decay rate $\frac{dN_2^{spon}(\lambda)}{dt}$ of the occupation number is proportional to the occupation number of the excited state 2: N_2.

The constant of proportionality is the Einstein coefficient A_{21} .

Planck's radiation law 7

stimulated emission of a photon:

$$\frac{dN_2^{stim}(\lambda)}{dt} = -u(\lambda)B_{21}(\lambda)N_2$$

$u(\lambda)$ denotes the spectral electromagnetic energy density

$$u(\lambda) = \frac{1}{V}\frac{\Delta E_\lambda}{\Delta\lambda}$$

ΔE_λ denotes the electromagnetic energy in the wavelength interval $\lambda \to \lambda \pm \Delta\lambda/2$ ($\Delta\lambda$ is the line width of the considered transition)

$B_{21}(\lambda)$ is the Einstein coefficient for stimulated emission

Planck's radiation law 7 Comment 1

The decay rate of the occupation number of the excited state 2 due to the stimulated emission of a photon is also proportional to the occupation number of the quantum state itself.

In addition, the change in the occupation number is proportional to the density of the photons that can stimulate the transition.

Due to the finite line width of the transition, the energy of the photons does not have to match the energy of the transition exactly.

Due to the uncertainty relations, these energies are not precisely defined.

Therefore, all photons within a certain energy interval can trigger the transition.

Planck's radiation law 7 Comment 2

The formula $u(\lambda) = \frac{1}{V}\frac{\Delta E_\lambda}{\Delta \lambda}$ defines the spectral energy density of these photons.

The constant of proportionality for the simulated emission is the Einstein coefficient B_{21}.

Planck's radiation law 8

Absorption:

$$\frac{dN_2^{abs}(\lambda)}{dt} = +u(\lambda)B_{12}(\lambda)N_1$$

$B_{12}(\lambda)$ is the Einstein coefficient for absorption

Planck's radiation law 8 Comment

A similar formula describes the absorption of a photon.

The occupation number of the excited state 2 increases due to absorption.

The constant of proportionality for the absorption is the Einstein coefficient B_{12}.

Planck's radiation law 9

The occupation numbers do not change in thermal equilibrium. E.g. for N_2 the equation results

$$\frac{dN_2}{dt} = 0 = \frac{dN_2^{abs}}{dt} + \frac{dN_2^{stim}}{dt} + \frac{dN_2^{spon}}{dt}$$

$$\boxed{0 = +u(\lambda)B_{12}N_1 - u(\lambda)B_{21}N_2 - A_{21}N_2}$$

spectral electromagnetic energy density is therefore

$$u(\lambda) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2} = \frac{A_{21}}{B_{12}\frac{N_1}{N_2} - B_{21}}$$

Planck's radiation law 9 Comment

The occupation numbers of the quantum states do not change in thermal equilibrium.

The formula framed in red results from Einstein's equations.

With this equation the spectral energy density $u(\lambda)$ can be calculated.

The energy density depends on the Einstein coefficients and the quotient of the occupation numbers.

Planck's radiation law 10

quotient of the occupation numbers (with $\nu\lambda = c$)

$$\frac{N_1}{N_2} = \exp\left(\frac{E_2 - E_1}{k_B T}\right) = \exp\left(\frac{h\nu}{k_B T}\right) = \exp\left(\frac{hc}{\lambda k_B T}\right)$$

spectral energy density in thermal equilibrium

$$\boxed{u(\lambda) = \frac{A_{21}}{B_{12} e^{hc/\lambda k_B T} - B_{21}}}$$

Planck's radiation law 10 Comment

The quotient of the occupation numbers is determined by the Boltzmann factor.

The energy difference in the exponent of the Boltzmann factor corresponds to the energy of the photons and can therefore be expressed in terms of the frequency or the wavelength of the radiation.

The formula outlined in red gives the spectral energy density, whereby the quotient of the occupation numbers is expressed by the Boltzmann factor.

Planck's radiation law 11

$$u(\lambda) = \frac{A_{21}}{B_{12}e^{hc/\lambda k_B T} - B_{21}}$$

Since the energy density diverges for $T \to \infty$

$$\underline{B_{12} = B_{21}}$$

Simulated emission is the inverse process of absorption

- Absorption does not destroys coherence
- Equally, stimulated emission cannot destroy the coherence of a light wave $\to$light amplification by stimulate emission of radiation

Planck’s radiation law 11 Comment

This formula leads to an important conclusion.

Since the energy density has to diverge to infinity with increasing temperature, the Einstein coefficients for absorption and stimulating emission must be the same.

