Modern Physics

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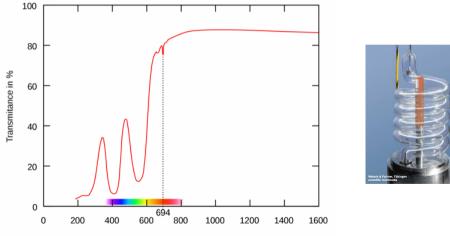
- 1 Classical Wave Phenomena
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- 5 Atoms
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Wave-particle dualism

- Thermal radiation
- Planck's radiation law
- Photoelectric effect
- Laser
- Compton effect
- Pair production
- Matter waves
- Uncertainty relations

Laser 8: Ruby-Laser ($Cr^{3+}:Al_2O_3$)

Absorption spectrum of ruby



Wavelength in nm

Comment

The first lasers were realized in 1960.

One laser was the ruby laser, the other the He-Ne laser.

The chemical formula of ruby is Al_2O_3 .

The AI_2O_3 crystal lattice is doped with Cr^{3+} ions.

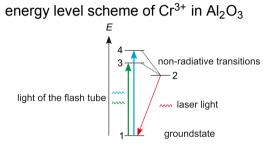
The spectrum shows the absorptions of the Cr^{3+} ions.

There are two strong absorptions in the blue and green regions of the spectrum and a tiny absorption in the red region.

The illustration on the right shows the ruby laser.

The ruby crystal is surrounded by a flash tube.

Laser 9: Ruby-Laser (Cr^{3+} :Al₂O₃)





Disadvantage: More than half of the atoms have to be excited from the ground state into state 2 for population inversion

 \rightarrow large excitation power

Comment 1

The figure shows the relevant energy levels of the Cr³⁺ ion.

The flash tube excites Cr³⁺ ions to energy levels 3 and 4.

The strong absorption lines indicate that this can be done efficiently through the flash tube.

The excited Cr³⁺ ions change quickly from energy level 3 and 4 to energy level 2.

The transitions are not due to electromagnetic radiation, but to lattice vibrations.

Instead of photons, the quantum particles of the lattice vibrations are emitted.

These quantum particles are called phonons in analogy to the photons of electromagnetic waves.

Comment 2

The transition between energy level 2 and the ground state only leads to a very weak absorption in the spectrum.

The transition probability between level 2 and the basic state is very small.

Therefore, population inversion can be achieved by a strong light pulse from the flash tube.

As soon as population inversion is reached, a short laser pulse is emitted and the Cr^{3+} ions relax back to the ground state.

The main disadvantage of this laser is that more than half of the Cr^{3+} ions have to be excited in order to produce population inversion.

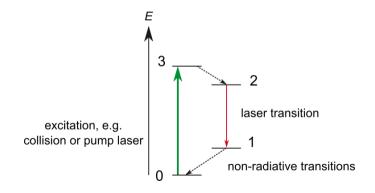
Therefore, the excitation power is large compared to the power of the laser pulse.

Comment 3

In addition, the energy level scheme cannot be used to build a permanent laser.

If a strong permanent light source is used to excite energy levels 3 and 4, the system becomes saturated, i.e. the occupation number is the same for all energy levels and a population inversion cannot be achieved.

4 level scheme



Comment 1

Usually a four-step scheme is used, as shown in the figure.

The laser medium (atoms, molecules, etc.) is excited from the ground state to the excited state 3.

There is a large transition rate from state 3 to state 2.

The laser transition between state 2 and 1 is indicated by a red arrow.

The laser transition does not connect state 2 with the ground state, but with the excited state 1.

If the lifespan in state 1 is very long, a laser pulse results, as with the ruby laser.

With a four-level scheme, however, half of the atoms or molecules no longer have to be excited if a population inversion is to be achieved.

Comment 2

Therefore, a small excitation power is sufficient to generate laser light.

If the lifespan of state 1 is very short, then energy level 1 is practically always unoccupied and the population inversion between states 1 and 2 can be set permanently.

In this case, a continuous wave laser can be implemented with the four-level scheme.

Laser 11: He-Ne Laser



(Gasentladungsroehre.mp4)



The He-Ne laser is a gas laser and is based on a discharge tube.

In a gas discharge tube, a high voltage is applied to two electrodes located in the tube.

