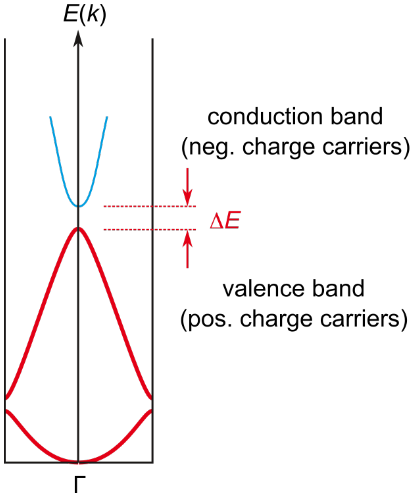
Solids

Semiconductors

Electrons in crystal lattices

Semiconductors 1



Semiconductors 1 Comment 1

The figure shows the energy bands of an insulator or semiconductor for one direction in the reciprocal lattice.

The occupied k states are shown in red.

In the 1st Brillouin zone of an insulator, all k states of an energy band for all directions of the wave vector $\vec{k}$ must be occupied by two electrons.

The solid is a metal if there are regions of an energy band with unoccupied k states in the 1st Brillouin zone.

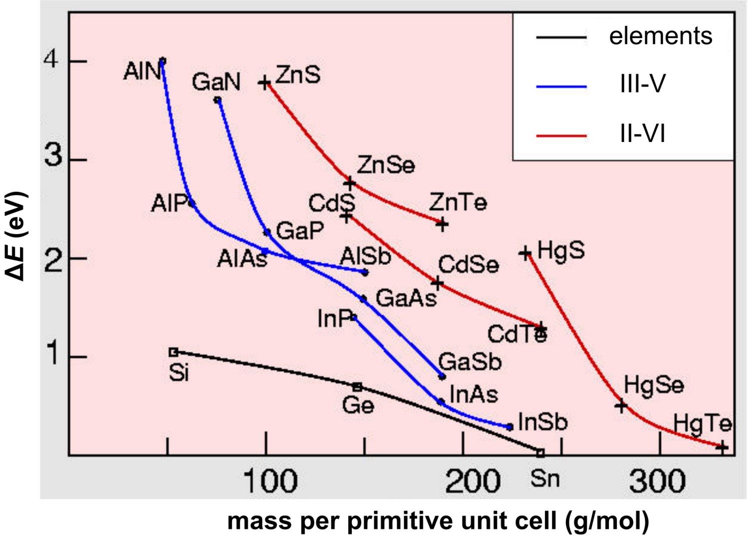
In a semiconductor, the highest occupied energy band is called the valence band and the lowest unoccupied band is called the conduction band.

Semiconductors 1 Comment 2

Because of the energy gap between the valence band and the conduction band, the electrical conductivity of a semiconductor approaches zero at zero temperature, i.e. $\sigma \to 0$ for $T \to 0$.

The electrical conductivity increases with increasing temperature, since more and more electrons can be excited from the valence band into the conduction band.

Semiconductors 2



Semiconductors 2 Comment

The figure gives an overview of the energy gaps of various semiconductors.

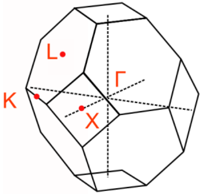
The black line connects the elements silicon, germanium and tin.

The blue line connects the III-V semiconductors and the red line the II-VI semiconductors.

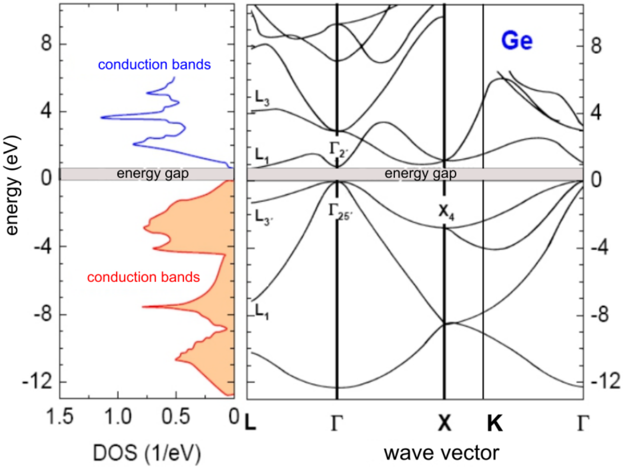
A look at the periodic table of the elements shows what is meant by a III-V or a II-VI semiconductor.

The combination of elements from different groups of the periodic table affects the size of the smallest energy gap between the valence and conduction bands.

Semiconductors 3



band structure of Germanium (electron configuration [Ar] $3d^{10}\ 4s^2\ 4p^2$)



Semiconductors 3 Comment

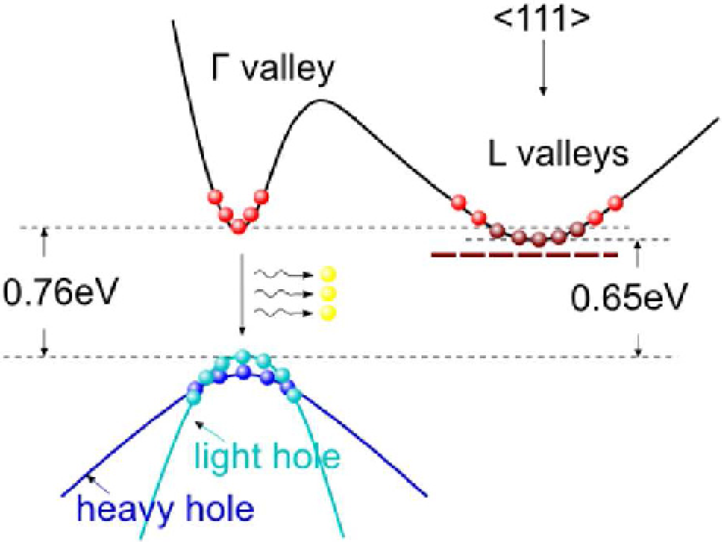
Germanium crystallizes in the fcc lattice of the diamond structure.

The figure on the left shows the 1st Brillouin zone of the fcc lattice.

The first panel shows the density of states and the second panel the energy bands for some selected directions of the wave vector.

The lowest energy of the conduction band is at the L point of 1st Brillouin zone.

Semiconductors 4



Semiconductors 4 Comment

The figure shows the conduction band and the valence band in more detail.

