# Modern Physics

**Contents:**


1. Classical Wave Phenomena
2. Essentials of Thermodynamics
3. Special Relativity
4. Wave-Particle Dualism
5. Atoms
6. Solids

# Matter waves

# Wave-particle dualism

- Thermal radiation
- Planck’s radiation law
- Photoelectric effect
- Laser
- Compton effect
- Pair production
- Matter waves
- Uncertainty relations

# Matter waves 1: classical mechanics and wave mechanics

Particle wave dualism for electromagnetic waves

energy and frequency

$$\boxed{E = h\nu = \hbar\omega}$$

momentum and wavelength

$$\boxed{p = \frac{h}{\lambda} \quad \text{and} \quad \vec{p} = \hbar\vec{k}}$$

special relativity

$$\boxed{E = cp} \quad \text{and} \quad \nu = \frac{c}{\lambda}$$

# Matter waves 1 Comment 1

During the 18th century, under the influence of Newton, light was considered a particle phenomenon.

In particular, the Newtonian particle image was confirmed by the aberration of the light from the stars.

In the 19th century, it was accepted that light is a wave phenomenon.

However, the aberration of the light from the stars made the theory of aether very complicated and contradictory.

By realizing that light is an electromagnetic wave and by the theory of special relativity, these problems were solved.

# Matter waves 1 Comment 2

With the explanation of the photoelectric effect, Albert Einstein showed that the energy of electromagnetic waves is quantized.

Arthur Compton showed that there is also a momentum associated with the energy quanta of electromagnetic radiation, as predicted by the special theory of relativity.

The energy quanta of electromagnetic radiation are particles that we call photons.

In 1924 Louis de Broglie postulated that the wave-particle dualism is a general phenomenon of nature.

With his excellent knowledge of classical mechanics, he was able to show how the properties of waves are related to the properties of the particles.

# Matter waves 1 Comment 3

The formulas summarize the wave-particle dualism of electromagnetic waves.

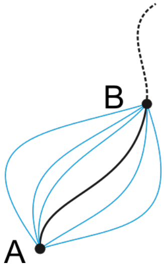
The first formula outlined in red gives Planck’s law for the energy of light quanta and the second formula outlined in red the relationship between the wavelength and the momentum of the particles.

In 1924 de Broglie showed that these formulas, which follows from the special theory of relativity in connection with electromagnetic waves, must generally apply to all types of particles.

The following slides outline the historical background.

# Matter waves 2: classical mechanics and wave mechanics

Fermat 1660: Light takes a path where the transit time takes an extreme value.



# Matter waves 2 Comment

Pierre de Fermat did not care about waves or particles but realized that light always takes the path that can be traversed within the shortest possible time.

Fermat was the first ever to use an extremal principle to derive a law of nature.

The figure illustrates the extremal principle.

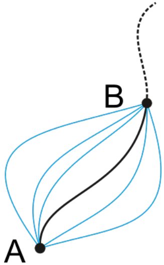
There are several possible paths between point A and B, but in nature only one path is used: the path of the shortest time.

In the 17th and 18th centuries the wave theory of Huygens (1650) and the corpuscle theory of light by Newton (1704) were discussed.

Independent of these ideas, Fermat’s theory makes it possible to describe the propagation of light in the context of geometrical optics.

# Matter waves 3: classical mechanics and wave mechanics

Lagrange 1788: Newtonian mechanics can be traced back to an extremal principle.




The action is extremal for the movement of a mass *m* between point A and B.

# Matter waves 3 Comment

Joseph-Louis Lagrange rediscovered Fermat's extremum principle and showed that Newton's mechanics can also be derived from an extremal principle.

In mechanics, however, it is not the time, but the action that becomes extremal for the path of a particle between point A and B.

The action *S* is a new physical quantity discovered by Lagrange.

# Matter waves 4: classical mechanics and wave mechanics

In 1834 showed Carl Jacobi that the theory of classical mechanics and geometrical optics are identical theories.

1924 Louis de Broglie wondered:

> Since geometrical optics is the limiting case of wave optics
>
> can it be that classical mechanics is the limiting case of a wave mechanics ?

