Modern Physics

Contents:

Elm. Waves

General Wave Phenomena

- Waves in one dimension
- Reflection and transmission
- Standing waves
- Waves in three dimensions
- Electromagnetic waves

Electromagnetic waves 1

Maxwell's equations in vacuum reduce to the classical wave equation for the electric $\vec{E}$ and magnetic field $\vec{B}$

$$\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2} = c^2 \nabla^2 \vec{E}(\vec{r},t)$$
$$\frac{\partial^2 \vec{B}(\vec{r},t)}{\partial t^2} = c^2 \nabla^2 \vec{B}(\vec{r},t)$$

The speed of light is a physical constant

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299\,792\,458 \ \mathrm{m/s} \approx 3 \cdot 10^8 \ \mathrm{m/s} = 300\,000 \ \mathrm{km/s}}$$

Electromagnetic waves 1 Comment 1

In 1831 Faraday showed that a time-dependent magnetic field induces a time-dependent electric field.

In 1864, Maxwell formulated his theory of electromagnetism based on the idea that a time-dependent electric field, similar to Faraday's law, induces a time-dependent magnetic field.

This allows electromagnetic fields to propagate through space.

The Maxwell equations in vacuum lead to the classical wave equations for electric and magnetic fields.

Heinrich Hertz succeeded in producing and detecting artificial electromagnetic waves for the first time in 1886.

Electromagnetic waves 1 Comment 2

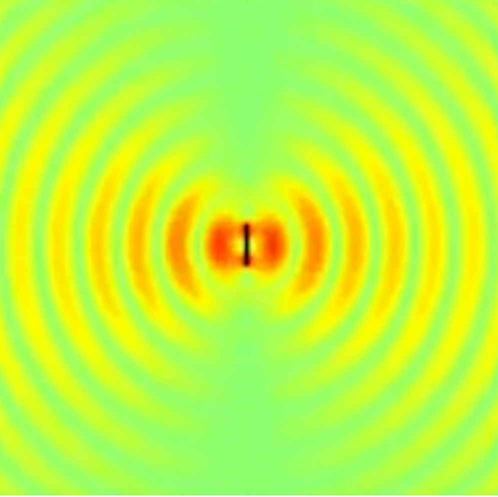
The speed of light is a natural constant in Maxwell's theory.

The speed of light is expressed in the MKSA system of units by the electric and magnetic field constant.

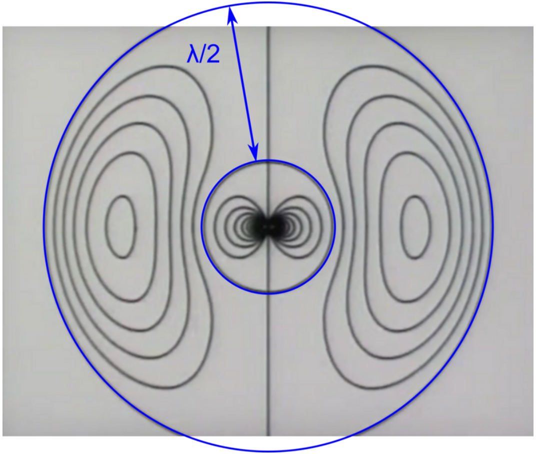
Albert Einstein recognized in 1905 that Maxwell's theory is a relativistic theory and that the principles of relativity must be applied to all areas of physics.

The formula outlined in red indicates the numerical value of the speed of light, which was defined in the International System of Units in 1983.

Electromagnetic waves 2



(DipolAntenne.mp4)



(HertzscheDipolAbstrahlung.mp4)

Electromagnetic waves 2 Comment 1

The two videos on this page explain the emission of electromagnetic radiation from a Hertzian dipole.

Positive and negative electrical charges move along a wire, creating a time-dependent electrical and magnetic field.

The animation on the left shows the separation of the electric field from the dipole.

The radiated electrical power is maximum perpendicular to the dipole axis and zero in the direction of the dipole axis.

The video on the right shows the separation of the electric field from the dipole in more detail.

In a wire a charge current wave can propagate. On the antenna a standing charge current wave is exited. Only the basic mode with $L = \lambda/2$ is considered in the animation.

Electromagnetic waves 2 Comment 2

Directly at the antenna, the electric and magnetic fields oscillate by 90° out of phase.

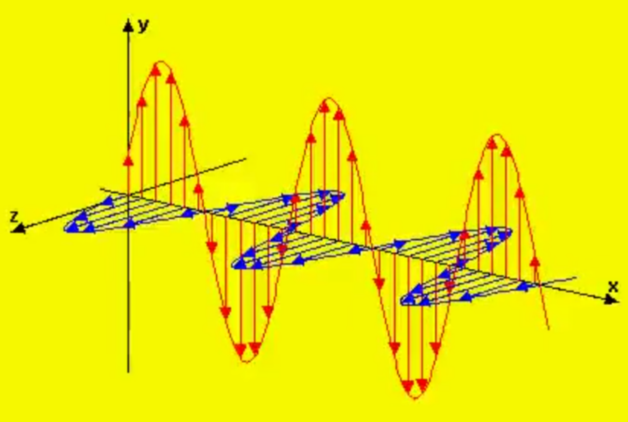
When the current and thus the *B*-field is maximum, the charges on the dipole compensate each other and the electric field disappears.

Conversely, the electric field is maximum when the current has disappeared and there is no magnetic field at the location of the dipole.

The phase shift between electric and magnetic fields disappears quickly with the distance to the dipole.

At a distance of one wavelength, the *E*-field and *B*-field are in phase.

Electromagnetic waves 3



(ElectromagnetischeWellen.mp4)

Electromagnetic waves 3 Comment

The video shows the animation of a plane electromagnetic wave according to Maxwell's theory, which propagates along the x-axis.

The orientation of the k-vector is parallel to the x-axis and the electric and magnetic field vectors are oriented within the planes of constant phase.