In contrast to the previous figure, the L point is plotted to the right of the Γ point.

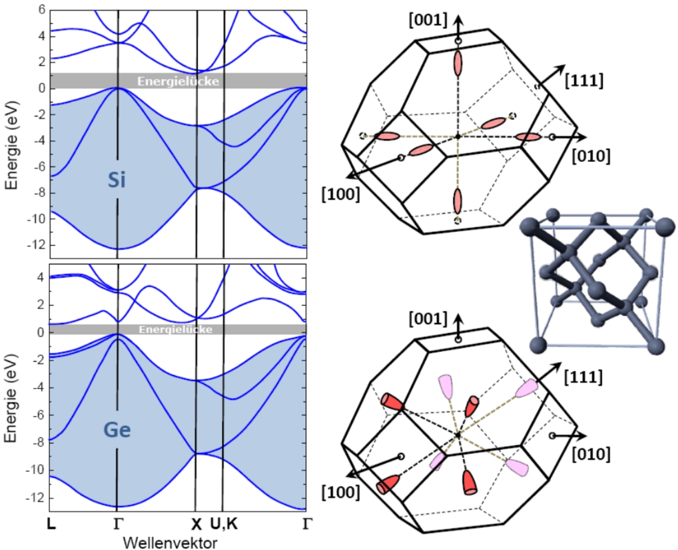
At the Γ point there are three valence bands that have exactly the same energy when the lattice symmetry is the undisturbed diamond structure.

The figure shows the case where the lattice is doped with impurity atoms that act as donor atoms.

The energy of the donor atoms is plotted as a dashed line just below the conduction band minimum.

The energy gap is so small that a large number of electrons can be thermally excited into the conduction band from the donor atoms.

Semiconductors 5



Semiconductors 5 Comment

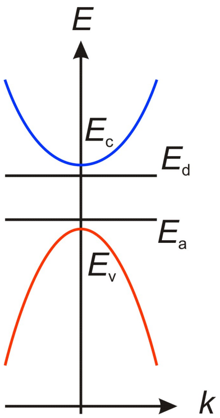
The figures compare the band structures of silicon and germanium.

The lowest energy of the silicon conduction band is near the X point of the 1^{st} Brillouin zone.

The areas drawn in red in the 1^{st} Brillouin zone of silicon and germanium show the surfaces of lowest energy of the conduction band.

The *k* states of these regions are first populated when electrons are excited into the conduction band.

Semiconductors 6



Semiconductors 6 Comment 1

The figure shows schematically the doping of semiconductors.

The horizontal bars show the relevant localized energy levels of the donor and acceptor atoms, E_d and E_a, respectively.

The electrons of the donator and acceptor atoms are localized and the corresponding energy band is simply a horizontal line.

The energy gap between the lowest energy of the conduction band and the relevant energy level of the donor atom is small, so the probability that an electron of the donor atom will be excited into the conduction band is very high.

Semiconductors 6 Comment 2

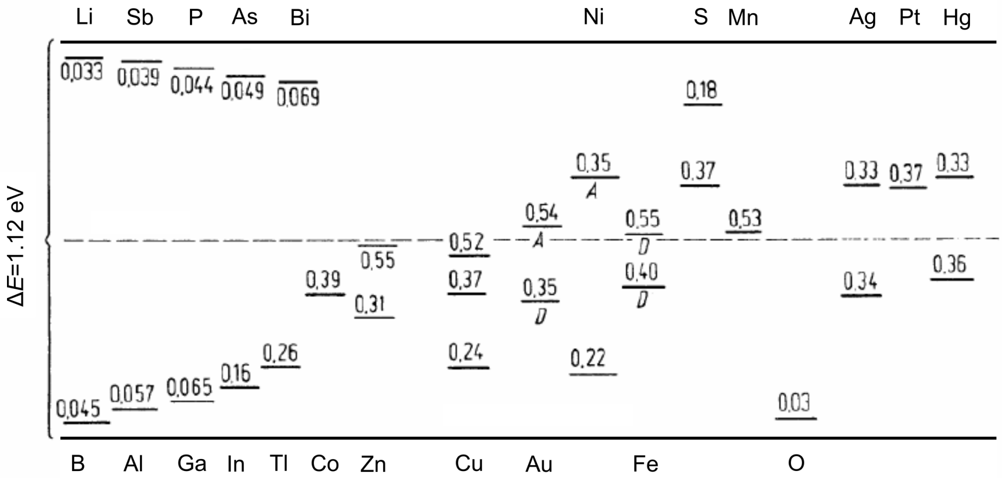
Also, the energy gap between the relevant energy levels of the acceptor atom and the highest energy of the valence band is small, so the probability that an electron of the valence band will be excited to an energy level of an acceptor atom is very large.

Since the effective mass of the electrons (quasiparticles) at the top of the valence band is negative, the electrical response to an applied electric and/or magnetic field can be described by positive charge carriers.

The positive charge carriers are commonly referred to as “electron holes”.

Semiconductors 7

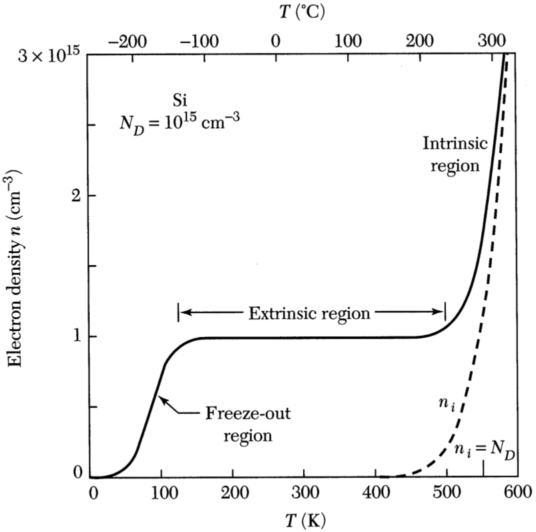
Energy levels of donor and acceptor atoms in the energy band gap of silicon



Semiconductors 7 Comment

The figure gives an overview of the energy levels of donor and acceptor atoms in the energy band gap of silicon.

Semiconductors 8



Semiconductors 9 Comment 1

The figure shows the electron density in the conduction band of silicon.

The dashed line shows the intrinsic electron density of pure silicon.

Because of the large energy gap, the electron density is very small for temperatures below 400 K.

The energy gap between the valence and conduction bands of silicon is quite large and a high temperature is necessary to excite electrons from the valence band into the conduction band.