# Matter waves 4 Comment

The mathematician Carl Jacobi showed in 1834 based on the work of Fermat and Lagrange that the theory of geometric optics and classical mechanics are identical theories.

90 years later, Louis de Broglie had the seminal idea.

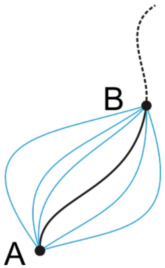
Geometric optics is clearly the limit case of wave optics when the wavelength is small compared to the dimensions of the optical devices.

Louis de Broglie wondered whether classical mechanics can be the limiting case of a wave mechanics that is valid in the microscopic world.

E.g. the quantized energies of atoms can be easily understood if the microscopic world is determined by waves.

The quantized energies of the atoms are then the result of standing waves.

# Matter waves 5: classical mechanics and wave mechanics




differential of the action

$$\boxed{dS = \vec{p}d\vec{r} - Hdt}$$

the Hamilton function $H$ corresponds to the energy $E = E_{kin} + E_{pot}$

but: The speed of the particle has to be replaced by the momentum $\vec{v} = \vec{p}/m$

$$\boxed{H = \frac{\vec{p}^{\,2}}{2m} + E_{pot}}$$

# Matter waves 5 Comment 1

De Broglie’s theory begins with the action of mechanics.

The action is extreme for the path between point A and B.

The action $S$ on the way between A and B changes if the end point B is moved in time $dt$ and space $d\vec{r}$.

The first formula outlined in red shows the change in the action $S$.

The differential of the action is given by the momentum multiplied with $d\vec{r}$ minus the Hamilton function multiplied with $dt$.

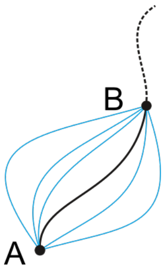
# Matter waves 5 Comment 2

The Hamilton function corresponds to the energy of the particle, e.g. the sum of the kinetic and potential energy.

The energy becomes the Hamilton function when the velocity is expressed in terms of momentum.

The second equation outlined in red gives the Hamilton function of a particle with kinetic and potential energy.

# Matter waves 6: classical mechanics and wave mechanics




necessary time that the light needs for the path $d\ell$

$$dt = \frac{d\ell}{c}$$

and with $c = \omega / k$

$$dt = \frac{1}{\omega} k d\ell$$

and

$$\boxed{dt \propto \vec{k} d\vec{r}}$$

# Matter waves 6 Comment

The underlined equation gives the additional time needed for light when endpoint B is shifted by $d\ell$.

If one replaces the speed of light by the quotient of angular velocity $\omega$ and the wave number $k$, one finds that $dt$ is proportional to the product of wave number and the shift $d\ell$.

The equation outlined in red results when the wave vector $\vec{k}$ and the spatial displacement vector $d\vec{r}$ are used.

The product of wave vector and $d\vec{r}$ is part of the phase of a wave.

# Matter waves 7: classical mechanics and wave mechanics

differential of the phase

$$d\varphi = \vec{k}d\vec{r} - \omega dt$$

differential of the action

$$dS = \vec{p}d\vec{r} - Hdt$$

de Broglie:

The phase of a matter wave is proportional to the action of the particle

# Matter waves 7 Comment

The first equation gives the differential of the phase of a plane wave.

The second equation gives the differential of the action of a particle.

de Broglie concluded that the action of the particle is proportional to the phase of the corresponding matter wave.

# Matter waves 8: classical mechanics and wave mechanics

With Planck's law $H = E = \hbar\omega$ one gets for the phase of a matter wave

$$\boxed{\varphi = \frac{S}{\hbar}}$$

and

$$\boxed{\vec{p} = \hbar\vec{k}}$$

de Broglie wavelength

$$\boxed{\lambda = \frac{h}{p}}$$

# Matter waves 8 Comment

Using the results for the wave-particle duality of electromagnetic waves, de Broglie suggests that the phase of a matter wave is equal to the action of the particle divided by Planck's constant $\hbar$.