The simulated emission of radiation and absorption are inverse processes.

Therefore, light can be amplified by stimulated emission of radiation.

Since absorption does not destroy the coherence of a light wave, the stimulated emission does not destroy the coherence of a light wave either.

A device that amplifies a wave of light through stimulated emission is called a laser.

Planck's radiation law 12

spectral energy density

$$u(\lambda) = \frac{A_{21}}{B_{21}} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

Planck's radiation law

$$\boxed{\frac{dP}{Ad\lambda} = 2\frac{hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}}$$

Wien's radiation law

$$\lambda \to 0 \quad \text{and} \quad \frac{dP}{Ad\lambda} = 2\frac{hc^2}{\lambda^5} e^{-hc/\lambda k_B T}$$

Rayleigh-Jeans law

$$\lambda \to \infty \quad \text{and} \quad \frac{dP}{Ad\lambda} = 2\frac{c}{\lambda^4} k_B T$$

Planck's radiation law 12 Comment

The intensity of the thermal radiation can be calculated with the spectral energy density.

The calculation itself is not very interesting and the equation outlined in red gives the result that has become famous as Planck's radiation formula.

In the Limes of small wavelengths, Wien's radiation law results and in the Limes of longer wavelengths, the Rayleigh-Jeans law.

According to the discussion of Wien's law, the blue factor can be traced back to the photon energy $h\nu = hc/\lambda$ and the number of k-modes that contribute.

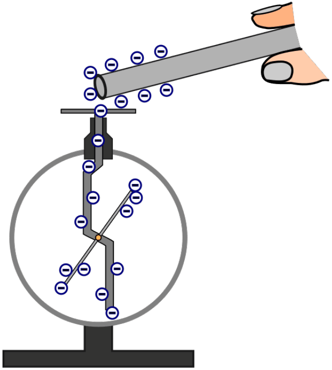
Photoelectric effect

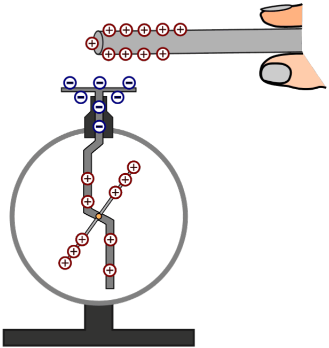
Wave-particle dualism

- Thermal radiation
- Planck’s radiation law
- Photoelectric effect
- Laser
- Compton effect
- Pair production
- Matter waves
- Uncertainty relations

Photoelectric effect 1

Demonstration of the photoelectric effect





(photoelectriceffect.mp4, PhotoeffektFreiburg.mp4)

Photoelectric effect 1 Comment 1

Max Planck's idea that electromagnetic radiation can only be absorbed and emitted in quanta is correct.

The analysis of thermal spectra is, however, time-consuming and not very useful in order to clearly demonstrate the effect.

When analyzing the photoelectric effect in 1905, Albert Einstein was able to show beyond doubt that Max Planck's ideas are correct.

Assuming that the energy of electromagnetic waves is quantized according to Max Planck's idea, he showed that the photoelectric effect can be understood in all its details.

Einstein was awarded the Nobel Prize in Physics in 1921 for this work in particular.

Photoelectric effect 1 Comment 2

The two figures show two charged electroscopes.

In the left figure, the electroscope is charged with electrons.

In the right figures, positive charges are used and electrons are removed from the electroscope.

The figure shows the effect of electrostatic induction.

The first video shows an electroscope being charged with electrons.

Ultraviolet light is used to discharge the electroscope.

The charge of the electroscope does not change when the electroscope is charged with positive charge carriers.

Photoelectric effect 1 Comment 3

In the second video, the electroscope is also charged with electrons.

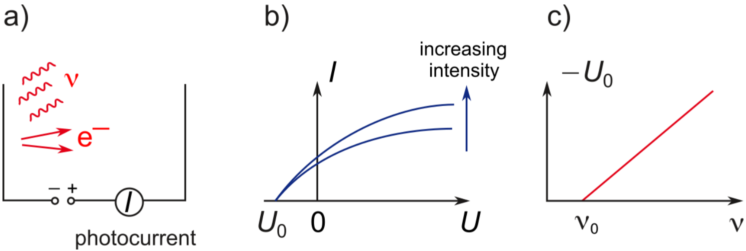
But now there is a piece of glass between the lamp and the electrode of the electroscope.

Despite the bright light, the charge cannot be removed from the electroscope.

These experiments show that electrons gain energy from electromagnetic radiation and overcome the binding energy of the metal.