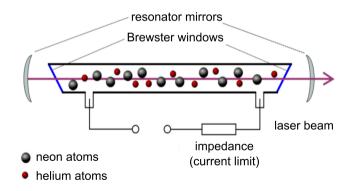
lonized atoms or molecules accelerate and collide with other gas particles, causing an avalanche of charged gas particles and free electrons.

The gas particles are excited by the collision. The gas particles give off the excitation energy again through collisions and through the emission of light

The video shows the light effects of a gas discharge tube.

Laser 12: He-Ne Laser

the basic set-up of a He-Ne laser





The figure shows the basic structure of a He-Ne laser.

The He-Ne laser is a continuous wave laser.

A gas discharge tube is filled with He and Ne atoms.

In the illustration, the gas discharge tube is located between two mirrors and the tube itself is closed off by so-called Brewster windows.

The gas discharge tube can also be closed directly by mirrors.

One of the mirrors is semi-transparent so that laser light can leave the discharge tube.

Laser 13: He-Ne Laser



(brewster.mp4)

Revision

Laser 13



The video demonstrates the effect of the Brewster windows.

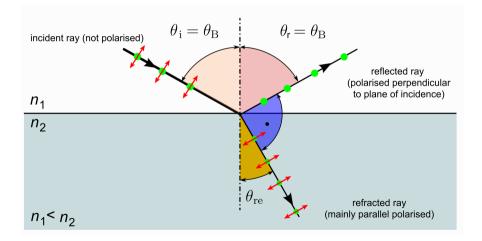
Only light that is polarized perpendicular to the direction of the beam and to the normal of the glass plate is reflected if the Brewster condition (Brewster angle) is met.

Light that is polarized in the plane that is formed by the laser beam and the normal of the window can pass through the window without loss.

The perpendicular polarization is greatly weakened by the reflection and can therefore not be amplified by stimulated emission.

The suppression of one polarization direction can be avoided if the mirrors are built directly into the end caps of the gas discharge tube.

Laser 14: Brewster's angle (1815)





The sketch illustrates the Brewster effect.

If the reflected beam and the refracted beam are perpendicular to each other, then only the component of the light that is polarized perpendicular to the plane of incidence is reflected.

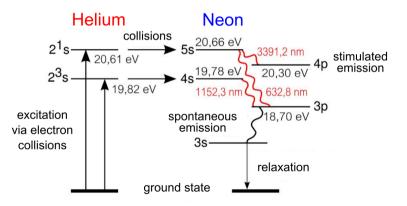
The Brewter effect is described by the Maxwell equations and in particular by the Fresnel equations.

The heuristic explanation is based on the Hertzian dipole.

With a Hertzian dipole, the emitted radiation disappears along the dipole axis and is maximal in the vertical direction.

If the Brewster condition is fulfilled, no light polarized in the plane of incidence can be reflected.

relevant energy levels of the He and Ne atoms





The figure shows the relevant energy levels of the He atom and some energy levels of the Ne atom.

There are many more energy levels in the neon atom than in the helium atom because the electron configuration of helium is made up of 2 electrons while the configuration of neon is made up of 10 electrons.

The two excited energy levels of the helium atom can be excited directly by collisions with free electrons.

The excitation energy of the helium atoms is transferred to neon atoms when helium atoms collide with neon atoms.

Comment

The 4s and 5s levels of the neon atom are excited, so that the population inversion condition is met for many transitions to lower energy states.

The transition between the 5s and 3p levels, which is often used in the He-Ne laser, is particularly strong.

The wavelength of the transition is in the red area of the visible spectrum.

Laser 16: He-Ne Laser



(HeNeFreiburg.mp4)



The video shows how a He-Ne laser works.

If the gas discharge tube is placed between two mirrors, the light amplification starts through stimulated emission.

The video also shows that many spectral lines are excited by the collisions of the atoms.

Only one transition is amplified by the stimulated emission.

Which transition this is can be set by adjusting the mirrors.

The emission of the red laser light is particularly robust, but the emission of other wavelengths is also possible.

Laser 17: N₂ Laser



(N2(Stickstoff)Laser.mp4)

Comment

The video shows the simple construction of a N_2 laser that can be operated directly with air.

It consists of two aluminum plates, a spark gap and a high voltage source.