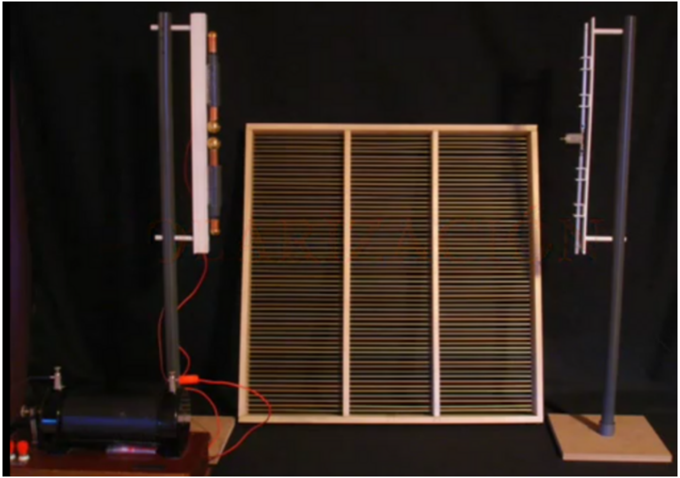
The red vectors indicate the electric field and the blue vectors the magnetic field.

The *k*-vector and the vectors of the electric and magnetic field form a right-handed system.

Electromagnetic waves are transverse waves as opposed to sound waves, which are longitudinal waves.

In the case of sound waves in a gas, the displacement vector is parallel to the *k*-vector.

Electromagnetic waves 4



(Polarisation.mp4)

Electromagnetic waves 4 Comment 1

The video demonstrates the polarisation of electromagnetic waves.

The electromagnetic waves are generated as Heinrich Hertz did it in 1886.

The spark inductor can be seen in the lower left corner of the picture, which generates the necessary high voltage.

The antenna consists of two metal rods with metal balls at the ends.

If the high voltage is applied to the two metal rods, an arc ignites between the neighbouring metal balls.

The two metal rods are connected to one another by a short circuit and the charges oscillate back and forth at a frequency that is determined by the capacitance and inductance of the two rods.

Electromagnetic waves 4 Comment 2

The frequency of the oscillation is very high because the capacitance and inductance of the metal rods are very small.

The oscillation of the charges is strongly dampened because electromagnetic waves are emitted.

If the arc between the two metal rods extinguishes due to the damping, it is re-ignited by the applied high voltage.

In this way, electromagnetic waves can be radiated permanently, although the amplitude of the waves cannot be stable.

The generation of perfect electromagnetic waves was a technical problem that was solved in the years after 1886.

Electromagnetic waves 4 Comment 3

The electromagnetic waves are detected by an antenna, which consists of two metal rods which are connected by a light bulb.

The light bulb glows when an electric current is induced in the antenna by an electromagnetic wave.

The metal rods of the detection antenna are oriented parallel to the metal rods of the antenna.

The electric field of the wave is parallel to the metal rods.

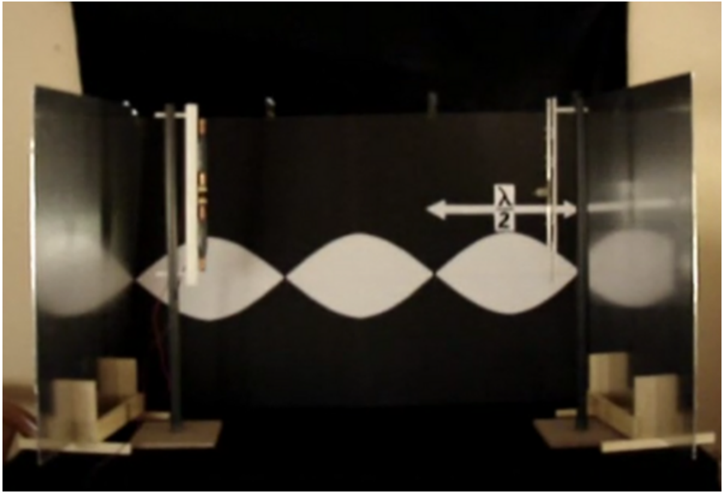
A grid of parallel wires is held between the two antennas.

The electromagnetic wave is absorbed by the wires if they are oriented parallel to the electric field strength.

Electromagnetic waves 4 Comment 4

The wave penetrates the grid of parallel wires when they are oriented perpendicular to the electric field.

Electromagnetic waves 5



(ElmStehendeWelle.mp4)

Electromagnetic waves 5 Comment

The video shows the formation of standing electromagnetic waves.

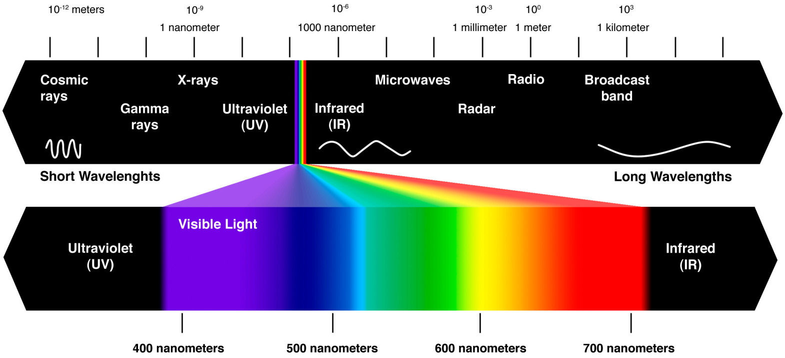
The waves are generated and detected with the same experimental setup that was used in the last video.

The electromagnetic waves are reflected back and forth between two parallel metal plates.

This traps the electromagnetic energy between the two metal plates and creates a standing wave for certain frequencies.

(Calculate the frequency of the wave assuming that the distance between the metal plates is 1 m.)

Electromagnetic waves 6



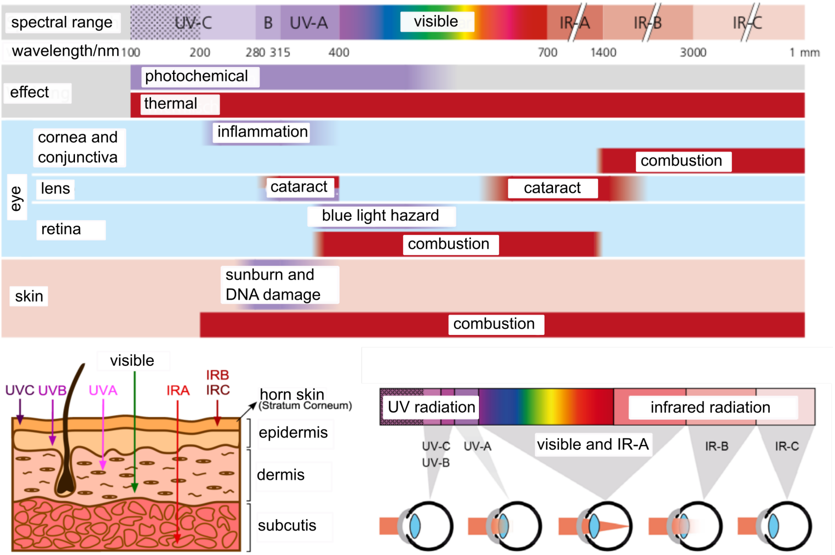
The electromagnetic spectrum

Electromagnetic waves 6 Comment

The figure illustrates the wide range of the electromagnetic spectrum.