If the semiconductor is doped with donor atoms, the energy gap is much smaller, so that electrons from the donor atom can be excited into the conduction band even at low temperatures.

Semiconductors 8 Comment 2

The electron density in the conduction band is the density of donor atoms over a wide temperature range.

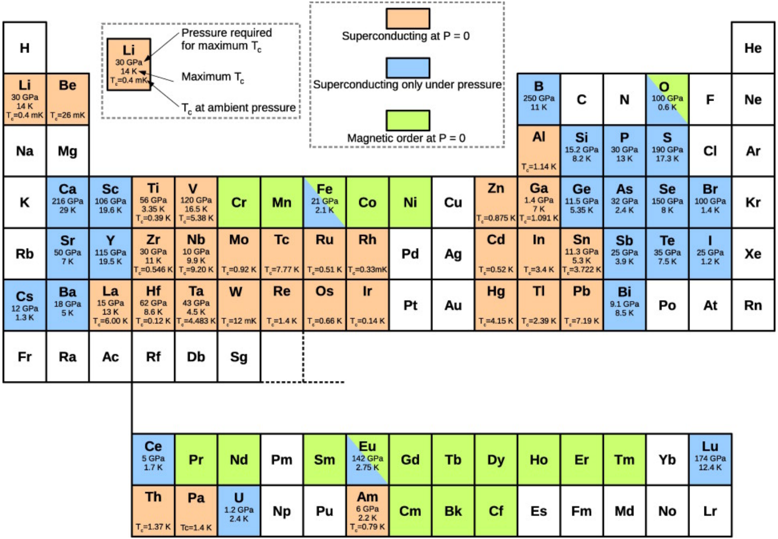
Only when the temperature becomes very high does the electron density increase again, since electrons can be excited from the valence band into the conduction band via the large energy gap.

Ferromagnetism

Electrons in crystal lattices

- Electrical conductivity and Ohm's law
- Drude model
- Sommerfeld model
- Bloch waves
- Energy bands and Fermi surfaces
- Photoemission Spectroscopy
- Semiclassical Electron Dynamics
- Semiconductors
- Ferromagnetism
- Superconductivity

Ferromagnetism 1



Ferromagnetism 1 Comment 1

The periodic table shows that most elements become superconducting at low temperatures.

Many elements spontaneously become superconducting at low temperatures.

For some elements, hydrostatic pressure must be applied before superconductivity can be observed.

Few elements show magnetism.

Chromium is antiferromagnetic below the so-called Néel temperature of 311 K.

Manganese is complicated because it forms four different crystal structures that have very different physical properties, e.g. α-Manganese is antiferromagnetic below the Néel temperature of 100 K.

Iron, cobalt and nickel are ferromagnetic below the Curie temperature.

Ferromagnetism 1 Comment 2

The Curie temperature of iron is 1000 K, of cobalt 1400 K and of nickel 630 K.

Magnetic order is also observed for the rare earth metals.

Complicated magnetic spiral structures are caused by the interplay between the localized 4f electrons and the conduction electrons of the 5d and 6s orbitals.

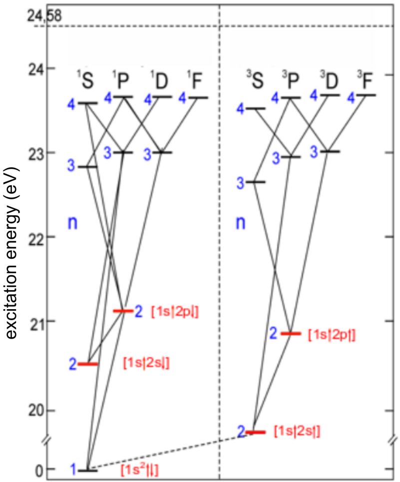
Magnetic order is only observed for a small number of elements.

However, the transition temperatures between the magnetically ordered phase and the paramagnetic phase are comparatively high.

This is in striking contrast to the usually very low transition temperatures of superconductivity.

Since the ferromagnetism of iron plays such a large role in daily life, it is worth commenting on the ferromagnetism of band electrons.

Ferromagnetism 2



- The exchange interaction can lead to ferromagnetism of the conduction electrons.

but in contrast to atoms

- Bloch waves extend over the whole crystal
- The excitation energy is extremely small
- There are many electrons involved

Stoner criterium $\boxed{ID(E_F)/2n > 1}$

Ferromagnetism 2 Comment 1

The figure shows the energy level scheme of helium.

When an electron is excited from the ground state, the spins of the two electrons can remain antiparallel or align in parallel.

If the spins are parallel, the excited electron cannot even virtually return to the ground state due to the Pauli principle.

Therefore, the Coulomb repulsion of the parallel spin configuration is smaller than that of the antiparallel configuration and the binding energy of the parallel spin configuration is therefore larger.

In contrast to the narrow atomic obitals, the conduction electrons occupy Bloch waves that extend throughout the entire crystal.

Nevertheless, the Coulomb repulsion keeps the electrons away from each other, and due to the Pauli principle, this effect increases with parallel electron spins.

Ferromagnetism 2 Comment 2

This increases the binding energy of the conduction electrons when they are ordered parallel to one another.

Edmund Stoner first described this effect by adding additional energy to an energy band proportional to the number of electrons with opposite spin.

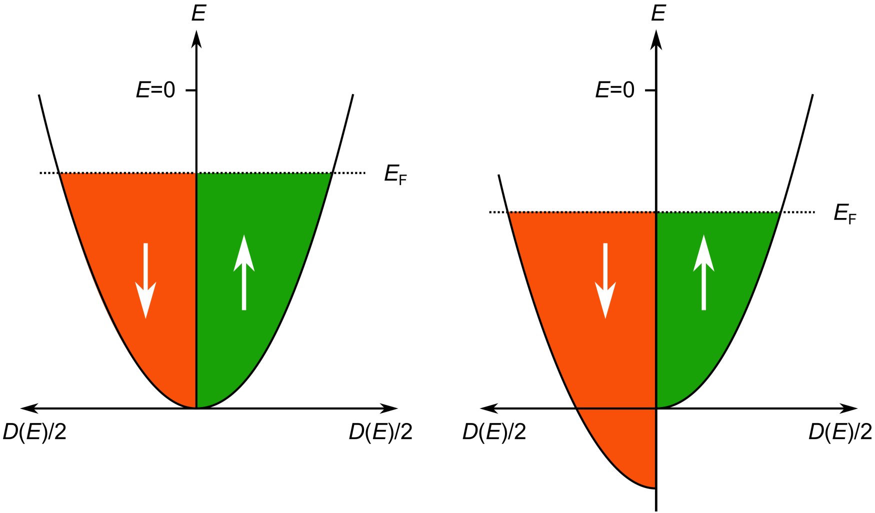
This additional energy is denoted by the letter *I*.