Since the frequency of an electromagnetic wave does not change on its way, de Broglie assumes that his result is correct at least if the energy of the particle is conserved.

The Hamilton function corresponds to the energy when the energy is conserved.

This also results in the relationship between the momentum of the particles and the wave number vector.

The wavelength of a matter wave is also known as the de Broglie wavelength.

# Matter waves 9

de Broglie (1924): Wave particle duality is a common property of nature

de Broglie wavelength

$$\boxed{\lambda = \frac{h}{p}}$$

First test (Davisson and Germer) in 1927: Diffraction of electron waves on nickel

# Matter waves 9 Comment

de Broglie concludes that wave particle duality is a general property of nature.

The de Broglie wavelength was first tested experimentally in 1927 by Davisson and Germer.

Davisson and Germer studied the scattering of electrons on nickel crystals.

They observed that the diffraction of electrons is similar to the diffraction of X-rays.

Louis Victor Pierre Raymond, 7th Duc de Broglie was awarded the Nobel Prize in 1929 “for his discovery of the wave nature of electrons”.

# Matter waves 10: Davisson Germer Experiment

**The momentum of the electrons:**

When electrons are accelerated by a voltage $U$, the kinetic energy is

$$\underline{E_{kin} = eU}$$

the total energy of the electron is

$$\underline{E = m_0 c^2 + E_{kin}}$$

with $E^2 - c^2 p^2 = m_0^2 c^4$ and $E^2 = m_0^2 c^4 + 2 m_0 c^2 E_{kin} + E_{kin}^2$ is the momentum

$$\boxed{c^2 p^2 = E_{kin}^2 + 2 m_0 c^2 E_{kin}}$$

small kinetic energy approximation $E_{kin} << m_0 c^2$ ( i.e. $U$ is very much smaller than 500 kV)

$$\boxed{p = \sqrt{2 m_0 E_{kin}}}$$

# Matter waves 10 Comment

The first underlined equation gives the kinetic energy of electrons that are accelerated by the voltage $U$.

For the total energy, the rest energy must be added.

The momentum can be calculated with the relativistic energy-momentum relation.

This results in the formula outlined in red.

The square of the kinetic energy can be neglected if the kinetic energy is much smaller than the rest energy.

In this case the equation on the last line results.

# Matter waves 11: Davisson Germer Experiment

de Broglie wavelength of electrons ($m_0c^2 = 500$ keV)

$$\boxed{\begin{aligned}\lambda &= \frac{h}{p} = \frac{h}{\sqrt{2m_0eU}} = \frac{hc}{\sqrt{2\,m_0c^2eU}}\\ &= \frac{4.14\cdot 10^{-15}\,\mathrm{eVs}\,3\cdot 10^{8}\,\mathrm{ms}^{-1}}{\sqrt{2\cdot 500\cdot 10^{3}\,\mathrm{eV}\cdot eU}}\\ &= 12.42\cdot 10^{-10}\,\frac{\sqrt{\mathrm{V}}\,\mathrm{m}}{\sqrt{U}}\end{aligned}}$$

Comparison with X-rays:

e.g. the $K_\alpha$-line of molybdenum: $\lambda = 0.71\cdot 10^{-10}$ m

the corresponding acceleration voltage for electrons is

$$\underline{U = 305\,\mathrm{V}}$$

# Matter waves 11 Comment

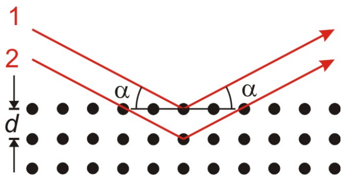
The small calculation in the box gives the de Broglie wavelength for electrons as a function of the acceleration voltage *U*.

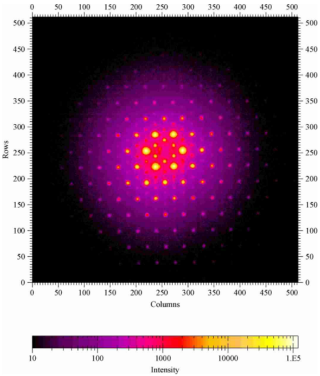
A moderate voltage of 305 V is required if the de Broglie wavelength of the electrons is to be equal to the wavelength of the X-rays of the $K_\alpha$ line of molybdenum.