The human eye can detect electromagnetic waves in the wavelength range between 400 and 700 nm.

Electromagnetic waves 7



Electromagnetic waves 7 — Comment 1

The sun’s light is most intense around 550 nm.

Therefore, the visible region and the adjacent regions of ultraviolet and infrared light are of utmost importance in daily life.

Depending on the penetration depth light, the ultraviolet and infrared spectrum is further divided into UV-C, UV-B, etc. ranges.

Photochemical reactions can be induced when the wavelength is less than about 550 nm.

High intensities deposit thermal energy across the entire spectral range and can therefore cause damage.

In the case of skin, UV-C is absorbed directly in the surface (horn skin).

Electromagnetic waves 7 Comment 2

UV-B and UV-A penetrate deeper and can cause sunburns and DNA damage. This radiation is responsible for skin cancer and premature skin aging.

The interaction of the eye with light is particularly important.

UV-C and UV-B are absorbed in the cornea. Too much intensity can cause inflammation of the conjunctiva.

UV-A is absorbed in the lens. There it can trigger photochemical reactions that, over time, can lead to ocular cataract.

Visible light and IR-A reach the retina. Blue light may cause macula degeneration and red light and IR-A are also responsible for ocular cataract.

Conclusion: Protect your skin and especially your eyes from too much light.

Reflection and Refraction

Modern Physics

Contents:

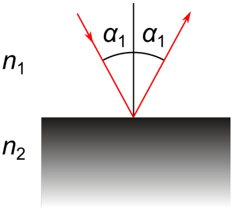
Modern Physics: Comment

The physical properties of atoms and solids are determined by electron waves.

Therefore, it makes sense to remember both general wave properties and the basics of classic wave optics.

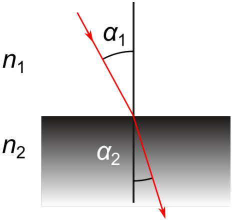
Classical wave optics

Reflection and Refraction 1



law of reflection

$$\boxed{\alpha_1 = \alpha_{1'}}$$



law of refraction

$$\boxed{n_1 \sin\alpha_1 = n_2 \sin\alpha_2}$$

law of refraction in the course of time

- 10th century: Ibn Sahl, rediscovered by Willebrord van Roijen Snell in 1620
- 1660 Pierre de Fermat
- 1678 Christian Huygens
- 1694 Isaac Newton
- 1886 Maxwell's electrodynamics

Reflection and Refraction 1 — Comment 1

It has been known in ancient times that light is reflected and refracted on glasses.

The first mathematically correct description was given by Ibn Sahl in Persia in the 10th century.

Even without the aid of mathematical formulas, lenses had been used in Europe as reading aids and magnifying glasses since the 13th century.

The work of Ibn Sahl was rediscovered in 1620 by Willebrord van Roijen Snell.

That is why the law of refraction is often called Snell’s law of refraction.

The first correct physical justification for the laws of reflection and refraction was worked out by Pierre de Fermat up to 1660.

Reflection and Refraction 1 — Comment 2

Fermat was the first to use an extremal principle to derive a law of nature.

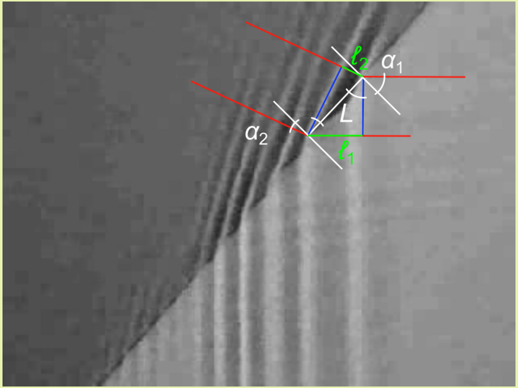
He postulates that light takes the path between two points for which the transit time is shortest.

Fermat had made a fundamental discovery, the deep meaning of which was only fully recognized more than 250 years later by Louis de Broglie.

Based on the extremal principle, de Broglie shows how the wave-particle dualism can be understood and thus founded modern physics.

Reflection and Refraction 1 — Comment 3

In 1678 Christian Huygens derived the law of reflection and refraction for waves.



(BrechungvonWasserwellen.mp4)

Huygens: The phase distance between two wave crests is 2π and thus

$$k_1 \ell_1 = k_2 \ell_2 = 2\pi$$

with

$$\sin \alpha_{1,2} = \frac{\ell_{1,2}}{L} = \frac{2\pi}{L} \frac{1}{k_{1,2}}$$

and $k \propto n$ results the law of refraction

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

Reflection and Refraction 1 — Comment 4

In 1694, Isaac Newton involved in astronomical considerations became interested in the problem of atmospheric refraction.

He assumed that light consisted of particles (corpuscles) and applied his mechanics to the problem.

For a single planar surface separating two regions of constant index of refraction, Newton's assumptions lead to Snell's law.

Newton's reflections are famous, although the work was never published. E.g. he concluded that the density of the atmosphere decreases exponentially with altitude.

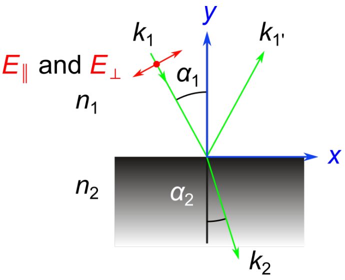
Reflection and Refraction 1 — Comment 5

Newton's theory of light was strongly supported by astronomical results first reported by James Bradley in 1725 (cf. the aberration of light) and gave physics the wave-particle problem of light which occupied physicists throughout the 18th and 19th centuries and which was only solved in the context of classical wave theory by Albert Einstein's special theory of relativity in 1905.