This creates spin-polarized energy bands and the density of states is no longer the same for the two spin orientations.

Stoner showed that ferromagnetism of conduction electrons is possible if the criterion outlined in red is met.

What is important is the additional energy due to the antiparallel spin orientation and, above all, the density of states at the Fermi energy.

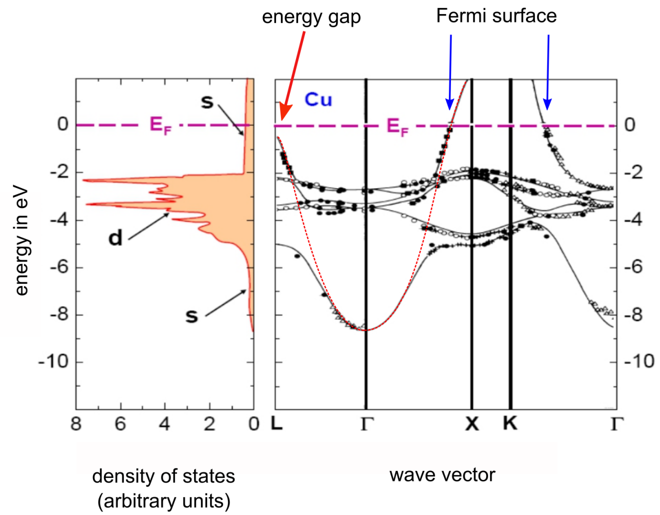
Ferromagnetism 3

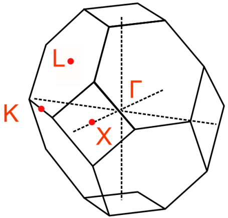


Ferromagnetism 3 Comment

The figure schematically illustrates the increase in binding energy due to the spin polarization of the energy band.

Ferromagnetism 3





Ferromagnetism 3 Comment 1

The figure shows once again the density of states and the band structure of copper.

The 4s electrons are the conduction electrons, which behave almost like quasi-free electrons.

Unlike the neighboring elements nickel, cobalt and iron, copper is not a magnet.

The main difference between nickel, cobalt, iron on the one hand and copper on the other hand is that the Fermi energy for nickel, cobalt and iron is in the region of the 3d bands, while the Fermi energy for copper is exclusively in the region of the 4s band.

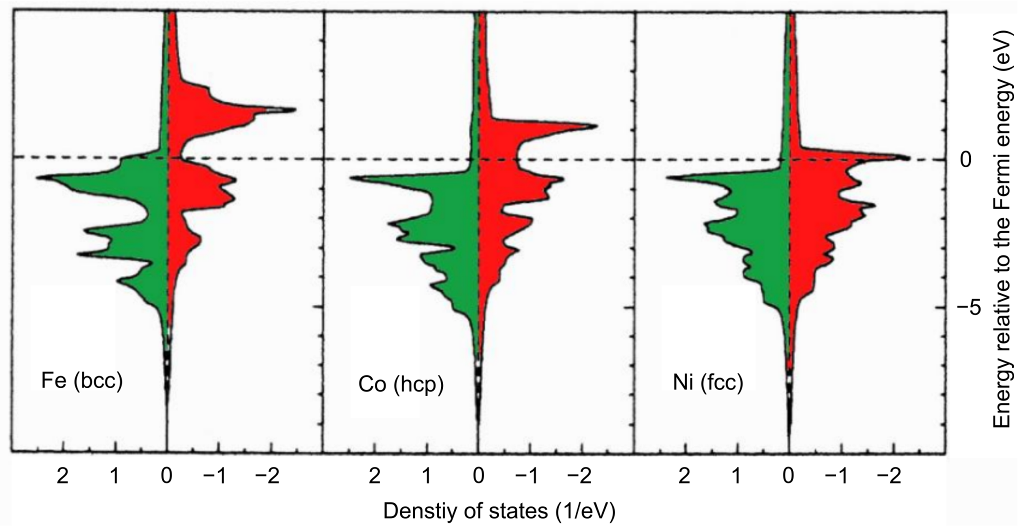
It is therefore interesting to note the differences between the 4s band and the 3d bands shown in the figure.

Ferromagnetism 3 Comment 2

The density of states of the 4s band is small compared to the density of states of the 3d bands.

The 3d bands are narrow and the density of states of the 3d bands is confined to an energy range that is small compared to the energy range of the 4s band.

Ferromagnetism 4



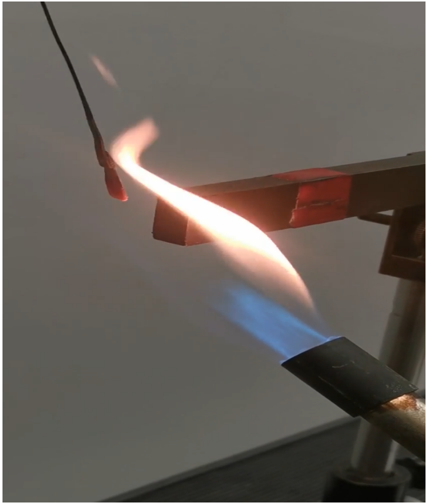
Ferromagnetism 4 Comment

The figure shows the density of states of iron, cobalt and nickel for up and down spin in the ferromagnetic phase.

The density of states is similar to the 3d density of states of copper with the difference that the Fermi energy is in the region of the 3d bands.

The exchange interaction induces a robust difference between the up and down spin states.

Ferromagnetism 5



(CurieEisen3.mp4)

Ferromagnetism 5 Comment

The video shows a small piece of iron attached to the end of a pendulum.

Since iron is ferromagnetic, the pendulum is attracted to a permanent magnet.

Iron loses its magnetic moment when heated above the Curie temperature of 758° C.

The video shows the alternating heating and cooling of the small piece of iron.

If the temperature of the piece of iron is below the Curie temperature, the pendulum is attracted to the permanent magnet.