The experiment with the glass plate shows that the intensity of the light is not the decisive physical quantity.

Photoelectric effect 2



$$\boxed{E_{kin} = |eU_0|}$$

Photoelectric effect 2 Comment 1

In a quantitative experiment, monochromatic light must be used.

The sketch on the left shows the experimental setup.

One electrode is illuminated while the photoelectrons are collected with a second electrode.

The photocurrent depends on the applied voltage.

The photocurrent becomes saturated when a high positive voltage is applied.

The maximum photocurrent increases with the intensity of the light.

Regardless of the intensity of the radiation, all photoelectrons can be stopped by the same delay voltage.

Photoelectric effect 2 Comment 2

The product of the stop voltage and the charge of an electron corresponds to the kinetic energy of the photoelectrons.

The equation framed in red indicates the kinetic energy of the photoelectrons.

There is a threshold frequency or maximum wavelength for which photoelectron emission is possible.

If the frequency of the radiation is less than ν_0, no photoelectrons can be emitted.

For most metals, this threshold frequency is in the ultraviolet region of the electromagnetic spectrum.

Since UV light is normally absorbed by normal glass, photoelectrons cannot be observed when the light is passed through an ordinary glass plate.

Photoelectric effect 3

kinetic energy of photoelectrons according to Einstein

$$\boxed{E_{kin} = h\nu - W_A}$$

W_A denotes the work function, i.e. the binding energy of the electrons

photoelectrons are only emitted when

$$\underline{h\nu > W_A}$$

threshold frequency and wavelength

$$h\nu_0 = \frac{hc}{\lambda_0} = W_A \rightarrow \boxed{\lambda_0 = \frac{hc}{W_A}}$$

Photoelectric effect 3 Comment

The kinetic energy of the photoelectrons is determined by the energy of the photons and reduced by the binding energy of the electrons.

Electrons can only leave the metal surface if the energy of the photons is greater than the work function, i.e. the binding energy.

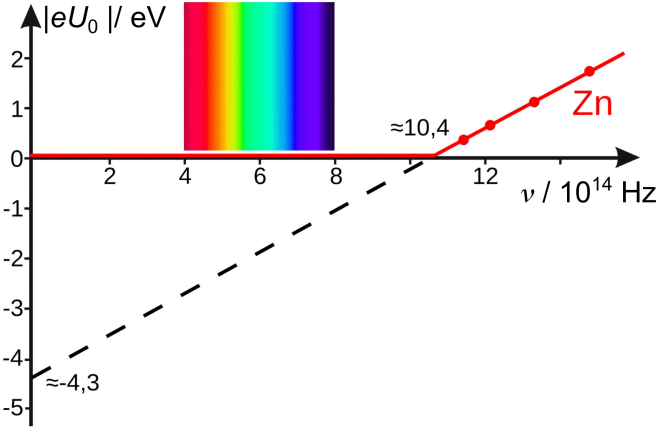
The linear dependence of the kinetic energy of the photoelectrons on the frequency of the electromagnetic wave is a clear proof of Planck’s law $E = h\nu$.

The intensity of the radiation determines the number of photons.

Therefore, the photocurrent becomes larger as the intensity of the light increases.

The last line shows the calculation of the threshold wavelength for the emission of photoelectrons.

Photoelectric effect 4



Photoelectric effect 4 Comment

The figure shows the experimental results for zinc.

The work function of zinc is ≈ 4.3 eV.

The cutoff frequency of $10.4 \cdot 10^{14}$ Hz corresponds to a wavelength of 286 nm.

In the second video, a zinc electrode was used and photoelectrons can only be emitted by ultraviolet light.

Apparently, ultraviolet light is effectively blocked by normal glass (not quartz glass).

The following table provides values for the work function and threshold wavelength of some elements.

Photoelectric effect 5

	W_A	$\lambda_0 = \frac{hc}{W_A}$
Cu	4.3 eV	289 nm
Ag	4.05 eV	307 nm
Al	4.0 eV	310 nm
Au	4.8 eV	259 nm
Pt	5.32 eV	233 nm
Zn	4.34 eV	286 nm
Ba	1.8 eV	690 nm

work function W_A and threshold wavelength λ_0

Laser

Wave-particle dualism

- Thermal radiation
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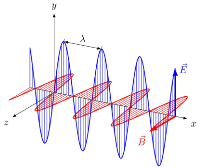
Laser 1

Light **a**mplification by **s**timulated **e**mission of **r**adiation

Amplification of a wave with a wave vector $\vec{k}$ and a small spectral width $\Delta\lambda$



spectral energy density

$$\boxed{u(\lambda) = \frac{1}{V}\frac{\Delta E_\lambda}{\Delta\lambda} = \frac{1}{V}\frac{1}{\Delta\lambda}\frac{hc}{\lambda}n_\lambda}$$

Laser 1 Comment 1

The stimulated emission of radiation is the crucial process of a laser.