It has been known since Heinrich Hertz's experiments in 1886 that electromagnetic waves explain the phenomenon of light.

Classical optics can be fully understood in the context of Maxwell's theory of electrodynamics.

Reflection and Refraction 2



perpendicular incidence

$$\boxed{r = \frac{E_r}{E_0} = \frac{n_1 - n_2}{n_1 + n_2}}$$

plane wave

$$\vec{E} = \vec{E}_0 \exp i(\vec{k}\vec{r} - \omega t)$$

the phase of all three waves must be the same at the interface for all position vectors $\vec{r}$ and times:

$$\underline{\vec{k}_1\vec{x} = \vec{k}_{1'}\vec{x} = \vec{k}_2\vec{x}}$$

- $\vec{k}_1$, $\vec{k}_{1'}$ and $\vec{k}_2$ lie in the same plane
- $k_{1,x} = k_{1',x} \quad \rightarrow \quad \underline{\alpha_1 = \alpha_{1'}}$
- $k_{1,x} = k_{2,x} \quad \rightarrow \quad \underline{n_1 \sin\alpha_1 = n_2 \sin\alpha_2}$

Reflection and Refraction 2 — Comment 1

The figure illustrates the reflection and transmission for electromagnetic waves.

A plane wave falls from the top left onto a surface that separates air and glass, for example.

The wave number vector $\vec{k}_1$ of this wave lies in the xy-plane.

The electric field of this wave can be divided into a component that vibrates in the xy plane and a component that vibrates perpendicular to the xy plane.

The vertical component is symbolized by a red point.

If one assumes that the phases of the incoming wave, the reflected wave and the transmitted wave are the same at the surface, the laws of reflection and refraction result.

Reflection and Refraction 2 — Comment 2

And vice versa, the law of reflection and refraction means that the phases of the three waves must be the same.

The solution of Maxwell’s equations with the boundary conditions for the electric and magnetic fields at the interface leads to the law of reflection and refraction and also gives the reflection and transmission coefficients for the electric and magnetic fields (see Fresnel’s formulas).

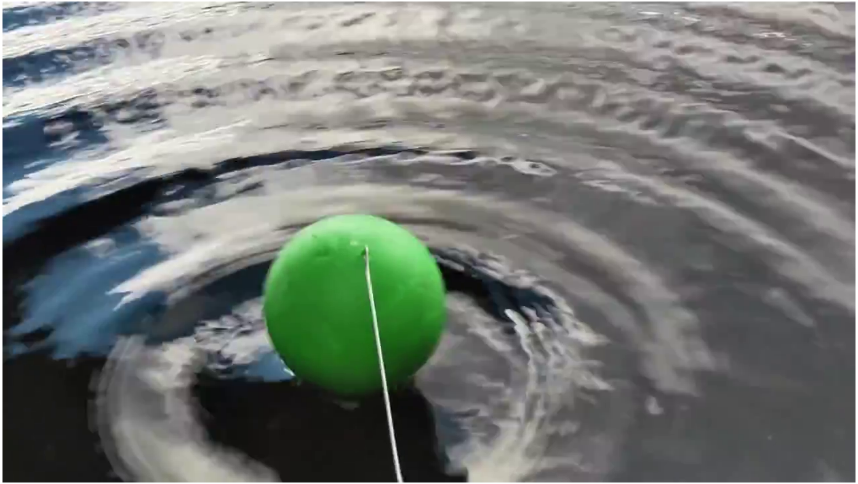
In the context of this lecture, only the special case of perpendicular incidence is required.

For the ratio of the reflected field strength E_r to the incident field strength E_0, the formula framed in red results for the reflection coefficient r.

Coherence

Classical wave optics

Coherence 1



(InterferenzWasserwellen.mp4)

Coherence 1 Comment 1

Classic wave optics deal with interference and diffraction.

Diffraction means that waves can propagate around corners.

This is evident for water and sound waves, but not for light waves.

The video shows the interference of water waves.

Interference is caused by the superposition of waves of the same wavelength.

Interferences in one dimension lead to standing waves.

Coherence 1 Comment 2

As with standing waves, there are areas where the superimposed waves cancel each other out.

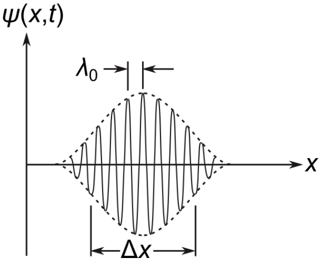
This is known as destructive interference.

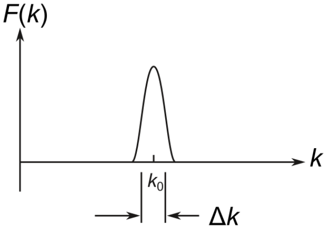
The amplitude is maximal for constructive interference.

Coherence 2

The description of waves by sine and cosine functions is an idealisation.

Waves have always a finite length $\rightarrow$ wave packets





Coherence 2 — Comment 1

In order to observe interference, light must be coherent.

The two spheres in the previous video have to excite the water surface with the same frequency so that an interference pattern can form.

In addition, the two balls must more or less simultaneously excite the surface of the water.

No interference pattern can form if the balls excite the water waves at very different times.

The excitation of the balls creates wave trains or wave packets of finite length.

An interference pattern can only form when the two wave packets overlap.

Coherence 2 Comment 2

This is evident with water waves as we can see the waves.

It is not obvious in visible light.

Instead of waves, we see colours..

The eye does not provide any information about the frequency or wavelength and the length of wave packets.

Unfortunately, the wave packets of natural light are very short, so it is not easy to observe interference.

It is therefore not obvious that visible light is a wave phenomena.

Around 1668 Newton was the first to observe interference in the form of Newton’s rings.

Coherence 2 — Comment 3

He was also the first to use Newton’s rings to assess the quality of lenses and spherical mirrors.

Newton made also an attempt to describe the phenomenon of interference in the context of a corpuscular theory of light.

However, this failed.

The wave packets must be long compared to the dimensions of the experiment so that the wave packets overlap and form an interference pattern.

The nature of wave packets can already be seen in the videos of the first lecture.

Coherence 2 Comment 4

Waves are excited on a water surface by a falling stone and spread from the point of impact.