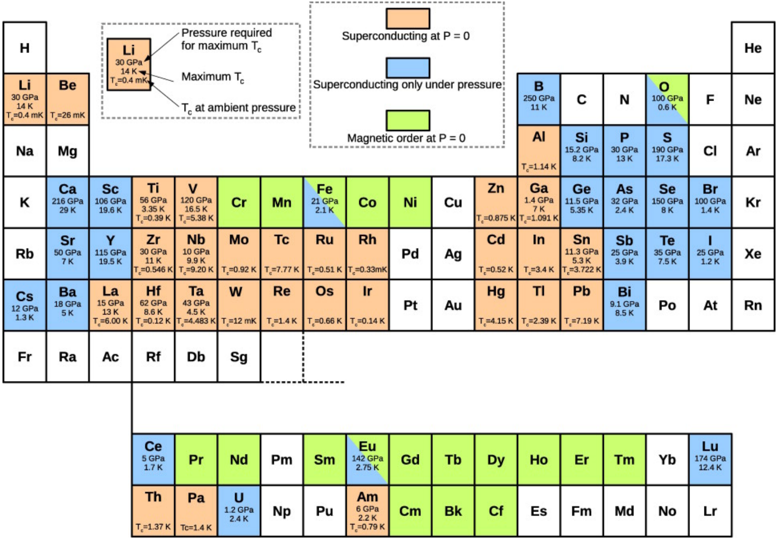
If the temperature of the piece of iron is greater than the Curie temperature, the pendulum can swing freely.

(The rather cool yellow flame is irrelevant to the experiment. It’s just a nasty imperfection of the Bunsen burner.)

Electrons in crystal lattices

- Electrical conductivity and Ohm’s law
- Drude model
- Sommerfeld model
- Bloch waves
- Energy bands and Fermi surfaces
- Electron dynamics and Electron hole bands
- Photoemission Spectroscopy
- Semi-classical electron dynamics
- Semiconductors
- Ferromagnetism
- Superconductivity

Superconductivity



Superconductivity Comment

Superconductivity is not an exotic phenomenon.

Many elements of the periodic table and countless alloys and chemical compounds become superconducting below a critical temperature.

The periodic table shows that superconductivity and magnetism are mutually exclusive.

Only a few elements (oxygen, iron and europium) not only order magnetically, but also become superconducting under pressure.

BCS theory

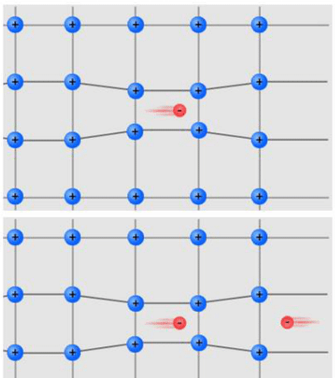
Superconductivity

- BCS theory
- London equations

BCS theory 1

John Bardeen, Leon Cooper, Robert Schriefer 1957 (NP 1972)
Cooper pairs

$$\boxed{\{+\vec{k}\uparrow, -\vec{k}\downarrow\}}$$



spacial extent of the Cooper pairs

$$\boxed{\xi_0 \approx 2 - 1000\,\mathrm{nm}}$$

macroscopic wave function

$$\boxed{\psi = \psi_0 e^{-iEt/\hbar}}$$

BCS theory 1 — Comment 1

Superconductivity was discovery by Kamelingh Onnes in 1911.

The first microscopic theory of superconductivity was published in 1957 by Bardeen, Cooper and Schriefer.

The theory is very successful and is called the BCS theory.

The basic idea of the BCS theory is that two counter-propagating electron waves with the same wave number, i.e. $\vec{k}$ and $-\vec{k}$, form a new quantum state with lower energy.

The figure illustrates the idea of the mechanism that couples the two electron waves.

Since atoms are not fixed in their lattice site but can oscillate freely around an equilibrium position, atoms can react to electrical charges flying past them.

BCS theory 1 Comment 2

The figure shows atoms moving in the direction of an electron flying through the crystal lattice.

The speed of the electrons is in the range of the Fermi velocity and therefore very high compared to the movement of the atoms.

The atoms remain in the deflected position on the electron time scale for a very long time, creating a channel that can be used by an electron propagating in the opposite direction to reduce its potential energy.

This gain in energy joins two electrons in a new shared quantum state.

Due to the Pauli principle, this new quantum state can be occupied by two electrons with opposite spin quantum numbers.

BCS theory 1 — Comment 3

This coupling mechanism of two electrons with opposite wave vectors $\vec{k}$ and $-\vec{k}$ was discovered by Leon Cooper in 1956.

This new quantum state is called a Cooper pair.

Electrons blocked in their k states contribute neither to electrical conductivity nor to superconductivity.

Only quasiparticles i.e. electrons whose energy is close to the Fermi energy can form Cooper pairs.

These quasiparticles are described by wave packets that have a certain spatial extent.

BCS theory 1 Comment 4

Cooper pairs can only form when wave packets with opposite wave vectors overlap.

The Cooper pairs therefore have a certain spatial extension, which is given by the correlation length ξ_0.

The correlation length ξ_0 is comparatively large, so that many electrons are in the range of a Cooper pair.

BCS theory 1 — Comment 5

Cooper pairs behave similar to bosons, since the electron spins of a Cooper pair add up to the total spin $S = 0$.

Similar to photons ($S = 1$), any number of Cooper pairs can occupy a single quantum state and thus form a macroscopic wave function that extends over the entire solid.

This is similar to the laser where many photons form a macroscopic electromagnetic wave.

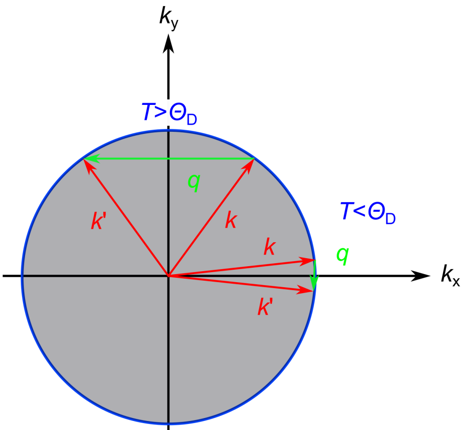
It is therefore possible to carry out interference experiments with these macroscopic wave functions.

Electrical circuits based on the interference of the macroscopic wave functions of a superconductor are called SQUIDs (Superconducting Quantum Interference Device).