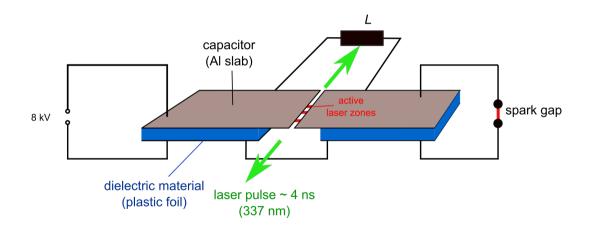
The laser generates pulses in the ultraviolet spectral range with a pulse duration of approximately 4 ns.

The ultra violet light is detected by the photoluminescence of paper that absorbs the laser light.

As with the He-Ne laser, the laser light is generated by a transition between excited states of the N_2 molecule.

In contrast to the He-Ne laser, the lifetime of the lower energy level is long, so that the condition of population inversion is quickly suppressed due to saturation.

Laser 18: N₂ Laser



Comment 1

The sketch shows the basic experimental setup.

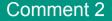
Two capacitors are separated by a small gap created by the edges of the Al plates.

The capacitors are connected by an inductance (or impedance).

High voltage is applied to one capacitor.

This creates a high electric field in the gap between the two capacitors.

lonized air molecules are accelerated, collide and create an avalanche of excited molecules.



The voltage applied to the second capacitor increases slowly due to the inductance.

If the voltage of the second capacitor reaches a critical value, the spark gap ignites and the two capacitors are discharged.

The first capacitor is immediately charged again by the high voltage source, so that a large electric field builds up again in the gap between the two aluminum plates.

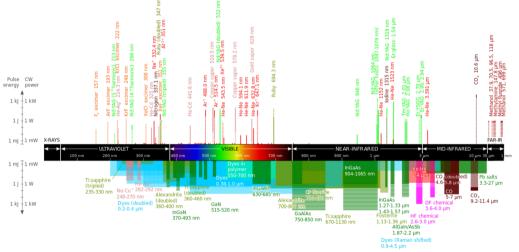
In this way, laser pulses are generated almost permanently.

In contrast to the He-Ne laser, this laser does not need a resonator.

Comment 3

However, only light is amplified that propagates essentially along the gap between the aluminum plates.

Perpendicular to the gap, the photon loss rate is obviously so large that no population inversion can be achieved in these directions.



Wavelengths of commercially available lasers. Laser types with distinct laser lines are shown above the wavelength bar, while below are shown lasers that can emit in a wavelength range. The height of the lines and bars gives an indication of the maximal power/pulse energy commercially available, while the color codifies the type of laser material. Most of the data comes from Weber's book Handbook of laser wavelengths.



The figure gives an overview of the lasers based on stimulated emission of radiation.

The wavelength ranges from infrared to ultraviolet.

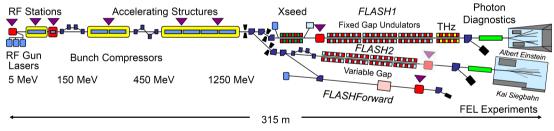
The shortest wave length is 157 nm.

Lasers with shorter wave length are no longer based on the stimulated emission of radiation.

Laser 20: FEL (Free Electron Laser)

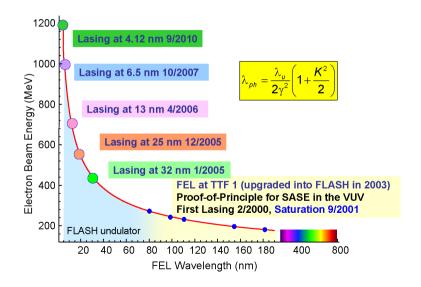


Laser 21: FEL (Free Electron Laser)

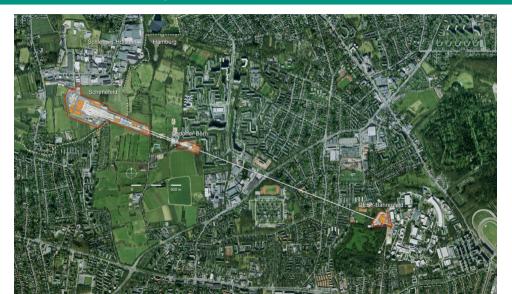


Schematic layout of FLASH (not to scale). Beam direction is from left to right, the total length is 315 m. (Image: DESY/Siegfried Schreiber)

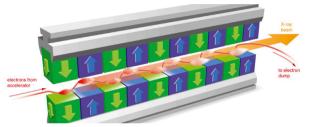
Laser 22: FEL



Laser 23: XFEL (X-Ray Free-Electron Laser, $\lambda \approx 0.05$ to 4.7 nm)



Laser 24: XFEL



In the undulator, the electron bunches are guided by structures with periodically arranged magnets (green and blue). This brings the electrons on a tight slalom course. In doing so, they emit X-ray light, which continues to intensify.