The water pulse generated by the stone quickly breaks down into its harmonic components due to dispersion.

The harmonic components propagate with the respective phase velocity, so that long water waves reach the shore faster than the waves with short wavelengths.

This experiment shows that each wave packet consists of a spectrum of harmonic waves with different wavelengths and wave numbers.

The left figure shows the wave packet.

The right figure shows the spectrum of wave numbers.

Coherence 2 — Comment 5

The length of a wave packet is proportional to the inverse width of the spectrum, i.e. the length of the wave packets increases as the width of the spectrum decreases.

The wave packet still resembles a harmonic wave if the width of the spectrum is not too large.

The length of the wave packets is called the coherence length.

For an ideal sine or cosine wave, the width of the spectrum is zero and the wave is infinitely long.

This ideal case cannot be realized in nature. This should not be forgotten, especially when it comes to the dynamics of electrons in solids.

Coherence 2 Comment 6

However, sine and cosine waves are used to describe experiments because of the simplicity of these functions.

Spectral lamps with narrow spectral lines emit light with long wave packets.

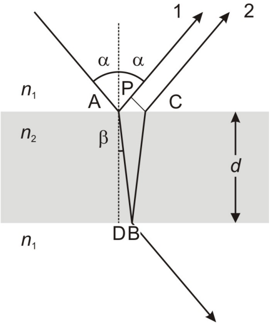
Continuous wave laser have been used since 1960 to produce light with long coherence lengths.

Thin films

Classical wave optics

Interference on thin films 1

Interference on dielectric thin films which are characterised by a refractive index.



Interference on thin films 1 Comment 1

The sketch shows the reflection on a thin film.

The rays run parallel to the *k*-vectors of the waves.

The wave fronts are not shown in the sketch.

An incident ray is reflected at point A in ray 1.

A fraction of the wave penetrates the film and is reflected at point B.

A fraction of this beam is transmitted in beam 2 at point C.

Interference on thin films 1 Comment 2

The distance between point A and P is generally different from the distance A, B, C.

Interference can be observed when the waves at point A and C come from the same wave packet.

Otherwise there is no fixed relationship between the phase of the two waves.

Since the wavelength of the wave packets of visible light is small, this type of interference can only be observed on thin layers.

Interference on thin films 1 Comment 3

In principle there is also a reflected ray at point C, which is not shown in the sketch.

This ray can usually be neglected.

This is due to the reflection coefficient of the electromagnetic waves.

In general, the reflection coefficients are given by complicated formulas which depend on the polarisation of the wave.

The simple formula for the reflection coefficient, which was given in the first lecture, results only for perpendicular incidence.

Interference on thin films 2

$$r = \frac{k_1 - k_2}{k_1 + k_2} = \boxed{r = \frac{n_1 - n_2}{n_1 + n_2}}$$

With $\nu\lambda = v$ and $v = c/n$

$$\lambda = \frac{c}{n\nu} \quad \text{and} \quad k = \frac{2\pi}{\lambda} = 2\pi\frac{n\nu}{c} = n\frac{2\pi\nu}{c} = n\frac{2\pi}{\lambda_0} = nk_0$$

λ_0 denotes the vacuum wavelength and $k_0 = 2\pi/\lambda_0$ the wave number in vacuum.

E.g. for the interface air ($n_1 = 1$) glass ($n_2 = 1.5$) is $r = -0.2$ and only 4% (i.e. r^2 !!) of the intensity is reflected

Interference on thin films 2 — Comment 1

The first formula was derived in the first lecture for the case of rope waves.

The formula framed in red results when the wave number is replaced by the refractive index.

Since the phase velocity decreases with increasing refractive index, the wavelength becomes shorter with increasing refractive index.

Consequently, the wavenumber is proportional to the refractive index.

The reflection coefficient at the air / glass interface is -0.2.

The reflection coefficient refers to the amplitude of the electric field strength of the wave.

Interference on thin films 2 — Comment 2

Since the reflected intensity is proportional to the square of the electric field, only 4% of the total intensity is reflected.

Although the exact value of the reflected intensity depends on the angle of incidence and the polarization, it is clear that most of the incident intensity is transmitted and not reflected.

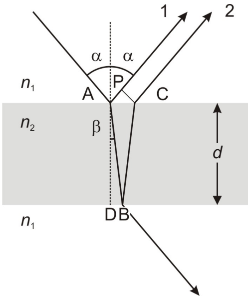
Most of the intensity is transmitted at points B and C.

The small reflected portion at point C can be neglected.

Therefore the intensity of ray 1 is roughly the same as the intensity of ray 2 and only these two rays need to be considered.

The intensity of all other rays is small and can be neglected.

Interference on thin films 3



For perpendicular incidence is the path difference $2d$ between ray 1 and 2. The additional phase of ray 2 is

$$\Delta\varphi_2 = k_2 \cdot 2d = n_2\, k_0 \cdot 2d = n_2 \frac{2\pi}{\lambda_0} 2d$$

For $n_1 < n_2$ is $r < 0$ and $\Delta\varphi_1 = \pm\pi$

Interference on thin films 3 Comment

At normal incidence, the path difference between ray 1 and ray 2 is twice the layer thickness.

The phase difference determines the amplitude of the superposition of beams 1 and 2.

The spatial part of the phase results from the product of the wave vector and the position vector.

The additional phase of ray 2 results from the wave number k_2 in the slice multiplied by twice the slice thickness.

The reflection coefficient at the air / glass interface is negative.

This corresponds to an additional phase of $\pm\pi$ for ray 1.

Interference on thin films 4

For perpendicular incidence is the phase difference between ray 2 and 1

$$\boxed{\Delta\varphi = \Delta\varphi_2 - \Delta\varphi_1 = n_2 \frac{2\pi}{\lambda_0} 2d \pm \pi}$$

The condition for constructive interference is

$$\underline{\Delta\varphi = 2\pi m \quad \text{and} \quad m = 0, 1, 2 \ldots}$$

The layer thickness is

$$\underline{d_m = \frac{1}{2n_2}\left(m\lambda_0 + \frac{\lambda_0}{2}\right) = (2m+1)\frac{\lambda_0}{4n_2}}$$

and with $\lambda_2 = \lambda_0 / n_2$

$$\boxed{d = \frac{\lambda_2}{4}, \; 3\frac{\lambda_2}{4} \ldots}$$

Interference on thin films 4 Comment

The first formula framed in red gives the phase difference between ray 1 and 2 for perpendicular incidence.