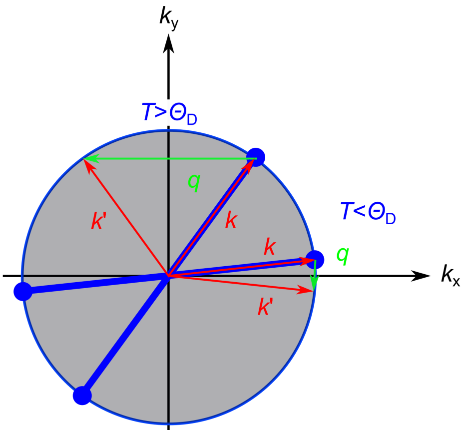
BCS theory 2

Cooper pairs cannot lose energy due to scattering

normal conductor $T>T_c$



superconductor $T<T_c$



BCS theory 2 — Comment 1

The figure on the left shows the electron-phonon scattering that leads to electrical resistance in metals.

An electron emits or absorbs a phonon and is thereby deflected in a different direction.

At low temperatures well below the Debye temperature, only acoustic phonons are excited, which can only slightly deflect electrons.

At temperatures above the Debye temperature, all phonon modes within the 1st Brillouin zone are excited and electrons can be strongly deflected by electron-phonon scattering.

Strong deflections also occur when an electron is scattered off an impurity atom or a crystal defect.

BCS theory 2 — Comment 2

The figure on the right shows the case that electrons form Cooper pairs.

These are represented in the figure by the blue dots connected by the blue line.

The scattering can only take place if the binding energy of the Cooper pair can be overcome by the scattering.

If the energy of a phonon is less than the binding energy, the Cooper pair cannot be broken by a phonon.

Cooper pairs move through the crystal lattice without being able to gain or lose energy from the crystal.

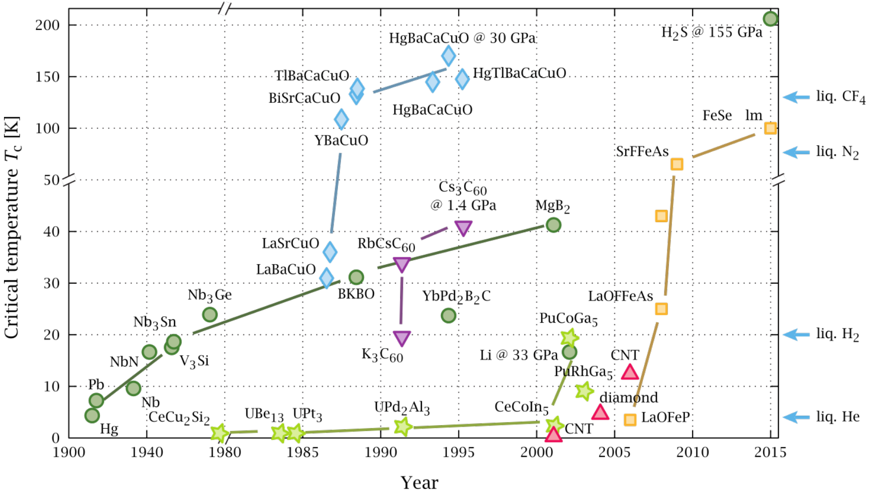
The superconducting current flows without resistance.

BCS theory 2 Comment 3

The binding energy of the Cooper pairs is usually very small, so that the transition temperatures T_c of superconductors are often very low.

The transition temperature of a superconductor is called the critical temperature.

BCS theory 3



Critical temperature T_c [K]
Year
HgBaCaCuO @ 30 GPa
TlBaCaCuO
BiSrCaCuO
HgTlBaCaCuO
HgBaCaCuO
YBaCuO
LaSrCuO
LaBaCuO
BKBO
Cs_3C_{60}
@ 1.4 GPa
$RbCsC_{60}$
K_3C_{60}
MgB_2
$YbPd_2B_2C$
H_2S @ 155 GPa
FeSe lm
SrFFeAs
LaOFFeAs
LaOFeP
$PuCoGa_5$
$PuRhGa_5$
$CeCoIn_5$
Li @ 33 GPa
CNT
diamond
CNT
Nb_3Ge
Nb_3Sn
V_3Si
NbN
Nb
Pb
Hg
$CeCu_2Si_2$
UBe_{13}
UPt_3
UPd_2Al_3
liq. CF_4
liq. N_2
liq. H_2
liq. He

BCS theory 3 Comment 1

Since the discovery of superconductivity in 1911, physicists and chemists have tried to find substances with higher transition temperatures.

The figure shows how substances with higher transition temperatures have been discovered over the years.

In the search for substances with higher transition temperatures, chance played a major role.

The BCS theory of superconductivity did not bring a breakthrough in 1957 either.

The dark green dots show superconductors that can be well understood with the BCS theory.

The green stars show so-called heavy fermion superconductors.

BCS theory 3 Comment 2

The Cooper pairs are bound by magnetic interactions.

Therefore, this class of superconductors is used as model systems to study unconventional superconductivity, although the critical temperatures are usually very small.

The light blue diamonds show the cuprate superconductors.

The discovery of cuprate superconductors in 1986 was a breakthrough because it made it possible to use liquid nitrogen as a coolant instead of expensive helium.

The cuprate superconductors are based on copper-oxygen planes and the Cooper pairs are believed to form due to the interaction of the electrons with strong two-dimensional magnetic correlations.

BCS theory 3 Comment 3

Another class of high-temperature superconductors are the iron pnictide superconductors discovered in 2006.

These superconductors are based on iron planes separated by pnictides.

Pnictides are the elements of the periodic table that are in the column below nitrogen, e.g. phosphorus or arsenic.

The phase diagrams of the iron pnictides are very similar to the phase diagrams of the heavy fermion and cuprate superconductors.

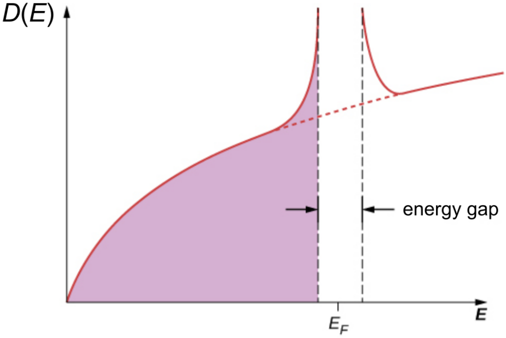
Finally, the figure also shows some carbon-based superconductors (red triangles).