Amplification by constructive interference: $n\lambda = L_u \sqrt{1 - \frac{v^2}{c^2}} \sqrt{\frac{1 - v/c}{1 + v/c}} = L_u (1 - v/c)$

$$\nu_n = \frac{c}{\lambda_n} = \frac{nc}{L_u} \frac{(1+v/c)}{(1-v/c)(1+v/c)} \approx \frac{nc}{L_u} 2\gamma^2 = \frac{2nc}{L_u} \left(\frac{E}{m_0 c^2}\right)^2$$

Laser 24: XFEL

Comment 1

The electron beam is forced into a zigzag path by an alternating magnetic field.

If the lateral acceleration is not too great, then electromagnetic radiation is emitted mainly in the direction of the electron beam.

The radiation is amplified by constructive interference when the wavelength is a multiple of the characteristic undulator length L_u . L_u denotes the period of the magnetic field modulation.

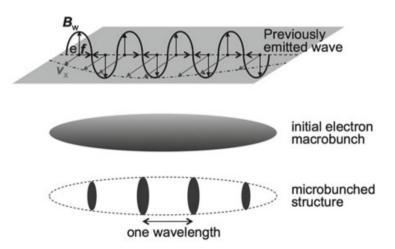
Since the electrons move almost at the speed of light, the length contraction must be taken into account in the coordinate system of an electron.

In addition, since the electrons are moving in the direction of the experiment, the Doppler effect must be taken into account and the formula underlined in red results.



Assuming an undulator period of $L_u = 10$ cm an electron energy of 15 GeV can be estimated for a wavelength of 0.05 nm.

Laser 25: SASE (Self-Amplified Spontaneous Emission)



Microbunching mechanism. Top the Lorentz force due to the interaction between the transverse B-field of previously emitted waves and the transverse velocity of electrons oscillating in an undulator pushes the electrons towards every other node in the wave. In this way, an initially unstructured macrobunch (middle) develops a structure (bottom) consisting of microbunches with the period of one wavelength.

Laser 25: XFEL



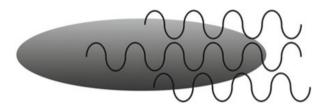
The upper picture shows an electron beam with the transverse velocity v_x , which is imposed on the beam by the magnetic field of the undulator.

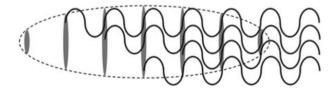
The solid line shows the magnetic field of an electromagnetic wave that is radiated by the accelerated electrons.

Due to the Lorentz force, the magnetic field of the wave drives the electrons to a zero crossing of the magnetic field strength in the manner outlined.

The initial bunch of electrons breaks up into a micro-bunched structure as shown in the lower picture.

Laser 26: XFEL





from: Synchrotron Radiation Settimio Mobilio, Federico Boscherini, Carlo Meneghini (Editors), Springer-Verlag Berlin Heidelberg 2015

Laser 26: XFFL



The upper figure shows the incoherent emission of radiation of the initial electron bunch.

The lower figure shows the coherent emission of radiation by the micro-bunched electrons.

Wave-particle dualism

- Thermal radiation
- Planck's radiation law
- Photoelectric effect
- Laser
- Compton effect
- Pair production
- Matter waves
- Uncertainty relations

Albert Einstein (1905)

the energy quanta of electromagnetic radiation are "localized in spatial points, move without division and can only be absorbed and generated as a whole"

Arthur Holly Compton (1922)

the energy quanta carry a momentum: $p = \frac{h}{\lambda}$ and $\vec{p} = \hbar \vec{k}$

$$\frac{E = h\nu = \frac{hc}{\lambda} = cp}{\lambda}$$

Comment 1

In 1900 Max Planck postulated that the energy of electromagnetic radiation can only be absorbed and emitted in quanta of $\hbar\omega$.

With this assumption, the spectrum of thermal radiation can be perfectly explained.

In 1905, Albert Einstein formulated Planck's assumption in explaining the photo effect more precisely: the energy quanta of electromagnetic radiation are "localized in spatial points, move without division and can only be absorbed and generated as a whole".