The stimulated emission for light amplification is possible from the microwave range to ultraviolet light.

The task of a laser is not to amplify light in general, but to amplify a certain wave, which is characterized by a wave vector and a narrow spectral width.

In some cases only one polarization of the wave is amplified.

The figure shows the classic image of an electromagnetic wave.

In order to amplify the wave, the number of photons that make up the wave must be increased.

Laser 1 Comment 2

For the discussion of thermal radiation, the spectral energy density $u(\lambda)$ was defined.

The spectral energy density of a laser is somewhat limited because the light from a laser only propagates along one axis.

The spectral energy density of a laser depends on the direction in which the photons are moving.

With lasers, the spectral energy density only relates to the photons that move along a certain axis.

Laser 2

Adaptation of Einstein's equations to the laser

$$\frac{dN_2^{abs}}{dt} = +\, u(\lambda) B_{21} N_1 \quad \rightarrow \quad \underline{\frac{d{n_\lambda}^{abs}}{dt} = -n_\lambda W_{21} N_1}$$

$$\frac{dN_2^{stim}}{dt} = -\, u(\lambda) B_{21} N_2 \quad \rightarrow \quad \underline{\frac{d{n_\lambda}^{stim}}{dt} = +n_\lambda W_{21} N_2}$$

$$\frac{dN_2^{spon}}{dt} = -\, A_{21} N_2$$

with

$$\boxed{W_{21} = \frac{1}{V} \frac{1}{\Delta\lambda} \frac{hc}{\lambda} B_{21}}$$

Laser 2 Comment 1

Einstein's equation for absorption and stimulated emission can be adapted for the laser.

Instead of the thermal occupation of the energy levels, the photon number n_λ is now the decisive variable.

The first underlined equation describes the influence of absorption on the number of the photons.

It is obvious that the influence of the absorption is proportional to the population of the lower quantum state N_1 and the number of the available photons.

The new proportionality constant W_{21} will be discussed in a moment.

Laser 2 Comment 2

Likewise, the influence of the stimulated emission on the photon number is proportional to the population of the excited quantum state N_2 and the number of the available photons.

Absorption decreases the photon number, while stimulated emission increases the photon number.

The third equation is Einstein's spontaneous emission equation.

The equation outlined in red shows the formula for the proportionality constant W_{21}.

W_{21} results from the definition of the spectral energy density.

Laser 3

Rate equation of the photon number n_λ

$$\boxed{\frac{dn_\lambda}{dt} = \frac{dn_\lambda^{abs}}{dt} + \frac{dn_\lambda^{stim}}{dt} + \frac{dn_\lambda^{spon}}{dt} + \frac{dn_\lambda^{loss}}{dt}}$$

$$\underline{\frac{dn_\lambda^{loss}}{dt} = -\frac{n_\lambda}{\tau}}$$

$$\frac{dn_\lambda^{stim}}{dt} >> \frac{dn_\lambda^{spon}}{dt}$$

There is no coherent light due to spontaneous emission $\frac{dn_\lambda^{spon}}{dt}$!

Laser 3 Comment 1

The formula outlined in red shows the rate equation for the photon number.

The photon number is reduced by absorption and increased by simulated emission.

The spontaneous emission leads to incoherent light and disrupts the coherent light of the laser.

The last term describes the loss of photons in the active laser area.

The underlined equation describes the loss term more precisely.

The loss is proportional to the photon number and is determined by a loss rate τ^{-1}.

The rate of loss is determined by the construction of the laser.

Laser 3 Comment 2

Spontaneous emission is important for starting the laser.

The inequality written in blue says that the contribution of the spontaneous emission must be very much smaller than the contribution of the stimulated emission when the laser is operated.

Laser 4

Comparison between stimulated and spontaneous emission in thermal equilibrium

$$\boxed{\begin{aligned} \frac{dN_2^{stim}}{dt} &= -u(\lambda)B_{21}(\lambda)N_2 \\ \frac{dN_2^{spon}}{dt} &= +A_{21}(\lambda)N_2 \end{aligned}}$$

with

$$u(\lambda) = \frac{A_{21}(\lambda)}{B_{21}(\lambda)} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

and

$$\frac{dN_2^{stim}}{dN_2^{spon}} = \frac{B_{21}(\lambda)}{A_{21}(\lambda)} u(\lambda) = \frac{1}{e^{hc/\lambda k_B T} - 1}$$

Laser 4 Comment

The first lasers were build in 1960.