They are based on buckminsterfullerenes (C_{60}) or carbon nanotubes (CNT).

Even heavily doped diamond becomes superconducting at a critical temperature in the range of 3K.

BCS theory 4

energy gap at the Fermi energy



BCS theory 4 Comment

The figure shows the density of states of quasi-free electrons.

The occupied states are marked in red.

An energy gap opens at the Fermi energy when the metal becomes superconducting below the critical temperature T_c.

The electrons occupying k states near the Fermi surface form Cooper pairs.

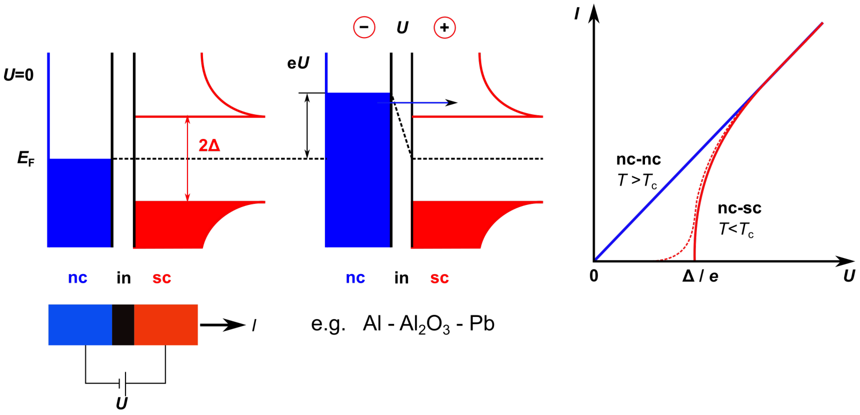
At least the energy of the superconducting energy gap is necessary to break the Cooper pairs.

The k states in the energy gap region are shifted to higher and lower energies as the energy gap opens.

Therefore, the density of states is increased just below and just above the superconducting energy gap.

BCS theory 5

Measurement of the energy gap with a tunnel junction between a normal conductor (e.g. Al $T_\mathrm{c} = 1.2\,\mathrm{K}$) and a superconductor (e.g. Pb: $T_\mathrm{c} = 7.2\,\mathrm{K}$)



BCS theory 5 — Comment 1

The superconducting energy gap can be measured with a tunnel junction between a normal conductor and a superconductor.

A nice example is the tunnel contact between aluminum and lead.

Both metals are separated by a thin layer of aluminum oxide.

Electrons can tunnel through this barrier.

The figure below left shows the tunnel junction with the current and the applied voltage.

The figure on the right shows the current-voltage curve of the tunnel junction.

BCS theory 5

Comment 2

If the current-voltage characteristic is measured at a temperature above the critical temperature of lead, the linear current-voltage characteristic of an ohmic resistor is obtained.

The resistance is mainly determined by the oxide barrier.

The figures on the left show the density of states at the Fermi energy when the temperature is below the critical temperature of lead but above the critical temperature of aluminum.

The figure on the left shows the case that no voltage is applied.

Because of the energy gap of lead, no electrons can tunnel from aluminum to lead, since there are no k states in the energy gap of lead.

BCS theory 5 Comment 3

If the potential energy of the electrons in aluminum is increased by an applied voltage, an electric current can flow as soon as the Fermi energy of aluminum is above the energy gap of lead.

Once this occurs, electrons can tunnel from the aluminum into the free k states of lead and an electric current begins to flow.

If the current-voltage characteristic of the tunnel junction is measured at a temperature below the critical temperature of lead but above the critical temperature of aluminum, the current-voltage characteristic shown in red is obtained.

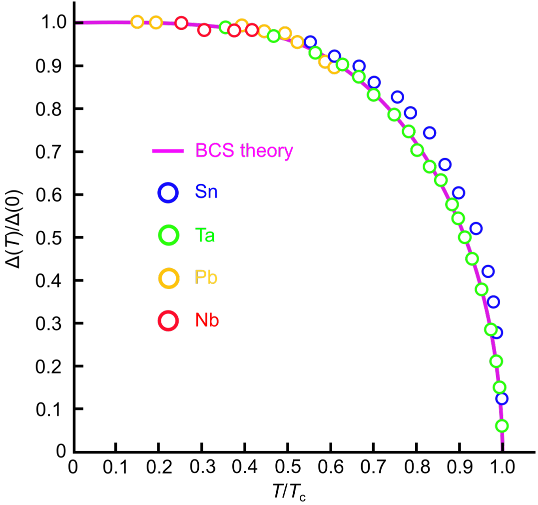
The current begins to flow when the applied voltage satisfies the condition $eU = \Delta$.

BCS theory 5 Comment 4

The solid red line shows the ideal case when thermal excitations of electrons across the tunnel junction can be neglected.

The dashed red line shows the more realistic current-voltage characteristic when thermal excitations cannot be neglected.

BCS theory 6



BCS energy gap

$$2\Delta(0) = 3.52\, k_B T_c$$

and

$$\Delta(T \to T_c) = \Delta(0)\sqrt{1 - \frac{T}{T_c}}$$

BCS theory 6 Comment

The figure shows the temperature dependence of the energy gap that occurs below the critical temperature.

The prediction of the BCS theory is confirmed by the experimental results.

The first formula on the right shows BCS theory’s prediction for the maximum energy gap at low temperatures.

The second formula gives the temperature dependence of the energy gap just below the critical temperature.

BCS theory 7

BCS energy gap

$$\boxed{2\Delta(0) = 3.52\, k_{\mathrm{B}} T_{\mathrm{c}}}$$

element	T_{c} [K]	$2\Delta(0)/k_{\mathrm{B}} T_{\mathrm{c}}$
Cd	0.56	3.2
Al	1.196	3.4
In	3.4	3.6
Sn	3.72	3.5
Ta	4.48	3.6
V	5.3	3.4
Pb	7.19	4.3
Nb	9.26	3.8

BCS theory 7

Comment

The boxed equation gives the relation between the superconducting energy gap and the critical temperature according to the BCS theory.

The table shows the experimental results for some elements which confirm the BCS theory.