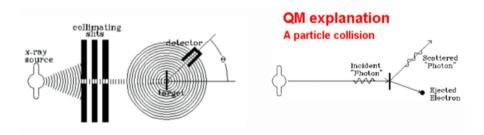
When Arthur Compton investigated the scattering of X-rays on graphite in 1922, he was able to show explicitly that the quanta of electromagnetic radiation behave like relativistic particles.

Comment 2

In 1927 Arthur Compton was awarded the Nobel Prize for "for his discovery of the effect named after him".

The relationship between energy and momentum corresponds to the formula that results from the special theory of relativity for a particle without rest mass.

Scattering of high-energy photons on quasi-resting electrons





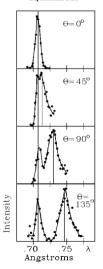
The illustration shows the experimental setup.

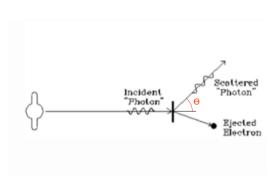
An X-ray beam is generated through several apertures.

The X-ray beam hits a sample and the scattered X-ray beam is measured at the angle θ with respect to the incident beam.

With the detector, the intensity and also the wavelength, i.e. the energy of the scattered X-rays can be measured.

Compton's Experimental Data







The left figure shows experimental results for different scattering angles θ .

The measurements show that there is an elastic component in the spectrum of the scattered X-rays.

In elastic scattering, the wavelength does not change when the scattering angle is changed.

The spectrum also contains an inelastic component, i.e. the wavelength changes when the scattering angle is changed.

The wavelength becomes larger with increasing angle θ and and the energy of the scattered photons is less than the energy of the incident photons.

Comment 2

The figure on the right illustrates the interpretation of the experimental results.

The incident photon transfers part of its energy and momentum to an electron.

If the binding energy and the kinetic energy of the electron can be neglected compared to the energy of the incident photon, then the effect can be described by the collision of a photon with a quasi-free electron that is at rest.

If the binding energy of the electron is very large compared to the energy of the incident photon, then there is an elastic collision of the photon with the atom to which the electron is bound.

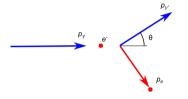
In the case of an elastic collision, no energy but only momentum is transferred to the atom (Think of the elastic reflection of a ball hitting a wall.).

Comment 3

The scattered photon has the same energy as the incident photon.

Only the direction in which the photon is moving has changed.

These photons result in the elastic peak in the spectrum.



energy conservation

$$m{E}_{\gamma}+m{m}_0m{c}^2=m{E}_{\gamma'}+m{E}_{e}$$

momentum conservation

$$ec{oldsymbol{
ho}}_{\gamma}=ec{oldsymbol{
ho}}_{\gamma'}+ec{oldsymbol{
ho}}_{oldsymbol{
ho}}$$

The energy and momentum of the electron can be eliminated with the help of $E_e^2 + c^2 \vec{p}_e^2 = m_0^2 c^4$ and $(\vec{p}_\gamma - \vec{p}_{\gamma'})^2 = \vec{p}_e^2$



Compton showed that the inelastic component can be traced back to the collision of a photon with an electron that is quasi at rest.

Quasi at rest means that both the kinetic energy and the binding energy of the electron can be neglected compared to the energy of the photon.

The illustration outlines this situation.

The law of conservation of energy states that the sum of the energy of the photon and the rest energy of the electron before the collision must be equal to the sum of the energy of the photon and the total energy of the electron after the collision.

Comment 2

The law of conservation of momentum states that the momentum of the incident photon must be equal to the sum of the momentum of the photon and that of the electron after the collision.

With the relativistic energy-momentum relationship, the energy and momentum of the electron can be eliminated.

$$\frac{1}{E_{\gamma'}} - \frac{1}{E_{\gamma}} = \frac{1}{m_0 c^2} (1 - \cos \theta)$$

with $E_{\gamma} = h\nu = hc/\lambda$

$$\frac{\lambda'}{hc} - \frac{\lambda}{hc} = \frac{1}{m_0 c^2} (1 - \cos \theta)$$

Compton formula

$$\lambda' - \lambda = \lambda_{C}(1 - \cos \theta)$$

Compton wavelength

$$\lambda_C = \frac{h}{m_0 c} = 2.43 \cdot 10^{-12} \,\mathrm{m}$$

Revision

Compton effect 5



A small calculation results in the first formula outlined in red.