# Matter waves 12: Davisson Germer Experiment

Bragg formula

$$n\lambda = 2d \sin \alpha_n$$

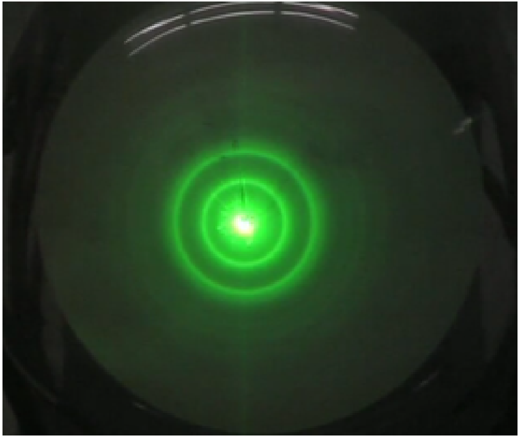


# Matter waves 12 Comment

The angles of the diffraction maxima can also be calculated for electron waves using the Bragg formula for X-rays.

# Matter waves 13: Davisson Germer Experiment



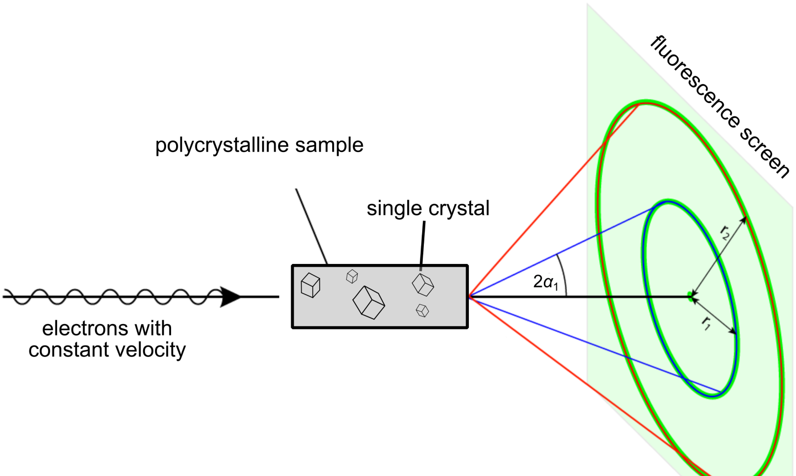
(elektronenbeugung.m4v)

# Matter waves 13 Comment

The video shows the diffraction of an electron beam on a polycrystalline sample of graphite.

# Matter waves 14: Davisson Germer Experiment

diffraction on a polycrystalline sample

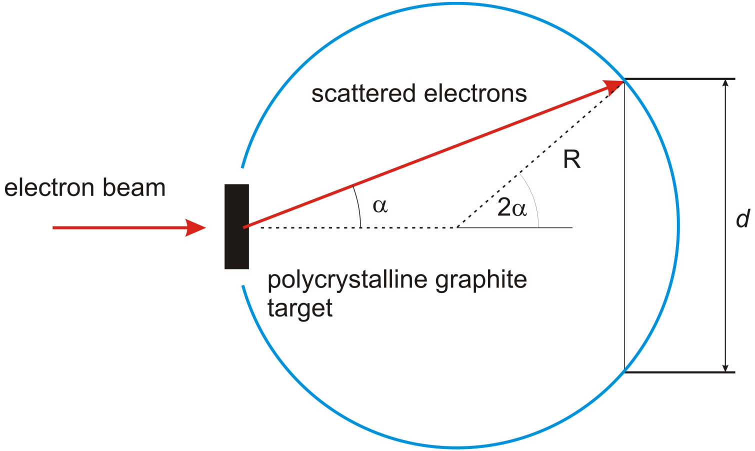


# Matter waves 14 Comment

If a polycrystalline sample is used, the diffraction peaks of the single crystals merge into diffraction rings due to the random orientation of the single crystals within the sample.