It does not matter whether the refractive index of the layer is larger or smaller than that of the surrounding material.

The amplitude is maximal when the phase difference between ray 1 and 2 is a multiple of 2π.

The layer thickness can be calculated with the condition for constructive interference.

The layer thickness must be an odd multiple of a quarter of the wavelength within the layer.

Interference on thin films 5

Condition for destructive interference

$$\underline{\Delta\varphi = \pi m \quad \text{and} \quad m = 1, 3, 5....}$$

The layer thickness is

$$\boxed{d_m = \frac{m\lambda_0}{2n_2} = m\frac{\lambda_2}{2} \quad \text{and} \quad m = 1, 2, 3, ...}$$

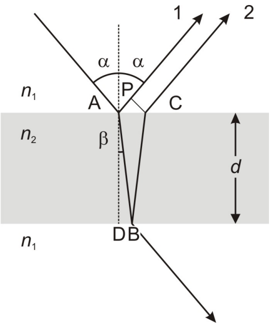
Interference on thin films 5

Comment

Destructive interference occurs when the phase difference between beam 1 and 2 is an odd multiple of π.

The layer thickness has to be a multiple of half the wavelength within the layer.

Interference on thin films 6



The phase difference between ray 2 and 1 is

$$\Delta\varphi = \Delta\varphi_2 - \Delta\varphi_1 = \frac{2\pi}{\lambda_2}\overline{ABC} - \frac{2\pi}{\lambda_1}\overline{AP} \pm \pi = \underline{\frac{2\pi n_1}{\lambda_0} 2d\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\alpha} \pm \pi}$$

Interference on thin films 6 Comment

If the beam falls obliquely on the thin layer, then the distance between points A and P and the distance between the points A and C must be calculated for ray 1 and ray 2, respectively.

The distance ABC must be multiplied with the wave number k_2 and the distance AP by the wave number k_1.

With a little geometry, the underlined formula results for the phase difference between ray 2 and ray 1.

Interference on thin films 7

The superposition of two waves with the same amplitude ψ_0 and the phase difference $\Delta\varphi$ is

$$
\begin{aligned}
\psi = \psi_1 + \psi_2 &= \psi_0 e^{i\vec{k}\vec{r}} + \psi_0 e^{i(\vec{k}\vec{r}+\Delta\varphi)} \\
&= \psi_0 e^{i\vec{k}\vec{r}} \left(1 + e^{i\Delta\varphi}\right) \\
&= \psi_0 e^{i\vec{k}\vec{r}} e^{i\Delta\varphi/2} \left(e^{-i\Delta\varphi/2} + e^{i\Delta\varphi/2}\right) \\
&= \psi_0 e^{i\vec{k}\vec{r}} e^{i\Delta\varphi/2} 2\cos(\Delta\varphi/2)
\end{aligned}
$$

The intensity is given by

$$
|\psi|^2 = \psi \cdot \psi^* = \boxed{I = 4I_0 \cos^2(\Delta\varphi/2)}
$$

Interference on thin films 7 Comment

It is not difficult to calculate the sum of two superimposed waves.

For the sake of simplicity, the same amplitude is assumed for both waves.

The intensity results from the square of the magnitude of the complex wave function.

The variation between the maxima and minima of the intensity is determined by the square of the cosine function.

Interference on thin films 8

Interference on thin films 8 — Comment

The picture shows the interference on soap bubbles.

The typical interference colours appear in daylight.

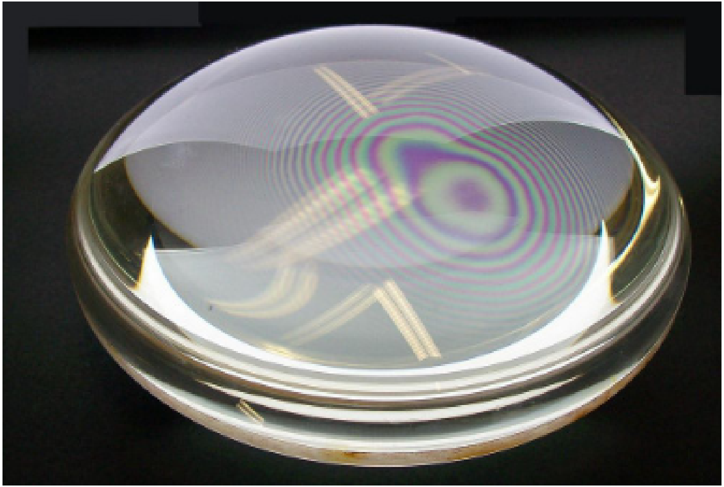
Some wavelengths of the white spectrum are eliminated by destructive interference.

The effect depends on the thickness of the soap bubble and the angle of incidence.

Usually these colours can also be observed on thin oil films on water.

Interference on thin films 9

Newton rings Interference on a thin air layer



Interference on thin films 9 Comment

The picture shows the Newton rings.

A lens is placed on a glass plate.

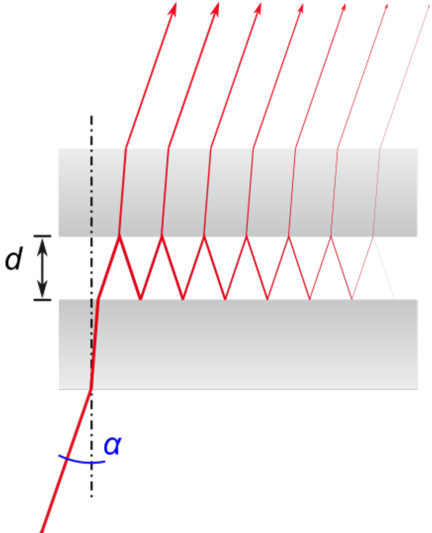
The interference is due to the small gap between the lens and the glass plate.