The Compton formula results from Planck's law $E = h\nu$.

In the forward direction, the wavelength of the scattered photon coincides with the wavelength of the incident photon.

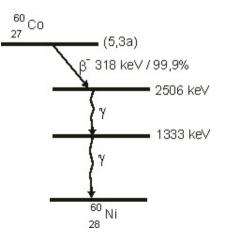
If the direction of observation deviates from the direction of incidence, the wavelength of the inelastically scattered photon increases and reaches the greatest value with backward scattering.

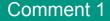
The strength of the effect is determined by the Compton wavelength.

The Compton wavelength is about five orders of magnitude below the wavelength of visible light.

Compton effect: Example 6

radioactive decay of ⁶⁰Co





The Compton effect is usually observed in the gamma decay of nuclei.

The figure shows the decay of cobalt to nickel.

The cobalt nucleus ejects an electron in the β decay and thus increases its nuclear charge by one elementary charge.

Cobalt turns into nickel.

However, the resulting nucleus of nickel is in an excited state and changes to the ground state through two successive gamma decays.

Gamma radiation is electromagnetic radiation.

Comment

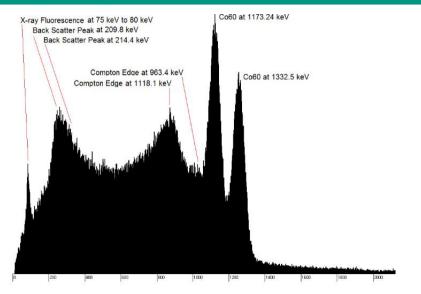
Compton effect 6

The energy of gamma particles is much greater than the typical energy of X-rays.

The first gamma particle has an energy of 1173 keV.

The second gamma particle has an energy of 1333 keV.

Compton effect: example 7



Comment 1

The figure shows the spectrum of gamma radiation when cobalt decays.

Since the nickel nuclei can emit the gamma particles in all directions, when the gamma spectrum is measured, a distribution over all angles θ in the Compton formula results.

The two peaks of the elastic scattering can be clearly seen in the spectrum.

The energy corresponds to the energy of the nuclear transition.

Then there is a broad spectrum that extends to the so-called backscatter peaks. The energy of these peaks is slightly greater than 200 keV.

The sharp peak at even lower energy is due to fluorescence.

The gamma quanta excite the electrons of the cobalt and nickel atoms and the excitation energy is released by the emission of fluorescence photons.

$$\frac{1}{E_{\gamma'}}-\frac{1}{E_{\gamma}}=\frac{1}{m_0c^2}(1-\cos\theta)$$

backscattering $\theta = 180^{\circ}$

$$rac{1}{E_{\gamma'}}-rac{1}{E_{\gamma}}=rac{2}{m_0c^2}$$

for $E_{\gamma} >> m_0 c^2$

$$E_{\gamma'}pprox rac{m_0 c^2}{2}=250\,{
m keV}$$

Comment 1

The energy of the backscatter can be understood with the formula outlined in red.

In the case of backward scattering, the right-hand side of the formula corresponds to the reciprocal value of half the rest energy of the electron, i.e. the reciprocal value of around 250 keV.

If the incident photon has a very high energy, then its contribution in the Compton formula can be neglected and the energy of the scattered photon tends towards half the rest energy of the electron.

If the energy of the incident photon cannot be neglected, then the reciprocal value of the energy of the incident photon is added to the reciprocal value of half the rest energy and the energy of the scattered photon is slightly smaller.



With the energy of the two photons of the cobalt decay, the values 210 and 214 keV result for the backscatter peaks.

Another distinctive feature in the Compton spectrum is the Compton edge.

The reason for the Compton edge lies in the high kinetic energy of the electrons, which are created when the photons are backscattered.

These electrons are strongly decelerated on their way through the material of the sample and emit photons, as can be expected according to Maxwell's electrodynamics.

One speaks of bremsstrahlung.



In the extreme case, an electron converts its entire kinetic energy into a photon.

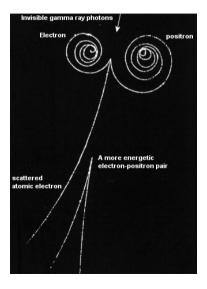
These photons, which absorb almost the entire kinetic energy of the electrons, which were created during the backward scattering, form the Compton edge.