This is known as powder diffraction.

# Materiewellen 15: Davisson Germer Experiment



# Matter waves 15 Comment

The illustration shows a sketch of the experiment.

The electron beam is formed in an electron tube that ends in a sphere.

The sample is placed at the entrance of the sphere.

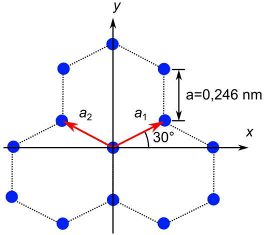
The diffracted electrons are stopped in a fluorescence layer on the inner side of the sphere.

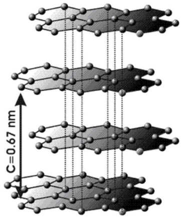
With the known radius of the sphere, the diffraction angle $\alpha$ can be easily calculated from the diameter of the diffraction rings.

The video shows that the radius of the rings increases as the accelerating voltage is reduced.

With Bragg's law, this result is evident since the wavelength increases as the voltage is decreased.

# Matter waves 16: Davisson Germer Experiment




$$\vec{a}_1 = a\left(\frac{\sqrt{3}}{2}\vec{x} + \frac{1}{2}\vec{y}\right)$$

$$\vec{a}_2 = a\left(-\frac{\sqrt{3}}{2}\vec{x} + \frac{1}{2}\vec{y}\right)$$

$$\vec{a}_3 = c\vec{z}$$

$$\vec{b}_1 = \frac{2\pi}{a}\left(\frac{1}{\sqrt{3}}\vec{x} + \vec{y}\right)$$

$$\vec{b}_2 = \frac{2\pi}{a}\left(-\frac{1}{\sqrt{3}}\vec{x} + \vec{y}\right)$$

$$\vec{b}_3 = \vec{z} = \frac{2\pi}{c}\vec{z}$$

# Matter waves 16 Comment

The two figures show the crystal structure of graphite.

The red vectors $\vec{a}_1$ and $\vec{a}_2$ in the hexagonal plane give the lattice vectors which, together with the vector $\vec{a}_3$ along the c-axis, span the primitive unit cell .

It is simple to calculate with the vectors $\vec{a}_{i=1,2,3}$ the basis vectors $\vec{b}_{i=1,2,3}$ of the reciprocal lattice.

$$\vec{b}_1 = \frac{2\pi}{V_{\text{Cell}}}(\vec{a}_2 \times \vec{a}_3)$$
$$\vec{b}_2 = \frac{2\pi}{V_{\text{Cell}}}(\vec{a}_3 \times \vec{a}_1)$$
$$\vec{b}_3 = \frac{2\pi}{V_{\text{Cell}}}(\vec{a}_1 \times \vec{a}_2)$$

# Matter waves 17: Davisson Germer Experiment

distance between the lattice planes

$$\boxed{K = \frac{2\pi}{d}} \quad \rightarrow \quad d = \frac{2\pi}{K}$$

$$|\vec{b}_3| = \frac{2\pi}{c} \quad \rightarrow \quad d = \frac{2\pi}{\frac{2\pi}{c}} = c = 0.67\,\mathrm{nm}$$

$$|\vec{b}_1| = |\vec{b}_2| = |\vec{b}_1 - \vec{b}_2| = \frac{2\pi}{a}\frac{2}{\sqrt{3}} \quad \rightarrow \quad d = \frac{2\pi}{\frac{2\pi}{a}\frac{2}{\sqrt{3}}} = a\frac{\sqrt{3}}{2} = 0.213\,\mathrm{nm}$$

$$|\vec{b}_1 + \vec{b}_2| = 2\frac{2\pi}{a} \quad \rightarrow \quad d = \frac{2\pi}{2\frac{2\pi}{a}} = \frac{a}{2} = 0.123\,\mathrm{nm}$$

Bragg's law $n\lambda = 2d\sin\alpha_n$

# Matter waves 17 Comment

The length of the reciprocal lattice vectors is inversely proportional to the distance between the Bragg planes.