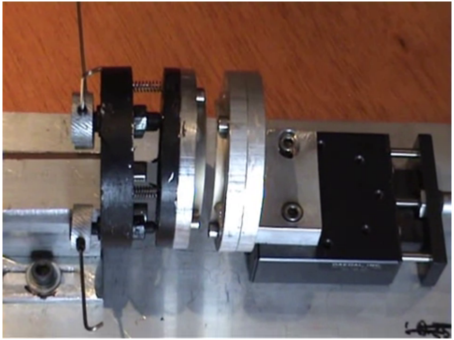
The thickness of the glass plate and the lens is usually very much larger than the coherence length, so that no interference can occur either in the lens or in the glass plate.

Fabry-Perot

Classical wave optics

Fabry-Perot Interferometer 1





(FabryPerotInterferometer.mp4)

Fabry-Perot Interferometer 1 Comment 1

The Fabry-Perot interferometer is a useful device for resolving the fine structure of spectral lines.

It consists of two mirrored glass plates aligned in parallel.

Only a small fraction of the intensity can pass through the mirrors.

The left figure shows the path of a beam that enters the interferometer at an angle α.

Light entering the interferometer is trapped in the gap between the two glass plates.

Only a small fraction can leave the interferometer.

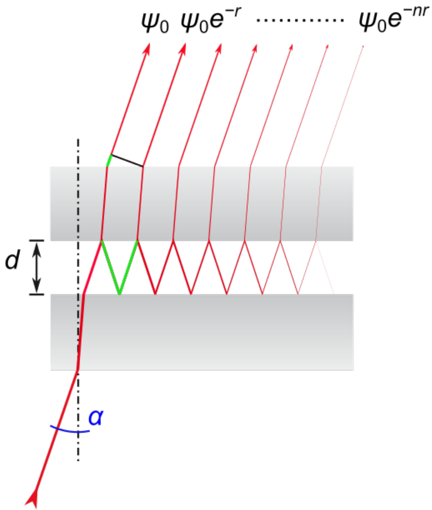
Fabry-Perot Interferometer 1 — Comment 2

The light rays leaving the interferometer can interfere if the coherence length of the light is large enough.

In contrast to the interference on thin layers, very sharp interference maxima can be observed.

The video shows the construction of a simple Fabry-Perot interferometer.

Fabry-Perot Interferometer 2



Phase difference between neighbouring rays

$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda} 2d\cos\alpha}$$

Condition for constructive interference

$$\Delta\varphi = 2\pi m \quad \rightarrow \quad \underline{m\lambda = 2d\cos\alpha_m}$$

Half width at half height of the maxima

$$\Delta\varphi_{1/2} = r$$

Fabry-Perot Interferometer 2 Comment 1

Unlike the interference in thin films, the phase difference between adjacent beams occurs entirely in air.

The refractive indices $n_{1,2}$ in the formula $\Delta\varphi = \frac{2\pi n_1}{\lambda_0} 2d\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\alpha}$ are 1, so that the formula for the phase difference framed in red results. (A phase shift due to reflection from the mirror is ignored for simplicity.)

The underlined formula for the path difference between two adjacent maxima results from the condition for constructive interference.

The path difference is proportional to the cosine of the angle of incidence.

The number m becomes smaller as the angle to the normal of the glass plate increases.

Fabry-Perot Interferometer 2 — Comment 2

The figure on the left shows the beam path in the interferometer again.

The amplitude of the rays leaving the interferometer is gradually reduced by a factor expressed by an exponential function.

The number r in the exponent equals half the width of the interference maxima at half the intensity.

The reduction of the amplitude should be as small as possible in order to achieve high resolution.

Therefore, the light should be trapped within the gap of the interferometer as long as possible.

Fabry-Perot Interferometer 2 — Comment 3

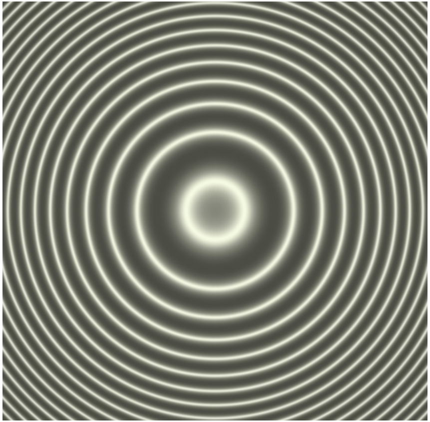
If the reduction factor is small enough, the number of interfering rays is limited by the coherence length of the light examined.

The Fabry-Perot interferometer only works with a large coherence length, i.e. with narrow spectral lines.

The condition for constructive interference also shows that the resolution of the interferometer is greatest for the smallest angle α_m.

Fabry-Perot Interferometer 3

Condition for constructive interference: $m\lambda = 2d\cos\alpha_m$



Fabry-Perot Interferometer 3 Comment

The figure shows the simulated output signal of a Fabry-Perot Interferometer.

The order of the interference fringes decreases with increasing radius of the rings.

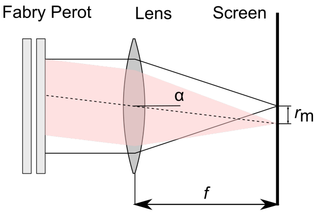
The radius of the rings also depends on the wavelength.

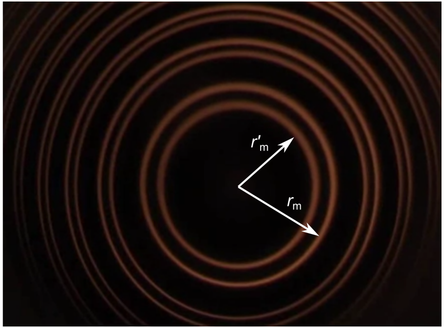
The radius of the rings decreases with increasing wavelength.

Fabry-Perot Interferometer 4: Splitting of the sodium D-lines

$D_1 : \lambda = 589.5924\,\text{nm}$ and $D_2 : \lambda = 588.9951\,\text{nm}$

$$\boxed{\frac{\Delta\lambda}{\lambda} = \frac{1}{2f^2}\left(r_m^2 - {r'}_m^2\right)}$$





Fabry-Perot Interferometer 4 Comment 1

The video shows the splitting of the yellow sodium line into two components.

The interference maxima show a double line structure.

The greatest effect is observed for the two inner rings, which correspond to the highest *m* number for the path difference between adjacent rays.

Since the angle α decreases with increasing wavelength (compare $m\lambda = 2d\cos\alpha_m$) the inner ring corresponds to the D_1-line ($\lambda = 589.5924\,\mathrm{nm}$) and the outer ring to the D_2-line ($\lambda = 588.9951\,\mathrm{nm}$).