The energy results from the energy of the original photon minus the energy of the backscattered photon.

This means that the energy of the Compton edges can be expected at 963 keV and 1173 keV, respectively

Wave-particle dualism

- Thermal radiation
- Planck's radiation law
- Photoelectric effect
- Laser
- Compton effect
- Pair production
- Matter waves
- Uncertainty relations



Comment 1

Arthur Compton's work showed that the quanta of electromagnetic radiation can be treated as particles that obey the laws of conservation of energy and momentum.

Patrick Blackett discovered the conversion of photons into electron-positron pairs while studying cosmic rays in 1933.

The positron is the antiparticle to the electron. Particles and antiparticles are exactly the same in all properties. Only in the case of the charge do the two particles differ in sign.

The illustration shows contrails caused by electrons and positrons in a cloud chamber. A cloud chamber is a chamber that is filled with saturated water vapor.

A magnetic field is applied to the cloud chamber, so that the path of charged particles is curved due to the Lorentz force ($\vec{F}_L = q\vec{v} \times \vec{B}$).

Comment 2

Pair production 1

Only charged particles lead to contrails.

The uncharged photons are not visible.

In the upper part of the figure, a photon decays into an electron-positron pair.

The decay takes place near an atom and part of the energy of the photon is transferred to an electron in the atom, which is knocked out of the electron shell.

A second event can be seen in the lower part of the figure.

During this decay, the entire energy of the photon is transferred to the electron-positron pair.

The curvature of the particle trajectories shows that the kinetic energy of the electron-positron pair is greater than the kinetic energy of the electron-positron pair that was created in the upper area of the image.

1st condition for pair production

 $h
u > 2m_0c^2$

 $m_0 c^2 \approx 500 \, \text{keV}$ denotes the rest energy of the electron



The 1st condition for pair formation is that the energy of the photon is greater than twice the rest energy of the electron.

2nd condition for pair production

$$ec{m{
ho}}_{\gamma}
eq m{0}$$

In the center-of-momentum frame of the electron-positron pair

$$ec{p}_{e}+ec{p}_{\overline{e}}=0$$

A free photon can not decay into an electron positron pair!

A photon can only decay if there is a partner with whom momentum can be exchanged



The momentum of a photon $E_{\gamma} = cp_{\gamma}$ is never zero.

The momentum of the electron positron pair is zero in the center-of-momentum frame.

Therefore the law of conservation of momentum cannot be fulfilled for the decay of a single photon as long as it moves freely through space.

Free photons cannot decay.

The decay of a photon is only possible when other particles (atoms or molecules) are nearby, so that the exchange of energy and momentum becomes possible.

In one electron-positron decay, which is shown in the cloud chamber image, so much momentum is transferred to an atom that an electron is knocked out of the electron shell.



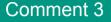
When an electron-positron pair annihilates, two photons are created.

The momentum is conserved during this process.

Radioactive tracers (β^+ -decay) are used for functional imaging in medicine.

The gamma radiation emitted during the pair annihilation is detected and forms an image analogous to X-ray tomography.

This is known as positron emission tomography (PET).



Electrons and positrons will immediately be called true particles.

Photons, on the other hand, are the quantum particles of electromagnetic radiation.

The conversion of photons into electron-positron pairs and vice versa the conversion of electron-positron pairs into photons shows that there is no differentiation between the quantum particles of a wave and apparently true particles.

It turns out that electrons and positrons are also the quantum particles of electron waves.

These waves are called matter waves to distinguish them from waves formed by quantum particles with no rest mass.

Revision

Summary in questions 1

- 1. Explain how the ruby laser works.
- 2. Explain the four-level scheme for generating laser light.
- 3. Explain the role of the helium atoms in the He-Ne laser.
- 4. Why is the light of some He-Ne lasers polarized?
- 5. What is the Compton Effect?
- 6. Write down the law of conservation of energy and momentum for the Compton effect.
- 7. Give the relationship between the energy of the incident photon and the scattered photon.

Summary in questions 2

- 8. Give the relationship between the wavelength of the incident photon and the scattered photon.
- 9. Give the order of magnitude of the Compton wavelength.
- 10. Sketch the Compton spectrum of a nuclear γ decay.
- 11. What denotes the Compton edge?
- 12. Give the energy of the Compton edge.
- 13. What is pair production?
- 14. Give the minimum energy of a photon for pair production.
- 15. Explain why a free photon is a stable particle.