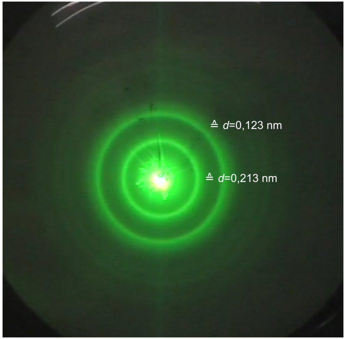
Conversely, the distance between the Bragg planes can be calculated from the length of the reciprocal lattice vectors.

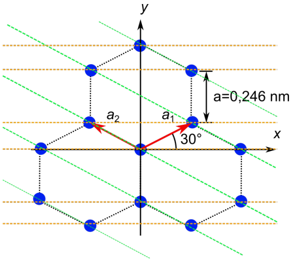
If one takes the vector $\vec{b}_3$ along the $z$-axis, the lattice parameter $c$ results for the distance between the Bragg planes, which is not surprising.

For the vectors $\vec{b}_1$, $\vec{b}_2$ and $\vec{b}_1 - \vec{b}_2$, the distance between the Bragg planes is 0.213 nm.

If one takes the sum $\vec{b}_1 + \vec{b}_2$, then the smaller distance between the Bragg planes of 0.123 nm results.

# Matter waves 18: Davisson Germer Experiment




brown: lattice planes for $\vec{b}_1 + \vec{b}_2$ ($d$=0.123 nm)

green: lattice planes for $\vec{b}_1$ ($d$=0.213 nm)

# Matter waves 18 Comment 1

The comparison with the Bragg formula shows that the diffraction angle decreases with increasing distance between the Bragg planes.

The evaluation of the experiment shows that the outer diffraction ring corresponds to the Bragg plane distance 0.123 nm and the inner ring corresponds to the Bragg plane distance 0.213 nm.

The right figure shows the orientation of some Bragg planes.

The vectors of the reciprocal lattice are always perpendicular to the Bragg planes.

According to its definition, the vector $\vec{b}_1$ is perpendicular to the vector $\vec{a}_2$ and forms an angle of 30° with the $y$-axis. This results in the Bragg planes drawn in green for the vector $\vec{b}_1$.

# Matter waves 18 Comment 2

The vector from the sum of $\vec{b}_1$ and $\vec{b}_2$ is oriented parallel to the $y$-axis and the brown-drawn Bragg planes result.

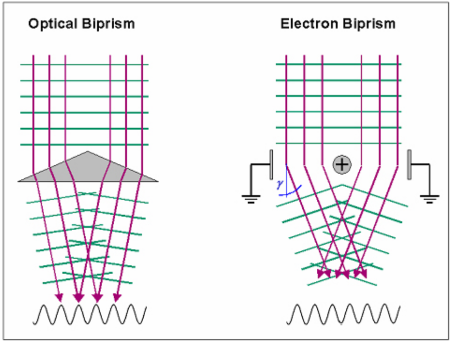
The Bragg planes perpendicular to the c-axis have larger distances and result in the non-resolvable intensity around the central bright point of the non-diffracted electrons.

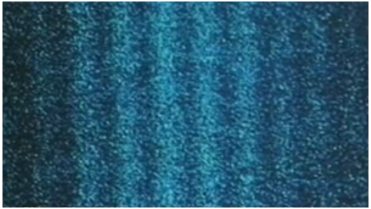
This demonstration experiment shows that electrons and other particles, similar to photons, can be used for diffraction experiments.

Each type of particle has its own special properties, which make it ideal for special applications.

E.g. neutrons have a magnetic moment and are therefore particularly suitable for the exploration of magnetic structures.

# Matter waves 19



(SingleElectronDoubleSlitWaveExperiment.mp4)

# Matter waves 19 Comment 1

The video shows an interference experiment with electrons, in which the electron current is so small that the arrival of individual electrons on the luminescent screen can be observed.

In the experiment, an electron beam is guided past a positively charged electrode on the left and right.

As a result, the electrons are deflected somewhat and interference occurs.