London equations

Superconductivity

- BCS theory
- London equations

London equations 1

Fritz and Heinz London (1935): equation of motion of superconducting charge carriers

$$m\frac{\partial \vec{v}}{\partial t} = q\vec{E}$$

with the superconducting current density

$$\vec{j}_s = q n_s \vec{v}$$

follows

$$\underline{\frac{m}{q^2 n_s}\frac{\partial \vec{j}_s}{\partial t} = \vec{E}}$$

definition of the London penetration depth

$$\boxed{\lambda_L^2 = \frac{m}{\mu_0 q^2 n_s}}$$

London equations 1 — Comment 1

The first theoretical approach to superconductivity was formulated by Fritz and Heinz London in 1935.

Their theory is known as the London theory of superconductivity.

The London theory assumes that there are superconducting charge carriers that react to an applied external electric field according to Peierl's equations of motion with an effective mass.

This assumption is formulated by the first equation.

The superconducting charge carriers are the Cooper pairs, i.e. $q = -2e$.

The main difference between electron dynamics and Cooper pair dynamics is that they are unaffected by the scattering processes.

London equations 1 — Comment 2

The second equation is the superconducting current density j_s, which results from the density n_s of the superconducting charge carriers, their charge $q = -2e$ and the drift velocity.

The underlined equation results from inserting the current density into the equation of motion.

Dividing the prefactor on the left side of the equation by the magnetic field constant $\mu_0 \approx 4\pi \cdot 10^{-7}\,\mathrm{Vs/Am}$ gives a quantity that is measured in square meters.

It turns out that this quantity is the square of the penetration depth of an external magnetic field into the superconductor.

The equation outlined in red gives the definition of the London penetration depth.

London equations 2

1st London equation

$$\boxed{\vec{E} = \mu_0 \lambda_L^2 \frac{\partial \vec{j}_s}{\partial t}}$$

with the 1st London equation and the 3rd Maxwell equation (induction law)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \underline{\mu_0 \lambda_L^2 \frac{\partial}{\partial t} \nabla \times \vec{j}_s}$$

results the

2nd London equation

$$\boxed{\vec{B} = -\mu_0 \lambda_L^2 \nabla \times \vec{j}_s}$$

London equations 2 Comment

The formula outlined in red now gives the first London equation with the London penetration depth.

If the first London equation is inserted into Faraday's law of induction, then the second London equation can be read from it.

The second formula outlined in red shows the second London equation.

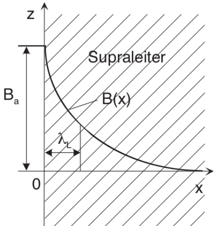
London equations 3

With the 2nd London equation and Ampere's law $\nabla \times \vec{B} = \mu_0 \vec{j}_s$ results

$$\vec{B} = \lambda_L^2 \nabla^2 \vec{B}$$

use $\nabla \vec{B} = 0$ (2nd Maxwell law) and $\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \vec{B}) - \nabla^2 \vec{B}$

in one dimension one gets $B(x) = \lambda_L^2 \frac{\partial^2 B(x)}{\partial x^2} \quad \rightarrow \quad B(x) = B_a e^{-x/\lambda_L}$



a magnetic field cannot penetrate a superconductor $\rightarrow$ Meissner effect

London equations 3 Comment 1

Substituting the 2nd London equation into Ampere's law gives the underlined equation.

The figure illustrates the meaning of this equation.

The magnetic field B_a is applied along the z-direction parallel to the surface of a superconductor.

The drop in the magnetic field within the superconductor perpendicular to the surface can be calculated using the differential equation.

The solution is a simple exponential decay and the decay length is the London penetration depth.

London equations 3 — Comment 2

The magnetic field can only penetrate the superconductor in a thin surface layer.

There is no magnetic field in the bulk of the superconductor.

This is the Meissner effect.

Shielding currents begin to circulate when a superconducting material is cooled below the critical temperature in a magnetic field.

If the temperature is above the critical temperature, the magnetic field can penetrate the material, since the shielding currents that occur due to Lenz's law are quickly reduced to zero by the ohmic resistance.

Shielding currents always suppress the magnetic field within the superconducting material when the temperature is below the critical temperature.

London equations 3 Comment 3

Therefore, shielding currents begin to flow even when a superconductor is cooled in a magnetic field below T_c.

The shielding currents lead to a strong magnetic moment in the superconducting material, which reacts very quickly to any change in the external magnetic field.

London equations 4

(Moebiusband.mp4)

London equations 4 Comment

The video shows a superconductor located at a certain distance above a magnetic rail.

Any change in distance changes the magnetic field strength, which leads to an immediate reaction of the shielding currents.

The magnetic moment of the superconductor is always set in such a way that the distance to the magnetic rail does not change.

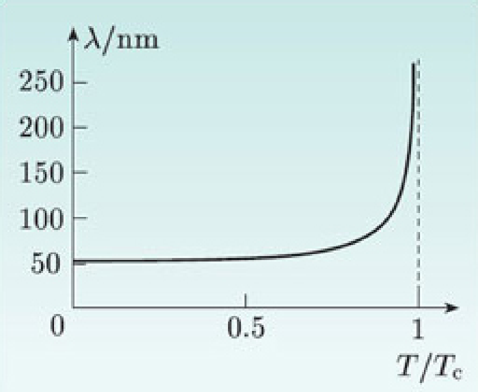
The video shows a small piece of an high temperature superconductor.

Liquid nitrogen is used to cool the superconductor below the critical temperatures.

The superconductor is placed in a small container made of foamed plastic for thermal insulation.

London equations 5

London penetration depth of various elements



element	T_c [K]	$\lambda_L(T \to 0)$ [nm]
Cd	0.56	110
Al	1.196	16
Sn	3.72	34
Pb	7.19	37
Nb	9.26	39

London equations 5 Comment

The London penetration depth is temperature dependent.

It is minimal for $T \to 0$ and diverges for $T \to T_c$.

$$\lambda_L(T) = \lambda_L(0)/\sqrt{1 - (T/T_c)^4}$$

The table shows the critical temperatures and the London penetration depth of some elements.

The London penetration depth is in the range of a few 10 nm.

Revision

Summary in Questions 1

1. Sketch the conductivity as a function of temperature for a metal and for a semiconductor.
2. Why do impurities affect the electrical conductivity of semiconductors so much?
3. How do donor atoms differ from acceptor atoms?
4. Sketch the conduction electron density of a semiconductor doped with donor atoms as a function of temperature.
5. There are three different temperature ranges. How do these temperature ranges differ?
6. Explain the exchange interaction for conduction electrons.
7. Which conditions must be fulfilled for conduction electrons to order ferromagnetically?

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Modern Physics (Winter Semester 2023/24)

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