The formula that can be used to calculate the wavelength difference $\Delta\lambda$ between the two spectral lines is outlined in red.

Fabry-Perot Interferometer 4 Comment 2

A sketch of the experimental setup is shown on the left. The parallel light rays leaving the interferometer are focused onto a screen by a lens with the focal length f.

The wavelength difference $\Delta\lambda$ can be calculated using the radii of the interference rings r_m and r'_m.

This can be done with the two rings of any order m.

In the figure, the radii of the inner rings of the highest order are marked by arrows.

Fabry-Perot Interferometer 4 Comment 3

Derivation of the formula is boring, but I show it because you may need this formula in one of our practicals (Hyperfine Interaction).

As shown in the sketch, the parallel rays leaving the interferometer are focused onto a screen by a lens with focal length *f*. The radius of the interference rings is

$$r = f \tan \alpha = f\alpha$$

Since α is small, $\tan \alpha$ can be replaced by α and $r/f = \alpha << 1$.

For small α a Taylor expansion can be applied to the interference condition

$$m\lambda = 2d \cos \alpha_m = 2d(1 - \frac{\alpha_m^2}{2}) = 2d(1 - \frac{r_m^2}{2f^2})$$

Fabry-Perot Interferometer 4 — Comment 4

This equation can be rearranged

$$\frac{2d}{\lambda} = \frac{m}{(1 - \frac{r_m^2}{2f^2})} \approx m(1 + \frac{r_m^2}{2f^2})$$

Now one likes to compare to wavelength λ and $\lambda' = \lambda + \Delta\lambda$ of the same order m, i.e.

$$\begin{aligned}\frac{2d}{\lambda} =& m(1 + \frac{r_m^2}{2f^2}) \\ \frac{2d}{\lambda'} =& m(1 + \frac{{r'}_m^2}{2f^2}) = \frac{2d}{\lambda + \Delta\lambda} = \frac{2d}{\lambda}\frac{1}{1 + \frac{\Delta\lambda}{\lambda}} \approx \frac{2d}{\lambda}(1 - \frac{\Delta\lambda}{\lambda})\end{aligned}$$

With this one can write

$$m(1 + \frac{{r'}_m^2}{2f^2}) = m(1 + \frac{r_m^2}{2f^2})(1 - \frac{\Delta\lambda}{\lambda})$$

and you see that the order m drops out!

Fabry-Perot Interferometer 4 Comment 5

One gets

$$(1+\frac{r'^2_m}{2f^2}) = (1+\frac{r^2_m}{2f^2}) - \frac{\Delta\lambda}{\lambda}(1+\frac{r^2_m}{2f^2})$$

Since

$$\frac{\Delta\lambda}{\lambda}\cdot\frac{r^2_m}{2f^2} << \frac{\Delta\lambda}{\lambda}$$

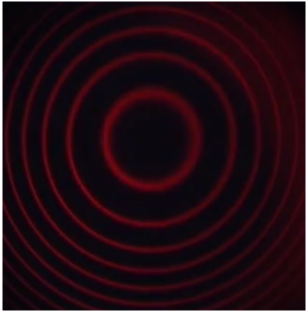
one can write

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{2f^2}(r^2_m - r'^2_m)$$

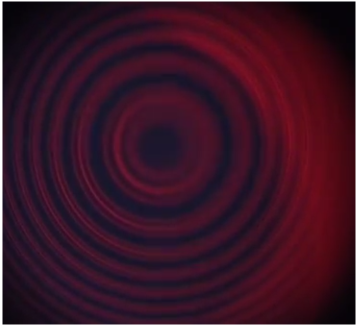
Fabry-Perot Interferometer 5

Splitting of the red cadmium line when a magnetic field is applied

without a magnetic field



with a magnetic field



(ZeemanCdLinie.mp4)

Fabry-Perot Interferometer 5 Comment

The video shows the splitting of the red cadmium line into three components when a magnetic field is applied.

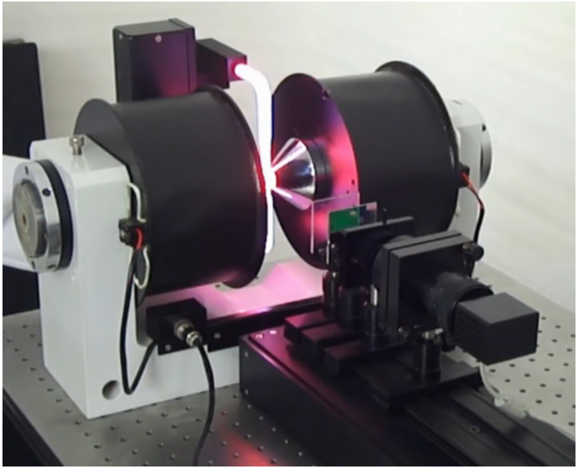
The splitting of the spectral lines in a magnetic field was discovered by Pieter Zeeman in 1896 and is called the Zeeman effect in honor of Pieter Zeeman.

Hendrik Lorentz concluded in 1899 from an analysis of the Zeeman effect that the spectral lines of atoms are caused by electrons.

Pieter Zeeman and Hendrik Lorentz were awarded the Nobel Prize in 1902 “in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena”

Fabry-Perot Interferometer 6

Splitting of the green Hg line in the magnetic field



(ZeemanGrueneHgLinie.mp4)

Fabry-Perot Interferometer 6 Kommentar

The figure shows the experimental setup.

The spectral lamp, which is filled with a gas of mercury atoms, stands between the poles of an electromagnet.

In front of it is the Fabry-Perot interferometer with the small camera for recording the interference image.

The video shows the experiment and the splitting of the interference fringes into three components when the magnetic field is switched on.

Revision

Summary in questions

1. What is the coherence length?
2. When is light called coherent?
3. What is the interference condition for constructive interference for a thin film in air and perpendicular incidence of light?
4. Write down the reflection coefficient for the electric field at normal incidence of light on a dielectric interface.
5. What is the Fraunhofer type of observation?
6. Sketch a Fabry-Perot interferometer.
7. What determines the resolution of the Fabry-Perot interferometer?
8. Sketch the interference pattern of a Fabry-Perot interferometer when it is illuminated with a monochromatic light wave.