The optical analog is a biprism shown on the left.

A light wave hits the two prisms, which deflect the light rays in such a way that interference occurs.

The astonishing result of the experiment is that an interference pattern forms even though the electrons fly past the electrode independently of one another.

# Matter waves 19 Comment 2

The electron wave can be used to calculate the probability of where the electrons will arrive on the screen.

The square of the magnitude of the wave function determines the particle density and thus the probability.

This is similar to electromagnetic waves, where the square of the electric and magnetic field strength determines the energy density of the wave, i.e. the photon density.

This interpretation of the experiment leads to the problem that Newton wanted to solve in explaining the Newton rings with his corpuscular theory of light.

How do the particles know that there are forbidden areas because they fly past the electrode independently of each other?

# Matter waves 19 Comment 3

The concept of the wave function seems to be an auxiliary mathematical construction which cannot answer the question of what determines the path of the particles.

The wave function is needed while the particle is traveling to calculate the probability of where the particle will end up on the screen.

If a luminous point on the screen signals that the electron has arrived, the concept of the wave function is instantly obsolete.

This property of a wave function is problematic. Since, if the wave function is to describe a physical phenomenon, the speed of light is the highest speed, so that the collapse of the wave function can propagate in space at the most at the speed of light.

# Matter waves 19 Comment 4

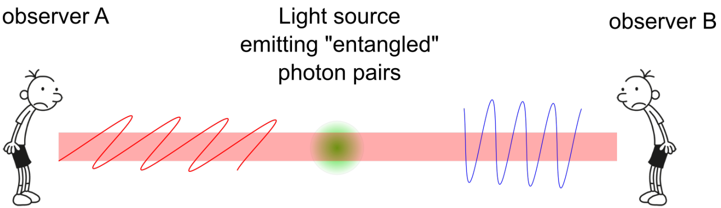
Einstein believed that the matter waves introduced by Louis de Broglie are preliminary and that there must be a more comprehensive theory that can actually explain the behavior of quantum particles.

Einstein summed up the problem by formulating the famous Einstein-Podolsky-Rosen Paradox (EPR Paradox) together with Podolsky and Rosen in 1935.

In this paradox it is shown that the concept of the wave function violates elementary physical principles that were generally accepted up to the point in that wave functions (among other things) enable a remote interaction in which information can be transmitted at infinite speed.

# Matter waves 20: Einstein Podolsky Rosen Paradox

- A non-linear light source emits pairs of photons that are polarized perpendicular to each other.
- Both photons of a pair are described by a common wave function.
- As soon as one observer receives a photon and determines the polarization, the wave function collapses and the result of the second observer's measurement is fixed.



# Matter waves 20 Comment 1

The paradox was formulated in various variants. In the following I am discussing a variant that is used in optics and that does not initially require any special knowledge of quantum physics.

The figure outlines the EPR paradox as it can be realized with a non-linear light source that emits pairs of photons polarized perpendicular to each other.

It is crucial that the light source emits a single wave function in which the polarization of the two photons is coded in such a way that every observer can measure one of the two polarizations with a 50 % probability.

Well noticed! The polarization of the photons is not determined in the light source. The light source only determines the wave function of the two photons.

# Matter waves 20 Comment 2

Only when one observer determines the polarization of his photon does the polarization of the other photon appear immediately, which the second observer can determine. This is the collapse of the wave function.

This information has to get from one observer to another at an infinitely high speed, which is paradoxical.

There is a second problem with this experiment. Usually a measurement can only determine what is actually there.

When measuring the polarization of the photon, however, the polarisation is determined by the measurement, since the light source does not determine which photon should receive which observer.

# Matter waves 20 Comment 3

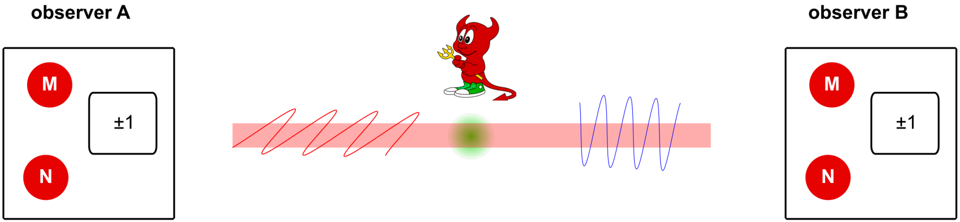
The paradox of Einstein, Podolski and Rosen shows that the concept of the wave function violates two fundamental and generally accepted principles.