To understand the challenge of building a laser, it is useful to consider the relationship between stimulated and spontaneous emission in thermal equilibrium.

The equations framed in red give the Einstein equations for the occupation numbers of the excited state 2.

The underlined equation indicates the spectral energy density in thermal equilibrium (compare Planck's radiation law).

The last line gives the quotient between stimulated and spontaneous emission.

The following example shows that stimulated emission is completely irrelevant in thermal equilibrium!

Laser 5

Example: $\lambda = 600$ nm and $T = 300$ K

$$\frac{dN_2^{stim}}{dN_2^{spon}} = \frac{1}{\exp\left(\frac{6.626 \cdot 10^{-34}\,\mathrm{Ws}^2\; 3 \cdot 10^8\,\mathrm{ms}^{-1}}{6 \cdot 10^{-7}\,\mathrm{m}\; 1.38 \cdot 10^{-23}\,\mathrm{WsK}^{-1}\; 300\,\mathrm{K}}\right) - 1} = \frac{1}{\exp(80)}$$

$$= \frac{1}{\underline{5.5 \cdot 10^{34}}}$$

Laser 6

rate equation for the relevant photon number n_λ

$$\boxed{\frac{dn_\lambda}{dt} = (N_2 - N_1)W_{21}n_\lambda - \frac{n_\lambda}{\tau}} = \left((N_2 - N_1)W_{21} - \frac{1}{\tau}\right) n_\lambda$$

solution of the differential equation (N_1, N_2 and τ are constant)

$$\underline{n_\lambda(t) = n_\lambda^0\, e^{At}}$$

$$A = \left((N_2 - N_1)W_{21} - \frac{1}{\tau}\right)$$

condition for amplification

$$A = (N_2 - N_1)W_{21} - \frac{1}{\tau} > 0 \quad \rightarrow \quad \text{population inversion} \quad \boxed{N_2 - N_1 \geq \frac{1}{W_{21}\tau}}$$

Laser 6 Comment 1

The formula outlined in red gives the rate equation of the photon number.

Only the photon number n_λ of the laser laser light has to be taken into account.

The spontaneous emission is not included in this equation.

Spontaneous emission is only important to start the laser process.

Once started, the laser process leads to an exponential increase in the photon number n_λ and the spontaneous emission can be neglected.

The solution of the rate equation is a simple exponential function as long as the occupation numbers and the loss rate do not depend on the photon number.

Laser 6 Comment 2

Due to the exponential increase, the laser photons dominate and the contribution of the spontaneous emission can be neglected.

The last equation gives the condition for the laser process.

Laser 7

$\rightarrow$ population inversion

$$\boxed{N_2 - N_1 > \frac{1}{W_{21}\tau}}$$

with

$$\frac{1}{W_{21}} = \frac{V\lambda\Delta\lambda}{hcB_{21}}$$

- small volume, i.e. high photon density
- small spectral width $\Delta\lambda$
- the wavelength λ should not be to small due $B_{21} \propto \lambda^2$
- for a continuous wave laser $W_{21}(N_2 - N_1) = \tau^{-1}$

Laser 7 Comment 1

In order to be able to amplify light through stimulated emission, the occupation number of the excited state 2 must be greater than the occupation number of the ground state 1 and one speaks of population inversion.

Population inversion never occurs in thermal equilibrium.

The underlined equation shows that the width of the transition used for the stimulated emission should be small.

At small wavelengths, amplification by stimulated emission becomes difficult because the Einstein coefficient B_{21} is proportional to the square of the wavelength.

Therefore, it becomes more difficult to generate coherent light by using the stimulated emission as the wavelength becomes shorter.

Laser 7 Comment 2

A distinction is made between pulsed and continuous wave lasers.

For a continuous wave laser, a certain ratio has to be established between the loss rate and the difference in occupation numbers.

Revision

Summary in questions 1

1. Explain the meaning of the Einstein coefficient A_{21}?
2. Explain the meaning of the Einstein coefficient B_{21}?
3. Explain the meaning of the Einstein coefficient B_{12}?
4. Why are the Einstein coefficients of absorption and stimulated emission the same?
5. Explain the basic assumptions of Wien's radiation law.
6. Explain the basis assumptions of Rayleigh-Jeans law.
7. Explain the basic assumptions of Planck's law of radiation.

Summary in questions 2

8. What does the acronym LASER mean?

9. State the basic condition for light amplification by stimulated emission of radiation.