The first is called the reality principle (Einstein asks: Is the moon in the sky even when nobody is looking?).

The second is called the locality principle (no signal propagation faster than the speed of light).

# Matter waves 21: Bell’s inequality

James Bell (1964)




E.g. button M of observer A gives the result $M_A = +1$ or $M_A = -1$, etc.

$$S = \langle M_A M_B - M_A N_B + N_A M_B + N_A N_B \rangle$$

# Matter waves 21 Comment 1

Einstein, Podolski and Rosen argue that there must be a “hidden” theory that already determines within the light source which polarization each observer will receive.

One can imagine a demon in the light source, which determines which polarization of the photon observers A and B will receive.

Instead of polarized photons, one can imagine the demon unpacking shoe boxes and sending one shoe to observer A and the other shoe to observer B.

If the demon throws the pairs of shoes randomly, then there is a 50 % probability that every observer will receive a left or right shoe.

The result corresponds exactly to the prediction of quantum mechanics, whereby - thanks to the demon - no laws of physics have to be disregarded.

# Matter waves 21 Comment 2

In 1964 James Bell found a way to test experimentally whether this hidden theory could exist. Thereby the details of the hidden theory not have to be known. The hidden theory only needs to satisfy two principles:

The first principle is the reality principle. The value of a measurand is independent of whether it is measured or not.

The second principle is the principle of locality. If two parts of an object are so far apart that communication even at the speed of light is impossible, the manipulation of one part cannot affect the other.

Quantum physics violates both principles to the maximum.

# Matter waves 21 Comment 3

The sketch shows an experiment proposed by James Bell to test the EPR paradox.

Each observer is equipped with an analyzer that has two setting options, which are marked M and N in the sketch.

If the polarization of the photon corresponds to the orientation of the analyzer, the device displays the number +1, or $-1$ if the polarization does not match.

The positions M and N of the analyzers can be chosen arbitrarily and the apparatus of the two observers do not have to be aligned relative to one another.

It is crucial that both observers analyze the same pair of photons.

The experimental difficulty lies in ensuring this condition in the experiment.

# Matter waves 21 Comment 4

The observers note their measurement results and come together after the experiment and use their joint results to calculate the correlation function *S*, underlined in red.

If, for example, both observers have pressed button M for a photon pair, then it gives a numerical value for the 1st term.

For example, if observer A pressed button M and observer B pressed button N, then it gives a numerical value for the second term, etc.

The results of all experiments are added up and then divided by the number of experiments.

In this way the number *S* is calculated.

# Matter waves 22: Bell's inequality

$$\boxed{S \leq 2}$$

**quantum mechanics**

$$\boxed{S > 2}$$

# Matter waves 22 Comment 1

James Bell showed that the value of $S$ must be less than or equal to 2 if there is a hidden theory.

If quantum theory is valid, then the numerical value of $S$ is always larger than 2.

The first convincing experimental results were published in 1982 by Alain Aspect.

The measurements show that the quantum mechanics describes the experiments correctly and that there is no hidden theory.

Since then, this type of experiment has been repeated in many variations, and the predictions of quantum physics have always been confirmed.

# Matter waves 22 Comment 2

In the meantime, people have come to terms with the fact that quantum physics disregards the plausible principles of reality and locality and recognized that this opens up completely new possibilities, e.g. quantum cryptography and quantum computing.

In 2022 the Nobel prize was awarded to Alain Aspect, John Clauser and Anton Zeilinger ”for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science”

# Revision

# Summary in questions

1. Give the formula for the de Broglie wavelength.
2. Calculate the momentum of an electron that is accelerated with the voltage $U$.
3. What does the collapse of the wave function mean?